Hybrid inflation

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Usually inflation ends either by a slow rolling of the inflaton field, which gradually becomes faster and faster, or by a first-order phase transition. We describe a model where inflation ends in a different way, due to a very rapid rolling ("waterfall") of a scalar field σ triggered by another scalar field ϕ. This model looks like a hybrid of chaotic inflation with $V(\phi) = \frac{m^2\phi^2}{2}$ and the usual theory with spontaneous symmetry breaking with $V(\sigma) = \frac{1}{4\lambda}(M^2 - \lambda\sigma^2)^2$. The last stages of inflation in this model are supported not by the inflaton potential $V(\phi)$ but by the "noninflationary" potential $V(\sigma)$. Another hybrid model to be discussed here uses some building blocks from extended inflation (Brans-Dicke theory), from new inflation (phase transition due to a nonminimal coupling of the inflaton field to gravity), and from chaotic inflation (the possibility of inflation beginning at large as well as at small σ). In the simplest version of this scenario inflation ends up by slow rolling, thus avoiding the big-bubble problem of extended inflation.

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I. INTRODUCTION

There exist three independent ways of classifying inflationary models. The first classification deals with the initial conditions for inflation. The old and new inflationary models were based on the assumption that the Universe from the very beginning was in a state of thermal equilibrium at an extremely high temperature, and that the inflaton field ϕ was in a state corresponding to the minimum of its temperature-dependent effective potential $V(\phi)$ [1, 2]. The main idea of the chaotic inflation scenario was to study *all* possible initial conditions in the Universe, including those which describe the Universe outside of the state of thermal equilibrium, and the scalar field outside of the minimum of $V(\phi)$ [3]. This scenario includes the possibility of new inflation from the state in a thermal equilibrium, but it contains many other possibilities as well. Therefore it can be realized in a much greater variety of models than the new inflationary universe scenario. In fact, at present the idea of thermal beginning is almost completely abandoned, and all realistic models of inflation from the point of view of the first classification are of the chaotic inflation type [4].

The second classification describes various regimes which are possible during inflation: quasiexponential inflation, power law inflation, etc. This classification is absolutely independent of the issue of initial conditions. Therefore it does not make any sense to compare, say, power law inflation and chaotic inflation, and to oppose them to each other. For example, in [5] it was pointed out that the chaotic inflation scenario, as distinct from the new inflationary universe scenario, can be realized in the theories with the effective potential $e^{\alpha\phi}$ for $\alpha \ll \sqrt{16\pi}$. Meanwhile, in [6] it was shown that this inflation is power law. Thus, the inflationary Universe scenario in the theory $e^{\alpha\phi}$ describes chaotic power law inflation.

Finally, the third classification is related to the way inflation ends. There are two possibilities extensively discussed in the literature: slow rollover versus the first-order phase transition. The models of the first class describe slow rolling of the inflaton field ϕ, which gradually becomes faster and faster. A particular model of this type is chaotic inflation in the theories ϕ^n. The models of the second class should contain at least two scalar fields: ϕ and σ. They describe a strongly first-order phase transition with bubble production which is triggered by the slow rolling of the field ϕ. One of the popular models of this type is the extended inflation scenario [7], which is a combination of the Brans-Dicke theory and the old inflationary scenario. There exist other versions of the first-order scenario with two scalar fields, which do not require any modifications of the Einstein theory of gravity; see, e.g., [8].

In the beginning it was assumed that the bubbles formed during the first-order phase transition could be a useful ingredient of the theory of the large scale structure formation. However, later it was realized that one should make considerable modifications of the original models in order to avoid disastrous consequences of the bubble production. According to the most recent modification [9], the bubble formation happens only after the end of inflation. In this case, the end of inflation occurs as in the standard slow-rollover scenario. Therefore it would be interesting to find out other possible ways in which inflation may end in the models with several different scalar fields. More generally, one may try to find out other qualitatively new inflationary regimes which may appear due to a combined evolution of several scalar fields.

Of course, one should not invent excessively compli-

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cated models without demonstrated need. However, sometimes qualitatively different inflationary regimes appear after minor modifications of the basic inflationary models, or after making their hybrids. For example, in [10] we proposed a very simple model of two interacting scalar fields where inflation may end by a rapid rolling of the field σ ("waterfall") triggered by the slow rolling of the field ϕ. This regime differs both from slow-rollover and first-order inflations. By changing parameters of this model one can continuously interpolate between these two regimes. Therefore some hybrid models of such type may share the best features of the slow-rollover and the first-order models.

This model was only briefly introduced in [10]. One of the purposes of the present paper is to discuss this model in a more detailed way. Another model to be discussed in this paper looks like a hybrid of the Brans-Dicke theory and new inflation. This model is similar to extended inflation, but is does not suffer from the big-bubble problem, since the phase transition which occurs in this model is second order. Finally, we will discuss a model which looks like a hybrid of the Brans-Dicke theory and chaotic inflation. In this model the Universe after inflation becomes divided into exponentially large domains with different values of the Planck mass M_P and of the amplitude of density perturbations $\frac{\delta\rho}{\rho}$.

II. HYBRID INFLATION MODEL

We begin with the discussion of the hybrid inflation model suggested in [10] in the context of the Einstein theory of gravity. The effective potential of this model is given by

$$V(\sigma,\phi) = \frac{1}{4\lambda}(M^2-\lambda\sigma^2)^2 + \frac{m^2}{2}\phi^2 + \frac{g^2}{2}\phi^2\sigma^2 \ . \quad (1)$$

Theories of this type were considered in [11–13]. The main difference between the models of Refs. [11–13] and our model is a specific choice of parameters, which allows the existence of the waterfall regime mentioned above. There is also another important difference: We will assume that the field σ in this model is the Higgs field, which remains a physical degree of freedom after the Higgs effect in an underlying gauge theory with spontaneous symmetry breaking. This field acquires only positive values, which removes the possibility of domain wall formation in this theory. Usually it is rather dangerous to take the inflaton field interacting with gauge fields, since its effective coupling constant λ may acquire large radiative corrections $\sim e^4$, where e is the gauge coupling constant. In our case this problem does not appear since density perturbations in our model remain small at rather large λ; see below.

The effective mass squared of the field σ is equal to $-M^2+g^2\phi^2$. Therefore for $\phi > \phi_c = M/g$ the only minimum of the effective potential $V(\sigma,\phi)$ is at $\sigma = 0$. The curvature of the effective potential in the σ direction is much greater than in the ϕ direction. Thus we expect that at the first stages of expansion of the Universe the field σ rolled down to $\sigma = 0$, whereas the field ϕ could remain large for a much longer time. For this reason we will consider the stage of inflation at large ϕ, with $\sigma = 0$.

At the moment when the inflaton field ϕ becomes smaller than $\phi_c = M/g$, the phase transition with symmetry breaking occurs. If $m^2\phi_c^2 = m^2M^2/g^2 \ll M^4/\lambda$, the Hubble constant at the time of the phase transition is given by

$$H^2 = \frac{2\pi M^4}{3\lambda M_P^2} \ . \quad (2)$$

Thus we will assume that $M^2 \gg \frac{\lambda m^2}{g^2}$. We will assume also that $m^2 \ll H^2$, which gives

$$M^2 \gg m M_P \sqrt{\frac{3\lambda}{2\pi}} \ . \quad (3)$$

One can easily verify, that, under this condition, the Universe at $\phi > \phi_c$ undergoes a stage of inflation. In fact, inflation in this model occurs even if m^2 is somewhat greater than H^2. Note that inflation at its last stages is driven not by the energy density of the inflaton field ϕ but by the vacuum energy density $V(0,0) = \frac{M^4}{4\lambda}$, as in the new inflationary Universe scenario. This was the reason why we called this model "hybrid inflation" in [10].

Let us study the behavior of the fields ϕ and σ after the time $\Delta t = H^{-1} = \sqrt{\frac{3\lambda}{2\pi}}\frac{M_P}{M^2}$ from the moment t_c when the field ϕ becomes equal to ϕ_c. The equation of motion of the field ϕ during inflation is $3H\dot\phi = m^2\phi$. Therefore during the time interval $\Delta t = H^{-1}$ the field ϕ decreases from ϕ_c by $\Delta\phi = \frac{m^2\phi_c}{3H^2} = \frac{\lambda m^2 M_P^2}{2\pi g M^3}$. The absolute value of the negative effective mass squared $-M^2+g^2\phi^2$ of the field σ at that time becomes equal to

$$M^2(\phi) = \frac{\lambda m^2 M_P^2}{\pi M^2} \ . \quad (4)$$

The value of $M^2(\phi)$ is much greater than H^2 for $M^3 \ll \lambda\, m M_P^2$. In this case the field σ within the time $\Delta t \sim H^{-1}$ rolls down to its minimum at $\sigma(\phi) = M(\phi)/\sqrt{\lambda}$, rapidly oscillates near it, and loses its energy due to the expansion of the Universe. However, the field cannot simply relax near this minimum, since the effective potential $V(\phi,\sigma)$ at $\sigma(\phi)$ has a nonvanishing partial derivative

$$\frac{\partial V}{\partial\phi} = m^2\phi + \frac{g^2\phi M^2(\phi)}{\lambda} \ . \quad (5)$$

One can easily check that the motion in this direction becomes very fast and the field ϕ rolls to the minimum of its effective potential within the time much smaller than H^{-1} if $M^3 \ll \sqrt{\lambda}\, g\, m M_P^2$. Thus, under the specified conditions inflation ends in this theory almost instantaneously, as soon as the field ϕ reaches its critical value $\phi_c = M/g$.

The amplitude of adiabatic density perturbations produced in this theory can be estimated by standard methods [4] and is given by

$$\frac{\delta\rho}{\rho} = \frac{16\sqrt{6\pi}}{5}\,\frac{V^{3/2}}{M_P^3\,\frac{\partial V}{\partial\phi}} = \frac{16\sqrt{6\pi}\left(\frac{M^4}{4\lambda}+\frac{m^2\phi^2}{2}\right)^{3/2}}{5M_P^3\, m^2\phi} \ . \quad (6)$$

In the case $m^2 \ll H^2$ the scalar field ϕ does not change substantially during the last 60 e-foldings (i.e., during the interval $\Delta t \sim 60H^{-1}$). In this case the amplitude of density perturbations practically does not depend on scale, and is given by

$$\frac{\delta\rho}{\rho} \sim \frac{2\sqrt{6\pi}gM^5}{5\lambda\sqrt{\lambda}M_P^3m^2} . \tag{7}$$

The definition of $\frac{\delta\rho}{\rho}$ used in [4] corresponds to Cosmic Background Explorer (COBE) data for $\frac{\delta\rho}{\rho} \sim 5 \times 10^{-5}$. Dividing it by (3) with an account taken of (7) gives $M^3 \ll 5 \times 10^{-5}\lambda g^{-1} m M_P^2$. This means that the "waterfall conditions" $M^3 \ll \lambda\ m M_P^2$ and $M^3 \ll \sqrt{\lambda} g\ m M_P^2$ automatically follow from the conditions $m^2 \ll H^2$ and $\frac{\delta\rho}{\rho} \sim 5 \times 10^{-5}$, unless the coupling constants λ and g are extremely small. Therefore the waterfall regime is realized in this model for a wide variety of values of parameters m, M, λ, and g which lead to density perturbations $\sim 5 \times 10^{-5}$.

To give a particular example, let us take $g^2 \sim \lambda \sim 10^{-1}$, $m \sim 10^2$ GeV (electroweak scale). In this case all conditions mentioned above are satisfied and $\frac{\delta\rho}{\rho} \sim 5 \times 10^{-5}$ for $M \sim 1.3{\times}10^{11}$ GeV. In particular, we have verified, by solving equations of motion for the fields ϕ and σ numerically, that inflation in this model ends up within the time $\Delta t \ll H^{-1}$ after the field ϕ reaches its critical value $\phi_c = M/g$. The value of the Hubble parameter at the end of inflation is given by $H \sim 7 \times 10^3$ GeV. The smallness of the Hubble constant at the end of inflation makes it possible, in particular, to have a consistent scenario for axions in inflationary cosmology even if the axion mass is much smaller than 10^{-5} eV [10]. This model has some other distinctive features. For example, the spectrum of perturbations generated in this model may look as a power law spectrum rapidly decreasing at a large wavelength l [16].

Indeed, at the last stages of inflation (for $\frac{M^4}{4\lambda} \gg \frac{m^2\phi^2}{2}$) the field ϕ behaves as

$$\phi = \phi_c \exp\left(-\frac{m^2(t-t_c)}{3H}\right) , \tag{8}$$

whereas the scale factor of the Universe grows exponentially, $a \sim e^{Ht}$. This leads to the following relation between the wavelength of perturbations l and the value of the scalar field ϕ at the moment when these perturbations were generated: $\phi \sim \phi_c \left(\frac{l}{l_c}\right)^{m^2/3H^2}$. In this case

$$\frac{\delta\rho}{\rho} = \frac{2\sqrt{6\pi}gM^5}{5\lambda\sqrt{\lambda}M_P^3m^2}\left(\frac{l}{l_c}\right)^{-\frac{m^2}{3H^2}} , \tag{9}$$

which corresponds to the spectrum index $n = 1 + \frac{2m^2}{3H^2} = 1 + \frac{\lambda m^2 M_P^2}{\pi M^4}$. Note that this spectrum index is greater than 1, which is a very unusual feature. For the values of m, M, λ, and g considered above, the deviation of n from 1 is vanishingly small (which is also very unusual). However, let us take, for example, $\lambda = g = 1$, $M = 10^{15}$ GeV (grand unification scale), and $m = 5 \times 10^{10}$ GeV. In this case the amplitude of perturbations at the end of inflation ($\phi = \phi_c$) is equal to 4×10^{-4}, $n \sim 1.1$, and the amplitude of the density perturbations drops to the desirable level $\frac{\delta\rho}{\rho} \sim 5 \cdot 10^{-5}$ on the galaxy scale ($l_g \sim l_c e^{50}$). One may easily obtain models with even much larger n, but this may be undesirable, since it may lead to formation of many small primordial black holes [14].

Note, that the decrease of $\frac{\delta\rho}{\rho}$ at large l is not unlimited. At $\frac{m^2\phi^2}{2} > \frac{M^4}{4\lambda}$ the spectrum begins growing again. Thus, the spectrum has a minimum on a certain scale, corresponding to the minimum of expression (6). This complicated shape of the spectrum appears in a very natural way, without any need to design artificially bent potentials.

As we have seen, coupling constants in our model can be reasonably large, and the range of possible values of masses m and M is extremely wide. Thus, our model is very versatile. One should make sure, however, that the small effective mass of the scalar field ϕ does not acquire large radiative corrections near $\phi = \phi_c$. Hopefully this can be done in supersymmetric theories with flat directions of the effective potential.

One can suggest many interesting generalizations of our model. For example, instead of the term $\frac{m^2\phi^2}{2}$ in (1) one can use the term $\frac{\lambda_\phi\phi^4}{4}$. In this case one may have two disconnected stages of inflation. The first stage occurs at large ϕ, as in the simplest version of chaotic inflation scenario. This stage ends at $\phi < M_P/3$, if $M^2 \ll \lambda_\phi M_P^2$. Then the field rapidly rolls down and oscillates until the amplitude of its oscillations becomes smaller than $\phi \sim \frac{M^2}{\lambda_\phi M_P}$. At this moment the frequency of oscillations $\sim \sqrt{\lambda_\phi}\phi$ becomes smaller than the Hubble constant, and the second stage of inflation begins. This stage of inflation ends with the waterfall at $\phi_c = M/g$. As was shown in [15], in the models with two stages of inflation with a break between them the spectrum of density perturbations may have a very rich and nontrivial structure.

III. BRANS-DICKE INFLATION

Our second hybrid inflation model is very similar to extended inflation, but it does not lead to the first-order phase transition with bubble formation, which is a definitive feature of the extended inflation scenario. The corresponding action is

$$S = \int d^4x\sqrt{-g}\ \Big[\ \frac{\phi^2 R}{8\omega} - \frac{\xi}{2}\sigma^2 R + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - V(\sigma)\Big] . \tag{10}$$

Here $\phi^2 = \frac{\omega}{2\pi}\Phi$ is the Brans-Dicke field; σ is the field which may exhibit spontaneous symmetry breaking. In the extended inflation scenario the second term in the action (10) was absent. However, we believe that if one of the two scalar fields is nonminimally coupled to gravity, one should allow for a similar coupling for another field as well. [1]

We will consider effective potentials of the standard type, $V(\sigma) = \frac{1}{4\lambda}(m^2 - \lambda\sigma^2)^2$. From a comparison with the standard Einstein theory it follows that the effective Planck mass in the theory (10) depends on ϕ and is given by

$$M_P^2(\phi) = 2\pi \left(\frac{\phi^2}{\omega} - 4\sigma^2\xi\right) . \quad (11)$$

We will try to find inflationary solutions in the theories with $w \gg 1$, and we will assume that initially $\sigma = 0$. The reasons why we take $\sigma = 0$ at the beginning of inflation will be discussed in the next sections. During inflation one can neglect $\frac{\dot\phi}{\phi}$ as compared with $H = \frac{\dot a}{a}$ [where $a(t)$ is the scale factor of the Universe] and $\ddot\phi$ as compared with $3H\dot\phi$ and $\dot\phi^2$. In such a case equations for a and ϕ in the theory (10) take a very simple form:

$$H = \frac{\dot a}{a} = \frac{2}{\phi}\sqrt{\frac{\omega V(0)}{3}} = \frac{m^2}{\phi}\sqrt{\frac{\omega}{3\lambda}} , \quad (12)$$

$$3H\dot\phi = \frac{4V(0)}{\phi} = \frac{m^4}{\lambda\phi} . \quad (13)$$

From these equations it follows that, at large t,

$$a(t) = a_0 t^\omega , \quad H(t) = \frac{\omega}{t} , \quad \phi(t) = \frac{m^2 t}{\sqrt{3\omega\lambda}} . \quad (14)$$

The effective mass squared of the field σ is given by

$$m_\sigma^2 = -m^2 + \xi R = -m^2 + 12\xi H^2 = -m^2 + \frac{12\xi\,\omega^2}{t^2} . \quad (15)$$

This means that at small t the effective potential including the term $-\frac{\xi}{2}\sigma^2 R$ has a minimum at $\sigma = 0$. Here one may consider two limiting possibilities depending on the value of the parameter ξ.

IV. LARGE ξ

Let us consider first the model with large ξ. In this case our model looks like a hybrid of the Brans-Dicke theory and new inflation. In the very early Universe the term $12\xi H^2$ determines the effective mass squared of the scalar field σ. This mass squared is much greater than H^2, which means that the field σ rapidly rolls down to the state $\sigma = 0$ (symmetry restoration), after which inflation begins. Analogy between this scenario and the new inflationary universe scenario becomes even more striking if one remembers that the term $12\xi H^2$ can be formally represented as $48\pi^2\xi T_H^2$, where $T_H = H/2\pi$ is the Hawking temperature in the inflationary universe.

One of the main problems of the new inflationary scenario is that inflation in this scenario may occur only at a density much smaller than the Planck density. In this case a typical closed Universe of the Planck size M_P^{-1} and with the Planck density M_P^4 collapses before inflation has any chance to occur. The probability of creation of a closed universe which can survive until the beginning of inflation is exponentially small [4]. In our scenario this problem is absent if the universe is created in a state with the fields ϕ and σ related to each other by the condition $M_P^4(\phi,\sigma) \sim V(\sigma)$; there is no exponential suppression of the probability of creation of the universe with such properties [8].

Thus, in this scenario inflation begins at $\sigma = 0$. Later on, when the time t becomes greater than t_c, where

$$t_c = wH_c^{-1} = \frac{2\omega\sqrt{3\xi}}{m} , \quad H_c = \frac{m}{2\sqrt{3\xi}} , \quad \phi_c = 2m\sqrt{\frac{\omega\xi}{\lambda}} , \quad (16)$$

the effective mass squared of the field σ becomes negative, and spontaneous symmetry breaking occurs.

In the large ξ limit, the absolute value of the (negative) effective mass squared of the field σ becomes greater than H_c^2 within the Hubble time $\Delta t = H_c^{-1}$ after t_c. At the moment $t = t_c + H_c^{-1}$ the effective mass squared of the field σ becomes equal to $m_\sigma^2 = 12\xi\omega^2(t^{-2} - t_c^{-2}) \approx -24\xi H_c^2/\omega$. This quantity is much greater than H_c^2 for $\xi > \omega/24$. Under this condition, the field σ begins growing with a very large speed, which suggests that inflation ends up almost instantaneously. However, a more detailed investigation of this question shows that the growth of the field σ later slows down. The reason for this effect is rather nontrivial. When the field σ grows, the effective Planck mass decreases; see Eq. (11). This leads to an increase of the Hubble constant H, which slows down the rolling of the field σ. This leads to existence of an additional stage of inflation, which occurs after the phase transition at $t = t_c$. In many cases this stage proves to be relatively short [17]. Thus, in this model one can also have a "waterfall" regime, but this waterfall is much slower than in the model (1).

This regime has a peculiar feature which deserves further investigation. It is well known that the value of the Brans-Dicke field almost does not change after inflation [18]. The change of this field during the last stage of inflation under the condition $\xi > \omega/24$ is also very small. This means that the present value of the field ϕ in the first approximation is equal to ϕ_c. On the other hand, the present value of the field σ is given by $\sigma_0 = m/\sqrt{\lambda}$. According to (11), (16), the contributions to M_P from the fields ϕ and σ in this approximation cancel each other, $M_P = 0$. Thus, in order to obtain a finite answer for the gravitational constant $G = M_P^{-2}$ one should calculate the small difference $\Delta\phi$ between ϕ_c and the present value of the field ϕ:

[1] After we proposed this model, we received a paper by Laycock and Liddle [17], where a similar model was invented. We are extremely grateful to these authors for the discussion of their results prior to publication. Our understanding of the large-ξ limit of this model strongly benefited from these discussions.

$$M_P = 4m\sqrt{\frac{\pi\xi}{\lambda}}\sqrt{\frac{\Delta\phi}{\phi_c}} \sim 4\sigma_0\sqrt{\pi\xi}\sqrt{\frac{\Delta\phi}{\phi_c}} . \tag{17}$$

Alternatively, one may use a different effective potential $V(\sigma)$, for which this peculiar cancellation does not occur. A more detailed investigation of the regime discussed above will be contained in the forthcoming paper by Laycock and Liddle [17].

If the stage of inflation after the phase transition is very short, one should take special care of production of topological defects in this model. Typically it is very difficult to create heavy strings and monopoles after inflation. However, it is possible to produce them during inflationary phase transitions [11, 19]. If, for example, the field σ leads to spontaneous breakdown of an Abelian symmetry, after the phase transition in our model many heavy cosmic strings will be created. In fact, cosmic strings in this model may be even too heavy. Indeed, according to (17), the scale of symmetry breaking is $\sigma_0 = \frac{M_P}{4}\sqrt{\frac{\phi_c}{\pi\xi\Delta\phi}}$. This is much greater than the desirable value $\sigma_0 \sim 10^{-3}M_P$ [20], unless the parameter ξ is extremely large. This means that if string production is possible in our scenario, they may give excessively large contribution to the post-inflationary density perturbations. There exist several different ways to reduce these perturbations to an acceptable level. One possibility is to consider the field σ of the type used in the standard theory of electroweak interactions, where neither stable strings nor stable domain walls or monopoles can be produced. One should keep in mind, however, that it may be necessary for the field σ to be a gauge singlet, since in this model, unlike in the first hybrid inflation model (1), the coupling constant λ should be very small unless one takes ξ extremely large [17]. The most radical way to get rid of topological defects is to consider models where the last stage of inflation is long enough. This can be achieved, e.g., in the model to be discussed below.

V. SMALL ξ

Let us consider the model (10) with $V(\sigma) = \frac{\lambda}{4}(\sigma^2 - \frac{m^2}{\lambda})^2$ in the limit of small ξ. In this limit the phase transition becomes irrelevant since the correction $\xi R \sim 12\xi H^2$ to the effective mass of the field σ always remains much smaller than H^2. Therefore these corrections cannot influence behavior of the field σ during inflation in a noticeable way.

For this reason we will take a step back and totally disregard the term $-\frac{\xi}{2}R\sigma^2$ in the action (10). At the first glance, we are returning to the standard extended inflation scenario. The difference, however, is in the choice of the effective potential $V(\sigma)$. In the extended inflation scenario the effective potential $V(\sigma)$ should have a local minimum at $\sigma = 0$. The simple potential we consider, which is a standard potential used in gauge theories with spontaneous symmetry breaking, does not have this extra minimum. In order to obtain such a minimum one should add some cubic or logarithmic corrections to $V(\sigma)$. Then one should tune these corrections to make the tunneling suppressed, but not too strongly, since this would make the Planck constant exponentially large and the bubbles exponentially big. And, after all, one should either introduce a potential for the Brans-Dicke field or considerably modify the interaction of this field with gravity [9]. No such modification is required in our scenario.

At the first stages of inflation in our scenario the field σ slowly rolls down from some initial value $\sigma_{\rm in} \ll \sigma_0$. Until this field grows up approximately to $\sigma_0/2$, the effective potential $V(\sigma)$ remains almost unchanged, and its derivative is given approximately by $-m^2\sigma$. Therefore at this stage Eqs. (12)–(15) remain valid, and equation for the field σ reads $3H\dot\sigma = \frac{3\omega}{t}\dot\sigma = m^2\sigma$, which gives

$$\sigma = \sigma_{\rm in}\exp\left(\frac{m^2t^2}{6\,\omega}\right) . \tag{18}$$

This stage ends up at the time $t \approx t_1$, when $\sigma(t_1) = \sigma_0/2$. This gives $t_1 = \frac{\sqrt{6\omega}}{m}\ln^{1/2}\frac{\sigma_0}{2\sigma_{\rm in}}$, $\phi_1 = \sqrt{2}\,\sigma_0\ln^{1/2}\frac{\sigma_0}{2\sigma_{\rm in}}$. Exact values of t_1 and ϕ_1 depend on $\sigma_{\rm in}$, which may take different values in different parts of the Universe. But the situation actually is even more complicated. As was shown in [21], an inflationary universe in the theories with the potential $V(\sigma) = \frac{1}{4\lambda}(m^2 - \lambda\sigma^2)^2$ enters regime of self-reproduction at $\sigma \le H$. This regime exists for $m^2 < H^2$. In the context of our model this regime leads to the formation of domains with all possible values of ϕ compatible with inflation at $\sigma = 0$. The upper boundary for inflation at small σ (the condition $m^2 < H^2$) is given by $\phi < m\sqrt{\frac{w}{\lambda}} = \sigma_0\sqrt{\omega}$. In such domains the stage of classical growth of the field is very short, and the field ϕ remains almost unchanged when the field σ grows up to $\sigma_1 \sim \sigma_0/2 = m/2\sqrt{\lambda}$. For a complete investigation of this question one should use the stochastic approach to inflation. We will return to this problem in [22]. At the present moment we will just keep in mind that at the end of the first stage of inflation the Brans-Dicke field may acquire different values in different parts of the Universe, in the range of $\sigma_0 < \phi_1 < \sigma_0\sqrt{\omega}$. We will describe this effect by introducing a phenomenological parameter C, such that $\phi_1 = C\sigma_0$, $1 < C < \sqrt{\omega}$.

This means that at the end of the first stage, when the field σ grows up to $\sigma \sim \sigma_0/2 = m/2\sqrt{\lambda}$, the square of the Hubble constant remains greater than the (positive) effective mass squared $m^2(\sigma_0) = 2m^2$ of the field σ near the minimum of its effective potential at $\sigma = \sigma_0$. Therefore at that time inflation still continues. The effective potential $V(\sigma)$ near its minimum can be represented as $V(\chi) = m^2\chi^2$, where we made an obvious change of variables, $\chi = \sigma_0 - \sigma$.

Fortunately, we already studied inflation in the Brans-Dicke theory with this potential, and all analytical solutions are known [23]:

$$\phi = A\sin\Big(B + \frac{m}{\sqrt{3\omega}}t\Big) ,$$

$$\chi \equiv \sigma_0 - \sigma = \frac{A}{\sqrt{2}}\cos\Big(B + \frac{m}{\sqrt{3\omega}}t\Big) , \tag{19}$$

$$a(t) = a(t_1)\left(\frac{\phi(t)}{\phi_1}\right)^{\omega} = a_0\left(\frac{\sin(B + \frac{m}{\sqrt{3\omega}}t)}{\sin B}\right)^{\omega} . \tag{20}$$

Here A and B are some constants determined by initial conditions. For $\frac{m}{\sqrt{3\omega}}t > B$, $\omega \gg 1$, the last equation describes the power law inflation, $a(t) \sim t^{\omega}$.

Initial conditions for this stage of inflation are determined by $\chi_1 \approx \sigma_0/2$ and $\phi_1 = C\sigma_0$. This gives $A = \sigma_0\sqrt{\frac{1+2C^2}{2}}$. Inflation in this model ends up at $\phi_e \approx A \sim \sqrt{6\omega}\chi_e$ [23]. Consequently, the total increase of the size of the Universe at this stage of inflation is given by

$$\frac{a(t_e)}{a(t_1)} \sim \left(\frac{\phi(t_e)}{\phi_1}\right)^{\omega} \sim \left(1 + \frac{1}{2C^2}\right)^{\omega/2} . \tag{21}$$

Thus, for large ω this stage of inflation can be very long, and perturbations generated at this stage will be responsible for the formation of the observable part of the Universe. The value of the Planck mass after inflation is given by

$$M_P = \sigma_0\sqrt{\frac{\pi(1+2C^2)}{\omega}} = m\sqrt{\frac{\pi(1+2C^2)}{\omega\lambda}} . \tag{22}$$

Perturbations of the field ϕ at the end of inflation at $\omega \gg 1$ are orthogonal to the classical trajectory of the fields $\phi(t), \chi(t)$ in the (ϕ, χ) space. Therefore the main contribution to density perturbations is given by perturbations of the field χ. The standard calculation gives $\frac{\delta\rho}{\rho} \sim \frac{m}{M_P}$, as in the ordinary theory $m^2\chi^2$ without any modification of general relativity. However, in our case the Planck mass is not a constant, but is related to m by Eq. (22). This gives $\frac{\delta\rho}{\rho} \sim \sqrt{\frac{\lambda\omega}{1+2C^2}}$. In different exponentially large parts of the Universe this quantity takes different values corresponding to $1 < C < \sqrt{\omega}$.

Taking into account our bounds on C, we see that from the point of view of density perturbations this model does not exhibit any improvement as compared with the usual theory $\lambda\chi^4$, where $\frac{\delta\rho}{\rho} \sim \sqrt{\lambda}$, and one needs to have $\lambda \sim 10^{-13}$ to satisfy all observational constraints [4]. In our model the best situation occurs in those domains where quantum fluctuations lead to $C \sim \sqrt{\omega}$, in which case inflation is long and $\frac{\delta\rho}{\rho} \sim \sqrt{\lambda}$. Thus, one still needs to have $\lambda \leq 10^{-13}$ to satisfy all observational constraints. Before considering this problem and its possible resolution, let us discuss some distinctive features of this model and some lessons which we learned when we were developing it.

First of all, inflation in this theory is possible for all values of the parameter σ_0; any potential $V(\sigma) = \frac{1}{4\lambda}(m^2 - \lambda\sigma^2)^2$ leads to inflation at small σ. In this respect the model we consider differs from the analogous model in the context of the Einstein theory of gravity, where inflation at small σ is possible only if $\sigma_0 > M_P$. Of course, we do not make any miracles: In our model the value of the effective Planck mass after inflation appears to be smaller than σ_0; see Eq. (22). The subtle but important difference is that in the Einstein theory inflation at small σ occurs only in the subclass of the models in which the two parameters σ_0 and M_P happen to be related to each other in the above mentioned way, whereas in the context of our model inflation at small σ occurs for all values of parameters. Moreover, as we already mentioned, inflation in our model does not suffer from the problem of initial conditions. This makes the existence of the inflationary regime more robust.

Another interesting feature of this model is the formation of different domains of the Universe with different values of the Planck mass and, correspondingly, with different amplitudes of density perturbations. According to our results, the range of possible variations of M_P and $\frac{\delta\rho}{\rho}$ is not very wide. However, in our investigation we considered only the regime when the field σ originally was small, $\sigma \ll \sigma_0$. Whereas at large ξ this was a reasonable assumption, at small ξ one should consider all other possibilities as well, including inflation beginning at very large σ. Indeed, at small ξ we do not have any unavoidable symmetry restoration in our model. Thus, at small ξ our model looks like a hybrid of the Brans-Dicke theory and chaotic inflation with the potential $V(\sigma)$ which at large σ behaves as $\frac{\lambda}{4}\sigma^4$. In this case there is no upper limit on possible initial values of σ and on the resulting Planck mass M_P, whereas the amplitude of density perturbations in the limit of large M_P does not depend on M_P and is proportional to $\sqrt{\lambda}$. The process of self-reproduction of the Universe in this scenario divides the Universe into many exponentially large domains where all possible values of M_P are represented [23].

The situation changes even more dramatically if one considers a hybrid of the Brans-Dicke theory and the simplest chaotic inflation model with $V(\sigma) = \frac{m^2\sigma^2}{2}$ [23]. In this case the Universe becomes divided into exponentially large domains with the values of the Planck mass taking all values from m to ∞, and with $\frac{\delta\rho}{\rho} \sim \frac{m}{M_P}$ varying from order 1 to 0. This opens a very interesting possibility of relating to each other the large value of the Planck mass and the small value of $\frac{\delta\rho}{\rho}$ in our part of the inflationary universe [22].

In the absence of any realistic model of elementary particle interactions on the energy scale discussed in the present paper, it is very hard to tell whether the models we are discussing are natural and realistic. However, it is very encouraging that by making simple hybrids of basic inflationary models one can obtain an extremely rich variety of inflationary theories with interesting and sometimes even very unusual properties. We believe that this enhances the possibility of finding a correct description of the observational data within the context of inflationary cosmology.

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