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Unitarity and causality in generalized quantum mechanics for nonchronal spacetimes

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Spacetime must be foliable by spacelike surfaces for the quantum mechanics of matter fields to be formulated in terms of a unitarily evolving state vector defined on spacelike surfaces. When a spacetime possesses nonchronal regions which cannot be foliated by spacelike surfaces, as in the case of spacetimes with closed timelike curves, a more general formulation of quantum mechanics is required. In such generalizations the transition matrix between alternatives on two spacelike surfaces lying in regions of spacetime where foliating families *can* be defined may be nonunitary if a nonchronal region lies between them. This paper describes a sum-over-histories generalized quantum mechanics whose probabilities consistently obey the rules of probability theory even in the presence of such nonunitarity. The usual notion of state on a spacelike surface is lost in this generalization. Anomalies such as nonconservation of energy or "Everett phones" that are exhibited by some generalizations of quantum mechanics are not found in this one. However, the generalization is acausal in the sense that the existence of nonchronal regions of spacetime in the future can affect the probabilities of alternatives today and signaling outside the light cone is possible. The detectability of nonunitary evolution and violations of causality in measurement situations are briefly considered.

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I. INTRODUCTION

Conventional formulations of the quantum mechanics of matter fields in a curved background spacetime require that this spacetime be foliable by a family of spacelike surfaces. A family of spacelike surfaces is needed just to define a state of the matter fields on a spacelike surface and the progress of this state into the future by either unitary evolution between spacelike surfaces or by "state vector reduction" on them. However, not all spacetimes admit a foliation by spacelike surfaces. For example, spacetimes with closed timelike curves, such as would be produced by the motion of wormhole mouths, permit no foliating family of spacelike surfaces [1]. The quantum mechanics of matter fields in spacetimes with such nonchronal regions therefore cannot be formulated in terms of the evolution of states on spacelike surfaces. Rather, a more general formulation of quantum mechanics is required. Generalizations based on the ideas of quantum computation have been described by Deutsch [2], generalizations based on the algebraic approach to field theory have been discussed by Yurtsever [3], and yet others may be possible [4] using Hawking's Euclidean approach to quantum field theory on curved backgrounds [5]. Here, we pursue another class of generalizations based on the (Lorentzian) sum-over-histories formulation of quantum theory. Generalizations of this kind have previously been discussed by Klinkhammer and Thorne [6], Friedman, Papastamatiou, and Simon [7], and the author [8]. Specifically, we explore the notions of unitarity and causality and the connections between them in this class of generalizations.

Feynman's sum-over-histories formulation of quantum mechanics is a natural route to a generalized quantum mechanics of matter fields in spacetimes with nonchronal regions because, with it, quantum mechanics may be cast into a fully spacetime form that does not employ a notion of state that evolves through a foliating family of spacelike surfaces [6–8]. For example, in the sum-over-histories formulation, quantum dynamics is expressed, not through a differential equation for such a state, but rather by giving the amplitude for a fine-grained field history—a four-dimensional field configuration $\phi(x)$. In Feynman's prescription this amplitude is proportional to

$$\exp\bigl(iS[\phi(x)]/\hbar\bigr), \tag{1.1}$$

where S is the action functional for the field. Quantum dynamics can be defined in this way even when spacetime contains nonchronal regions. The alternatives potentially assigned probabilities by quantum theory can also be described four dimensionally as partitions, or coarse grainings, of the fine-grained field histories into an exhaustive set of exclusive classes. For instance, the four-dimensional field histories could be partitioned by the values of the field configurations $\phi(\mathbf{x})$ on a spacelike surface σ. The amplitudes for such alternatives define state functionals on σ in familiar quantum theories formulated in terms of states on spacelike surfaces. Even in nonchronal regions, where there are no foliating families of spacelike surfaces, meaningful coarse grainings of four-dimensional field configurations may still be defined. For example, the field histories may be partitioned by

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the value of a field averaged over a region of spacetime deep inside a wormhole throat. A decoherence functional defining the interference between such individual alternatives may be defined in terms of a density matrix representing the initial condition and operators representing the individual alternatives. From this probabilities for decohering sets of alternatives may be calculated. In this way the quantum theory of fields may be put into fully four-dimensional form free from the need of a foliating family of spacelike surfaces [8–11].

If the nonchronal regions of spacetime are bounded, then the spacetime contains regions before and after the nonchronal one in which familiar alternatives of the spatial field configurations can be defined on spacelike surfaces (Fig. 1). Transition probabilities between such alternatives are of interest. Transition amplitudes between a definite spatial field configuration $\phi'(\mathbf{x})$ on an initial spacelike surface σ' and a configuration $\phi''(\mathbf{x})$ on a final surface σ'' are given by a sum-over-histories expression of the form

$$\langle \phi''(\mathbf{x}), \sigma'' | \phi'(\mathbf{x}), \sigma' \rangle = \int_{[\phi', \phi'']} \delta\phi \; \exp\{iS[\phi(x)]/\hbar\} \; . \tag{1.2}$$

The sum is over four-dimensional field configurations between σ' and σ'' that match the prescribed spatial configurations on those surfaces. By such methods, for example, an S matrix for scattering through spacetime regions with closed timelike curves could be defined and calculated.

When spacetime can be foliated by a family of spacelike surfaces, Eq. (1.2) coincides with the unitary evolution operator generated by the Hamiltonian for the family. That is because, as Dirac [12] and Feynman [13] showed, when two spacelike surfaces are close together, the matrix elements of the operator effecting unitary evolution between them is proportional to $\exp(iS)$ where S is the action of the classical field history interpolating between the two. Explicitly in the case of two constant-time surfaces in Minkowksi space,

$$\begin{aligned} &\exp\left\{iS\left[\phi''(\mathbf{x}), t''; \phi'(\mathbf{x}), t'\right]/\hbar\right\} \\ &\quad \propto \langle \phi''(\mathbf{x})| \exp\left[-iH(t''-t')/\hbar\right] |\phi'(\mathbf{x})\rangle \; . \end{aligned} \tag{1.3}$$

However, in the absence of a connection such as (1.3), or even a well-defined meaning for its right-hand side, there is no particular reason to expect a construction such as (1.2) to yield a unitary transition matrix.[1] Calculations by Klinkhammer and Thorne [6] in nonrelativistic quantum mechanics first suggested that the evolution defined by (1.2) might be nonunitary. General results of Friedman, Papastamatiou, and Simon [7,14] in field theory, and explicit examples of Boulware [15] and Politzer [16], show the following: The scattering matrix constructed from the sum over histories (1.2) is unitary for free field theories in spacetimes with closed timelike curves, but not, in general for interacting theories, order by order in perturbation theory.[2] This paper discusses the implications of this nonunitarity.

Even were spacetime foliable by spacelike surfaces it would still be difficult to reconcile nonunitary evolution with the notion of state on a spacelike surface. The reasons, stated clearly by Jacobson [19], are reviewed in Sec. II. However, a generalized quantum mechanics neither requires nor does it always permit a notion of "state on a spacelike surface." In Sec. III we spell out enough details of the generalized sum-over-histories quantum mechanics sketched above to show how it consistently incorporates nonunitary evolution represented by the transition matrix (1.2) without employing a notion of state on a spacelike surface. The price for this generalization is not only the absence of a notion of state on a spacelike surface, but also a violation of causality that is discussed in Sec. IV. The existence of future nonchronal regions of spacetime will influence probabilities in the present. A theory of the future geometry of spacetime, as well as of the initial condition of the closed system and the geometry up to the present, is thus required for present prediction.

The theory of laboratory scattering measurements in the presence of nonchronal regions is developed in Sec. V and used to give a preliminary discussion of how violations of unitarity and causality might be detected. Section VI shows that some anomalies such as nonconservation of energy and communication between noninterfering branches that exist in some other generalizations of quantum mechanics are absent from this one. Section VII contains some brief conclusions.

II. NONUNITARITY AND THE QUANTUM MECHANICS OF STATES

In its simplest interpretations, nonunitary evolution of a quantum state defined on spacelike surfaces is either inconsistent or, as shown by Jacobson [19], dependent on the choice of spacelike surfaces. This section briefly reviews these arguments.

We consider a fixed background spacetime containing a bounded, nonchronal region NC, as shown in Fig. 1. Consider an initial state $|\psi(\sigma')\rangle$ on a spacelike surface σ' before[3] NC. Suppose its evolution to a spacelike surface σ'' after NC is given by a nonunitary evolution operator X:

$$|\psi(\sigma'')\rangle = X|\psi(\sigma')\rangle \; . \tag{2.1}$$

We now consider the calculation of the probabilities of

[1]A conclusion also reached by Deutsch [2] from the point of view of quantum computation.

[2]Goldwirth, Perry, and Piran [17] concluded that even free theories were nonunitary. This was corrected in [18], where some of the results of Klinkhammer and Thorne [6] for free theories are included.

[3]We shall be more precise about the meanings of "before" and "after" in Sec. III.

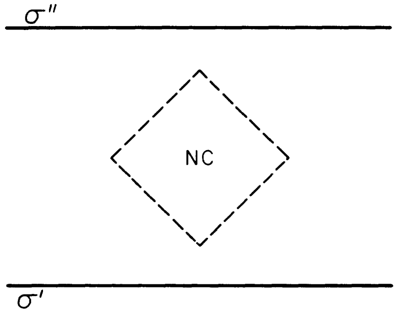

FIG. 1. A compact nonchronal region of spacetime NC with spacelike surfaces σ' and σ'' before and after. Alternative field configurations may be defined on these spacelike surfaces, but the transition matrix between them defined by a sum over intermediate field configurations is not necessarily unitary if the field is interacting.

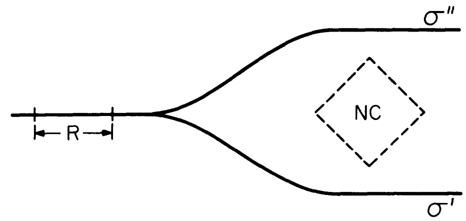

FIG. 2. A local piece of a spacelike surface R that is spacelike separated from a nonchronal region NC. R may be regarded either as lying on a spacelike surface σ' before NC or as lying on a spacelike surface σ'' after NC. If quantum mechanics is to be consistently formulated in terms of states on spacelike surfaces, then a prescription must be given for whether to compute the probabilities of alternatives confined to R with σ' or σ'' if the evolution through NC is not unitary, for the results are not the same.

an exhaustive set of exclusive alternatives on this spacelike surface represented by a set of (Schrödinger-picture) projection operators satisfying

$$\sum\nolimits_{\alpha} P_\alpha = 1 \quad , \quad P_\alpha P_\beta = \delta_{\alpha\beta} P_\beta \ . \tag{2.2}$$

What rule should be used to calculate these probabilities?

The usual prescription for the probability of the alternative corresponding to P_α on σ is

$$p(\alpha;\sigma) = \parallel P_\alpha |\psi(\sigma)\rangle \parallel^2 \ . \tag{2.3}$$

If $|\psi(\sigma')\rangle$ is normalized so that

$$\sum\nolimits_{\alpha} p(\alpha;\sigma') = 1 \ , \tag{2.4}$$

then (2.1) will imply for the probabilities of the same alternatives on the later surface

$$\sum\nolimits_{\alpha} p(\alpha;\sigma'') = \langle \psi(\sigma')|X^\dagger X|\psi(\sigma)\rangle \neq 1 \ . \tag{2.5}$$

When X is not unitary, probability is not conserved and the prescription (2.3) for assigning probabilities is thus inconsistent.

The generalization of (2.3),

$$p(\alpha;\sigma) = \frac{\parallel P_\alpha |\psi(\sigma)\rangle \parallel^2}{\parallel |\psi(\sigma)\rangle \parallel^2} \tag{2.6}$$

suggests itself as a way of maintaining the requirement that the probabilities of an exhaustive set of alternatives sum to unity. However, Jacobson [19] showed that this rule is not covariant with respect to the choice of spacelike surfaces. Consider a set of alternatives $\{P_\alpha(R)\}$ that distinguish only properties of fields on a spacelike surface that are restricted to a region R that is spacelike separated from a nonchronal region NC. For example, an exhaustive set of ranges of the average of a field over R defines one such set of alternatives. Since R is spacelike to NC it may be considered either as a part of a spacelike surface σ' that is before NC or as part of a spacelike surface σ'' that is after NC (Fig. 2). According to (2.6) and (2.1), the probabilities for the alternatives $\{\alpha\}$ evaluated on σ' would be

$$p(\alpha;\sigma') = \frac{\langle\psi(\sigma')|P_\alpha(R)|\psi(\sigma')\rangle}{\langle\psi(\sigma')|\psi(\sigma')\rangle} \ , \tag{2.7}$$

while on σ'' they would be

$$p(\alpha;\sigma'') = \frac{\langle\psi(\sigma')|X^\dagger P_\alpha(R) X|\psi(\sigma')\rangle}{\langle\psi(\sigma')|X^\dagger X|\psi(\sigma')\rangle} \ . \tag{2.8}$$

These must be equal since the alternatives are the same.

A state on σ' that is an eigenvector of the field configuration $\phi(\mathbf{x})$ with $\mathbf{x} \in R$, evolves into a state on σ'' that is also an eigenvector of $\phi(\mathbf{x})$, with $\mathbf{x} \in R$, having the same eigenvalue. Thus,

$$[X, P_\alpha(R)] = 0 \ . \tag{2.9}$$

Were X unitary, (2.9) would imply the equality of the numerators in (2.7) and (2.8) and of the denominators as well. However, when X is nonunitary the expressions (2.7) and (2.8) cannot be equal for all states $|\psi(\sigma)\rangle$. Nonunitary evolution therefore implies that the probabilities for the alternatives $\{P_\alpha(R)\}$ are different on σ' and σ''. Quantum mechanics defined by the rule (2.6) is not covariant with respect to the choice of spacelike surfaces unless the evolution is unitary. This is the essence of Jacobson's argument.

Thus a nonunitary transition matrix, say, constructed by a sum over histories as in (1.2), cannot be used to construct a quantum mechanics in which probabilities are computed from a "state of the system on a spacelike surface" using either of the prescriptions (2.1) or (2.6) if we insist on covariance with respect to the choice of spacelike surfaces. In the next section we shall show how a generalized quantum mechanics that avoids this problem can be constructed incorporating such nonunitary transition matrices. Such generalizations will not, of course, admit a notion of "state on a spacelike surface" in any of the senses discussed in this section.

III. GENERALIZED QUANTUM MECHANICS

In this section we will spell out more concretely some details of a sum-over-histories generalized quantum theory that consistently incorporates nonunitary evolution. We have described the principles of generalized quantum mechanics elsewhere [8,11] and do not review them here. The discussion is aimed at setting a formal framework for a generalized quantum mechanics of matter fields that incorporates nonunitary evolution and not at issues concerning the mathematical definition of the elements of such a theory, their finiteness, regularization, etc.[4]

We are concerned most generally with the quantum mechanics of a closed system containing both observers and observed, both measuring apparatus and measured subsystems. In the present investigation, the closed system is an interacting quantum field theory in a *fixed* given, background spacetime geometry. To keep the notation manageable we shall consider a single, scalar field $\phi(x)$. We shall assume that spacetime outside a bounded region NC is foliable by spacelike surfaces (see Fig. 3). Thus we can identify an initial region $\mathcal{IN}$(NC) outside of NC, no point of which can be reached from any point of NC by a timelike curve that is future pointing outside of NC. $\mathcal{IN}$(NC) is foliable by spacelike surfaces. Similarly we can define a final region $\mathcal{FN}$(NC) [generally overlapping $\mathcal{IN}$(NC)] that is foliable by spacelike surfaces. We will loosely refer to $\mathcal{IN}$(NC) as "before" NC and $\mathcal{FN}$(NC) as "after" NC. The initial condition of the closed system is specified by a density matrix ρ defined on a spacelike surface in $\mathcal{IN}$(NC). Throughout we adopt the point of view of quantum cosmology, in which there is one fixed initial condition, most generally that for the universe as a whole. The two fixed inputs which must be supplied before the theory will yield predictions are therefore the geometry of the background spacetime and the density matrix specifying the initial condition.

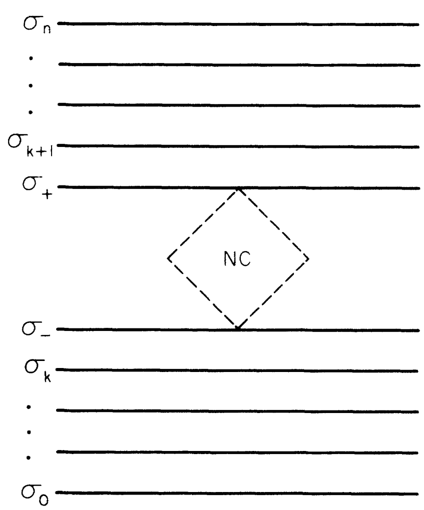

FIG. 3. A spacetime with a single nonchronal region NC. Before NC there is an initial region that can be foliated by spacelike surfaces some of which are illustrated. Afterwards there is a similar final region. The text describes a generalized quantum mechanics for computing the probabilities of decoherent sets of histories of alternatives defined on these surfaces even when the evolution through NC is nonunitary.

The most general objective of a quantum theory for a closed system is the prediction of the probabilities of the individual histories in an exhaustive set of alternative, coarse-grained histories of the system. In a sum-over-histories formulation of quantum fields in a background spacetime, sets of coarse-grained histories are most generally defined by partitions of the four-dimensional field configurations (the fine-grained histories) into an exhaustive set of exclusive classes. The individual classes are the individual coarse-grained histories in the set. A familiar way of partitioning the set of four-dimensional field configurations is by the values of *spatial* field configurations on one or more spacelike surfaces. In an operator formulation of field theory such partitions correspond to alternative values of the field operators on the spacelike surfaces. We cannot expect to define such alternatives inside the nonchronal regions where there are no spacelike surfaces, but we can define such alternatives both before and after the nonchronal region NC. Transition amplitudes between such alternatives on two spacelike surfaces far in the past and far in the future, for instance, specify scattering theory. Such transition amplitudes are defined by sums over the field histories in between the surfaces as in (1.2). When one surface is before NC and the other after, the results of Friedman, Papastamatiou, and Simon [14] show that this transition amplitude will be nonunitary for interacting field theories.

In this paper we will restrict attention to histories that are sequences of alternatives defined on spacelike surfaces before and after the nonchronal region. This may seem a limited class from the point of view of a generalized quantum mechanics which can deal with more general spacetime alternatives[5] that are the only ones possible *inside* NC. However, we make this restriction for three reasons. (1) Consideration of alternatives on spacelike surfaces before and after NC is sufficient to illustrate the issues of unitarity and causality with which we will be concerned. (2) Alternatives on spacelike surfaces can be represented by sets of projection operators in the usual Hilbert space of field theory in a familiar way, and the sums over histories such as (1.2) may be taken to define the matrix elements of transition *operators* in this Hilbert space, so that the resulting framework has many similarities with the Hilbert space formulation of usual quantum

[4] Such problems arise mostly from the behavior of the elements of the theory on small scales will certainly be no better in the present framework but also possibly no worse. For discussion of the standard approaches to these issues in quantum field theory in curved spacetime see, e.g., [20,21].

[5] For discussion of these more general classes of spacetime alternatives see, e.g., [8,9,22].

mechanics (but also, of course, some differences).[6] (3) The resulting framework will be largely independent of the specific mechanism of nonunitarity (1.2) and suitable for treating histories of alternatives on spacelike surfaces outside of NC whatever the origin of nonunitary transitions across it. It is therefore potentially applicable in a wider range of formulations than just the sum-over-histories one when restricted in this way.

In the Schrödinger picture, an exhaustive and exclusive set of alternatives defined on a spacelike surface corresponds to a set of projection operators $\{P_\alpha\}$ satisfying (2.2). The P_α, for example, might be projections onto ranges of values a field averaged over a spatial region R in the surface. Specifying (generally different) sets of alternatives $\{P^1_{\alpha_1}\}, \{P^2_{\alpha_2}\}, \ldots, \{P^n_{\alpha_n}\}$ on a sequence of nonintersecting spacelike surfaces $\sigma_1, \ldots, \sigma_n$ defines a set of coarse-grained alternative histories for the system. A particular history corresponds to a particular sequence of alternatives $\alpha_1, \ldots, \alpha_n$, that we shall often abbreviate by a single index, viz., $\alpha = (\alpha_1, \ldots, \alpha_n)$. The exhaustive set of histories consists of all possible sequences $\{\alpha\}$. The histories are coarse grained because not all information is specified that could be specified: Alternatives are not specified at each and every time, and the alternatives that are specified do not correspond to a complete set of states unless all the P's are one dimensional.

A quantum theory of a closed system does not assign probabilities to every set of coarse-grained histories of a closed system. In the two-slit experiment, for example, we cannot assign probabilities to the alternative histories in which the electron went through one slit or the other and arrived at a definite point on the detecting screen. It would be inconsistent to do so because, as a consequence of quantum mechanical interference, these probabilities would not correctly sum to the probability to arrive at the designated point on the screen. The quantum mechanics of closed systems assigns probabilities only to the members of sets of alternative, coarse-grained histories for which there is negligible interference between the individual histories in the set as a consequence of the system's dynamics and boundary conditions [23–25]. Such sets of histories are said to *decohere*. In a generalized quantum theory, the interference between histories in a set is measured by a decoherence functional incorporating information about the system's dynamics and initial condition. The decoherence functional $D(\alpha', \alpha)$ is a complex function of pairs of histories satisfying certain general conditions that we shall describe below. The set decoheres if $D(\alpha', \alpha)$ is sufficiently small for *all* pairs of *different* histories in the set $\{\alpha\}$. When that is the case, the probabilities of the individual histories $p(\alpha)$ are the diagonal elements of $D(\alpha', \alpha)$. The rule both for when probabilities may be assigned to a set of coarse-grained histories and what these probabilities are may thus be summarized by the fundamental formula

$$D(\alpha', \alpha) \approx \delta_{\alpha'\alpha} p(\alpha) \ . \tag{3.1}$$

When spacetime is completely foliable by spacelike surfaces, the decoherence functional of familiar Hamiltonian quantum mechanics is given by

$$\begin{aligned} D(\alpha', \alpha) = \mathrm{Tr}\Big[& P^n_{\alpha'_n} U(\sigma_n, \sigma_{n-1}) P^{n-1}_{\alpha'_{n-1}} U(\sigma_{n-1}, \sigma_{n-2}) \cdots P^1_{\alpha'_1} U(\sigma_1, \sigma_0) \\ & \times \rho U(\sigma_0, \sigma_1) P^1_{\alpha_1} \cdots U(\sigma_{n-2}, \sigma_{n-1}) P^{n-1}_{\alpha_{n-1}} U(\sigma_{n-1}, \sigma_n) P^n_{\alpha_n} \Big] \end{aligned} \tag{3.2}$$

where ρ is the density matrix describing the initial condition of the system of fields on an initial spacelike surface, σ_0, and $U(\sigma'', \sigma')$ is the unitary evolution operator between spacelike surfaces σ' and σ''.

Generalizing the form of the decoherence functional (3.2) generalizes Hamiltonian quantum mechanics. A wide class of generalizations, called generalized quantum theories [8,11], have decoherence functionals that (i) are Hermitian: $D(\alpha, \alpha') = D^*(\alpha', \alpha)$; (ii) are normalized: $\Sigma_{\alpha\alpha'} D(\alpha, \alpha') = 1$; (iii) have positive diagonal elements: $D(\alpha, \alpha) \geq 0$; and, most importantly, (iv) obey the principle of superposition in the following sense. A coarse graining of the set $\{\alpha\}$ means a partition of that set into a new set of (generally larger) exhaustive and exclusive classes $\{\bar\alpha\}$. A decoherence functional satisfies the principle of superposition when

$$D(\bar\alpha', \bar\alpha) = \sum_{\alpha' \in \bar\alpha'} \sum_{\alpha \in \bar\alpha} D(\alpha', \alpha) \tag{3.3}$$

for all coarse grainings $\{\bar\alpha\}$ of $\{\alpha\}$. When a set of histories decoheres, and probabilities are assigned according to the fundamental formula (3.1), the numbers $p(\alpha)$ lie between 0 and 1 and satisfy the most general form of the probability sum rules

$$p(\bar\alpha) = \sum_{\alpha \in \bar\alpha} p(\alpha) \ . \tag{3.4}$$

The decoherence functional of Hamiltonian quantum mechanics, (3.2), is easily seen to satisfy (i)–(iii), and satisfies (iv) because the projections of a coarser-grained set are *sums* of the projections in the fine-grained set.

Suppose we consider a spacetime and a single

[6]Thus the sum-over-histories formulation of quantum mechanics is not in some sense opposed to Hilbert space as a mathematical notion, for sums over histories can be used to define matrix elements of operators in Hilbert space, and operator methods can sometimes be used to define functional integrals. However, what does not necessarily emerge from a sum-over-histories formulation is any notion of a *Hilbert space of states on a family of spacelike surfaces.* There will be no such concept here for the reasons discussed in Sec. II.

nonchronal region NC and restrict attention to alternatives defined on spacelike surfaces either entirely in the region $\mathcal{IN}(\mathrm{NC})$ "before" NC or in the region $\mathcal{FN}(\mathrm{NC})$ "after" it. Suppose the evolution between a spacelike surface σ_- before NC and a spacelike surface σ_+ after NC is not described by a unitary matrix U, but by a nonunitary matrix X_S. The decoherence functional (3.2) with U replaced by X_S no longer satisfies the general requirements (i)–(iv). However, the following generalization does satisfy them:

$$D(\alpha',\alpha) = N\,\mathrm{Tr}\Big[P^n_{\alpha'_n}U(\sigma_n,\sigma_{n-1})\cdots P^{k+1}_{\alpha'_{k+1}}U(\sigma_{k+1},\sigma_+)X_S U(\sigma_-,\sigma_k)P^k_{\alpha'_k}\cdots U(\sigma_2,\sigma_1)P^1_{\alpha'_1}U(\sigma_1,\sigma_0)$$
$$\times\rho\, U(\sigma_0,\sigma_1)P^1_{\alpha_1}U(\sigma_1,\sigma_2)\cdots P^k_{\alpha_k}U(\sigma_k,\sigma_-)X_S^\dagger U(\sigma_+,\sigma_{k+1})P^{k+1}_{\alpha_{k+1}}\cdots U(\sigma_{n-1},\sigma_n)P^n_{\alpha_n}\Big], \qquad (3.5)$$

where

$$N^{-1} = \mathrm{Tr}\left(X\rho X^\dagger\right) \qquad (3.6)$$

and $\sigma_1,\cdots,\sigma_k$ lie before σ_- in $\mathcal{IN}(\mathrm{NC})$ while $\sigma_{k+1},\cdots,\sigma_n$ lie after σ_+ in $\mathcal{FN}(\mathrm{NC})$.

The expression (3.5) may be simplified by introducing a kind of Heisenberg picture with operators

$$P^i_{\alpha_i}(\sigma_i) = U^{-1}(\sigma_i,\sigma_0)P^i_{\alpha_i}U(\sigma_i,\sigma_0), \quad \sigma<\sigma_-, \qquad (3.7a)$$

$$P^i_{\alpha_i}(\sigma_i) = U^{-1}(\sigma_i,\sigma_f)P^i_{\alpha_i}U(\sigma_i,\sigma_f), \quad \sigma>\sigma_+, \qquad (3.7b)$$

and

$$X = U^{-1}(\sigma_+,\sigma_f)X_S U(\sigma_-,\sigma_0)\ , \qquad (3.8)$$

where σ_f is a final surface in the far future. Then (3.5) is

$$D(\alpha',\alpha) = N\,\mathrm{Tr}\Big[P^n_{\alpha'_n}(\sigma_n)\cdots P^{k+1}_{\alpha'_{k+1}}(\sigma_{k+1})X\ P^k_{\alpha'_k}(\sigma_k)\cdots P^1_{\alpha'_1}(\sigma_1)\rho P^1_{\alpha_1}(\sigma_1)\cdots P^k_{\alpha_k}(\sigma_k)X^\dagger \times P^{k+1}_{\alpha_{k+1}}(\sigma_{k+1})\cdots P^n_{\alpha_n}(\sigma_n)\Big]\ . \qquad (3.9)$$

The expression can be written even more compactly if we introduce the notation

$$C_\alpha = P^k_{\alpha_k}(\sigma_k)\cdots P^1_{\alpha_1}(\sigma_1) \qquad (3.10)$$

for a chain of projections on spacelike surfaces before σ_- and

$$C_\beta = P^n_{\beta_n}(\sigma_n)\cdots P^{k+1}_{\beta_{k+1}}(\sigma_{k+1}) \qquad (3.11)$$

for a chain on spacelike surfaces after σ_+. Then

$$D(\beta',\alpha';\beta,\alpha) = \frac{\mathrm{Tr}\left(C_{\beta'}XC_{\alpha'}\rho C^\dagger_\alpha X^\dagger C^\dagger_\beta\right)}{\mathrm{Tr}\left(X\rho X^\dagger\right)}\ . \qquad (3.12)$$

This compression of notation has emphasized the similarity of the formula for the decoherence functional (3.12) with that of usual quantum mechanics in the Heisenberg picture. The expression (3.12) incorporates a Heisenberg picture initial density matrix and the projection operators defining alternatives evolve according to Heisenberg equations of motion before the nonchronal region and also after it. However, it would be misleading to say that the generalized quantum mechanics is simply the familiar Heisenberg picture with the only novelty being the nonunitary evolution of the projection operators defining alternatives. These operators *cannot* be evolved *through* the nonchronal region by anything like the usual Heisenberg equation of motion

$$P_\alpha(\sigma'') = \mathcal{U}P_\alpha(\sigma')\mathcal{U}^\dagger, \quad \sigma'<\sigma_+<\sigma_-<\sigma'', \qquad (3.13)$$

with some nonunitary $\mathcal{U}$ (say, $\mathcal{U}=UX$, in the above notation). Were $\mathcal{U}$ *nonunitary*, (3.13) would be inconsistent because it does not preserve the relations (2.2). There are thus no Heisenberg equations of motion connecting alternatives before the nonchronal region to alternatives afterwards. Further, as we have discussed, alternatives inside the nonchronal region cannot even be defined by projection operators on foliating families of spacelike surfaces because no such foliating families exist and the inclusion of such alternatives makes the difference from the usual Heisenberg picture even more apparent.

The decoherence functional (3.12) thus defines a quantum mechanics that reduces to the usual one (3.2) when the evolution is unitary, but generalizes it when it is not. It consistently assigns probabilities to decoherent sets of histories. There is no issue of the violation of a probability sum rule such as (2.5) here. All probability sum rules (3.4) are satisfied as a consequence of decoherence, including the elementary requirement that the probabilities of an exhaustive set of alternatives sum to 1. Neither is there hypersurface dependence of local probabilities as with Jacobson's rule (2.6). From (3.12) it follows that the probability of a set of alternatives $P_\alpha(R)$ that distinguish only field values on a local piece of a spacelike surface R that is everywhere spacelike separated from the nonchronal region NC is

$$p(\alpha,\sigma') = N\,\mathrm{Tr}\left[XP_\alpha(R)\rho P_\alpha(R)X^\dagger\right], \qquad (3.14)$$

when R is considered part of a spacelike surface σ' to the before NC, and given by

$$p(\alpha;\sigma'') = N\,\mathrm{Tr}\left[P_\alpha(R)X\rho\, X^\dagger P_\alpha(R)\right], \qquad (3.15)$$

when R is considered part of a spacelike surface after NC. However, since $P_\alpha(R)$ and X commute [cf. Eq. (2.8)], Eqs. (3.14) and (3.15) are equivalent. The rule (2.6) includes or does not include the nonunitary evolution operator X depending on which surface is chosen. By contrast the rules (3.14) and (3.15) both include X. The order of the X with respect to projection operator representing the alternative in R is different depending on whether R is considered a part of σ' or σ'', but that order is immaterial since the operators commute. The generalized quantum mechanics defined by the decoherence functional (3.12) is thus consistent with elementary requirements.

This generalized quantum mechanics obeys the principle of superposition in the sense that the decoherence functional satisfies (3.3). However, probabilities of decoherent sets of histories are not linear in the initial density matrix ρ because of the normalizing factor in the denominator of (3.12). That does not correspond to an observable nonlinearity because, as stressed above, we are considering the quantum mechanics of a closed system, containing both measured subsystems and measuring apparatus, both human observers and what they observe. There is only one initial condition for such a closed system and issue of superposing several of them does not arise. Thus it would be incorrect simply to apply the framework to a measured subsystem, with a ρ representing one of a variety of prepared states of the subsystem, without further analysis. As should be clear from the discussion in Sec. II and will be made more explicit in the following section, the theory does not generally permit a notion of states on a spacelike surface much less of their superposition. However, we shall provide such an analysis and discuss measurement situations and approximate formulas for the probabilities of their outcomes in Sec. V. For these we shall see that there is a sense in which the approximate quantum mechanics of subsystems is nonlinear in the initial state.

We now turn to the important issue of causality in the generalized quantum mechanics just constructed.

IV. CAUSALITY

The past influences the future but the future does not influence the past; that is the essence of causality. A fixed spacetime geometry whose causal structure defines "future" and "past" as needed just to ask whether or not a theory is consistent with causality. A fixed background spacetime *has* been assumed for the field theories that are the concern of this paper, but the future and past cannot be unambiguously distinguished for points *inside* nonchronal regions connected by closed timelike curves. However, we *can* ask whether the probabilities of a set of alternatives defined entirely *outside* such regions are independent of the geometry of spacetime to their future. It is straightforward to see that the generalized quantum mechanics of matter fields described in the previous section is *not* causal in this sense if the evolution through nonchronal regions is not unitary.

Suppose that spacetime contains a single nonchronal region that is to our future and we are concerned with the probabilities of a chain of alternatives C_α all occurring *before* the nonchronal region. If these alternatives decohere, then their probabilities $p(\alpha)$ are given, according to (3.1) and (3.12), by

$$p(\alpha) = N\,\mathrm{Tr}\big(XC_\alpha\rho C_\alpha^\dagger X^\dagger\big), \tag{4.1}$$

where X describes the evolution through the nonchronal region and $N^{-1} = \mathrm{Tr}(X\rho X^\dagger)$. Were X unitary, the cyclic property of the trace could then be used to show

$$p(\alpha) = \mathrm{Tr}\big(C_\alpha\rho C_\alpha^\dagger\big)\ . \tag{4.2}$$

Equation (4.2) could then be written out in the Schrödinger picture using (3.5). Since only $U(\sigma,\sigma_0)$ for values of σ less than the last σ_n occur in the chain C_α, there is no dependence on the geometry of spacetime to the future of the surface σ_n, whether or not it contains nonchronal regions. In this sense, unitary evolution leads to causality.

If X is not unitary, then the probabilities defined by (4.1) depend on the future geometry of spacetime. Experiments could, in principle, detect the existence of nonchronal regions in our future by testing whether present data is better fit by (4.2) or (4.1) with the appropriate X. We shall return to some simple considerations of such experiments in Sec. V.

Another way of seeing that information about the future is required to calculate present probabilities is to write $\rho_f = X^\dagger X$ and use the cyclic property of the trace to reorganize (4.1) as

$$p(\alpha) = N\,\mathrm{Tr}\big(\rho_f C_\alpha\rho C_\alpha^\dagger\big), \tag{4.3}$$

where now $N^{-1} = \mathrm{Tr}(\rho_f\rho)$. Equation (4.3) is the formula for the probabilities of a generalized quantum mechanics with both an initial condition ρ and a final condition ρ_f. Such generalizations were discussed in [23] and [26] for the quantum mechanics of closed systems. Information about *both* the future and the past is required to make predictions in the present. In the example under discussion, that information concerns the failure of unitarity in the future arising from nonchronal regions of spacetime.

The notion of state of the system on a spacelike surface provides the most familiar expression of causality in usual quantum mechanics. From a knowledge of the state in the present, all future probabilities may be predicted. Thus the present determines the future. We next show that the acausal generalized quantum mechanics under discussion does not contain such a notion of state.

When the probability formula is the usual (4.2), it is straightforward to reformulate it in terms of states on spacelike surfaces. Let σ denote the spacelike surface defining the present; let C_α denote a history of alternatives that have already happened and C_β a history of future alternatives whose probabilities we wish to predict. The conditional probability for the future alternatives given the past ones is

$$p(\beta|\alpha) = p(\beta,\alpha)/p(\alpha)\ . \tag{4.4}$$

If the joint probabilities on the right-hand side of (4.4) are given by (4.2), then $p(\beta|\alpha)$ can be written in terms of an effective density matrix $\rho_{\rm eff}$ defined on σ as

$$p(\beta|\alpha) = \mathrm{Tr}\big[C_\beta\rho_{\rm eff}(\sigma)C_\beta^\dagger\big]\ , \tag{4.5}$$

where

$$\rho_{\rm eff}(\sigma) = \frac{C_\alpha\rho C_\alpha^\dagger}{\mathrm{Tr}\big(C_\alpha\rho C_\alpha^\dagger\big)}\ . \tag{4.6}$$

The density matrix $\rho_{\rm eff}(\sigma)$ is the usual notion of state on a spacelike surface. As σ advances, $\rho_{\rm eff}(\sigma)$ is constant in time in this Heisenberg picture until the time of a new alternative is reached at which point it is "reduced" by the addition of a new projection to the chain C_α. The conditional probabilities of future decoherent alternatives continue to be given by (4.5) with the new $\rho_{\rm eff}(\sigma)$.

If the probability formula is (4.1) or (4.3), then it is *not* possible to construct a $\rho_{\rm eff}$ on a spacelike surface from which alone future probabilities can be predicted.[7] Additional information about the existence of future nonchronal regions summarized by ρ_f in (4.2) is required. There is thus no notion of the state of the system on a spacelike surface in this generalized quantum mechanics.

The existence of nonunitary evolution in the future not only acausally affects the probabilities of present alternatives, it also affects their decoherence. Consider, for example, a set of histories defined by alternatives $\{P_\alpha(\sigma)\}$ a single spacelike surface that is before any nonchronal region. The decoherence functional according to (3.14) is

$$D(\alpha', \alpha) = N\,{\rm Tr}\big[XP_{\alpha'}(\sigma)\rho P_\alpha(\sigma)X^\dagger\big]\ . \qquad (4.7)$$

Were X unitary, *any* set of alternatives *automatically* decoheres because of the cyclic property of the trace. If X is nonunitary, then only certain sets of P_α will decohere. Decoherence is therefore acausally affected by the spacetime geometry of the future. However, typical mechanisms of decoherence that involve the rapid dispersal of phase information among ignored variables that interact with those of interest operate essentially locally in time.[8] Such mechanisms may be essentially unaffected by nonunitary evolution in the future. We may, for example, continue to expect the decoherence of alternatives that define the present quasiclassical domain of familiar experience even in the presence of a modest number of future nonchronal regions.

As we have seen, generalized quantum mechanics with nonunitary evolution violates causality because information about the future is required to calculate the probabilities in the present. However, it is important to stress that it is only information about the future geometry of spacetime, which enters as a fixed input in this quantum field theory in curved spacetime, that is required to calculate the probabilities of present alternatives. We do not need to know which of any set of future alternatives actually occurs to calculate present probabilities. That is guaranteed because (1) we are dealing with a quantum mechanics of a closed system and (2) because of the consistency of the probability sum rules which follow from decoherence. To illustrate, let $\{\alpha\}$ denote a set of alternatives accessible to us and $\{\beta\}$ another set in the future such that $\{\alpha, \beta\}$ decoheres. One can find the conditional probability $p(\alpha|\beta)$ of present events given a future alternative β. We could use those to calculate the probabilities of present events, but, since which of the future alternatives occurs is unknown, we should first multiply $p(\alpha|\beta)$ by the probabilities $p(\beta)$ of the future alternatives and sum over the possibilities β. These probabilities are themselves predicted by the theory because the system is closed. Because the alternatives $\{\alpha, \beta\}$ decohere, this sum gives the same answer as a calculation of the probabilities $p(\alpha)$ of the present alternatives using (4.1) directly without consideration of *any* future alternatives [cf. (3.4)]:

$$p(\alpha) \approx \sum_\beta p(\alpha, \beta) = \sum_\beta p(\alpha|\beta)p(\beta)\ . \qquad (4.8)$$

For example,[9] suppose a nonchronal region of spacetime exists in the future but is contained inside an impenetrable box with a door. Observers in the future may open the door to let fields propagate into this region or leave it closed and prevent fields from interacting with it. In the absence of information about which they choose, present probabilities are affected by the existence of such a region, whether the door is opened or not, provided the probability for the observers to open the box is nonzero. Since the future observers are themselves described by quantum fields in this closed system, it is in principle possible to predict the probabilities of whether the observers will open the box or leave it shut from the prescribed initial condition. One would expect the influence of future nonchronal regions on present observations to differ between an initial condition in which the probability of opening the box is high and one where that probability is low. However, we do not need to know the specific decision the future observers take in order to predict present probabilities.

In a quantum mechanics based on the decoherence of coarse-grained sets of alternative histories, probability sum rules such as (4.8) hold in much wider circumstances than those described above. For example, the probabilities of alternatives $\{\alpha\}$ in the present are independent of specific unknown alternatives $\{\beta\}$ whether these are in the future, past, or spacelike separated from $\{\alpha\}$ provided the joint set of alternatives decoheres. Even in a quantum theory of gravity where spacetime geometry is a dynamical variable, and we could envision observers in the future deciding whether to create nonchronal regions or not, similar results would be expected to hold.

V. TESTING NONUNITARY EVOLUTION AND CAUSALITY VIOLATION

How might the nonunitary evolution and causality violation of the present generalized quantum mechanics of nonchronal spacetimes be tested in the laboratory? This section offers a preliminary discussion.

[7]This is evident from the formulas (4.1) and (4.3) and has been widely discussed in quantum mechanics in various contexts. See [27,28,25] for recent examples.

[8]See, for example, the discussion in [25] and the references therein.

[9]Suggested to the author by J. Friedman.

We begin with a simple model quantum cosmology which describes the scattering through a nonchronal region of spacetime. More specifically, we imagine that a small nonchronal region of spacetime has been located and that we direct particle beams so as to interact in that region, measuring their asymptotic states by apparatus that does not itself interact with the nonchronal region to a good approximation. The incoming particles are specified by a pure initial state and final pure states are detected by the apparatus. This is certainly not the most general measurement situation that can be envisioned but gives a simple illustration of the effects of nonunitarity.[10]

We suppose the Hilbert space of the closed system factors into a tensor product $\mathcal{H}_s \otimes \mathcal{H}_r$ of a Hilbert space $\mathcal{H}_s$ describing the scattering particles and a Hilbert space $\mathcal{H}_r$ describing the rest, including the apparatus for preparation and detection. We consider an initial state of this model universe ρ at time t_1 corresponding to the preparation of the experiment as described above. We consider the alternatives for the scattering particles in which they are prepared and detected as members of complete sets of states $\{|\alpha, t\rangle\}$ in $\mathcal{H}_s$ at various times (e.g., wave packets with approximately definite momentum). These alternatives are represented as a set of projection operators

$$S_\alpha(t) = |\alpha, t\rangle\langle\alpha, t| \otimes I_r \tag{5.1}$$

in the Heisenberg-like picture represented in Sec. III.

The histories describing the scattering process are represented by the chains $S_\beta(t_2)S_\alpha(t_1)$ where t_1 and t_2 are the initial and final times of the scattering process. The decoherence functional for these histories is then [cf. (3.9)]

$$D\,(\beta', \alpha'; \beta\alpha) = N\,\mathrm{Tr}\big[S_{\beta'}(t_2) X S_{\alpha'}(t_1) \rho S_\alpha(t_1) X^\dagger S_\beta(t_2)\big], \tag{5.2}$$

where N is given by (3.6).

The defining assumption of the model is that, with an initial ρ appropriate to the experimental setup described above, the nonunitary evolution operator X effectively acts only on $\mathcal{H}_s$. Noting that the decoherence of the alternatives β at time t_2 is automatic because of the cyclic property of the trace, we may then write

$$\begin{aligned} D\,(\beta', \alpha'; \beta, \alpha) = \delta_{\beta'\beta} N\, &\langle \beta, t_2|X|\alpha', t_1\rangle \\ &\times \langle\alpha', t_1|\mathrm{Sp}(\rho)|\alpha, t_1\rangle\, \langle\alpha, t_1|X^\dagger|\beta, t_2\rangle\ , \end{aligned} \tag{5.3}$$

where $\mathrm{Sp}(\rho)$ is the operator on $\mathcal{H}_s$ that is the trace of ρ over $\mathcal{H}_r$ and X is the restriction of the nonunitary evolution to $\mathcal{H}_s$.

[10]A more general measurement theory may be exhibited along the lines described in Ref. [8], Sec. II.10. It is subject to limitations of the character described in Ref. [9] regarding the influence of the measuring apparatus on the probabilities of the measured alternatives.

The decoherence of the measured alternatives α at the initial time t_1 is not automatic. However, in this model of a measurement we *assume* that the interaction of the particles with the apparatus effects the decoherence of the alternative initial states of the particles, say, by correlation with an independent, persistent record of their initial state (as in Ref. [8], Sec. II.10). Effectively we assume

$$\langle\alpha', t_1|\mathrm{Sp}(\rho)|\alpha, t_1\rangle \propto \delta_{\alpha'\alpha}\ . \tag{5.4}$$

The joint probabilities for this now decoherent set of histories are

$$p(\beta, \alpha) = N|\,\langle\beta, t_2|X|\alpha, t_1\rangle\,|^2\,\langle\alpha, t_1|\mathrm{Sp}(\rho)|\alpha, t_1\rangle\ . \tag{5.5}$$

In scattering experiments it is not so much the joint probability $p(\beta, \alpha)$ of both initial and final states that is of interest, but rather the conditional probability $p(\beta|\alpha)$ of a final state β, given an initial one α. This is constructed in the standard way [cf. (4.4)] with the result

$$p(\beta|\alpha) = \frac{|\,\langle\beta, t_2|X|\alpha, t_1\rangle\,|^2}{\Sigma_\beta|\,\langle\beta, t_2|X|\alpha, t_1\rangle\,|^2}\ . \tag{5.6}$$

All reference to the external apparatus has canceled from this effective expression for conditional probabilities. Except for the normalizing denominator, (5.6) is the usual expression the probability of a scattering process. Indeed, were X unitary, the denominator would be unity. The net effect of the nonunitarity of X has simply been to normalize the usual quantity $|\langle\beta t_2|X|\alpha t_1\rangle|^2$ so that the probability sum rule $\Sigma_\beta p(\beta|\alpha) = 1$ is satisfied. This normalizing factor means that the probabilities for the outcomes of the measurements of a final state are not linearly related to the density matrix $|\alpha, t_1\rangle\langle\alpha, t_1|$ that describes the initial state. Experiments which test this nonlinearity would be interesting to investigate.

Were small size nonchronal regions widespread in spacetime, the difference between the probabilities predicted by (5.6) and a standard formula for the same scattering in flat spacetime would be both a means of detecting such nonchronal regions and verifying the nonunitarity of evolution through them. In the absence of estimates of the sizes and density of nonchronal regions and of the X's which describe the evolution through them, we cannot provide estimates of the effect of nonunitary evolution here. It is through (5.6), however, that such effects would be calculated [29].

In a similar way, we could estimate the acausal effects of future nonchronal regions on present experiments. We would compare standard flat space formulas for probabilities with those computed from formulas such as (3.14) with a nonunitary X describing the effect of nonchronal regions in the future on present measurement situations. We would then be led to formulas such as (5.3) or (5.6) for the probabilities of the measured subsystem but with the nonunitary X to the *future* of the projections describing the measurement. The measurement situation would have a chance of detecting a departure from strict causality only if there were a significant probability that the subsystem under study interacted with a nonchronal region subsequent to the time of the experiment. If nonchronal regions are sparse in the future history of spacetime, then we might expect this probability to be very, very small and the resulting violation of causality negligible. If, however, there were a roiling sea of nonchronal regions near a generic final singularity, then the probabilities for causality violation might be more interesting. The present generalized quantum mechanics provides a way of estimating them.

VI. CONSERVATION OF ENERGY, THE ABSENCE OF EVERETT PHONES, AND SIGNALING FASTER THAN LIGHT

Generalized quantum mechanics is a modest generalization of familiar quantum theory that retains the principle of the linear superposition of amplitudes in the form (3.3). Generalized quantum theories, such as the one under discussion, may have a notion of a Heisenberg-like state that specifies the initial and final conditions, but will not always permit a notion of an evolving state on a spacelike surface. Various other generalizations of quantum mechanics have been proposed that retain the notion of a state on a spacelike surface but abandon or modify the principle of superposition in some way. Recent examples, are the work of Banks, Peskin, and Susskind [30] and Srednicki [31] in which pure density matrices evolve into mixed ones, and Weinberg's nonlinear quantum mechanics [32]. The generalization of Banks, Peskin, and Susskind suffers from energy nonconservation while that of Weinberg can permit communication between alternative branches of the universe in situations that have been called the "Everett phone" by Polchinski [33].

The above generalizations of quantum mechanics are nonlinear because they incorporate a nonlinear law of evolution for states on a spacelike surface. The generalized quantum mechanics for nonchronal spacetimes under discussion in this paper cannot be characterized as linear or nonlinear in this way because it does not generally permit a notion of state on a spacelike surface much less a discussion of the law for its evolution. This generalization might be said to be linear in the sense that it respects the linear principle of superposition in the sense of (3.3). However, it might also be said to be nonlinear because, as a consequence of the normalization factor (3.6), probabilities are not quadratically related to a pure state vector describing the initial condition as they would be in usual quantum mechanics. For this reason it is prudent to examine the present generalized quantum mechanics for energy nonconservation, Everett phones, and signaling faster than light. In this section we shall show that decoherence prohibits energy nonconservation and Everett phones. Our arguments apply to all generalized quantum theories although we shall describe them here for the particular case of the generalized quantum mechanics of fields in nonchronal spacetimes. We also describe how signaling faster than light is possible.

A. Energy conservation

When spacetime geometry is time dependent we do not expect conservation of the total energy of matter fields moving in it, even classically. Where there are compact nonchronal regions of spacetime, the geometry will certainly be time dependent. However, we can still analyze the question of energy conservation in those regions where spacetime *is* locally time independent. Specifically, consider a region of spacetime that is foliable by spacelike surfaces labeled by a coordinate t such that $\partial/\partial t$ is a Killing vector which asymptotically corresponds to a time translation in some Lorentz frame. We can then define the energy-momentum four-vector of the matter fields on a spacelike surface of constant t as

$$P^\alpha(t) = \int_t d\Sigma^\beta T^\alpha_\beta, \tag{6.1}$$

where T^α_β is the stress energy of the matter fields and $d\Sigma^\beta$ is an element of the surface of constant t. In particular $P^t \equiv H$ is the total energy of the matter fields and the corresponding quantum mechanical operator is the generator of translations in t. The total energy is conserved between surfaces of constant t because $\partial/\partial t$ is a Killing vector, which means that the operator H is independent of time.

Whether energy is conserved quantum mechanically is a question of the probabilities for the correlation of the values of H on two different surfaces of constant t. A specific example of the calculation of such probabilities will illustrate all the features of the general case. Consider a single nonchronal region as discussed in Sec. III, and suppose that before the nonchronal region there is a region of spacetime with a time-translation symmetry in the sense discussed above. Let $\{P^H_\alpha(t)\}$ denote a set of projections onto an exhaustive set of ranges $\{\Delta_\alpha\}$ of the total energy in matter fields, H, in the Heisenberg-like picture specified by (3.7). Since H is independent of t before the nonchronal region, the projections $\{P^H_\alpha(t)\}$ are also.

Now consider a set of histories which contain the projections $\{P^H_\alpha(t)\}$ at two different times t_1 and t_2 in the region of time-translation symmetry. The chain of projections before σ_- [cf. (3.10)] would have the form

$$C_\alpha = C^c_{\alpha_c} P^H_{\alpha_2}(t_2) C^b_{\alpha_b} P^H_{\alpha_1}(t_1) C^a_{\alpha_a}, \tag{6.2}$$

where $C^a_{\alpha_a}$, $C^b_{\alpha_b}$, and $C^c_{\alpha_c}$ are themselves chains of projections. Suppose the set of alternative histories defined by C_α before σ_- and C_β after σ_+ decoheres. The joint probabilities for the individual histories may be calculated from (3.1) and (3.12). Conservation of energy would mean

$$p\left(\beta, \alpha_c, \alpha_2, \alpha_b, \alpha_1, \alpha_a\right) \propto \delta_{\alpha_1\alpha_2} \tag{6.3}$$

for any choice of the other alternatives $\alpha_a, \alpha_b, \alpha_c$, and β. Equation (6.3) is not a consequence of any operator identity since $C^b_{\alpha_b}$ in (6.2) need not commute with H. However, it is a consequence of decoherence.[11] Decoher-

[11] The argument appears to be part of the lore of consistent histories. The author learned it from R.W. Griffiths. For a more detailed discussion including a consideration of approximate decoherence see [34].

ence guarantees the consistency of probability sum rules. Thus, in particular,

$$p(\alpha_2,\alpha_1) = \sum_{\beta,\alpha_c,\alpha_b,\alpha_a} p(\beta,\alpha_c,\alpha_2,\alpha_b,\alpha_1,\alpha_a) \ . \quad (6.4)$$

However, the history which consists just of alternative values of the energy at time t_1 and t_2 is represented by the chain

$$P^H_{\alpha_2}(t_2)P^H_{\alpha_1}(t_1) \propto \delta_{\alpha_2\alpha_1} \ . \quad (6.5)$$

This chain vanishes unless $\alpha_1 = \alpha_2$ because the projections onto the values of a conserved quantity are independent of time and projections for different alternatives are orthogonal. Thus, $p(\alpha_2,\alpha_1) \propto \delta_{\alpha_1\alpha_2}$. Since the right-hand side of (6.4) is a sum of positive numbers, (6.3) follows also.

Obvious extensions of this argument show that, in general, decoherence guarantees the conservation of energy in regimes of spacetime that possess a time-translation symmetry in the sense described above.

B. Everett phones

In his analysis of Weinberg's nonlinear quantum mechanics, Polchinski [33] has given an example of a kind of "communication" between different branches of the wave function that he dubbed the "Everett phone." Specifically, he considers sets of histories of a spin-$\frac{1}{2}$ ion in a Stern-Gerlach apparatus and a "macroscopic observer." At time t_1, the z component of the spin is determined by the splitting of the Stern-Gerlach beams. At time t_2, if the spin was up, no action is taken by the observer. If the spin was down, it is either left alone or flipped with some probability. At time t_3, if the spin was up at time t_1, the z component of the spin is again determined. In Weinberg's nonlinear quantum mechanics, the probability of the measurement of the spin at time t_3, in the branch where the spin was up at time t_1, depends on whether the observer did or did not flip the spin in the *alternative branch* where the spin was down at time t_1 and the measurement at t_3 does not occur. That is the "Everett phone." In the language of the quantum mechanics of closed systems this is simply an inconsistent set of histories.

These histories of the closed system spin and observer are represented by a sequence of three branch-dependent sets of projections at the times t_1, t_2, and t_3. They are branch dependent because, whether the alternatives {flip, no flip} or the trivial unit projection are used at time t_2 depends on the specific alternatives {up, down} at time t_1. Similarly the sets of projections used at t_3 depend on the specific alternatives at time t_1. Let $\{P_\uparrow(t), P_\downarrow(t)\}$ be the projections representing whether the spin is up or down at time t. Let $\{P_f(t_2),\ (P_{\bar f}(t_2)\}$ represent the alternatives that the spin was flipped or not flipped. The four histories in the set described by Polchinski would be represented by the chains

$$\begin{aligned} &P_\uparrow(t_3)I(t_2)P_\uparrow(t_1) \quad , \quad I(t_3)P_f(t_2)P_\downarrow(t_1) \ , \\ &P_\downarrow(t_3)I(t_2)P_\uparrow(t_1) \quad , \quad I(t_3)P_{\bar f}(t_2)P_\downarrow(t_1) \ , \end{aligned} \quad (6.6)$$

where trivial unit projections have been included for clarity and vanishing chains have been omitted.

Branch dependence is not an obstacle to defining the decoherence of a set of histories [24,25,35]. If the above set decohered in the presence of a nonchronal region to the future of the experiment, the probabilities of the individual histories would be given by

$$p(\alpha) = N\,\mathrm{Tr}\left(XC_\alpha\rho C^\dagger_\alpha X^\dagger\right) , \quad (6.7)$$

where C_α, $\alpha = 1,2,3,4$, is one of the four chains in (6.6). The probabilities of histories in which the spin is up or down at t_3 are independent of whether the spin was flipped or not flipped at t_2 simply because the corresponding chains contain neither the projection P_f nor $P_{\bar f}$.

The above is a specific example of a general situation. Consider a decoherent set of branch-dependent histories $\{\alpha\}$. Partition this set of histories into the class consisting of a single history α and the class $\neg\alpha$ consisting of all other histories. That partition is a coarse graining of the set $\{\alpha\}$ and so is also decoherent. In the coarser-grained set, the probability of α remains $p(\alpha)$. The probability of $\neg\alpha$ is

$$p(\neg\alpha) = \sum_{\beta\neq\alpha} p(\beta) \ . \quad (6.8)$$

Thus both $p(\alpha)$ and $p(\neg\alpha)$ are manifestly independent of alternatives in the other branches. There are no "Everett phones" in generalized quantum mechanics. Decoherence guarantees the independence of individual branches.

C. Signaling faster than light

In a theory which permits signals to travel backward in time along closed timelike curves, it is perhaps not surprising that it is possible to signal outside the light cone. What is less evident is that in the present generalized quantum mechanics this can be done utilizing observers who are outside the nonchronal region as the following example of Friedman and Papastamatiou [36] shows.

Consider the model described in Sec. IV in which a nonchronal region NC is contained in a box that is inpenetrable to quantum fields before time $t = 0$ in some Lorentz frame. A $t = 0$ the box may either open to allow fields to interact with NC or, alternatively, remain closed so that fields never interact with NC. To make this more concrete consider an idealized case in which a single, two-valued degree of freedom, separate from the field degrees of freedom, controls whether the box is open or shut. Then the operators ρ and X would have the form $\mathrm{diag}(\rho_{\mathrm{open}}, \rho_{\mathrm{shut}})$ and $X = \mathrm{diag}(X_{\mathrm{open}}, I)$ in the two-dimensional (open, shut) space where $\rho_{\mathrm{open}}, X_{\mathrm{open}}$, etc., are operators acting on the Hilbert space of spatial field configurations. Next, consider a set of decohering alternatives $\{\alpha\}$ describable in terms of fields restricted to a spacetime region R that is spacelike separated from the volume of the box at $t = 0$. In the histories in which the box opens, the joint probabilities for the alternatives are

$$p(\alpha, \text{open}) = N \,\text{tr}(X_{\text{open}} P_\alpha \rho_{\text{open}} P_\alpha X^\dagger_{\text{open}}), \qquad (6.9\text{a})$$

while in the histories in which the box remains shut the joint probabilities are

$$p(\alpha, \text{shut}) = N \,\text{tr}(P_\alpha \rho_{\text{shut}} P_\alpha), \qquad (6.9\text{b})$$

where tr denotes a trace in the Hilbert space of the field degrees of freedom and N is the usual normalizing factor. Evidently these probabilities will be different even if $\rho_{\text{open}} \propto \rho_{\text{shut}}$. One expects that an observer in R who considers many identical measurement situations localized in R could use the frequencies of measured outcomes to determine these probabilities to be one or the other of the possibilities (6.9), and thereby determine whether the box is open or shut.[12] Therefore, by extension, it seems that an observer who could open or shut such a box could signal to one at a spacelike separation.

VII. CONCLUSION

The familiar quantum mechanics of a state vector that evolves unitarily through a foliating family of spacelike surfaces depends centrally on the existence of a fixed background spacetime geometry with a well-defined causal structure that allows the surfaces on which the state must be defined. When spacetime geometry is not fixed, as in quantum gravity, or when it is fixed but not foliable by spacelike surfaces, some modification of familiar quantum theory seems inevitable. Generalized quantum theory provides a broad framework for constructing extensions of familiar quantum theory that can apply when spacetime is not fixed [11] or when it is fixed but not foliable by spacelike surfaces. Such theories are unlikely to permit a notion of a unitarily evolving state on a spacelike surface or possess familiar notions of causality.

In this paper we have discussed a particular generalized quantum mechanics for matter fields in background spacetimes with nonchronal regions. There may well be others. In this one, the geometry is fixed and given for all time. The matter fields do not modify it. Alternatives are defined four dimensionally as partitions of spacetime field configurations—a notion general enough to describe alternatives in the nonchronal regions which are not foliable by spacelike surfaces. Transition amplitudes between alternatives on spacelike surfaces outside the nonchronal regions are defined by sums of $\exp[i(\text{action})]$ over intermediate field configurations. The nonunitarity of such transition amplitudes can be incorporated into generalized quantum theory through an appropriately defined notion of decoherence. All probability sum rules are satisfied for decoherent alternatives because decoherence implies them.

This generalized quantum theory of fields in spacetimes with nonchronal regions does not display a number of familiar features of quantum theory in flat background spacetime. Most importantly the theory cannot be reformulated in terms of states on spacelike surfaces. That is not surprising since the spacetime itself does not possess a foliating family of spacelike surfaces. Lost with this notion of state is the familiar idea of causality in the sense that the entire four-dimensional spacetime geometry, past, present, and future, must be known to establish the decoherence and predict the probabilities of alternatives in the present. Whether the predictions of quantum mechanics are consistent with a notion of causality is, of course, an empirical question that is accessible to experimental test. This generalized quantum mechanics permits definite predictions of the magnitude of any causality violation once the the initial condition is given and the background spacetime geometry is specified. Violations of causality may not be so very large if the number and volume of nonchronal regions in our future is small.

Fundamentally spacetime geometry is not fixed but variable quantum mechanically. Quantum fluctuations in spacetime geometry are central to a discussion of nonchronal regions because it is only through the intervention of quantum gravity that spacetimes with nonchronal regions could ever evolve [38,39]. The present generalized quantum mechanics of matter fields in a fixed background spacetime is thus only a model or an approximation to a more general quantum mechanics including the gravitational field. It serves to illustrate, however, how much of the structure of familiar quantum mechanics is tied to assumptions concerning the character of spacetime geometry and what departures from this structure we may expect in a generalized quantum mechanics of geometry as well as matter fields.

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[12]This could be stated more precisely using the operator corresponding to the frequency of occurrences of an outcome in an ensemble as in [37].

[1] M. Morris, K.S. Thorne, and U. Yurtsever, Phys. Rev. Lett. **61**, 1446 (1988); F. Echeverria, G. Klinkhammer, and K.S. Thorne, Phys. Rev. D **44**, 1077 (1991); J.L. Friedman and M. Morris, Phys. Rev. Lett. **66**, 401 (1991); J. Friedman, M. Morris, I.D. Novikov, F. Echeverria, G. Klinkhammer, K.S. Thorne, and U. Yurtsever, Phys. Rev. D **42**, 1915 (1990).
[2] D. Deutsch, Phys. Rev. D **44**, 3197 (1991).
[3] U. Yurtsever, Class. Quantum Grav. **11**, 999 (1994).
[4] S.W. Hawking (private communication).
[5] S.W. Hawking, Commun. Math. Phys. **87**, 395 (1982).
[6] G. Klinkhammer and K.S. Thorne (unpublished).
[7] J.L. Friedman, N.J. Papastamatiou, and J.Z. Simon, Phys. Rev. D **46**, 4456 (1992).
[8] J.B. Hartle, in *The Quantum Mechanics of Cosmology*, in *Quantum Cosmology and Baby Universes: Proceedings of the 1989 Jerusalem Winter School for Theoretical Physics*, edited by S. Coleman, J.B. Hartle, T. Piran, and S. Weinberg (World Scientific, Singapore, 1991), pp. 65–157.
[9] J.B. Hartle, Phys. Rev. D **44**, 3173 (1991).
[10] J.B. Hartle, Vistas Astron. **37**, 569 (1993); or in *Topics on Quantum Gravity and Beyond (Essays in Honor of Louis Witten on His Retirement)*, edited by F. Mansouri and J.J. Scanio (World Scientific, Singapore, 1993).
[11] J.B. Hartle, in *Gravitation and Quantizations,* Proceedings of the Les Houches Summer School, Les Houches, France, 1992, edited by B. Julia and J. Zinn-Justin, Les Houches Summer School Proceedings Vol. LVII (North-Holland, Amsterdam, 1993).
[12] P.A.M. Dirac, Phys. Z. Sowjetunion **3**, 64 (1933).
[13] R.P. Feynman, Rev. Mod. Phys. **20**, 267 (1948).
[14] J.L. Friedman, N.J. Papastamatiou, and J.Z. Simon, Phys. Rev. D **46**, 4441 (1992).
[15] D. Boulware, Phys. Rev. D **46**, 4421 (1992).
[16] H.D. Politzer, Phys. Rev. D **46**, 4470 (1992).
[17] D. Goldwirth, M. Perry, and T. Piran, Gen. Relativ. Gravit. **25**, 7 (1993).
[18] D. Goldwirth, M. Perry, T. Piran, and K.S. Thorne, report (unpublished).
[19] T. Jacobson, in *Conceptual Problems of Quantum Gravity*, edited by A. Ashtekar and J. Stachel (Birkhäuser, Boston, 1991).
[20] N. Birrell and P.C.W. Davies, *Quantum Fields in Curved Spacetime* (Cambridge University Press, Cambridge, England, 1982).
[21] S. Fulling, *Aspects of Quantum Field Theory in Curved Spacetime* (Cambridge University Press, Cambridge, England, 1989).
[22] N. Yamada and S. Takagi, Prog. Theor. Phys. **85**, 985 (1991); *ibid.* **87**, 77 (1992).
[23] R. Griffiths, J. Stat. Phys. **36**, 219 (1984).
[24] R. Omnès, J. Stat. Phys. **53**, 893 (1988); **53**, 933 (1988); **53**, 957 (1988); **57**, 357 (1989); Rev. Mod. Phys. **64**, 339 (1992).
[25] M. Gell-Mann and J.B. Hartle, in *Complexity, Entropy, and the Physics of Information, SFI Studies in the Sciences of Complexity*, Vol. VIII, edited by W. Zurek (Addison-Wesley, MA, 1990) Reading, or in *Proceedings of the 3rd International Symposium on the Foundations of Quantum Mechanics in the Light of New Technology,* edited by S. Kobayashi, H. Ezawa, Y. Murayama, and S. Nomura (Physical Society of Japan, Tokyo, 1990).
[26] M. Gell-Mann and J.B. Hartle, in *Proceedings of the NATO Workshop on the Physical Origins of Time Asymmetry, Mazagon, Spain, September 30–October 4, 1991*, edited by J. Halliwell, J. Perez-Mercader, and W. Zurek (Cambridge University Press, Cambridge, England, 1994).
[27] W. Unruh, in *New Techniques and Ideas in Quantum Measurement Theory*, Ann. N.Y. Acad. Sci. Vol. 480, edited by D.M. Greenberger (New York Academy of Sciences, New York, 1986).
[28] Y. Aharonov and L. Vaidman, J. Phys. A **24**, 2315 (1991).
[29] S. Rosenburg (unpublished).
[30] T. Banks, L. Susskind, and M. Peskin, Nucl. Phys. **B244**, 125 (1984).
[31] M. Srednicki, Nucl. Phys. **B410**, 143 (1993).
[32] S. Weinberg, Ann. Phys. (N.Y.) **194**, 336 (1989); Phys. Rev. Lett. **62**, 485 (1989).
[33] J. Polchinski, Phys. Rev. Lett. **66**, 397 (1991).
[34] R. Laflamme and J.B. Hartle (unpublished).
[35] M. Gell-Mann and J.B. Hartle, Phys. Rev. D **47**, 3345 (1993).
[36] J. Friedman and N. Papastamatiou (unpublished).
[37] J.B. Hartle, Am. J. Phys. **36**, 704 (1968).
[38] G. Klinkhammer, Phys. Rev. D **43**, 2542 (1991).
[39] R.M. Wald and U. Yurtsever, Phys. Rev. D **44**, 403 (1991).