Cosmic walls from gravitational collapse

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The formation of plane cosmic walls of negligible thickness from the collapse of smooth inhomogeneous plane-symmetric distributions of matter is considered. Two models with different asymptotic behaviors far from the wall in formation are constructed. In the first, the fluid far from the wall is anisotropic, with pressures proportional to density. The second model describes an asymptotically isotropic ideal gas in isentropic flow. Even though both models start from matter distributions with positive density and pressures everywhere, it is found that, during the collapse, negative pressures (tensions) appear within the wall in formation.

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I. INTRODUCTION

In grand unified theories (GUT's) of the early Universe, it is usually considered that domain walls arise from spontaneous symmetry breaking during a first-order phase transition [1]. Since the width of a wall is inversely proportional to the energy scale of the symmetry breaking, such GUT domain walls are often idealized as having zero thickness. When gravitational effects are taken into account, a zero-thickness wall becomes a topological defect of space-time. The gravitational field of the wall is then determined by solving the Einstein field equations with a prescribed energy-momentum tensor (EMT) which is proportional to $\delta(z)$, where z is a coordinate normal to the wall. The tension in any direction tangential to the wall is assumed to be equal to the wall energy density, and the normal stress component is assumed to vanish. This form of the stress tensor may be obtained as a limit of the static plane-symmetric solution for a domain wall of finite thickness in Minkowski space, for which the above relations between energy density, tension, and normal stress are exactly satisfied [2]. Static wall solutions can be obtained only in the case of plane symmetry, since curved walls tend to collapse due to surface tension. The simplest solutions of this kind describe static thin walls in vacuum, and have been found by Vilenkin [3] and Ipser and Sikivie [4]. These solutions have been generalized in several directions, including thin walls where the ratio of tension to energy density is not unity [4] and multiple domain walls [5]. The interaction of plane thin walls with gravitational waves and matter fields has been studied by Wang [6,7]. The head-on collision of plane domain walls has been simulated numerically (in the context of $\lambda\phi^4$ theory) by Anninos *et al.* [8] and analytically (using the thin-wall approximation in the context of general relativity) by Letelier and Wang [9].

Recently, a new scenario of galaxy formation involving low-mass domain walls [10] has stimulated research on the dynamics of thick domain walls. For this more general case the condition of zero normal stress is not consistent with a static solution of the Einstein equations [11]. However, static plane-symmetric solutions with tension equal to energy density and nonvanishing normal stress can be found: Goetz [12] has obtained simple analytic solutions by choosing a special form for the potential $V(\Phi)$ of the scalar field Φ which assumes different expectation values on both sides of the wall, and Tomita [13] has found a wall without reflection symmetry by assuming a polynomial ansatz for $\ln|\det(g_{\mu\nu})|$ and solving for the potential $V(\Phi)$. In the case of planar walls, the thin-wall approximation has been recovered by Widrow [14] as a regular limit of the Einstein scalar equations if the wall thickness approaches zero (in this limit, the normal stress vanishes). A more general argument for walls of arbitrary shape and sufficiently low curvature has been advanced by Garfinkle and Gregory [2] by expanding the solution of the Einstein scalar equations in powers of the wall thickness.

Thus far, most studies on domain walls have focused on static solutions, which are assumed to form by symmetry breaking. However, it would be relevant to know if static walls can be obtained as the final result of a dynamical evolution governed by appropriate gravitational and field equations. In this paper, we undertake a first step in this direction, by considering some classes of metrics which tend to the metric of a planar thin domain wall in the limit $t\rightarrow\infty$. Since wall solutions have been found for several different potentials [12,13] we will not consider any specific potential. Instead, we will work in the spirit of Synge's g method [15] by searching for metrics whose associated EMT tensor $T_{\mu\nu}=-(1/8\pi)G_{\mu\nu}$ describes "ordinary matter" (at least initially), and then tends to the distributional EMT of a thin domain wall. By "ordinary matter" we mean a fluid with non-negative energy density and pressure, although not necessarily isotropic. It is known that the Einstein equations with certain fluid sources are equivalent to the same equations coupled to a scalar field [16–19]. For instance, the case of a stiff perfect fluid corresponds to a real scalar field [16] and aniso-

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tropic fluids with stiff equation of state in two directions correspond to a complex scalar field [17]. Fields with SU(2) structure can also be used to describe anisotropic fluids [18]. Thus, in principle, it is possible to interpret the time-dependent EMT's that we find here in terms of specific field theory; this interpretation is a nontrivial task, and will be worked out elsewhere. On the other hand, we want to remark that the results of this paper simulate dynamical features of wall formation which may be common to different field theories in which plane domain walls can form.

Since the domain wall has tangential tension and zero normal stress, it is clear that if we start with positive pressures everywhere, some kind of transition must occur in the fluid at some later time, in order that the pressures become tensions. It is also necessary that the fluid becomes highly anisotropic at large times. In constructing the classes of metrics that we will present below, we have tried to ensure that the above-mentioned transition and the high anisotropies be confined to a small vicinity of the wall in formation, wherein the largest densities and pressures occur. Far from the wall, the EMT tends to zero, and in this region the pressure is positive for all times. As $t\rightarrow\infty$, this outer region becomes empty and the region of high densities becomes the zero-thickness wall. Thus, the models suggest that the transition from pressures to tensions can be interpreted as some phase transition which occurs at sufficiently high pressure and density. We remark that the process of formation of the wall is reminiscent of plane gravitational collapse, which has been studied in the case of zero pressure (dust collapse) by De [20] and Liang [21].

The plan of the paper is as follows. In Sec. II, we review the Einstein equations for a plane-symmetric space-time and present expressions for the eigenvalues of the associated EMT. In Sec. III, we construct a class of models in which the "outer region" far from the wall is filled with an anisotropic fluid whose equations of state are given asymptotically by $p_\perp/\rho=$const, $p_\parallel/\rho=$const ($p_\perp$ and $p_\parallel$ are the pressures in the directions normal and tangential to the wall, respectively, and ρ is the density). In Sec. IV, we construct a similar but more realistic model, in which the "outer region" is filled with an asymptotically isotropic classical gas in isentropic flow [22].

II. PLANE-SYMMETRIC SPACE-TIMES

We will consider here the formation of a vacuum domain wall with metric

$$ds^2=\exp(-4\pi\sigma|z|)\times[dt^2-dz^2-\exp(4\pi\sigma t)(dx^2+dy^2)] \tag{2.1}$$

and EMT [3]

$$T_\mu^\nu=\text{diag}(\sigma,0,\sigma,\sigma)\delta(z)\ , \tag{2.2}$$

starting from a general plane-symmetric metric [22]

$$ds^2=e^F dt^2-e^G dz^2-e^H(dx^2+dy^2)\ , \tag{2.3}$$

where F, G, and H are functions of (t,z). The Einstein equations, $G_{\mu\nu}=-8\pi T_{\mu\nu}$, give us

$$\begin{aligned}
T_{00}&=\frac{1}{32\pi}[H_t^2+2H_tG_t+e^{F-G}(F_z^2-F_zG_z-4H_{zz}-3H_z^2+2H_zG_z)]\ ,\\
T_{01}&=-\frac{1}{16\pi}(2H_{tz}-H_tF_z-H_zG_t+H_tH_z)\ ,\\
T_{11}&=\frac{1}{32\pi}[e^{G-F}(2H_tF_t-4H_{tt}-3H_t^2)-F_z^2+F_zG_z+H_z^2+2H_zF_z]\ ,\\
T_{22}&=T_{33}=\frac{1}{32\pi}e^H[e^{-F}(H_tF_t-H_tG_t-2G_{tt}-2H_{tt}-H_t^2-G_t^2+G_tF_t)+e^{-G}(H_zF_z-H_zG_z+2F_{zz}+2H_{zz}+H_z^2)]\ .
\end{aligned} \tag{2.4}$$

The pressure and density fields are the eigenvalues of the EMT (2.4). The tangential pressure is given by the tetradic component

$$p_\parallel=e^{-H}T_{22}\ . \tag{2.5}$$

The density and the normal pressure are the eigenvalues of the block (T_{ij}), $i,j=0,1$, which satisfy the characteristic equation

$$\det(T_{ij}-\lambda g_{ij})=0\ . \tag{2.6}$$

The roots of (2.6) for the metric (2.3) are

$$\lambda=\lambda_\pm=\tfrac{1}{2}[(e^{-F}T_{00}-e^{-G}T_{11})\pm\Delta^{1/2}]\ , \tag{2.7}$$

$$\Delta=(e^{-F}T_{00}+e^{-G}T_{11})^2-4e^{-F-G}T_{01}^2\ , \tag{2.8}$$

and the eigenvectors $\xi_\pm=(\xi_\pm^t,\xi_\pm^z)$ have the form

$$\xi_\pm^t=c_\pm T_{01}\ ,\quad \xi_\pm^z=-c_\pm(T_{00}-\lambda_\pm e^F)\ , \tag{2.9}$$

where $c_\pm$ are constants. We will consider here only the case $\Delta>0$; if $\Delta<0$, (T_{ij}) can be reduced to the canonical form

$$\begin{bmatrix}\alpha & \beta\\ \beta & -\alpha\end{bmatrix}\ ,$$

$$\alpha=\tfrac{1}{2}e^{-F}(T_{00}-T_{11})\ ,\qquad \beta=\tfrac{1}{2}e^{-F}(-\Delta)^{1/2}\text{sgn}\,T_{01}\ , \tag{2.10}$$

which says that $p_\perp=-\rho=\alpha$, and that there is a heat flux

β in the positive z direction. Since in the limit $t\rightarrow\infty$ we want $p_\perp \ll \rho$, we can disregard the case $\Delta<0$. The case $\Delta=0$ may be disregarded by the same reason.

If $\Delta>0$, we can decide which eigenvalue must represent the normal pressure and which the density, by computing the scalar product of the corresponding eigenvectors:

$$\langle \xi_\pm,\xi_\pm\rangle = e^F c_\pm^2 T_{01}^2 - e^G c_\pm^2 (T_{00}-\lambda_\pm e^F)^2$$
$$= c_\pm^2 e^F A_\pm \ , \tag{2.11}$$

$$A_\pm = -\tfrac{1}{2}e^G \Delta^{1/2}[e^F\Delta^{1/2} \mp (T_{00}+e^{F-G}T_{11})] \ . \tag{2.12}$$

Defining

$$s=\operatorname{sgn}\operatorname{Tr}(T_{ij})=\operatorname{sgn}(e^{-F}T_{00}+e^{-G}T_{11}) \ , \tag{2.13}$$

we conclude from (2.12) and (2.8) that $s>0$ implies $\operatorname{sgn} A_\pm = \pm 1$, and that the opposite is true if $s<0$. Hence, ξ_s is the time like eigenvector, and ξ_{-s} is the spacelike eigenvector. Consequently, we interpret λ_s as the density ρ and $-\lambda_{-s}$ as the normal pressure $p_\perp$.

III. ANISOTROPIC FLUID MODEL OF DOMAIN-WALL FORMATION

In order to study the formation of the wall described by (2.1), we will look for plane-symmetric metrics (2.3), of class C^∞, such that

$$F,G\rightarrow -4|z| \ , \quad H\sim 4(t-|z|) \tag{3.1}$$

as $t\rightarrow\infty$. (The t and z coordinates have been rescaled by $\pi\sigma t\rightarrow t$, $\pi\sigma z\rightarrow z$.) One possibility is to take

$$F=G=-4N(t,z) \ , \quad H=4[t-N(t,z)] \ , \tag{3.2}$$

where

$$\lim_{t\rightarrow\infty} N(t,z)=|z| \tag{3.3}$$

and N is a C^∞ function of (t,z). With this choice of the metric the EMT (2.4) reduces to

$$\begin{aligned} T_{00} &= N_{zz}+1-N_z^2-4N_t+3N_t^2 \ , \\ T_{01} &= N_{tz}+2N_tN_z \ , \\ T_{11} &= N_{tt}-3(1-N_z^2)+4N_t-N_t^2 \ , \\ T_{22} &= T_{33}=e^{4t}(N_{tt}-N_{zz}+N_z^2-1+2N_t-N_t^2) \end{aligned} \tag{3.4}$$

[the components of the energy-stress tensor have been rescaled by $(2/\pi\sigma^2)T_{\mu\nu}\rightarrow T_{\mu\nu}$].

In order to obtain density and pressure fields which decay exponentially with $|z|$ as $|z|\rightarrow\infty$, we require that

$$N(t,z)-|z| = O(\exp[-\lambda(t)|z|]) \quad [\lambda(t)>0] \tag{3.5}$$

as $|z|\rightarrow\infty$. A relatively simple function with this property is

$$\begin{aligned} N(t,z) &= (1/\varepsilon)\ln[2\cosh(\varepsilon z)] \\ &= \int_0^z \tanh(\varepsilon z')dz' + (\ln 2)/\varepsilon \ , \end{aligned} \tag{3.6}$$

$$\varepsilon=\varepsilon(t)\rightarrow\infty \ \text{ as } t\rightarrow\infty \ .$$

In order to determine the asymptotic behavior of $T_{\mu\nu}$ as $t\rightarrow\infty$ and $z\neq 0$, we need the asymptotic form of N and its derivatives:

$$\begin{aligned} N &\sim |z| + (1/\varepsilon)\exp(-2\varepsilon|z|) \ , \\ N_z &= \tanh(\varepsilon z) \sim [1-2\exp(-2\varepsilon|z|)]\operatorname{sgn} z \ , \\ N_t &= (\dot\varepsilon/\varepsilon)[z\tanh(\varepsilon z)-N] \\ &\sim -2(\dot\varepsilon/\varepsilon)|z|\exp(-2\varepsilon|z|) \ , \\ N_{zz} &= \varepsilon\,\mathrm{sech}^2(\varepsilon z) \sim 4\varepsilon\exp(-2\varepsilon|z|) \ , \\ N_{tz} &= \dot\varepsilon z\,\mathrm{sech}^2(\varepsilon z) \sim 4\dot\varepsilon z\exp(-2\varepsilon|z|) \ , \\ N_{tt} &= (\ddot\varepsilon/\varepsilon - 2\dot\varepsilon^2/\varepsilon^2)[z\tanh(\varepsilon z)-N] + (\dot\varepsilon^2/\varepsilon)z^2\mathrm{sech}^2(\varepsilon z) \\ &\sim (1/\varepsilon)[4\dot\varepsilon^2 z^2 - 2|z|(\ddot\varepsilon - 2\dot\varepsilon^2/\varepsilon)]\exp(-2\varepsilon|z|) \ . \end{aligned} \tag{3.7}$$

Substituting these expressions in (3.4), and assuming that $\dot\varepsilon/\varepsilon \ll \varepsilon$ as $t\rightarrow\infty$, we obtain

$$\begin{aligned} & T_{00}\sim 4\varepsilon\exp(-2\varepsilon|z|) \ , \quad T_{01}\sim 4\dot\varepsilon z\exp(-2\varepsilon|z|) \ , \\ & T_{11}\sim 2[2(\dot\varepsilon^2/\varepsilon)z^2 - (\ddot\varepsilon/\varepsilon - 2\dot\varepsilon^2/\varepsilon^2)|z| \\ & \qquad -6-4(\dot\varepsilon/\varepsilon)|z|]\exp(-2\varepsilon|z|) \ , \\ & p_\parallel = e^{-H}T_{22} = [4(\dot\varepsilon^2/\varepsilon)z^2 - 2(\ddot\varepsilon/\varepsilon - 2\dot\varepsilon^2/\varepsilon^2)|z| \\ & \qquad -4\varepsilon]\exp[2(2-\varepsilon)|z|] \ . \end{aligned} \tag{3.8}$$

The particular cases in which ε increases linearly or exponentially with t serve to illustrate the asymptotic properties of the model under consideration:

Case 1. $\varepsilon=at$, $a=$const. In this case, $T_{01},T_{11}\ll T_{00}$ we have $\Delta^{1/2}\sim T_{00}$, and hence

$$\rho\sim T_{00}e^{-F}\sim 4at\exp[-2(at-2)|z|]>0 \ , \tag{3.9}$$

which decays exponentially with time for a fixed z. The tangential pressure is asymptotically equal to $-\rho$, and $p_\perp = O(T_{01}e^{-F}, T_{11}e^{-F}) \ll \rho$. Therefore, even if the initial state is such that $\rho,p_\parallel,p_\perp>0$, we would eventually have $p_\parallel<0$ in the low-density regions far from $z=0$. A more realistic model should have $p_\parallel<0$, $p_\perp<0$ only in a neighborhood of $z=0$, where the largest densities are reached.

Case 2. $\varepsilon=ae^{bt}$, $a,b=$const. In this case,

$$(T_{00},T_{01},T_{11})\sim 4a(1,bz,b^2z^2)\exp(-2ae^{bt}|z|+bt) \ , \tag{3.10}$$

$$\Delta^{1/2}\sim 4a|1-b^2z^2|\exp[-2(ae^{bt}-2)|z|+bt] \ , \tag{3.11}$$

$$\rho\sim 2a[(1-b^2z^2)]+|1-b^2z^2|\exp[-2(ae^{bt}-2)|z|+bt] \ . \tag{3.12}$$

Therefore, $\rho>0$ for $|z|<b^{-1}$; for $|z|>b^{-1}$, (3.24) gives $\rho\sim 0$, which indicates that the density in this region is asymptotically small compared to the density for $|z|<b^{-1}$. From (2.7), (3.8), (3.10), and (3.11), it may be seen that $p_\perp$, $p_\parallel>0$ for $|z|>b^{-1}$; clearly, in this region we will have $p_\parallel,p_\perp \gg \rho$, which is an unphysical situation.

A model possessing the features mentioned in the discussion of case 1 above can be obtained by considering functions $N(t,z)$ with the asymptotic behavior

$$N(t,z) \sim |z| + k(t)\exp(-\lambda|z|)\ , \quad k(t) \to 0\ , \qquad (3.13)$$

for $t \to \infty$, and sufficiently large $|z|$; λ is a constant. The asymptotic form of the derivatives of N is

$$\begin{aligned} &N_z \sim [1-\lambda k \exp(-\lambda|z|)]\mathrm{sgn} z\ , \quad N_t \sim \dot{k}\exp(-\lambda|z|)\ , \\ &N_{tz} \sim -\lambda\dot{k}\exp(-\lambda|z|)\mathrm{sgn} z\ , \\ &N_{tt} \sim \ddot{k}\exp(-\lambda|z|)\ , \\ &N_{zz} \sim \lambda^2 k \exp(-\lambda|z|)\ , \end{aligned} \qquad (3.14)$$

inserting these expressions in (3.4), we obtain

$$\begin{aligned} &T_{00} \sim (\lambda^2 k + 2\lambda k - 4\dot{k})\exp(-\lambda|z|)\ , \\ &T_{01} \sim (2-\lambda)\dot{k}\exp(-\lambda|z|)\mathrm{sgn} z\ , \\ &T_{11} \sim (\ddot{k} + 4\dot{k} - 6\lambda k)\exp(-\lambda|z|)\ , \\ &p_{\parallel} \sim (\ddot{k} + 2\dot{k} - \lambda^2 k - 2\lambda k)\exp[-(\lambda-4)|z|]\ . \end{aligned} \qquad (3.15)$$

The influence of the rate of decay of $k(t)$ on the behavior of the model can be verified by considering the cases $k = at^{-\omega}$, $(a,\omega = \text{const})$, and $k = a\exp(-\nu t)$ $(a,\nu = \text{const})$.

Case 1. $k = at^{-\omega}$. Here $\ddot{k} \ll \dot{k} \ll k$; therefore $T_{01}/(T_{00}+T_{11}) = O(\dot{k}/k) \ll 1$, and hence $\Delta^{1/2} \sim |T_{00}+T_{11}|$,

$$\rho \sim T_{00}e^{-F} \sim \lambda(\lambda+2)k\exp[-(\lambda-4)|z|] > 0 \qquad (3.16)$$

if $k > 0$; but then $p_{\parallel} \sim -\rho < 0$ and

$$p_{\perp} \sim T_{11}e^{-F} \sim -6\lambda k \exp[-(\lambda-4)|z|] < 0\ . \qquad (3.17)$$

Therefore, the properties of this model are similar to those of case 1 discussed above.

Case 2. $k = a\exp(-\nu t)$. In this case the EMT is given asymptotically by

$$\begin{aligned} &T_{00} \sim a(\lambda^2 + 2\lambda + 4\nu)\exp(-\nu t - \lambda|z|)\ , \\ &T_{01} \sim a\nu(\lambda-2)\exp(-\nu t - \lambda|z|)\mathrm{sgn} z\ , \\ &T_{11} \sim a(\nu^2 - 4\nu - 6\lambda)\exp(-\nu t - \lambda|z|)\ , \\ &p_{\parallel} \sim a(\nu^2 - \lambda^2 - 2\nu - 2\lambda)\exp[-\nu t - (\lambda-4)|z|]\ . \end{aligned} \qquad (3.18)$$

The pressure $p_{\parallel}$ will decay as $|z| \to \infty$ if

$$\lambda - 4 > 0\ , \qquad (3.19)$$

in this case, the condition $\Delta > 0$ (which is equivalent to $|T_{00}+T_{11}| > 2|T_{01}|$) may be written as

$$\nu^2 + \lambda(\lambda-4) > 2\nu(\lambda-2)\ , \qquad (3.20)$$

which will be satisfied for

$$\nu > \lambda \quad \text{or}\ 0 < \nu < \lambda - 4\ . \qquad (3.21)$$

The tangential pressure will be positive if $a > 0$ and

$$\nu^2 - 2\nu - \lambda(\lambda+2) > 0\ , \qquad (3.22)$$

that is, if

$$\nu > \lambda + 2\ . \qquad (3.23)$$

Note that (3.21) will be automatically satisfied if (3.23) is valid. The asymptotic forms of the density and normal pressure fields will be

$$\begin{aligned} &\rho \sim \tfrac{1}{2}a[\lambda^2 - \nu^2 + 8(\lambda+\nu) + \hat{\Delta}^{1/2}] \\ &\qquad \times \exp[-\nu t - (\lambda-4)|z|]\ , \\ &p_{\perp} \sim \tfrac{1}{2}a[\nu^2 - \lambda^2 - 8(\lambda+\nu) + \hat{\Delta}^{1/2}] \\ &\qquad \times \exp[-\nu t - (\lambda-4)|z|]\ , \end{aligned} \qquad (3.24)$$

where

$$\begin{aligned} \hat{\Delta} &= [\nu^2 + \lambda(\lambda-4)]^2 - 4[\nu(\lambda-2)]^2 \\ &= (\nu-\lambda)(\nu+\lambda)(\nu-\lambda+4)(\nu+\lambda+4)\ . \end{aligned} \qquad (3.25)$$

The conditions $\rho, p_{\perp} > 0$ are equivalent to

$$\hat{\Delta}^{1/2} > |\lambda^2 + 8\lambda - \nu^2 + 8\nu| = (\nu+\lambda)|\nu - \lambda - 8|\ , \qquad (3.26)$$

taking the square of this inequality, and using (3.25), we get

$$(\nu-\lambda)(\nu-\lambda+4)(\nu+\lambda-4) > (\nu+\lambda)(\nu-\lambda-8)^2\ , \qquad (3.27)$$

which is equivalent to

$$2\nu^2 + (\lambda-10)\nu - 3\lambda(\lambda+2) > 0\ . \qquad (3.28)$$

The solution of (3.28) such that $\nu > 0$ for $\lambda > 4$ is

$$\begin{aligned} \nu &> \tfrac{1}{4}[(10-\lambda) + (25\lambda^2 + 28\lambda + 100)^{1/2}] \\ &\sim \lambda + \tfrac{16}{5} \end{aligned} \qquad (3.29)$$

for $\lambda \to \infty$. Since the lower bound of ν in (3.29) is always greater than $\lambda + 2$, we can summarize the above deduced conditions for λ and ν by (3.19) and (3.29).

A final condition that we would expect from the matter in the low-density region far from $z = 0$ is $p_{\parallel}, p_{\perp} < \rho$. In view of (3.24), the condition on $p_{\perp}$ says that

$$\lambda^2 + 8\lambda - \nu^2 + 8\nu > 0\ , \qquad (3.30)$$

whose solution is

$$\nu < \lambda + 8\ . \qquad (3.31)$$

Using (3.18) and (3.24), the condition on $p_{\parallel}$ reduces to

$$3(\nu^2 - 4\nu - \lambda^2 - 4\lambda) < \hat{\Delta}^{1/2}\ , \qquad (3.32)$$

which splits into the two possibilities

$$\nu < \lambda + 4 \qquad (3.33)$$

[if the left-hand side of (3.32) is negative] or

$$\begin{aligned} &\nu > \lambda + 4\ , \\ &\nu^4 - 9\nu^3 - 2(\lambda^2 + 5\lambda - 10)\nu^2 + 9\lambda(\lambda+4)\nu + \lambda^2(\lambda^2 + 10\lambda + 16) \\ &\qquad = (\lambda+\nu)[\nu^3 - (\lambda+9)\nu^2 - (\lambda^2 + \lambda - 20)\nu + \lambda(\lambda^2 + 10\lambda - 16)] < 0\ . \end{aligned} \qquad (3.34)$$

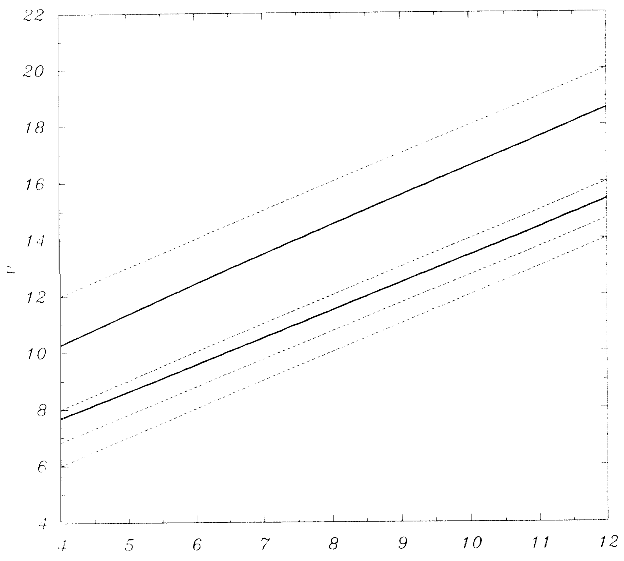

FIG. 1. Curves in the $(\lambda\nu)$ plane which arise in the discussion of the model of Sec. III. From bottom to top: $\nu=\lambda+2$ (where $p_{\parallel}\sim 0$), $\nu=\nu_3(\lambda)$, $\nu=(1/4)[(10-\lambda)+(25\lambda^2+28\lambda+100)^{1/2}]$ (where $p_{\perp}\sim 0$), $\nu=\lambda+4$, $\nu=\nu_1(\lambda)$ (where $p_{\parallel}\sim\rho$), and $\nu=\lambda+8$ (where $p_{\perp}\sim\rho$). The strip between thick lines is the solution of the inequality (3.41).

The roots of the cubic polynomial in ν which appears in (3.34) are

$$\begin{aligned} \nu_1 &= s_1+s_2-\tfrac{1}{3}a_2 , \\ \nu_{2,3} &= -\tfrac{1}{2}(s_1+s_2)-\tfrac{1}{3}a_2\pm\tfrac{1}{2}\sqrt{3}i(s_1-s_2) , \end{aligned} \tag{3.35}$$

where

$$\begin{aligned} s_{1,2} &= [r\pm(q^3+r^2)^{1/2}]^{1/3} , \quad q=\tfrac{1}{3}a_2-\tfrac{1}{9}a_1^2 , \\ r &= \tfrac{1}{6}(a_1a_2-3a_0)-\tfrac{1}{27}a_2^3 , \end{aligned} \tag{3.36}$$

and a_2, a_1, and a_0 are the coefficients of ν^2, ν, and 1, respectively, in the cubic polynomial. By direct calculation, one verifies that

$$\begin{aligned} q &= -\tfrac{1}{9}(4\lambda^2+21\lambda+21)<0 , \\ r &= -\tfrac{1}{54}(16\lambda^3+126\lambda^2+45\lambda+162)<0 , \\ D &= q^3+r^2 \\ &= -\tfrac{1}{108}(292\lambda^4+2328\lambda^3+3313\lambda^2+3576\lambda+400) \\ &<0 , \end{aligned} \tag{3.37}$$

therefore,

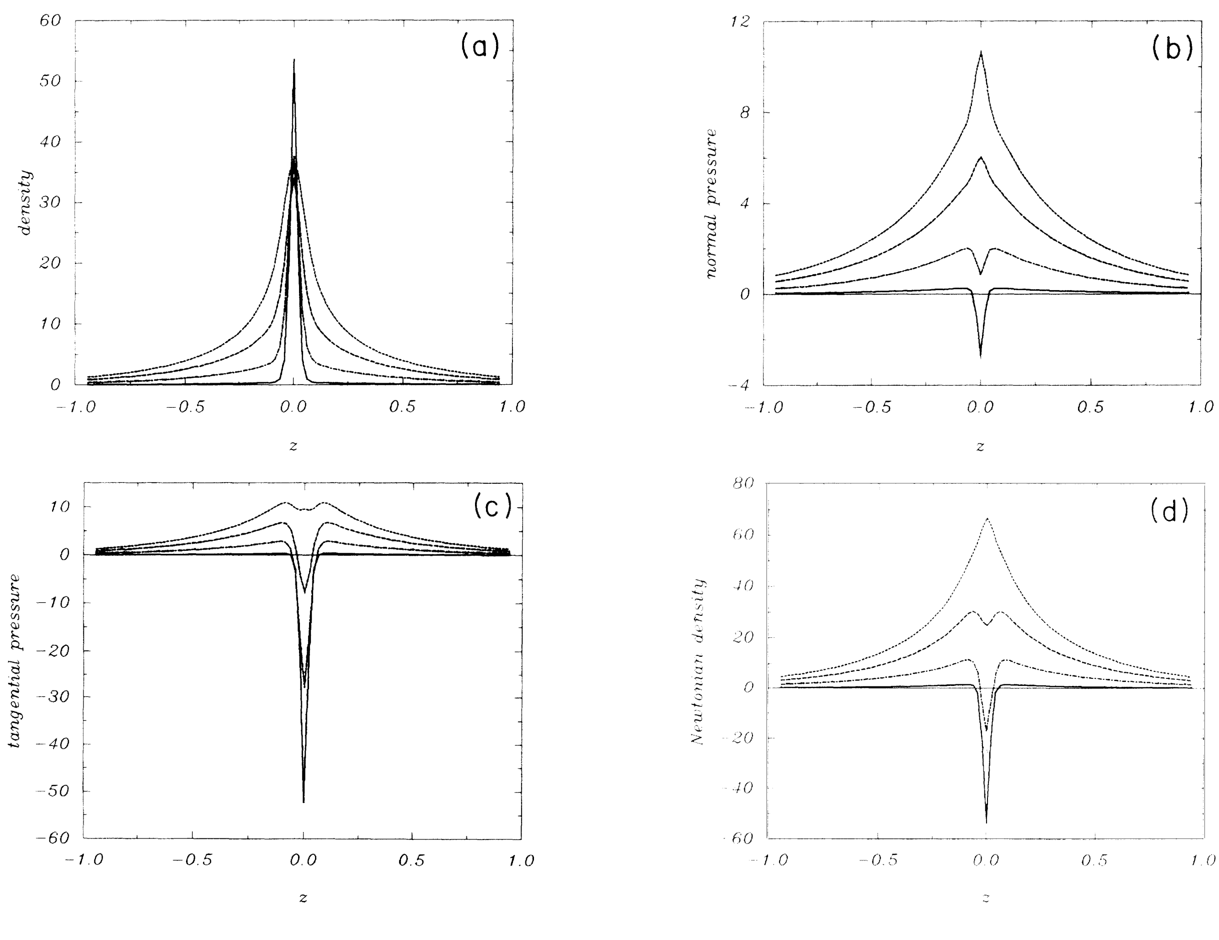

FIG. 2. Evolution of density, pressures, and Newtonian density in the model of Sec. III with $\nu=2\lambda=12.73$, $a=3.142$, and $\varepsilon=101.3tg$. The displayed curves are for $t=0.242$ (dashed), $t=0.273$ (long-dashed), $t=0.336$ (dot-dashed), and $t=0.493$ (solid).

$$s_{1,2}=[r\pm i\sqrt{-D}\,]^{1/3} = \sqrt{-q}\,\exp\left[\pm\frac{i}{3}\left[\frac{\pi}{2}-\arctan\frac{r}{\sqrt{-D}}\right]\right], \quad (3.38)$$

and

$$\nu_1=2\sqrt{-q}\,\cos\left[\frac{\pi}{6}-\frac{1}{3}\arctan\frac{r}{\sqrt{-D}}\right]-\frac{1}{3}a_2\,,$$
$$\nu_{2,3}=-2\sqrt{-q}\,\cos\left[\frac{\pi}{6}-\frac{1}{3}\arctan\frac{r}{\sqrt{-D}}\mp\frac{\pi}{3}\right]-\frac{1}{3}a_2\,. \quad (3.39)$$

It may be verified numerically that $\nu_2<0$ for $\lambda>4$, while $\nu_1>\lambda+4>\nu_3>0$. Hence, (3.34) will be satisfied if

$$\lambda+4<\nu<\nu_1(\lambda)\,. \quad (3.40)$$

The region of the (λ,ν) plane where all the above-deduced inequalities are valid is defined by (see Fig. 1)

$$\lambda>4\,,$$
$$\tfrac{1}{4}[(10-\lambda)+(25\lambda^2+28\lambda+100)^{1/2}]<\nu<\nu_1(\lambda)\,, \quad (3.41)$$

it can be easily shown that the asymptotic form of $\nu_1(\lambda)$ for large λ is $\nu_1(\lambda)\sim\lambda+(19+\sqrt{73})/4\approx\lambda+6.88$.

Figure 1 suggests that on the strip defined by (3.41) we will always have $p_\perp<p_\parallel$, i.e., that the model under consideration cannot represent a fluid which is asymptotically isotropic as $|z|\to\infty$. In fact, by (3.18) and (3.24), the condition $p_\perp\sim p_\parallel$ says that

$$\hat{\Delta}^{1/2}=\nu^2+4\nu-\lambda^2+4\lambda=(\lambda+\nu)(\nu-\lambda+4)\,, \quad (3.42)$$

taking the square of (3.42), and using (3.25), one concludes that $\nu=0$.

In conclusion, the model described by (3.13) with $k(t)=a\exp(-\nu t)$ presents the more acceptable features among all the models analyzed above. Far from the wall, the density and pressure fields behave like plane waves, propagating without change of shape towards the plane $z=0$. In the following, we look for C^∞ functions with the asymptotic behavior (3.13).

One possibility is

$$N=N_1+k\exp(-\lambda N_1)\,,$$
$$N_1=(1/\varepsilon)\ln[2\cosh(\varepsilon z)]\,,\quad \lim_{t\to\infty}\varepsilon(t)=+\infty\,. \quad (3.43)$$

It is easy to verify that (3.13) will be valid provided

$$k\varepsilon\exp[(2\varepsilon-\lambda)|z|]\gg 1\,. \quad (3.44)$$

Numerical experiments with (3.43) (Fig. 2) indicate that it is possible to choose λ, $\varepsilon(t)=\varepsilon_0 t$, $k(t)=a\exp(-\nu t)$, in such a way that ρ, $p_\parallel,p_\perp>0$ for all z at an initial time $t_0>0$. At a later time t_1, negative pressures appear in a small neighborhood of $z=0$; the collapsing fluid seems to undergo a phase transition in which an equation of state with pressures changes to one with tensions. In the initial stage of the collapse, the pressure gradient force points generally outward from the wall, indicating that the fluid pressure tends to slow down the collapse. At later times, the pressure gradient force changes its sense, pointing inward to the wall and therefore accelerating the collapse. In Fig. 2, we also show the "Newtonian density" [3]

$$\rho_N=\rho+2p_\parallel+p_\perp\,. \quad (3.45)$$

During the collapse, a repulsive layer ($\rho_N<0$) develops near the plane of symmetry, while the outer regions remain attractive ($\rho_N>0$) for all times. The density ρ remains positive for all (t,z), and the EMT tends to the distributional form (2.2) as $t\to\infty$.

IV. DOMAIN-WALL FORMATION FROM AN ASYMPTOTICALLY ISENTROPIC FLOW

The flow described by the model of the previous section has $p_\parallel/\rho\sim$const, $p_\perp/\rho\sim$const far from the wall. These relations between the density and the pressures are formally similar to the relation $p/\rho=$const, which holds when a classical ideal gas undergoes an isothermal process. However, since the collapse of the wall occurs rapidly, it is reasonable to expect that a model for which the flow from the wall is asymptotically isentropic (adiabatic) will better represent the physics of wall formation. The asymptotic relation between pressure and density in such a model is

$$p\rho^{-\gamma}=\text{const}\,, \quad (4.1)$$

where $\gamma=c_p/c_v$ is the adiabatic index. In an isentropic process at low densities, we have $p\ll\rho$, since the constant γ in (4.1) lies between 1 and 2 in physically relevant gases. The asymptotic form of the metric employed in Sec. III implies that the fields p and ρ decay with the same spatial and time scales, and is therefore unable to describe this kind of process.

Here we consider the general metric (2.3), where the asymptotic behavior of the coefficients is chosen as

$$F=-4M(t,z)\,,\quad G=-4P(t,z)\,,\quad H=4[t-N(t,z)]\,,$$
$$M\sim|z|+A_1e^{-\phi}+A_2e^{-\psi}\,, \quad (4.2)$$
$$N\sim|z|+\omega A_1e^{-\phi}+\zeta A_2e^{-\psi}\,,$$
$$P\sim|z|+\varphi A_1e^{-\phi}+\xi A_2e^{-\psi}$$

for $z\neq0$, $t\to\infty$, A_1A_2, ω, φ, ζ, ξ are constants and

$$\phi=\nu t+\lambda|z|\,,\quad \psi=\eta t+\mu|z| \quad (4.3)$$

$(\nu,\lambda,\eta,\mu=\text{const})$.

If we assume that $\nu\neq\eta$, $\lambda\neq\mu$, we hope that it will be possible to choose the constants in (4.2) and (4.3) in such a way that the fluid is asymptotically isotropic and p and ρ decay with different spatial and time scales, as required by (4.1). Without loss of generality, we can assume that $\nu<\eta$ and $\lambda<\mu$. In order that the leading-order terms in the asymptotic expansions for the density and the pressures $p_\perp$ and $p_\parallel$ be $O(e^{-\phi},e^{-\psi})$, we must ensure that the terms $O(e^{-m\phi-n\psi})$ with $m+n>1$ are negligible with respect to $e^{-\psi}$. Clearly, this condition is satisfied if and only if

$$\lambda<\mu<2\lambda\,,\quad \nu<\eta<2\nu\,; \quad (4.4)$$

in this case we will have

$$e^{-\phi} \gg e^{-\psi} \gg e^{-2\phi} \gg e^{-\phi-\psi} \gg e^{-2\psi} \gg \cdots \; . \tag{4.5}$$

The asymptotic expansions to $O(e^{-\psi})$ for the pressures and the density will have the form

$$\begin{aligned} &\exp(-4|z|)\rho \sim \rho_0 e^{-\phi} + \rho_1 e^{-\psi} + \cdots \; , \\ &\exp(-4|z|)p_\parallel \sim p_{\parallel 0} e^{-\phi} + p_{\parallel 1} e^{-\psi} + \cdots \; , \\ &\exp(-4|z|)p_\perp \sim p_{\perp 0} e^{-\phi} + p_{\perp 1} e^{-\psi} + \cdots \; , \end{aligned} \tag{4.6}$$

where the coefficients depend on (ν,λ,η,μ) and possibly on the other parameters appearing in (4.2). We will try to choose the parameters so that $p_{\parallel 0} = p_{\perp 0} = 0$. If, with this constraint, it is still possible to choose the remaining free parameters so that $p_{\parallel 1} = p_{\perp 1} > 0$, $\rho_0 > 0$, then the fluid will be asymptotically isotropic, with positive pressure and density. In this case, the classical equation (4.1) implies that

$$\eta = \gamma\nu \; , \quad \mu - 4 = \gamma(\lambda - 4) \; . \tag{4.7}$$

It can easily be verified that (4.7) implies (4.4) if $1 < \gamma < 2$ and $\lambda > 4$. In this case, it is known that the speed of sound in the gas is always less than the speed of light [22].

Inserting (4.2) in the expressions (2.4) for $T_{\alpha\beta}$, it is found that as $t \to \infty$ ($z \neq 0$) we have

$$\begin{aligned} &\exp(-4|z|)(e^{-F}T_{00} - e^{-G}T_{11}) \sim Q_0 e^{-\phi} + Q_1 e^{-\psi} + \cdots \; , \\ &\exp(-4|z|)(e^{-F}T_{00} + e^{-G}T_{11}) \sim \tau_0 e^{-\phi} + \tau_1 e^{-\psi} + \cdots \; , \\ &\exp(-8|z|)\Delta \sim \Delta_0 e^{-2\phi} + \Delta_1 e^{-\phi-\psi} + \cdots \; , \end{aligned} \tag{4.8}$$

where

$$\begin{aligned} Q_0 &= -A_1[(\nu^2-\lambda^2)\varphi + 2(1-\omega-4\varphi)\nu - 8\varphi\lambda + 16(\omega-1)] \; , \\ Q_1 &= -A_2[(\eta^2-\mu^2)\xi + 2(1-\zeta-4\xi)\eta - 8\xi\mu + 16(\zeta-1)] \; , \\ \tau_0 &= A_1[(\nu^2+\lambda^2)\varphi + 2(1+\omega-2\varphi)\nu - 2(\omega+1)\lambda + 8(\omega-1)] \end{aligned} \tag{4.9}$$

(we omit the rather long expressions for Δ_0 and Δ_1, which are more conveniently obtained by use of a computer algebra system). The first coefficients in the expansion for $p_\parallel$ may similarly be found:

$$\begin{aligned} p_{\parallel 0} &= \tfrac{1}{2}A_1[(\nu^2-4\nu-4\lambda)\varphi + (\nu^2-2\nu+2\lambda+8)\omega + (2\nu-\lambda^2-2\lambda-8)] \; , \\ p_{\parallel 1} &= \tfrac{1}{2}A_2[(\zeta+\xi)\eta^2 - \mu^2 + 2(1-2\xi-\zeta)\eta - 2(1+2\xi-\zeta)\mu - 8(1-\zeta)] \; . \end{aligned} \tag{4.10}$$

The first coefficients in the expansions of $p_\perp$ and ρ are obtained from (2.7), (2.13), and (4.8):

$$\begin{aligned} &\rho_0 = \tfrac{1}{2}(s\Delta_0^{1/2} + Q_0) \; , \quad p_{\perp 0} = \tfrac{1}{2}(s\Delta_0^{1/2} - Q_0) \; , \\ &p_{\perp 1} = \tfrac{1}{2}(s\Delta_1/2\Delta_0^{1/2} - Q_1) \; , \end{aligned} \tag{4.11}$$

where

$$s = \mathrm{sgn}\,\mathrm{Tr}(T_{ij}) \sim \mathrm{sgn}\tau_0 \; . \tag{4.12}$$

The conditions $p_{\parallel 0} = p_{\perp 0} = 0$ read

$$(\nu^2-4\nu-4\lambda)\varphi + (\nu^2-2\nu+2\lambda+8)\omega + (2\nu-\lambda^2-2\lambda-8) = 0 \; , \tag{4.13}$$

$$s\Delta_0^{1/2} - Q_0 = 0 \; , \tag{4.14}$$

this last equation may be replaced by the equivalent conditions

$$\Delta_0 - Q_0^2 = 0 \; , \quad \mathrm{sgn}Q_0 = s \tag{4.15}$$

(we assume that $\Delta_0 > 0$, which will be verified below). The first equation in (4.15) may be written in full as

$$\begin{aligned} &2(\nu^3+2\nu^2\lambda-6\nu^2-\nu\lambda^2-16\nu\lambda-10\lambda^2-2\lambda^3)\varphi^2 + (2\nu^3+3\nu^2\lambda-16\nu^2-12\nu\lambda+48\nu-\lambda^3+12\lambda^2+64\lambda)\varphi\omega \\ &-(4\nu^2+2\nu\lambda-24\nu-\lambda^2+8\lambda+48)\omega^2 + (8\nu^2-\nu^2\lambda-2\nu\lambda^2+12\nu\lambda-48\nu-\lambda^3-4\lambda^2-64\lambda)\varphi \\ &+2(2\nu^2+2\nu\lambda-16\nu+\lambda^2+48)\omega + 2(4\nu-\nu\lambda+4\lambda-24) - 3\lambda^2 = 0 \; . \end{aligned} \tag{4.16}$$

Solving (4.13) for ω, and substituting the result in (4.16), we get a quadratic equation for $\varphi = \varphi(\nu,\lambda)$:

$$B_0\varphi^2 + B_1\varphi + B_2 = 0 \; , \tag{4.17}$$

where

$$B_0=\lambda\nu^4(\nu+\lambda)(\nu-3\lambda)+\nu^2[2\nu(\lambda^3+2\nu^3)-\lambda(\nu^2\lambda+8\nu^3+18\lambda^3)]$$
$$-8[4\nu^4(\nu+\lambda)+3\lambda^4(\lambda-\nu)+\nu^2\lambda^2(5\nu+9\lambda)]-16\lambda[\nu^2(9\lambda+4\nu)+2\lambda^2(4\lambda+\nu)]-128\lambda^3 ,$$
$$B_1=(\lambda-\nu)B_1' ,$$
$$B_1'=\lambda\nu^2(\lambda+\nu)(\nu^2-\lambda^2)-2\lambda[\lambda(\lambda^3+\nu^3)-5\nu^2(\lambda^2-\nu^2)] \quad (4.18)$$
$$+16[\lambda^2(\lambda^2+\nu^2)-2\lambda\nu(\lambda^2-\nu^2)-\nu^4]+32[4\nu^2(\lambda+\nu)+\lambda^2(3\nu+5\lambda)]+128\lambda(\nu+2\lambda) ,$$
$$B_2=\lambda(\lambda-2\nu)(\lambda^2-\nu^2)^2+16\nu\lambda(\lambda-\nu)(\lambda^2-\nu^2)+16[(\lambda^2+\nu^2)(\nu^2-4\nu\lambda+\lambda^2)-4\lambda^4]-128(\lambda-\nu)(\lambda^2-\nu^2) .$$

Therefore,

$$\varphi=\varphi_\pm=(-B_1\pm\sqrt{D})/2B_0 , \quad D=B_1^2-4B_0B_2 , \quad (4.19)$$
$$\omega=\omega_\pm=[(\lambda^2+2\lambda-2\nu+8)-(\nu^2-4\nu-4\lambda)\varphi_\pm]/(\nu^2-2\nu+2\lambda+8) . \quad (4.20)$$

Remarkably, the polynomial D may be factored as

$$D=\lambda^2(\lambda-\nu)^2(\lambda+\nu-4)D' , \quad (4.21)$$

where

$$D'(\lambda,\nu)=\nu^4(\lambda+\nu)(\lambda^2-\nu^2)^2-4\nu^2(\lambda+\nu)^2[2\nu(\lambda^3+\nu^3)-\lambda^2(\lambda^2+\nu^2)]$$
$$+4(\lambda+\nu)[5\nu^5(\lambda+\nu)+\lambda(\lambda^5+\nu^5)+7\lambda^2\nu^2(\lambda^2-\nu^2)-2\nu\lambda^2(\lambda^3+2\nu^2\lambda+\nu^3)]$$
$$-16[(\lambda^2+\nu^2)(\nu^4-3\lambda^4)+4\nu^3(\nu^3-\lambda^3)+18\lambda\nu^4(\lambda+\nu)+2\nu(\lambda^5+\nu^5)]$$
$$+64[\lambda^2(\lambda^2-\nu^2)(5\lambda-3\nu)+2\nu^3(\nu^2+3\lambda^2)]-256[5\lambda^2(\nu^2-\lambda^2)+2\nu^2(2\nu^2+\nu\lambda+3\lambda^2)]$$
$$+1024[2\lambda^2(\lambda+\nu)+\nu^2(\nu-3\lambda)]-4096\nu^2 . \quad (4.22)$$

Inspection of the terms of degree 9 in D' shows that

$$\lim_{\lambda\to\infty} D'(\lambda,k\lambda)=+\infty , \quad k>0 , \quad k\neq 1 , \quad (4.23)$$

while for $k=1$, the terms of degree 8 imply that

$$\lim_{\lambda\to\infty} D'(\lambda,\lambda)=-\infty . \quad (4.24)$$

These results suggest that D' has at least two roots $\nu=\nu_\pm(\lambda)$ with the asymptotic behavior

$$\nu_\pm\sim\lambda+a_\pm\lambda^{1/\tau_\pm} , \quad \lambda\to\infty , \quad \tau_\pm>1 . \quad (4.25)$$

Taking into account the terms of degrees 8 and 9 in D', we obtain immediately

$$D'[\lambda,\nu_\pm(\lambda)]\sim\lambda^4(2\lambda)(-2a_\pm\lambda^{1+1/\tau_\pm})^2$$
$$-4\lambda^2(2\lambda)^2(4\lambda^4-2\lambda^4)+\cdots$$
$$=8a_\pm^2\lambda^{7+2/\tau_\pm}-32\lambda^8+\cdots . \quad (4.26)$$

The two leading terms in (4.26) will cancel out if $\tau_\pm=2$, $a_\pm=\pm 2$, (4.25) then suggests that the complete expansion of $\nu_\pm(\lambda)$ has the form

$$\nu_\pm\sim\lambda\pm 2\lambda^{1/2}+a_0+a_1\lambda^{-1/2}+a_2\lambda^{-1}+\cdots$$
$$=\delta^{-2}(1\pm 2\delta+a_0\delta^2+a_1\delta^3+\cdots) , \quad (4.27)$$

where $\delta=\lambda^{-1/2}$ is a small parameter. Inserting this expansion in (4.22), we get

$$D'=\delta^{-15}(c_0+c_1\delta+c_2\delta^2+\cdots)=0 , \quad (4.28)$$

the conditions $c_0=0$, $c_1=0,\ldots,$ determine $a_0,a_1,\ldots$ successively. With the aid of a computer algebra system, we have obtained the first eight coefficients a_k:

$$a_0=3 , \quad a_1=\pm\tfrac{1}{4} , \quad a_2=4 , \quad a_3=\tfrac{319}{64} , \quad (4.29)$$
$$a_4=-13 , \quad a_5=\pm\frac{4033}{512} , \quad a_6=4 , \quad a_7=\mp\frac{1\,182\,853}{16\,384} .$$

The accuracy of (4.27) has been numerically verified for $\lambda\geq 4$: the largest relative errors occur at $\lambda=4$ (about 5% for ν_+ and 1% for ν_-), and the error decreases rapidly for larger values of λ.

We have verified numerically that $\nu_\pm(\lambda)$ are the only roots of $D'(\lambda,\nu)$ in the region $\lambda>4$, $\nu>0$ (Table I). By (4.23), we must then have $D'>0$ for $\lambda>4$ and $\nu>\nu_+(\lambda)$ or $0<\nu<\nu_-(\lambda)$. The functions $\varphi_\pm(\lambda,\nu)$ will only be real for (λ,ν) satisfying these conditions.

Next, we need to verify the second condition in (4.15) for solutions (4.19) and (4.20).

Case 1. $\varphi=\varphi_+$, $\omega=\omega_+$. Let us first analyze the limit $\nu,\lambda\to\infty$, $\nu\sim k\lambda$ $(k>0)$:

$$B_1\sim-\lambda\nu^2(\nu^2-\lambda^2)^2 , \quad B_0\sim\lambda\nu^4(\lambda+\nu)(\nu-3\lambda) ,$$
$$D'\sim\nu^4(\lambda+\nu)(\nu^2-\lambda^2)^2 , \quad D\sim\lambda^2(\lambda-\nu)^2(\lambda+\nu)D' ,$$
$$\varphi_+\sim\frac{(\lambda-\nu)^2(\lambda+\nu)}{\nu^2(\nu-3\lambda)}\sim\frac{(1-k)^2(1+k)}{k^2(k-3)} , \quad (4.30)$$
$$\omega_+\sim(\lambda/\nu)^2-\varphi_+\sim[1-(1-k)^2(1+k)/(k-3)]/k^2 ,$$
$$s\sim\operatorname{sgn}\tau_0\sim\operatorname{sgn}A_1\varphi_+(\lambda^2+\nu^2)=\operatorname{sgn}A_1\varphi_+ ,$$
$$Q_0\sim A_1\varphi_+(\lambda^2-\nu^2)\sim A_1\varphi_+(1-k^2)\lambda^2 .$$

TABLE I. Difference ($\Delta\nu_{\pm}$) between the expansion (4.28) and the true root of $D'(\lambda,\nu)$. The largest power of δ included in the evaluation of (4.28) is shown in parentheses.

λ	$\Delta\nu_+$ (δ^7)	$\Delta\nu_-$ (δ)	$\Delta\nu_-$ (δ^7)
4	-0.45	0.08	0.03
5	-0.2	-0.001	-0.02
6	-0.1	-0.05	-0.02
7	-0.05	-0.09	-0.02
10	-0.01	-0.14	-0.008
20	-0.001	-0.12	-0.0008
30	-0.0002	-0.09	-0.0002
50	$< 10^{-4}$	-0.06	$< 10^{-4}$

Therefore, the second condition in (4.15) will be asymptotically satisfied for $k<1$; this suggests that φ_+,ω_+ will be a solution of (4.14) in the region $\lambda>4$, $0<\nu<\nu_-(\lambda)$. This conjecture has been verified numerically, together with the necessary condition $\Delta_0>0$.

When (4.14) holds, we have

$$\text{sgn}\rho\sim\text{sgn}\rho_0=\text{sgn}Q_0\ , \tag{4.31}$$

in the limit $\lambda,\nu\to\infty$, $0<\nu<\lambda$, we have $\varphi_+<0$, by (4.30). Therefore, in order to have $Q_0>0$ we must choose $A_1<0$. With this choice, it can be verified numerically that $Q_0>0$ for $\lambda>4$, $0<\nu<\nu_-(\lambda)$.

Case 2. $\varphi=\varphi_-$, $\omega=\omega_-$. Here, a simple modification of the argument of case 1 shows that in the limit $\lambda,\nu\to\infty$, $\nu\sim k\lambda$ we have $\varphi\to 0$. Therefore, to determine the asymptotic behavior of φ, we must take into account the terms of degree 6 in B_1 and the second term in the expansion of $\sqrt{D}$:

$$\begin{aligned}
&B_1\sim-\lambda(\lambda-\nu)\{\nu^2(\lambda+\nu)(\lambda^2-\nu^2)+2[\lambda(\lambda^3+\nu^3)-5\nu^2(\lambda^2-\nu^2)]+O(\lambda^3)\}\ ,\\
&\sqrt{D}\sim\lambda\nu^2(\lambda^2-\nu^2)^2-2\lambda(\lambda+\nu)[\nu^2(\lambda-\nu)^2+2\nu(\lambda^3+\nu^3)-\lambda^2(\lambda^2+\nu^2)]+O(\lambda^5)\ ,\\
&\varphi_-\sim\frac{(\lambda-\nu)[\lambda(\lambda^2-\lambda\nu+\nu^2)-4\nu^2(\lambda-\nu)]+2\nu(\lambda^3+\nu^3)-\lambda^2(\lambda^2+\nu^2)}{\nu^4(\nu-3\lambda)}\\
&\quad\sim-\frac{(2k^2-7k+3)}{k^2(k-3)\lambda}=\frac{(1-2k)}{k^2\lambda}\ ,\\
&\omega_-\sim(\lambda/\nu)^2-\varphi_-\sim 1/k^2\ ,\\
&s\sim\text{sgn}\tau_0\sim\text{sgn}A_1[\varphi_-(\nu^2+\lambda^2)+2(1+\omega_-)\nu-2(1+\omega_-)\lambda]\\
&\quad\sim\text{sgn}A_1[(1-2k)(1+k^2)/k^2+2(1+1/k^2)(k-1)]=-\text{sgn}A_1\ ,\\
&Q_0\sim-A_1[\varphi_-(\nu^2-\lambda^2)+2(1-\omega_-)\nu]\\
&\quad\sim A_1\lambda[(1-k^2)(1-2k)/k^2-2k(1-1/k^2)]=A_1\lambda(1-k^2)/k^2\ .
\end{aligned}\tag{4.32}$$

Therefore, (4.15) will hold asymptotically for $k>1$; we have verified numerically that φ_-, ω_- satisfy (4.15) and $\Delta_0>0$ in the region $\lambda>4$, $\nu>\nu_+(\lambda)$. As in case 1, (4.31) is valid; if $k>1$, (4.32) implies that we must take $A_1<0$ as before in order to have $\rho>0$. We have verified numerically that $Q_0>0$ for $\lambda>4$, $\nu>\nu_+(\lambda)$.

In conclusion, for any (λ,ν) in the region $\lambda>4$, $0<\nu<\nu_-(\lambda)$ or $\nu>\nu_+(\lambda)$, we obtain a model where $\rho>0$ and $p_\perp,p_\parallel\ll\rho$ hold in the limit $t\to\infty$, $z\neq 0$. In view of the acceptable domains of definition of the branches φ_+ and φ_-, we can write down a single expression for φ,

$$\varphi=\frac{\lambda-\nu}{2B_0}[\lambda\sqrt{(\lambda+\nu-4)D'}-B_1']\ . \tag{4.33}$$

The asymptotic behavior of the pressures as $t\to\infty$ will be determined by the coefficients

$$\begin{aligned}
&p_{\parallel 1}=\tfrac{1}{2}A_2[(\zeta+\xi)\eta^2-\mu^2+2(1-2\xi-\zeta)\eta-2(1+2\xi-\zeta)\mu-8(1-\zeta)]\ ,\\
&p_{\perp 1}=(\Delta_1-2Q_0Q_1)/4Q_0\ ,
\end{aligned}\tag{4.34}$$

where we have used (4.14). With the help of (4.7), (4.34) becomes

$$\begin{aligned}
p_{\parallel 1}=\tfrac{1}{2}A_2\{&\xi[\gamma^2\nu^2-4\gamma(\nu+\lambda)+16(\gamma-1)]+\zeta[\gamma^2\nu^2-2\gamma(\nu-\lambda)-8(\gamma-2)]\\
&-[\gamma^2(\lambda-4)^2-2\gamma(\nu-5\lambda+20)-32]\}\ ,
\end{aligned}\tag{4.35}$$

$$
\begin{aligned}
p_{\perp 1} = (A_1 A_2/Q_0)(\xi\{ & 2\gamma[(1+\gamma)\varphi+\gamma\omega]\nu^3+\gamma[4(1+\gamma)\varphi+\gamma(3\omega-1)]\nu^2\lambda-2\gamma[(1+\gamma)\varphi+\gamma]\nu\lambda^2 \\
& -\gamma[4(1+\gamma)\varphi+\gamma(1+\omega)]\lambda^3+4[2(2\gamma-1)(\gamma-4)\varphi-\gamma(5\gamma-1)\omega+\gamma(\gamma+1)]\nu^2 \\
& +4[4(2\gamma^2-7\gamma+1)\varphi-3\gamma(\omega-1)]\nu\lambda+4[2(\gamma-2)(4\gamma+1)\varphi+\gamma(5\gamma-2)\omega-\gamma^2]\lambda^2 \\
& -16[2(\gamma-1)(3\gamma-7)\varphi-(7\gamma-4)\omega-(2\gamma-1)(\gamma-4)]\nu \\
& -16[2(2\gamma-7)(\gamma-1)\varphi+(7\gamma^2-12\gamma+1)\omega-(5\gamma^2-10\gamma+1)]\lambda \\
& -64(3\gamma-7)(\gamma-1)(1-\omega)\} \\
& +\zeta\{\gamma\varphi(\nu+\lambda)^2(2\nu-\lambda)-4[2(\gamma+1)\varphi+\gamma(2\omega-1)]\nu^2-4\gamma(3\varphi+\omega-1)\nu\lambda \\
& \quad +2[2(\gamma+2)\varphi+\gamma(\omega+1)]\lambda^2-16[(\gamma-4)\varphi-(2\gamma+1)\omega+(\gamma+1)]\nu \\
& \quad -16(\gamma\omega-4\varphi)\lambda-32(1-\omega)(\gamma-4)\} \\
& +\{-\gamma\varphi\lambda(\nu+\lambda)^2+4\gamma(2\varphi+\omega)\nu^2+4[(7\gamma-4)\varphi-\gamma(1-\omega)]\nu\lambda \\
& \quad +2[2(3\gamma-4)\varphi+\gamma(\omega-3)]\lambda^2-16[(\gamma+2)\varphi+\gamma(2\omega-1)]\nu \\
& \quad -16[2(\gamma+1)\varphi+(\gamma-1)\omega-(2\gamma-1)]\lambda-32(\gamma+2)(1-\omega)\})\ .
\end{aligned}
\tag{4.36}
$$

The condition that the fluid be asymptotically isotropic, $p_{\perp 1}=p_{\parallel 1}$, can be written explicitly as

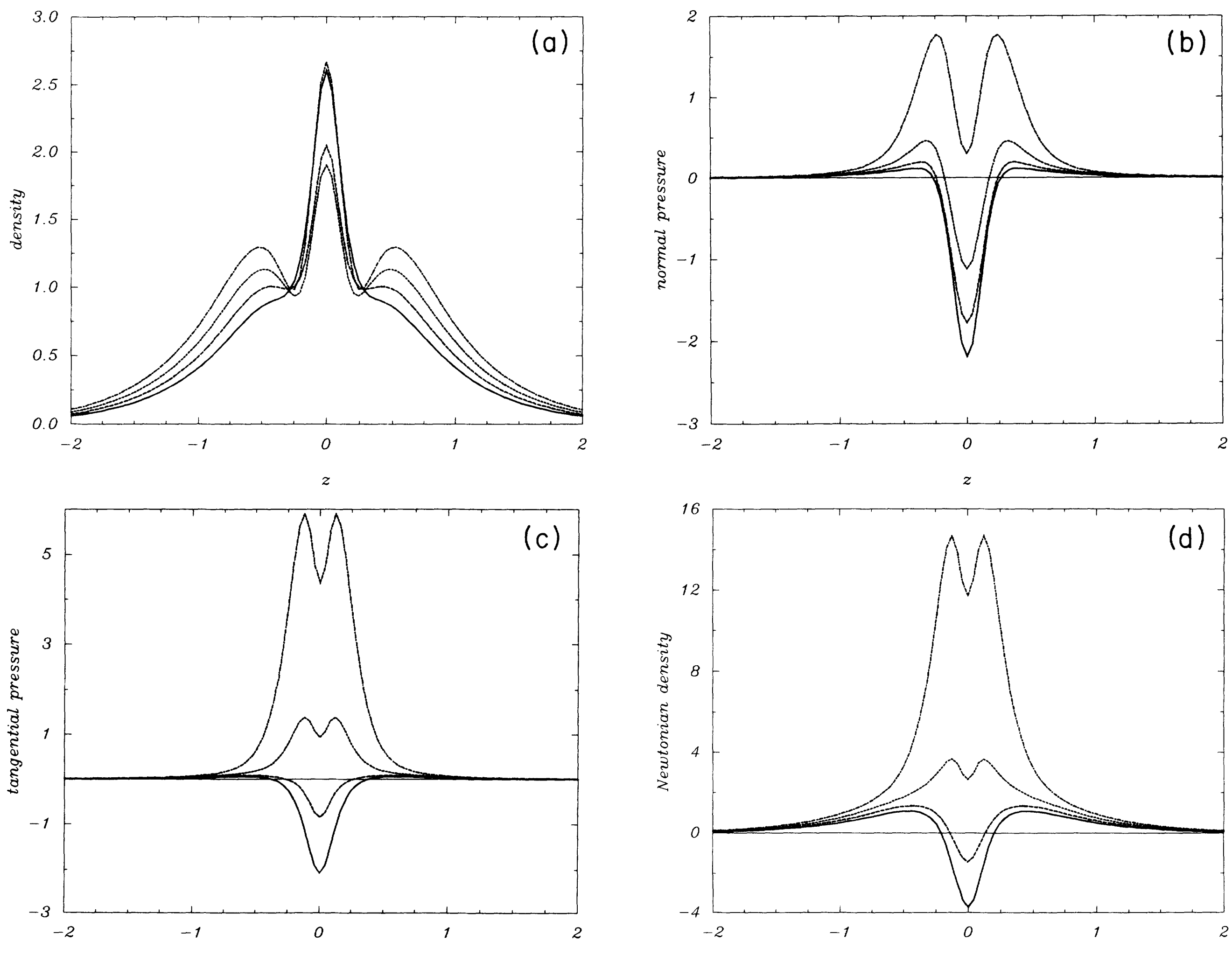

FIG. 3. Evolution of density, pressures, and Newtonian density in the model of Sec. IV with $\gamma=\frac{4}{3}$, $\lambda=6$, $\nu=2$, $A_1=-0.7$, $A_2=-0.05$, $\xi=0.8$, $\varepsilon=5t+7$ [these values of the parameters imply, by (4.7), (4.33), (4.20), and (4.37), that $\mu=\frac{20}{3}$, $\eta=\frac{8}{3}$, $\varphi=0.072\,25$, $\omega=2.701$, and $\zeta=0.5105$]. The displayed curves are for $t=0.2$ (dash-dotted), $t=0.3$ (dashed), $t=0.4$ (long-dashed), and $t=0.5$ (solid).

$$\xi\{\varphi\gamma^2\nu^2(\nu^2-\lambda^2)+2\gamma^2(\omega-2\varphi+1)\nu^3+2\gamma[2\varphi+\gamma(3\omega-1)]\nu^2\lambda-4\gamma^2(1+\varphi)\nu\lambda^2$$
$$-2\gamma[2(1+2\gamma)\varphi+\gamma(1+\omega)]\lambda^3+8[2(2\gamma^2-6\gamma+3)\varphi-\gamma(3\gamma-2)\omega-\gamma^2]\nu^2$$
$$+16[2(2\gamma^2-5\gamma+1)\varphi+\gamma(1-\omega)]\nu\lambda+8[2(4\gamma^2-6\gamma-1)\varphi+\gamma(5\gamma-2)\omega-\gamma^2]\lambda^2$$
$$+32[-2(\gamma-1)(3\gamma-5)\varphi+(4\gamma-3)\omega+(2\gamma^2-6\gamma+3)]\nu$$
$$+32[-2(\gamma-1)(2\gamma-5)\varphi-(7\gamma^2-10\gamma+1)\omega+(5\gamma^2-8\gamma+1)]\lambda$$
$$-128(\gamma-1)(3\gamma-5)(1-\omega)\}$$
$$+\zeta\{\gamma^2\varphi\nu^2(\nu^2-\lambda^2)-2\gamma[(4\gamma-1)\varphi-\gamma(1-\omega)]\nu^3-2\gamma\varphi\lambda[4(\gamma-1)\nu^2-\nu\lambda+2\lambda^2]$$
$$+4\gamma[-2\varphi+(4\gamma-3)\omega-(4\gamma-1)]\nu^2-12\gamma(2\varphi+\omega-1)\nu\lambda+4\gamma(1+\omega)\lambda^2+16\gamma(2\varphi+3\omega-1)\nu$$
$$+32\gamma(2\varphi-1)\lambda+64\gamma(1-\omega)\}$$
$$+\{\gamma^2\varphi\lambda^2(\lambda^2-\nu^2)+2\gamma\varphi[\nu^3+2(2\gamma-3)\nu^2\lambda+4\lambda^3]+2\gamma[(4\gamma-3)\varphi-\gamma(1-\omega)]\nu\lambda^2$$
$$-4[2(2\gamma^2-5\gamma+4)\varphi-\gamma(1+\omega)]\nu^2-4[2(8\gamma^2-15\gamma+4)\varphi+\gamma(4\gamma-7)(\omega-1)]\nu\lambda$$
$$-4\gamma[4(3\gamma-4)\varphi+(4\gamma-1)\omega-(4\gamma-3)]\lambda^2$$
$$+16[2(\gamma-2)(4\gamma-3)\varphi+(2\gamma^2-7\gamma+4)\omega-(2\gamma^2-5\gamma+4)]\nu$$
$$+32[2(2\gamma^2-6\gamma+3)\varphi+(4\gamma^2-6\gamma+1)\omega-(4\gamma^2-7\gamma+1)]\lambda$$
$$+64(\gamma-2)(4\gamma-3)(1-\omega)\}=0\ . \quad (4.37)$$

In summary, in order to obtain a model of domain-wall formation with the desired properties far from the wall (isotropic gas in isentropic flow), one can begin by choosing (λ,ν) satisfying the inequalities $\lambda>4$, $\nu>\nu_+(\lambda)$, or $0<\nu<\nu_-(\lambda)$. The constants φ and ω are then determined by (4.33) and (4.20). The values of $A_1<0$ and $A_2\neq 0$ may be chosen arbitrarily; ξ and ζ must be chosen (if possible) so that $p_{\perp 1}=p_{\parallel 1}=p>0$. Clearly, it is possible to specify ξ, say, arbitrarily, and then to find ζ from (4.37). The condition $p>0$ may then be verified by evaluating (4.35) and (4.36).

Examples of C^∞ metrics having the asymptotic behavior (4.2) can be constructed in analogy to (3.43) and (3.44); we have studied numerically the properties of the metric (2.3) where

$$-\tfrac{1}{4}F=M=N_1(t,z)+A_1\exp[-\nu t-\lambda N_1(t,z)]+A_2\exp[-\eta t-\mu N_1(t,z)]\ ,$$
$$-\tfrac{1}{4}G=P=N_1(t,z)+\varphi A_1\exp[-\nu t-\lambda N_1(t,z)]+\xi A_2\exp[-\eta t-\mu N_1(t,z)]\ , \quad (4.38)$$
$$t-\tfrac{1}{4}H=N=N_1(t,z)+\omega A_1\exp[-\nu t-\lambda N_1(t,z)]+\zeta A_2\exp[-\eta t-\mu N_1(t,z)]\ ,$$

and N_1 is given by (3.44) with $\varepsilon(t)=\varepsilon_0 t+\varepsilon_1$, $\varepsilon_1=$const. As in Sec. II, we find positive density and pressures at an initial stage of the collapse (Fig. 3). Later, a transition to negative pressures takes place at $z=0$. It should be noted that initially the pressures are almost isotropic everywhere (not only as $|z|\to\infty$). The evolution of the Newtonian density follows a pattern similar to the one found in the model of Sec. III, with the appearance of a repulsive layer around $z=0$.

V. CONCLUDING REMARKS

Following Synge's g method, we have constructed dynamical models of the formation of a plane, reflection-symmetric domain wall of negligible thickness. Our models start from inhomogeneous plane-symmetric distributions of matter, whose density and pressures decay exponentially (in a special system of coordinates) at sufficiently large distances from the plane of symmetry $z=0$. This matter distribution collapses, causing the density and pressures to decrease exponentially with time for $z\neq 0$. Such spatial and temporal behavior of the models has been chosen to ensure that the matter far from $z=0$ has low (positive) density and pressure.

In the first model we have considered, the metric has the form (2.3) with $F=G$, and its asymptotic behavior is given by (3.2) and (3.13), with $k(t)=ae^{-\nu t}$ $(\nu>0)$. This particular form cannot represent an asymptotically isotropic fluid, but by suitably restricting the free parameters λ and ν (Fig. 1), one can obtain equations of state of the form $p_\parallel/\rho\sim$const and $p_\perp/\rho\sim$const. A metric with the required asymptotic properties is given by (3.43) and (3.44), where the C^∞ function $N_1(z,t)$ plays the role of a "smoothed version" of $|z|$, and will, in the limit $t\to\infty$, contribute with the distributional EMT of the domain wall. During the collapse, a phase transition seems to take place, with the appearance of tensions at the high-density regions around $z=0$. The tension distribution around the collapsing wall is such that the pressure gradient forces tend to speed up the collapse (Fig. 2).

The second model includes certain physically interesting features, such as the asymptotic isotropy and isentropy far from the wall in formation. This is achieved at the expense of a more complex model, based on both the general metric (2.3) and the general asymptotic form (4.2) and (4.3). The wall collapse in this model is qualitatively similar to the collapse of the previous model. Finally, we remark that a more realistic model of domain-wall formation should include some mechanism of energy dissipation, such as the emission of electromagnetic radiation.

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