PHYSICAL REVIEW D VOLUME 49, NUMBER 12 15 JUNE 1994

Strings, black holes, and Lorentz contraction

Leonard Susskind*
Department of Physics, Stanford University, Stanford, California 94305
(Received 14 February 1994)

Consistency of quantum mechanics in black hole physics requires unusual Lorentz transformation properties of the size and shape of physical systems with momentum beyond the Planck scale. A simple parton model illustrates the kind of behavior which is needed. It is then shown that conventional fundamental string theory shares these features.

PACS number(s): 04.70.Dy, 11.25.Mj

I. INTRODUCTION

't Hooft has argued that the consistency of quantum mechanics in black hole evaporation will constrain high-energy physics so much that it will determine most of its features [1]. In this paper I will show that black hole complementarity [2] implies a radical revision of the usual kinematics of systems with very high energy. In particular, the usual Lorentz contraction of particles must saturate when their momenta approach the Planck scale. In other words, the physically measurable longitudinal size of a particle must tend to a constant and not decrease like its inverse momentum. Furthermore, the transverse size of boosted objects must grow with momentum. These requirements would certainly seem unbelievable if it were not for one circumstance. They are found to be true for the propagation of relativistic strings.

The plan of this paper is as follows. In this section I will make some preliminary remarks about Lorentz boosts and the behavior of the physically measurable dimensions of systems. Then I will define some concepts which will be useful in discussing the spatial localization of information.

In Sec. II some parton models illustrating possible behaviors of boosted particles are presented. One of the models is especially interesting because it necessarily contains a massless graviton.

Section III reviews the principle of black hole complementarity and the concept of the thermally excited stretched horizon. We describe how complementarity requires information, nearing the horizon, to spread when viewed by an observer outside the black hole. The same information does not spatially spread when viewed by an observer in free fall alongside the particle. The difference between these perceptions of events can be traced back to the difference in "resolving time" of the apparatuses available to the two observers. The requirements of black hole complementarity are satisfied in the model of Sec. II which requires the existence of gravitons.

In Sec. IV it is shown that string theory has exactly the properties required by black hole complementarity [3]. While this does not prove that string theory is the only possible description which allows black holes to be consistent with quantum mechanics, it is very suggestive. Finally, in Sec. V, I discuss the conclusions and philosophical implications of the paper.

In classical field theory an object is described by giving the values of certain local fields which are assumed to transform as tensors under Lorentz transformations. It follows straightforwardly that if the contours of constant field strength (scalars) form spatial spheres at rest in one frame, then in a boosted frame they form ellipsoids. The transverse size of the ellipsoid is unchanged by the boost and its longitudinal size is contracted according to the famous formula of Lorentz and Fitzgerald. In conventional quantum field theory the situation is more complicated for a number of reasons, including the uncertainty between position and velocity and the inability to localize a particle within its Compton wavelength. Quantum gravity may introduce other complications of an unknown kind. Therefore I am going to introduce a definition of size by operational procedures which could, in principle, be used to measure it. We will primarily be interested in objects moving with velocity near the speed of light.

Consider an object moving along the z axis with a velocity $v \approx 1$. We also consider an apparatus at rest which consists of an idealized surface of sensitive detectors such as a fluorescent screen or a photographic plate. I assume that the grain size and spacing is much smaller than the object. When the object strikes the plate, it leaves a mark. By the transverse size of the object I will mean the size of the mark that is left. Defined in this way, it is not clear that the transverse size remains constant as $v \rightarrow 1$. For example, it is widely believed that the cross section for proton-proton scattering logarithmically increases with energy. The damage left by a high-energy proton on a plate would also grow. The only requirement of Lorentz invariance is that, if both the object and the apparatus are boosted by a common angle, the transverse size not change. Similar conclusions can be drawn about the longitudinal size of objects.

I believe there is a sense in which the transverse size of

*Electronic address: susskind@dormouse.stanford.edu

an object does approach a limit as $v \to 1$ in ordinary relativistic field theory. Consider the difference between the spots left by protons and neutrons. To be more exact, consider a large number of marks left by protons and a similar ensemble left by neutrons under otherwise identical circumstances. Careful examination of the two ensembles will reveal differences in the statistical properties of p marks and n marks. In ordinary quantum field theory (QFT) we do not have to study the entire area occupied by each mark in order to distinguish the ensembles. Even assuming the mark size grows as $v \to 1$, the region which contains the relevant distinctions between particle types does not. Furthermore, the longitudinal size occupied by these distinctions Lorentz contracts, although the full physical extension may not.

To give an idea of what I mean by the transverse size occupied by information, consider a conventional field theory. Assume that the cutoff length is much smaller than the size of the objects being studied. Now consider two orthogonal states $|A\rangle$ and $|B\rangle$, which I will call particles but which could be more general. Let us suppose their transverse centers of mass are localized at the same place. Now partition space into two regions. The first region I is an infinite solid cylinder of radius R located at the same transverse position as the centers of mass of $|A\rangle$ and $|B\rangle$. The second region II is its complement.

The statistical results of all measurements within region I can be described by a density matrix in which the degrees of freedom in II are traced over. Thus we define

$$\rho_A^{\rm I} = {\rm Tr}^{\rm II}|A\rangle\langle A| , \tag{1.1}$$

$$\rho_B^{\rm I} j = {\rm Tr}^{\rm II}|B\rangle\langle B| , \tag{1.2}$$

where ${\rm Tr}^{\rm II}$ indicates a sum over a complete basis of states in II. A measure of the orthogonality of ρ_A and ρ_B is defined by

$$D_R^{\rm I}(A,B) = \frac{{\rm Tr}\rho_A^{\rm I}\rho_B^{\rm I}}{[{\rm Tr}(\rho_A^{\rm I})^2\,{\rm Tr}(\rho_B^{\rm I})^2]^{1/2}} . \tag{1.3}$$

As $R \to \infty$, $D_R^{\rm I}(A,B)$ must tend to zero, indicating that the two states are orthogonal and fully distinguishable. Furthermore, as $R \to 0$, $D_R^{\rm I}$ will tend to unity since the ultraviolet behavior of all states must be the same as the vacuum. We can therefore define a radius R_{AB} which characterizes the transverse region in which the information distinguishing A and B is localized. For example, R_{AB} could be defined by setting $D_{R_{AB}}(A,B)$ equal to $\frac{1}{2}$.

Similarly, given a collection of states $A,B,C,\ldots$, we can ask for the smallest region that needs to be investigated in order to distinguish these states. A simple definition would be obtained by requiring the largest of the quantities $D_R^{\rm I}(A,B), D_R^{\rm I}(A,C), D_R^{\rm I}(B,C),\ldots$ to be $\frac{1}{2}$.

We can also use such density matrices to define the size of an object. Consider the density matrix of the outer region II when a particle A is present and when the state is pure vacuum. Define them by

$$\rho_A^{\rm II} = {\rm Tr}^{\rm I}|A\rangle\langle A| , \tag{1.4}$$

$$\rho_0^{\rm II} = {\rm Tr}^{\rm I}|0\rangle\langle 0| . \tag{1.5}$$

The quantity

$$D_R^{\rm II}(A,0) = \frac{{\rm Tr}\rho_A^{\rm II}\rho_0^{\rm II}}{[{\rm Tr}(\rho_A^{\rm II})^2\,{\rm Tr}(\rho_0^{\rm II})^2]^{1/2}} \tag{1.6}$$

measures the distinguishability of the vacuum and particle A in the outer region II. The size of A can be defined by requiring

$$D_R^{\rm II}(A,0) = \tfrac{1}{2} . \tag{1.7}$$

Generally, there is no reason the size of the particles should be the same as the size of the regions carrying the information distinguishing particles. For example, if all particles in a certain class had some sort of long-range field with equal strength, then the distinction between particle types would be localized well within their full sizes.

II. PARTON MODELS

To understand how an observer outside a black hole describes the behavior of matter near the horizon, it is essential to first understand ordinary field theory in the light-cone frame [4]. Let us introduce Cartesian coordinates (x,y,z,t) into flat Minkowski space. The x,y directions will be called the transverse directions and indicated by $(X_\perp)$. The combinations $\tau = (t-z)/\sqrt{2}$ and $X^+ = (t+z)/\sqrt{2}$ are called the light-cone time and the longitudinal direction, respectively. The conjugates to τ and X^+ are called the light-cone Hamiltonian and longitudinal momenta H,P. For a free particle, the light-cone Hamiltonian is

$$H = \frac{q_\perp^2 + m^2}{2P} \tag{2.1}$$

(note that P is always positive), where m is the particle's rest mass and $q_\perp$ is the transverse momentum. Light-cone physics can also be thought of as the limiting description of matter which has been boosted to very large momentum.

The space of states of light-cone field theory is the Fock space describing particles with transverse position $X_\perp$ and longitudinal momentum P. The states are generated by applying creation operators $a^+(X_\perp,P)$ on a vacuum $|0\rangle$ which is annihilated by $a^-(X_\perp,P)$.

Notice that quanta can have large energy either because $q_\perp$ is large or because P is small. For the moment let us ignore the possibility that $q_\perp$ is large. Assume that fluctuations in transverse momenta and the mass m are of some common order of magnitude that characterizes the theory.

Let us suppose we are not interested in, or cannot resolve, processes taking place on time scales shorter than a *resolution time* $\delta\tau = \epsilon$. It is then appropriate to integrate out all degrees of freedom with energy greater than $1/\epsilon$. According to (2.1), this means we integrate out quanta with

$$P < m^2\epsilon . \tag{2.2}$$

The effective description has no quanta of longitudinal momenta less than $m^2\epsilon$. Furthermore, in the description of a system with total longidinal momentum $P_{\rm tot}$, there can be no quanta with $P > P_{\rm tot}$.

Under a longitudinal boost the P value of each parton rescales by a common amount. For example, a Lorentz boost which doubles $P_{\rm tot}$ also doubles each constituent P. However, if the resolving time ϵ is kept fixed, the lower cutoff in P is not doubled. This means that in the boosted system there will be no partons in the allowed region $m^2\epsilon < P < 2m^2\epsilon$. The partons in the region $P \sim m^2\epsilon$ must be dealt with separately from the rest when a boost is performed. They can be thought of as new partons which come into existence solely by virtue of boosting the system. Feynman called these the "wee partons."

In certain very well behaved and uninteresting field theories, the parton distribution rapidly diminishes toward low P once $P_{\rm tot} \gg m$. In that case essentially no new partons are created by increasing $P_{\rm tot}$. In these theories the boost properties of objects are very conventional.

In more interesting theories such as QCD the parton distribution is singular near $P=0$. In these cases, boosting a system requires adding partons at low P. If those partons are located at an ever increasing transverse distance, then the transverse size appears to grow with $P_{\rm tot}$.

Similarly, the longitudinal spread of the object will fail to Lorentz contract because the constantly renewed partons are of low momentum [5]. Nevertheless, there is a sense in which particle properties behave conventionally under boosts. The size and shape of the regions which contain the information necessary to distinguish particles undergo Lorentz contraction and no transverse spread. This is because the distinctions are carried by the high momentum partons which carry finite fractions of the total momentum. The low momentum cloud is universal.

The transverse size of an object depends both on its longitudinal momentum and the resolution time. However, Lorentz invariance requires it to depend only on the combination $P_{\rm tot}/\epsilon$.

We have ignored effects having to do with large transverse momenta. These effects are interesting but do not lead to further momentum dependence in the size of objects. Instead, they introduce fine detail in the structure of the partons themselves [6].

As we shall see, quantum gravity requires an altogether different description when the momenta of particles begin to exceed the Planck mass. The new type of behavior can be illustrated by a simple model. Let us suppose that a particle with longitudinal momentum P can be described as a bound state of two quanta when the resolution time $\delta\tau$ is of order P in some natural units. For simplicity, the quanta can be assumed to have approximately equal longitudinal momentum and a transverse separation of order unity. If the parent particle has longitudinal momentum P, the constituents have $P/2$. The configuration is described by a wave function

$$\psi=\psi(X_1-X_2)\delta(P_1-P/2)\delta(P_2-P/2)\delta(X_1+X_2)\ , \tag{2.3}$$

where X_1 and X_2 represent the transverse locations of the constituents.

Suppose that when the resolution time is decreased by a factor of 2, each constituent is itself resolved into a pair of new constituents with the same wave functions ψ except that the constituents now have longitudinal momentum $P/4$:

$$\begin{aligned}\psi(y_1,&+y_2,y_3,y_4,Q_1,Q_2,Q_3,Q_4)\\ &=\psi\left[\frac{y_1+y_2}{2}-\frac{y_3+y_4}{2}\right]\psi(y_1-y_2)\psi(y_3-y_4)\\ &\quad\times\delta\left[Q_1-\frac{P}{4}\right]\delta\left[Q_2-\frac{P}{4}\right]\delta\left[Q_3-\frac{P}{4}\right]\delta\left[Q_4-\frac{P}{4}\right]\\ &\quad\times\delta(y_1+y_2+y_3+y_4)\ .\end{aligned} \tag{2.4}$$

The first factor represents the original wave function (2.3) with X_1 and X_2 replaced by $(y_1+y_2)/2,(y_3+y_4)/2$, respectively. The second and third factors represent the compositeness of the original constituents.

Let us continue this process so that each time we improve the resolution by a factor of 2. The previous constituents are resolved into pairs with the wave function ψ. After n iterations, the resolving time is

$$\epsilon \sim P\tfrac{1}{2}^n\ , \tag{2.5}$$

the number of constituents is 2^n, and the longitudinal momentum of each is $P/2^n$.

As the resolving time decreases, the transverse spread of the configuration tends to a Gaussian probability distribution for finding a constituent at a given transverse distance. The width of the Gaussian grows like the square root of the number of iterations. Thus the transverse size R is given by

$$R\left[\frac{P}{\epsilon}\right] \sim \left[\ln\frac{P}{\epsilon}\right]^{1/2}\ . \tag{2.6}$$

This formula describes a growth which is similar to that of the proton that I described before. However, this time the information is spread over the entire area. To see this, we can consider constructing a second state, replacing (2.3) by

$$\psi'=\psi'(X_1-X_2)\delta(P_1-P_2)\delta\left[P_2-\frac{P}{2}\right]\delta(X_1+X_2)\ , \tag{2.7}$$

where ψ' is orthogonal to ψ. In each iteration we still replace each constituent by a pair with the original wave function ψ. After any number of iterations, the two wave functions are orthogonal. However, the density matrices for bounded regions of fixed size R_0 are indistinguishable as $P/\epsilon \to \infty$. To detect the orthogonality of the two states, a region of size $R(P/\epsilon) \sim \ln(P/\epsilon)$ must be inspected. As we shall see, this is a fundamental property that quantum gravity must have if black hole evaporation is to be consistent with quantum mechanics.

The longitudinal size of the distribution can also be estimated. Since the individual constituent longitudinal momenta are of order the resolving time ϵ [7], the uncertainty principle suggests that the longitudinal size Δz satisfies $\Delta z \sim 1/\epsilon$. In conventional terms this is equivalent to an absence of longitudinal Lorentz contraction as $P \to \infty$ with fixed ϵ.

I will now argue that if such a model can be consistent with special relativity, it must contain a graviton. To see this, let us consider the scattering of two particles. We take one particle to be at rest and one moving along the z axis with large momentum P. The fast particle has longitudinal momentum P and the scattering can resolve internal motions with $\delta\tau \approx 1$ so that the fast particle must be described as a number $N \sim P$ constituents. Now consider the low-momentum-transfer elastic amplitude. Let q be the transverse momentum transfer. Such the target can scatter off any of the constituents, the amplitude will be proportional to N. Furthermore, since the spatial distribution of constituents is Gaussian with width of order $(\ln N)^{1/2}$, we find the amplitude to be

$$A(q^2) \sim e^{-(\ln N)q^2} N \sim P^{1-q^2} . \tag{2.8}$$

The reader will recognize this as a Regge behaved scattering amplitude corresponding to a linear Regge trajectory,

$$J(q^2) = 2 - q^2 ,$$

from which one deduces the existence of a massless spin-2 particle.

The above argument is not meant to be a serious mathematical proof. It is a paraphrasing of string theory which we will see has the properties of the model. The main features to remember in this model are that the spatial extension of the cloud of information carried by a particle has longitudinal and transverse extension which depends on the ratio of the longitudinal momentum and resolution time. The pattern of transverse growth that occurs as the resolution time is decreased is a common feature of many systems and is called *branching diffusion*.

III. IMPLICATIONS OF BLACK HOLE COMPLEMENTARITY

Consider an object falling toward the horizon of a black hole. From the view point of fiducial observers at fixed static position, the momentum of the object increases without bound and its internal motions slow indefinitely. In effect, the fiducial observers outside the black hole see the object with increasing powers of resolution. To follow this process into the stretched horizon at a few Planck lengths from the event horizon, the Lorentz boost properties of matter must be thoroughly understood.

The black hole can be described by external observers in terms of tortoise coordinates which cover only the exterior region. Tortoise time is identical to Schwarzschild time and the tortoise radial coordinate r^* is defined by

$$r^* = r + 2m \ln(r - 2m) . \tag{3.1}$$

Far from the horizon, the metric has the flat space form

$$ds^2 = dt^2 - (dr^*)^2 - (r^*)^2 d\Omega^2 . \tag{3.2}$$

Near the horizon, it locally behaves like

$$ds^2 = \left[\frac{e^{r^*/2m}}{2m} \right] [-(dr^*)^2 + dt^2] - dX_\perp^2 , \tag{3.3}$$

where $X_\perp$ are Cartesian coordinates transverse to the radial direction.

As the particle falls toward the horizon, its longitudinal momentum increases like $\exp(t/4m)$. If the system behaves like a conventional classical object, it will appear to have fixed transverse size and Lorentz contracted longitudinal extension. The center of the object will move on a trajectory which approaches

$$r^* + t = 0 \tag{3.4}$$

as $t \to \infty$, and its longitudinal extension Δr^* will satisfy

$$e^{r^*/4m} \Delta r^* \approx e^{-t/4m} , \tag{3.5}$$

or

$$\Delta r^* \sim 1 . \tag{3.6}$$

Eventually, the particle and all its structure disappears to $r^* = -\infty$ and is lost. At best the information can be retrieved at the very end of the Hawking evaporation. It is this picture that is implicit when conventional quantum field theory is studied in curved space-time.

Black hole complementarity requires a different behavior from the viewpoint of the external observer. The information carried by the object should get deposited in a layer called the stretched horizon which is located near $r^* = 0$. This is the layer where the local temperature of the Unruh radiation is of Planckian magnitude. Furthermore, the information is supposed to be distributed among the hypothetical stretched horizon degrees of freedom as if it were being thermalized. In the final state of thermal equilibrium, the information should be delocalized over the entire horizon. A reasonable guess is that the information diffuses so that its transverse spread grows like

$$R^2 \sim \frac{t}{4M} ; \tag{3.7}$$

we use $t/4M$ because the proper time on the stretched horizon is red-shifted relative to Schwarzschild time by a factor of order $4M$.

The longitudinal spread of the information implied by complementarity is also unconventional. The region occupied by the system must continue to overlap the stretched horizon near $r^* = 0$. This requires (3.5) and (3.6) to be replaced by

$$\Delta r^* \sim e^{t/4M} . \tag{3.8}$$

This is equivalent to the condition that no Lorentz contraction takes place once the particles falling into the black hole reach momenta of order the Planck mass. In other words, the longitudinal extension Δz should satisfy

$$|\Delta z| \sim 1 , \tag{3.9}$$

in Planck units.

The conditions (3.8) and (3.9) are just those satisfied by the branched diffusion model of Sec. III. Of course, the model was cooked up for just this purpose. However, it is interesting that is also leads to the existence of a massless spin-2 graviton.

IV. STRINGS NEAR A HORIZON

String propagation in a Schwarzschild background has not been completely analyzed. However, the region near the event horizon of a very large black hole is enough like Minkowski space that much of what we need to know can be determined. In particular, as long as the region under study is small in all its dimensions compared with the Schwartzschild radius, the external region of the horizon is isomorphic to Rindler space.

In the previous sections we assumed that the standard laws of physics hold down to the Planck scale. In string theory, however, the new physics begins at the *string scale* which differs from the Planck scale by factors of the dimensionless coupling constant g. If l_P is the Planck length and l_s is the string length, then

$$l_P = g l_s \ , \tag{4.1}$$

so that if g is very small, the new physics begins at length scales appreciably larger than l_P. In this case the stretched horizon should be placed a distance $\sim l_s$ from the event horizon. At this distance the Unruh temperature is the same as the Hagedorn temperature and the properties of the vacuum become markedly different [8] from the zero-temperature vacuum seen by a freely falling Minkowski observer. This is also the place where the standard rules of Lorentz contraction begin to fail and where information begins to transversely spread [3]. The Planck length is where perturbative string theory fails but the interesting physics seen by the outside observers takes place between l_s and l_P.

Let us compute the properties of free strings in the light-cone frame. Points of the string are described by a transverse location $X_\perp(\sigma)$ and whatever internal degrees of freedom are implied by supersymmetry and compactification. In the light-cone frame the internal degrees of freedom are decoupled from $X_\perp(\sigma)$. Thus the normal-mode decomposition of $X_\perp$ is the same as for free bosonic strings:

$$X_\perp(\sigma) = X_\perp(\text{c.m.}) + \sum_l \frac{X(l)}{l} e^{il\sigma} + \frac{\tilde{X}(l)}{l} e^{-il\sigma} \ . \tag{4.2}$$

The transverse size of the string can be estimated by computing the quantity

$$R_\perp^2 = \langle [X_\perp(\sigma) - X_\perp(\text{c.m.})]^2 \rangle \ , \tag{4.3}$$

where the expectation value is calculated in whatever state is under consideration. From the ground state one easily finds

$$R_\perp^2 = \sum_l \frac{1}{l} \ , \tag{4.4}$$

which diverges logarithmically. This divergence is the key to understanding the curious properties of strings under Lorentz boosts.

The divergence in $R_\perp^2$ is due to a summation over modes of arbitrarily high frequency. The frequency of the lth normal mode in light-cone time τ is

$$\nu_l = \frac{l}{P_{\text{tot}}} \ , \tag{4.5}$$

where P_{tot} is the longitudinal momentum of the string. In (4.5) the frequency is defined in string units. If an experiment is performed by an observer with a resolution time ϵ, then the modes with $\nu > 1/\epsilon$ should be cut off. Hence we define a resolution-dependent size $R_\perp^2(\epsilon)$ by

$$R_\perp^2(\epsilon) = \sum_{l=1}^{P_{\text{tot}/\epsilon}} \frac{1}{l} \approx \ln \frac{P_{\text{tot}}}{\epsilon} \ . \tag{4.6}$$

Evidently, the transverse size grows exactly as in the branched diffusion model of Sec. II [3,9]. The phenomenon of transverse growth with decreasing resolution time has been noted previously but its connection with black hole horizons has not. The longitudinal behavior as a function of resolution has not, to my knowledge, received any attention. To compute the mean longitudinal spread Δz, we use the constraint equation

$$\frac{\partial X^+}{\partial \sigma} = \frac{\partial X_\perp}{\partial \sigma} \frac{\partial X_\perp}{\partial \tau} + I \ , \tag{4.7}$$

where I represents the contribution from compactified modes, fermionic modes, etc. We can rewrite (4.7) in terms of Virasoro generators:

$$\frac{\partial X^+}{\partial \sigma} = \sum_l L(l) e^{il\sigma} - \tilde{L}(l) e^{-il\sigma} \ , \tag{4.8}$$

which can be integrated to give

$$X^+ = X^+(\text{c.m.}) + \sum \frac{L(l) e^{il\sigma}}{il} + \frac{\tilde{L}(l)}{il} e^{-il\sigma} \ . \tag{4.9}$$

Using the standard Virasoro algebra, one finds

$$\langle 0 | [X^+(\sigma) - X^+(\text{c.m.})]^2 | 0 \rangle \sim \sum_l l \ , \tag{4.10}$$

which diverges quadratically. The cure is as before. Averaging X^+ over a resolution time ϵ cuts off the sum at $l \sim P/\epsilon$. Thus,

$$\langle 0 | [X^+(\sigma) - X^+(\text{c.m.})]^2 | 0 \rangle \sim \frac{P^2}{\epsilon^2} \ , \tag{4.11}$$

or

$$|\Delta X^+| \sim \frac{P}{\epsilon} \ . \tag{4.12}$$

Equation (4.12) indicates that no Lorentz contraction of the string distribution takes place. The two properties (4.6) and (4.12) are precisely what is needed in order that an infalling string appear to spread over the stretched horizon without escaping to $r^* = -\infty$.

The spreading process begins to occur when the string reaches the stretched horizon at distance l_s from the event horizon. The process is very similar to the stochastic evolution of a scalar field in an inflating universe. In both cases more and more modes enter the description with time. These modes enter with random phase and amplitude. In each case the growth and spreading over the *target space* can be described by stochastic interactions with a heat bath. In the string case the heat bath is provided by the Unruh effect.

If no other effects take place, the string would grow to a size comparable to the Schwarzschild radius in a time given by

$$t = g^2 M^3 , \tag{4.13}$$

in Planck units. If g is small, this is a short time by comparison with the evaporation time of the black hole.

As the string replicates, its transverse density increases. At the center of the distribution the average number of strings N passing through a region of area A (measured in string units) is of order $\exp(R_\perp^2) \sim e^{t/4M}$. However, this enormous density of string certainly leads to new effects once it becomes of order $1/g^2$. At this time the probability for string interactions becomes unity and perturbation theory breaks down. One attractive possibility is that the growth of string density is cut off at this point. The result would be that the density grows until there is about one string per unit Planck area. This is also suggested by the fact that the entropy of a black hole is proportional to its area as measured in Planck units.

Perhaps the most remarkable aspect of the above description is that none of it is seen by an observer who falls through the horizon with the string. Such an observer sees the string with a fixed time resolution and therefore sees a constant transverse and longitudinal size as the horizon is crossed.

V. PHILOSOPHICAL IMPLICATIONS

Black hole complementarity and its realization in string theory imply profound changes in our current views of matter and space-time. These concepts further erode the classical realism of the Newtonian picture of the universe. They entail a new degree of relativity and observer dependence of reality. The special theory of relativity destroyed the invariant meaning of simultaneity. Quantum theory introduced the idea of incompatible measurements and eliminated the classical concept of a well-defined trajectory. What was left intact is the invariant event, occurring in a well-defined space-time location even if that event can only be predicted statistically. Now, however, even that can no longer be relied upon. Consider, for example, that the destruction of an individual falling into a black hole takes place in a space-time region and in a manner which appears entirely different to observers in free fall and those supported outside the horizon. To those in free fall, the individual easily survives the passage through the horizon but is destroyed by infinite tidal forces much later. The outside observer witnesses death by heat at the stretched horizon. Which is correct? In my view there is no more an answer to this question than to whether two events really are simultaneous or to which of the two paths a photon traveled.

All of this is possible only because matter is not anchored in space-time as in classical or quantum field theory. The more precisely one tries to resolve the location of the constituents of matter, the more they fluctuate to large distances. Probing strings with infinite time resolution reveals that each bit of string fills space out to infinite distances. Only because finite energy implies finite time resolution do we see localized matter.

ACKNOWLEDGMENTS

The author thanks A. Peet and L. Thorlacius for help in calculating the longitudinal spread of strings. He also thanks A. Mezhlumian for informative discussions about branched diffusion processes, and J. Susskind for technical assistance. The work was supported in part by NSF Grant No. PHY 89 17438.

[1] G. 't Hooft, Nucl. Phys. **B335**, 138 (1990).
[2] L. Susskind, L. Thorlacius, and J. Uglum, Phys. Rev. D **48**, 3743 (1993); L. Susskind and L. Thorlacius, *ibid.* **49**, 966 (1994); very similar ideas have been independently advocated by E. and H. Verlinde [H. Verlinde (private communication)].
[3] L. Susskind, Phys. Rev. Lett. **71**, 2367 (1993).
[4] J. Kogut and L. Susskind, Phys. Rep. **8C**, 75 (1973).
[5] I first heard of this breakdown of Lorentz contraction from J. D. Bjorken (private communication).
[6] J. Kogut and L. Susskind, Phys. Rev. D **9**, 697 (1974); A. Casher, J. Kogut, and L. Susskind, *ibid.* **9**, 706 (1974).
[7] Parton models in which all the constituent momenta remain small are also discussed by C. B. Thorn, in the Proceedings of the Sakharov Memorial Lectures in Physics, Moscow, 1991 (unpublished), Vol. 1, pp. 447–453; Florida Report No. UFIFT-HEP-91-23 (unpublished).
[8] J. Atick and E. Witten, Nucl. Phys. **B310**, 291 (1988).
[9] M. Karliner, I. Klebanov, and L. Susskind, Int. J. Mod. Phys. A **3**, 1981 (1988).