Multiple field scalar-tensor theories of gravity and cosmology

Andrew L. Berkin* and Ronald W. Hellings†

Jet Propulsion Laboratory, California Institute of Technology, 4800 Oak Grove Drive, Pasadena, California 91109

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We consider multiple scalar fields coupled to gravity, with special attention given to two-field theories. First, the conditions necessary for these theories to exactly meet solar system tests are given. We find, in particular, that these constraints require that some scalar kinetic terms be non-positive-definite. Next, we investigate the cosmological evolution of the fields to see if these conditions can be met. Solutions are found in the dust era, as well as radiation- and cosmological-constant-dominated epochs. The possibility of inflation in these theories is discussed. While power law growth of the scalar fields can yield the appropriate conditions to meet solar system constraints, these solutions are unstable.

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I. INTRODUCTION

Scalar-tensor (ST) theories are alternative models of gravity which provide a theoretical framework within which general relativity (GR) may be tested. Many important tests so far have used the post-Newtonian approximation, and have relied upon corrections to dynamics in the solar system. Such tests force ST theories to limits where they greatly resemble GR. For example, the current limit on Brans-Dicke theories [1] requires $\omega > 500$ [2], where $\omega \rightarrow \infty$ recovers GR and $\omega \approx 1$ would be a natural value to expect *a priori*.

However, despite the strong resemblance to GR in our solar system, gravity may be radically different in other regimes. The gravitational field in our solar system is weak, severely limiting the parameter space of gravity tested. Theories which differ from GR in strong gravitational fields, but agree with solar system constraints, are needed to further test GR. Recently, a class of ST theories with multiple scalar fields has been proposed [3]. These theories can satisfy the solar system criteria to arbitrary accuracy, but still diverge from GR in other limits, for example, in the strong field regime around binary pulsars. Thus, such theories provide an important test of GR in a previously sparsely tested regime.

Aside from their importance as generalizations of standard ST theories, multiple scalar field theories have a second motivation. Such additional scalars coupled to gravity appear in Kaluza-Klein [4] and string theories [5] which seek to unify gravity with other forces. Tests of the correctness of GR thus provide constraints on such theories. Furthermore, ST theories combined with grand unified theories can provide adjustments to inflationary models of the early Universe [6] which allow the phase transition to complete in old inflation [7] and remove the fine-tuning of new and chaotic models [8]. The inflationary Universe not only solves several long-standing cosmological problems, but also provides the only currently known source of seed density fluctuations which obey the magnitude and spectrum of the microwave background found by the Cosmic Background Explorer (COBE) [9]. While these "extended" versions of old inflation are severely constrained by the solar system tests [10], multiple ST models may offer successful conditions for inflation which still obey all observational tests. Thus, we seek the viability of inflation in multiple field models where the solar system constraints may be avoided.

In this paper we consider a particular set of multiple scalar ST theories which are a straightforward generalization of nonminimally coupled models. The one-field nonminimally coupled model is related to the Jordan-Brans-Dicke (JBD) theory by a redefinition of the scalar field. Our starting action is thus different from those used previously [3], where the action was taken after a conformal transformation had been made so that the gravity sector appeared normal. We initially consider a model with an arbitrary number N of scalar fields, and derive the conditions necessary to meet the solar system constraints. Two constraints result from these conditions. First, the scalar field kinetic terms must be non-positive-definite. Second, the scalars must take on the correct asymptotic values far from the solar system. Previous work used the ansatz that these values were constants which satisfy all the equations. However, the actual values of these fields should be set by their cosmological evolution.

In order to study their cosmological evolution we first find a generic set of solutions which exhibit power-law behavior, and give exact solutions in the dust-, radiation-, and cosmological-constant-dominated eras. Next, we specialize to a two-field model, which elucidates the main features of the behavior. The stability of these solutions in time is then examined, and small deviations are found to diverge from the exact power law behavior as the Universe expands. Hence, while solutions of multiple field ST theories are possible which meet the solar system constraints and field equations for cosmological evolution, they are unstable. This is not surprising, given the

*Electronic address: berkin@krazy.jpl.nasa.gov

†Electronic address: rwh@grouch.jpl.nasa.gov

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need for negative kinetic terms. This finding calls into question the ability of these models to provide a true test of GR. We close with comments on the possibilities of success of these theories and their potential role in inflation.

II. *N* FIELD MODELS

We begin by considering the action

$$S=\int d^4x\sqrt{-g}\,[f(\phi)R-G_{AB}g^{\alpha\beta}\phi_{A,\alpha}\phi_{B,\beta}-2U(\phi)+16\pi L_m]\ , \tag{2.1}$$

where ϕ is an N component scalar field whose individual components will be denoted with capital Roman letters, U is a potential, G_{AB} describes the kinetic coupling of the fields, L_m is the Lagrangian for other matter and summation over A and B is implied. The scalar field enters the Lagrangian with metric coupling to matter. By an appropriate linear transformation and rescaling of the ϕ, G_{AB} may be taken to be diagonal with entries of ±1 for each scalar field. All other conventions are identical to those of Misner, Thorne, and Wheeler [11]. $f(\phi)=G\equiv1/\kappa^2$, where G is Newton's constant, recovers GR with scalar fields, while $f(\phi)=\xi\phi^2$ with ϕ a singlet is the standard nonminimally coupled model. Making the field redefinition $\Phi=\xi\phi^2$ for singlet ϕ places the action into standard Brans-Dicke form.

Varying this action with respect to the metric gives the gravitational field equations

$$\tfrac{1}{2}g_{\mu\nu}(-fR+G_{AB}g^{\alpha\beta}\phi_{A,\alpha}\phi_{B,\beta}+2U)-f_{;\mu\nu}+g_{\mu\nu}\Box f+fR_{\mu\nu}-G_{AB}\phi_{A,\mu}\phi_{B,\nu}=8\pi T_{\mu\nu}\ , \tag{2.2}$$

with $T_{\mu\nu}$ the energy-momentum tensor corresponding to L_m. The trace of this equation is

$$-fR+G_{AB}g^{\alpha\beta}\phi_{A,\alpha}\phi_{B,\beta}+4U+3\Box f=8\pi T\ . \tag{2.3}$$

Varying with respect to ϕ gives the scalar field equations

$$f_AR+2G_{AB}\Box\phi_B-2U_A=0\ , \tag{2.4}$$

with a subscript A referring to the partial derivative $\partial/\partial\phi_A$. In order to be viable, metric theories such as these must satisfy the solar system experimental constraints. Of these, we concentrate in the next section on the effects arising from first-order space curvature [parametrized post-Newtonian (PPN) parameter γ].

A. Solar system constraints

To lowest order in the post-Newtonian approximation, the metric around a test body of mass M in the solar system is [11]

$$ds^2=-\left[1-\frac{2M}{r}\right]dt^2+\left[1+2\gamma\frac{M}{r}\right][dx^2+dy^2+dz^2]\ , \tag{2.5}$$

where $\gamma=1$ recovers the value in GR. In the JBD theory, $\gamma=(1+\omega)/(2+\omega)$ and current tests from the Viking lander give $\omega>500$ [2].

We will use the (00) and trace equations to determine the expression for γ in the theory of Eq. (2.1). Note that

$$\Box f=g^{\alpha\beta}(f_{AB}\phi_{A,\alpha}\phi_{B,\beta}+f_A\phi_{A;\alpha\beta})\approx f_A\Box\phi_A\ . \tag{2.6}$$

To obtain this last approximation we removed the first term by using the fact that $\nabla\phi=O(M)$ and keeping only lowest order in M. Then, making use of Eq. (2.4) we find

$$\Box f=-\tfrac{1}{2}G^{-1}_{AB}f_Af_BR+U_Af_BG^{-1}_{AB}\ . \tag{2.7}$$

The matter source for the solar system bodies will be taken to be pressureless dust, with delta function stress-energy terms. Making use of the representation of the delta function by $\nabla^2(1/r)$, we therefore write

$$T_{00}=-\frac{1}{4\pi}\nabla^2\left[\frac{M}{r}\right],\quad T_{ij}=0,\quad T=\frac{1}{4\pi}\nabla^2\left[\frac{M}{r}\right]. \tag{2.8}$$

Again, to lowest order, the necessary curvature components are

$$R_{00}=-\frac{1}{2}\nabla^2g_{00}=-\nabla^2\left[\frac{M}{r}\right],\qquad R=2(1-2\gamma)\nabla^2\left[\frac{M}{r}\right]. \tag{2.9}$$

The (00) equation then implies

$$-(f+f_AG^{-1}_{AB}f_B)(1-2\gamma)\nabla^2\left[\frac{M}{r}\right]+f\nabla^2\left[\frac{M}{r}\right]+U+U_AG^{-1}_{AB}f_B=2\nabla^2\left[\frac{GM}{r}\right], \tag{2.10}$$

where G is Newton's constant. If we take the case $U=0$, which we will consider throughout the rest of this paper and define

$$C\equiv f_AG^{-1}_{AB}f_B\ , \tag{2.11}$$

then we find

$$C-2\gamma(f+C)=-2G\ . \tag{2.12}$$

Similarly, the trace equation yields

$$(2f+3C)(1-2\gamma)=-2G\ . \tag{2.13}$$

Combining these two equations then gives

$$\gamma=\frac{f+C}{f+2C}\ . \tag{2.14}$$

This theory will be identical to GR for solar system tests if $\gamma=1$, which then implies

$$C\equiv f_AG^{-1}_{AB}f_B=0\ . \tag{2.15}$$

As long as the coupling satisfies this last relationship, multiple ST theories can exactly replicate GR in the solar system without being identical to GR. However, theories which satisfy (2.15) may still be quite different in other gravitational regimes. This behavior is in contrast with single field ST gravity, where the solar system constraints force the entire theory to be indistinguishable from GR.

Of course, γ need not be exactly unity, but need only satisfy the current experimental limits. Indeed, nature may dictate that γ is not 1 but only some value close to 1, in which case GR would fail. If this should be the case, both single and multiple ST theories could meet the new constraint with a C that is a small but nonzero value, while higher order effects would cause a larger discrepancy with GR in other regimes.

Whether the condition of Eq. (2.15) can be met depends on the values of the scalar fields. Because G_{AB} may be diagonalized and rescaled, the vanishing of C implies either the f_A are all 0 in the solar system, or that some of the scalars have negative kinetic coupling. Also, if $C=0$, then Eqs. (2.12) and (2.13) demand that f be equal to the gravitational constant, setting another condition. For a large number of models, including the one here, setting the background value of the fields to zero certainly meets these conditions. In many other models, such as that of [3], values constant in time will suffice. However, these values are actually determined by the cosmological evolution of ϕ, to which we next turn our attention.

B. Cosmological evolution

For cosmology we use a spatially flat Robertson-Walker universe, with metric given by

$$ds^2=-dt^2+a^2(t)(dx^2+dy^2+dz^2) , \qquad (2.16)$$

where $a(t)$ is the time-dependent scale factor. From the (00) component of the gravitational field equations (2.2) we get

$$3H^2 f-\tfrac{1}{2}G_{AB}\dot{\phi}_A\dot{\phi}_B+3H\dot{f}-U=8\pi\rho , \qquad (2.17)$$

where $H\equiv\dot{a}/a$ is the Hubble parameter, ρ is the energy density, and an overdot indicates differentiation with respect to time. The trace equation gives

$$f(6\dot{H}+12H^2)+G_{AB}\dot{\phi}_A\dot{\phi}_B-4U+3\ddot{f}+9H\dot{f}=8\pi(\rho-3P) , \qquad (2.18)$$

where P is the pressure of matter, which is assumed to be a perfect fluid. Finally, the scalar field equations (2.4) yield

$$\Box\phi_A=-\tfrac{1}{2}G^{-1}_{AB}f_B R+U_B G^{-1}_{AB}=0 , \qquad (2.19)$$

with the Ricci scalar given by

$$R=6\dot{H}+12H^2. \qquad (2.20)$$

One equation of the set (2.17)–(2.19) is extraneous due to the symmetries of the spacetime and the contracted Bianchi identities.

1. Dust era

In the current dust-dominated universe, with $U=0$, the constraint for the solar system (2.15) must be satisfied. Contracting Eq. (2.19) with f_A gives

$$f_A\Box\phi_A=-\tfrac{1}{2}f_A G^{-1}_{AB}f_B R=0 , \qquad (2.21)$$

a rather strong constraint on the behavior of the scalar fields in a dust Universe. The limits imposed by the null summation (2.21) will be more obvious when we consider a two-field case in the next section.

To better understand this constraint we consider particular choices for $f(\phi)$ of the form

$$f(\phi)=f_c+F_{AB}\phi_A\phi_B , \qquad (2.22)$$

with f_c and F_{AB} constant. This model is a straightforward generalization of the standard one-field nonminimal coupling. There are no added dimensional coupling constants. Only f_c, which corresponds to a modified gravitational constant, has dimensions.

Because these equations are nonlinear, general solutions will be difficult, if not impossible, to obtain. We therefore look for power-law solutions, by making the ansatz

$$a(t)=a_c\left[\frac{t}{t_c}\right]^p , \quad \phi_A=b_A\left[\frac{t}{t_c}\right]^q , \qquad (2.23)$$

where a_c, t_c, b_A, p, and q are all constants. Although this power-law assumption restricts the generality of our solutions, the problem does become tractable. The dependence of the solar system constraints upon cosmological evolution is manifested, a feature missing from just a constant ϕ solution. For GR we know that the cosmological expansion is power law, with p being $\frac{2}{3}$ and $\frac{1}{2}$ in the dust and radiation regimes, respectively. Thus, physically, our solutions allow easy comparison with the standard cosmological behavior.

With the above simplifications, Eq. (2.21) gives

$$F_{AB}b_A b_B\frac{t^{2q-2}}{t_c^{2q}}(q^2-q+3pq)=0 . \qquad (2.24)$$

This equation is solved either by $q=0$, $q=1-3p$, or $F_{AB}b_A b_B=0$. The last condition causes f to be constant: $f=f_c=G$. The gravitational constant does not evolve. The first condition, $q=0$, corresponds to constant fields, while the middle condition relates the evolution of the fields to that of the scale factor.

We now examine the scalar field equations (2.19) which imply

$$b_A(q^2-q+3pq)=G^{-1}_{AB}F_{BC}b_C(-6p+12p^2) . \qquad (2.25)$$

If q equals either 0 or $1-3p$, then the left-hand side must be 0, giving $p=0$ or $p=\frac{1}{2}$. The first case corresponds to a static Universe, in conflict with observation, and will be ignored. We thus retain only $p=\frac{1}{2}$.

Next consider the (00) equation, which for power-law behavior becomes

$$\frac{3p^2}{t^2}\left[f_c+F_d\left[\frac{t}{t_c}\right]^{2q}\right]-\frac{q^2G_d}{2t^2}\left[\frac{t}{t_c}\right]^{2q}+\frac{6pqF_d}{t^2}\left[\frac{t}{t_c}\right]^{2q}=\frac{8\pi\rho_d}{a_c^3}\left[\frac{t}{t_c}\right]^{-3p} , \qquad (2.26)$$

with the definitions

$$F_d \equiv F_{AB} b_A b_B, \quad G_d \equiv G_{AB} b_A b_B , \tag{2.27}$$

in the dust era. As noted from Eq. (2.25), if $q=0$ or $q=1-3p$, then $p=\frac{1}{2}$. However, plugging these values into Eq. (2.26) shows that the (00) equation cannot be solved, because the powers of time will not balance. Therefore, of the three possible solutions to Eq. (2.24), only the last one, $F_d=0$, has the possibility of being consistent with both an expanding universe and the other field equations.

Now, Eq. (2.25) implies that

$$G_d = \frac{-6p+12p^2}{q^2-q+3pq} F_d = 0 . \tag{2.28}$$

Using the fact that both F_d and G_d are zero, the (00) equation simplifies to

$$\frac{3p^2}{\kappa^2 t^2} = \frac{8\pi\rho_d}{a_c^3} \left[\frac{t}{t_c} \right]^{-3p} , \tag{2.29}$$

identical with the GR case. Thus, because of the cancellation necessary to meet the solar system constraint (2.15), the dust-dominated Universe with power-law expansion resembles GR in all cosmological aspects. However, this does not mean that other cosmologies, such as a radiation-dominated Universe, necessarily resemble GR, as we shall see below.

2. *Radiation era*

For a radiation-dominated Universe the energy density is given by

$$\rho(t) = \rho_r a^{-4}(t) , \tag{2.30}$$

with ρ_r a constant, and $P=\rho/3$. Since the current Universe is not radiation dominated, the solar system constraint, Eq. (2.15), need not apply. Because the stress energy tensor for radiation is traceless, the trace equation

$$\left[\frac{1}{\kappa^2} + F_r \left[\frac{t}{t_r} \right]^{2q} \right] \frac{-6p+12p^2}{t^2} + \frac{q^2 G_r}{t^2} \left[\frac{t}{t_r} \right]^{2q} + \frac{6q(2q-1)F_r}{t^2} \left[\frac{t}{t_r} \right]^{2q} + \frac{18pqF_r}{t^2} \left[\frac{t}{t_r} \right]^{2q} = 0 \tag{2.31}$$

is most convenient to consider first. Here t_r is a constant, and F_r and G_r correspond to the quantities (2.27) in the radiation era.

Balancing the powers of time then requires that $p=\frac{1}{2}$, exactly as in the standard GR case. With this value, the first term in Eq. (2.31) vanishes, and the resulting time independent part is solved by

$$q=0 \quad \text{or} \quad q = \frac{-3F_r}{G_r+12F_r} . \tag{2.32}$$

We then plug this result into the (00) equation, which is identical with equation (2.26) except with a right-hand side

$$\frac{8\pi\rho_r}{a_r^4} \left[\frac{t}{t_c} \right]^{-4p} . \tag{2.33}$$

If $q=0$, then the fields do not evolve, and

$$\frac{3}{4} \left[\frac{1}{\kappa^2} + F_r \right] = \frac{8\pi\rho_r}{a_r^4} t_r^2 . \tag{2.34}$$

This is similar to the Einstein GR case, except that the gravitational constant is shifted by F_r.

When $q=-3F_r/(G_r+12F_r)$, the (00) equation that results is solved by any one of three conditions. These are

$$\begin{aligned} &F_r = 0 \Longrightarrow q = 0 , \\ &G_r = 0 \Longrightarrow q = -\tfrac{1}{4} , \\ &G_r = -6F_r \Longrightarrow q = -\tfrac{1}{2} . \end{aligned} \tag{2.35}$$

This first case just reproduces the $q=0$ case above. The latter two conditions cause cancellations such that

$$\frac{3}{4\kappa^2} = \frac{8\pi\rho_r}{a_r^4} t_r^2 , \tag{2.36}$$

which is exactly the same as in GR. All of the above solutions also satisfy the scalar field equation.

Much as with the dust case, these solutions strongly resemble GR, due to cancellations of the scalar field terms. If the scalars are constant during the radiation era, the gravitational constant may be shifted. This may have observable consequences on nucleosynthesis, for example. Deviations from exact Robertson-Walker behavior, caused by primordial black holes or density fluctuations, could also lead to regimes where the scalar fields play a more dynamic role.

3. *Cosmological constant era*

When the Universe is dominated by a cosmological constant, the matter is given by

$$\rho = \rho_0 = \text{const}, \quad P = -\rho . \tag{2.37}$$

Such a situation arises, for example, in the inflationary Universe [6], where ρ_0 is the energy density of a scalar field either trapped in a false minimum or slowly rolling down a potential. The (00) equation again is similar to (2.26), except that the matter side is now given by $8\pi\rho_0$, and F_d and G_d are replaced with the appropriate F_0 and G_0 in this regime. Examining the powers of time, there are three, namely, -2, $2q-2$, and 0. Unlike the previous matter conditions, where the right-hand side had p dependence, these three powers cannot in general be matched.[1] However, we may find approximate solutions

[1]There is one exception. An exact solution can be found if the terms of power $2q-2$ in the (00) equation cancel. In this case, Eq. (2.38) is again valid, and exponential solutions for the scale factor $a(t)$ and the ϕ are found. By using the (00) and ϕ equations, these conditions can all consistently be met if $G_0=-4F_0$ and $q=H(-3\pm\sqrt{3})/2$. However, this exact solution is really a special case of our first approximation, (2.38)–(2.41) when the terms ignored exactly cancel.

to the general case by assuming that one of the two terms dominates gravity.

If the fields are small, then the gravitational constant may dominate the field contribution, $1/\kappa^2 \gg F_{AB}\phi_A\phi_B$. Neglecting terms involving the ϕ then gives the standard GR equation

$$3H^2 = 8\pi\kappa^2\rho_0 \; , \tag{2.38}$$

which is solved by

$$a(t) = a_0 \exp\left[\left[\frac{8}{3}\pi\kappa^2\rho_0 \right]^{1/2} (t - t_0) \right] . \tag{2.39}$$

This exponential growth corresponds to an inflationary Universe. The scalar field equations give

$$6H^2 F_{AB}\phi_B - G_{AB}(\ddot{\phi}_B + 3H\dot{\phi}_B) = 0 \; . \tag{2.40}$$

Contracting with ϕ_A and solving the resulting equation gives

$$\phi_A = b_A e^{q(t-t_0)} \; , \qquad q = \frac{-3HG_0 \pm \sqrt{9H^2G_0^2 + 24H^2F_0G_0}}{2G_0} \; , \tag{2.41}$$

as a solution. Thus, for the small field approximation, the ϕ may grow, decay, or oscillate.

If the ϕ are large compared to the gravitational constant, then a power-law ansatz for the growth of the scale factor and fields gives for the (00) equation

$$\left[3p^2F_0 - \frac{q^2}{2}G_0 + 6pqF_0 \right] t^{2q-2} = 8\pi\rho_0 t_0^{2q} \; . \tag{2.42}$$

Equating powers of time demands $q = 1$, so the fields grow linearly, and remain dominant over the GR term. Combining the field equations with the time-independent part of the (00) equation gives two possibilities:

$$p = 0 \Longrightarrow G_0 = -16\pi\rho_0 t_0^2 \; , \qquad p = \frac{1}{2} + \frac{G_0}{4F_0} \Longrightarrow \frac{15}{4}F_0 + \frac{7}{4}G_0 + \frac{3}{16}\frac{G_0^2}{F_0} = 8\pi\rho_0 t_0^2 \; . \tag{2.43}$$

The first case gives a static Universe, while the second has the power of expansion depending on the parameters of the theory. For $p > 1$, the second could give rise to extended type inflation [7] if the phase transition is first order, or soft inflation [8] if the potential is slowly rolling of either the new- or chaotic-type.

An exact solution can be found if the terms of power $2q - 2$ in the (00) equation cancel. In this case, Eq. (2.38) is again valid, and exponential solutions for the scale factor $a(t)$ and the ϕ are found. By using the (00) and ϕ equations, these conditions can all consistently be met if $G_0 = -4F_0$ and $q = H(-3 \pm \sqrt{3})/2$. In fact, this exact solution is really a special case of our first approximation, when the terms ignored exactly cancel.

In the inflationary scenario, the potential of a scalar field acts as an effective cosmological constant to drive the expansion. For $G_0 > 2F_0$, Eq. (2.43) gives power-law inflation. If the potential is of the new [12] or chaotic [13] type, then this solution is a generalization of the soft inflationary scenario [8]. In soft inflation, modifications of Einstein gravity can remove the fine-tuning of potential parameters found in regular gravity. The same situation can arise here, with the added advantage of avoiding the solar system constraints.

More interesting is if the potential is of the old inflation [14] type, giving rise to a first-order phase transition. Simple nonminimal coupling allows the phase transition to complete [7]. However, to avoid an overproduction of big bubbles of true phase, which would produce an excess of microwave background anisotropy, requires parameters which violate the solar system constraint. Our theory can avoid this problem, and can do it with parameters which exactly reproduce GR in the solar system regime. Unfortunately, as shown in the next section, the models which reproduce GR are unstable in the dust era.

III. TWO FIELD CASE

Studying the N field case has the advantage not only of being general, but also of allowing compact notation. However, the interrelationship of parameters in these models can become obscured. We next consider the simpler case of a two field model to better elucidate such features, which include some stability problems. The action for two fields, in analogy with Eq. (2.1), is

$$S = \int d^4x \sqrt{-g}\,[f(\phi,\psi)R - \frac{\omega}{2}(\nabla\phi)^2 - \frac{\eta}{2}(\nabla\psi)^2 + 16\pi L_m] \; , \tag{3.1}$$

$$f(\phi,\psi) = \frac{1}{\kappa^2} + h_1\phi^2 + h_2\psi^2 + h_3\phi\psi \; . \tag{3.2}$$

A derivative cross term could also exist, but a proper rotation of the fields will eliminate this term, so we set it to 0 without loss of generality.

The solar system constraint, Eq. (2.15), now becomes

$$(h_3^2\omega + 4h_1^2\eta)\phi^2 + 4h_3(h_1\eta + h_2\omega)\phi\psi + (h_3^2\eta + 4h_2^2\omega)\psi^2 = 0 \; , \tag{3.3}$$

which may be rewritten as

$$\eta(h_3\psi + 2h_1\phi)^2 + \omega(h_3\phi + 2h_2\psi)^2 = 0 \; . \tag{3.4}$$

Therefore, ω and η must be opposite signs. We choose ω to be positive. Furthermore, by rescaling the fields in the original action, these kinetic constants may be chosen to be $\omega = 1$, $\eta = -1$, without any loss of generality. Thus, one important fact clearly shown in the two field case is that multiple scalar tensor theories must have negative kinetic terms if they are to agree with post-Newtonian tests of general relativity.

From the relation (3.4) one can also see the interplay between the kinetic terms ω and η, the coupling to gravity through the h_i, and the values of the fields themselves. Again, the asymptotic values of these fields are determined by cosmology, and specifically, by their evolution in the present dust era. We now study that evolution to derive further constraints on the parameters of the theory.

In a Robertson-Walker universe, the field equations become

$$(2h_1\phi+h_3\psi)(3\dot{H}+6H^2)-(\ddot{\phi}+3H\dot{\phi})=0 \ ,$$
$$(2h_2\psi+h_3\phi)(3\dot{H}+6H^2)+(\ddot{\psi}+3H\dot{\psi})=0 \ . \quad (3.5)$$

Again, we make the assumption of power-law behavior:

$$a(t)=a_c\left[\frac{t}{t_c}\right]^p, \quad \phi=b\left[\frac{t}{t_c}\right]^q, \quad \psi=c\left[\frac{t}{t_c}\right]^q . \quad (3.6)$$

The field equations (3.5) then combine to give

$$h_3=\frac{-2(h_1+h_2)bc}{b^2+c^2} . \quad (3.7)$$

Substituting this relationship back into the solar system constraint (3.4) implies

$$b=\pm c \ , \quad (3.8)$$

which also yields the relationship between the h_i:

$$h_3=\mp(h_1+h_2) \ . \quad (3.9)$$

These conditions also mandate that $f=1/\kappa^2$ in the dust era, as noted earlier for N fields. The (00) and trace equations for the two field case become

$$3H^2f+3H\dot{f}-\tfrac{1}{2}(\dot{\phi}^2-\dot{\psi}^2)=8\pi\rho \ , \quad (3.10)$$

$$(6\dot{H}+12H^2)f+3\ddot{f}+9H\dot{f}+\dot{\phi}^2-\dot{\psi}^2=8\pi(\rho-3P) \ , \quad (3.11)$$

respectively. As in the N field case, power-law solutions exist. In dust, the Universe will expand with power $\frac{2}{3}$ just as in GR, and the same relation for the energy density, (2.29), still holds. The power q at which the fields evolve is

$$q=\tfrac{1}{2}\{-1\pm[1-\tfrac{8}{3}(h_2-h_1)]^{1/2}\} \ . \quad (3.12)$$

The solutions for two fields in radiation and cosmological constant dominated universes also proceed in analogous fashion, and are easily obtainable from the more general N field case by substituting the appropriate expressions for F and G.

IV. STABILITY ANALYSIS

While the above solutions are exact, a crucial aspect of the system's behavior is whether these solutions are stable. Only exceptionally fine-tuned solutions will start with exactly the initial conditions to fit the form of the above solutions. Further, any slight physical fluctuation can move the system away from the exact solutions, and the solar system constraints will be violated. Only solutions which return to the exact behavior when deviating slightly will likely satisfy the observable tests of these theories. We therefore examine the evolution of initially small perturbations about our power-law solutions for the two field case in the current dust universe.

We start by writing the dust solutions for the case $b=c$, $h_3=-(h_1+h_2)$ as

$$a(t)=a_0\left[\frac{t}{t_0}\right]^{2/3}[1+\epsilon_1(t)] \ ,$$
$$\phi(t)=b\left[\frac{t}{t_0}\right]^q[1+\epsilon_2(t)] \ , \quad (4.1)$$
$$\psi(t)=c\left[\frac{t}{t_0}\right]^q[1+\epsilon_3(t)] \ ,$$

where $\epsilon_i \ll 1$ and q is given by Eq. (3.12) above. The scalar field equations then become

$$\left[\frac{4h_1}{3}-q(q+1)\right]\frac{\epsilon_2}{t^2}-2(q+1)\frac{\dot{\epsilon}_2}{t}-\ddot{\epsilon}_2-\frac{2(h_1+h_2)}{3}\frac{\epsilon_3}{t^2}+[8(h_1-h_2)-3q]\frac{\dot{\epsilon}_1}{t}+3(h_1-h_2)\ddot{\epsilon}_1=0 \ , \quad (4.2)$$

$$\left[-\frac{4h_2}{3}-q(q+1)\right]\frac{\epsilon_3}{t^2}-2(q+1)\frac{\dot{\epsilon}_3}{t}-\ddot{\epsilon}_3+\frac{2(h_1+h_2)}{3}\frac{\epsilon_2}{t^2}+[8(h_1-h_2)-3q]\frac{\dot{\epsilon}_1}{t}+3(h_1-h_2)\ddot{\epsilon}_1=0 \ , \quad (4.3)$$

while the trace and (00) equations give

$$\left[\left[12q^2+6q+\frac{4}{3}\right](h_1-h_2)+2q^2\right]\frac{\epsilon_2-\epsilon_3}{t^2}+[6(2q+1)(h_1-h_2)+2q]\frac{\dot{\epsilon}_2-\dot{\epsilon}_3}{t}$$
$$+3(h_1-h_2)(\ddot{\epsilon}_2-\ddot{\epsilon}_3)+\frac{1}{b^2\kappa^2}\left[\frac{t}{t_0}\right]^{-2q}\left[\frac{4\epsilon_1}{t^2}+\frac{16\dot{\epsilon}_1}{t}+6\ddot{\epsilon}_1\right]=0 \ , \quad (4.4)$$

$$(6q^3+7q^2+2q)\frac{\epsilon_2-\epsilon_3}{t}+(3q^2+2q)(\dot{\epsilon}_2-\dot{\epsilon}_3)+\frac{1}{b^2\kappa^2}\left[\frac{t}{t_0}\right]^{-2q}\left[\frac{4\epsilon_1}{t}+4\dot{\epsilon}_1\right]=0 \ . \quad (4.5)$$

First consider the case of GR, with $\phi=\psi=0$. The trace equation then gives

$$\frac{2}{t^2}\epsilon_1+\frac{8}{t}\dot{\epsilon}_1+3\ddot{\epsilon}_1=0 \ . \quad (4.6)$$

This equation is solved by

$$\epsilon_1 = \frac{A_1}{t} + \frac{A_2}{t^{2/3}} , \tag{4.7}$$

with A_1 and A_2 constant. The perturbation to the scale factor decays in time, and hence GR is stable, as expected.

Now, consider the perturbations of the scalar fields. Subtracting one scalar field equation from the other gives

$$\ddot{\epsilon}_{23} + \frac{2}{t}(q+1)\dot{\epsilon}_{23} = 0 , \tag{4.8}$$

where $\epsilon_{23} \equiv \epsilon_2 - \epsilon_3$ and Eq. (3.12) was used to remove the ϵ_{23} term. This equation is solved by

$$\begin{aligned} &\epsilon_{23} = Bt^{-2q-1} = Bt^{\pm\sqrt{1-(8/3)(h_2-h_1)}}, \quad q \neq -\tfrac{1}{2} , \\ &\epsilon_{23} = B \ln t, \quad q = -\tfrac{1}{2} , \end{aligned} \tag{4.9}$$

with B a constant. For the $q = -\frac{1}{2}$ case the perturbation grows logarithmically, and the scalar fields are unstable. When $q \neq -\frac{1}{2}$ there are two roots, one of which grows in time with positive power. In general, the scalar fields will pick up a combination of these two roots, with coefficients depending on initial conditions in the dust era. Thus, the perturbation ϵ_{23} will generally have a component which grows in time, and hence the scalar fields are unstable.

That the solutions are unstable is not a surprise, for one of the fields has a negative kinetic term. Such kinetic terms cause instability by allowing for infinitely many negative energy states when the system is quantized. They also can lead to the classical instability observed here. In addition, our power-law solutions include the case where the fields are zero far from the solar system, so that these values too will be unstable to perturbations as the Universe evolves. Hence, while multiple ST theories can satisfy the solar system constraints, the requisite values will be unstable, and hence are unlikely to actually occur.

There are possible modifications to evade this instability. One of the motivations for multiple ST theories is that they arise in string theories and other models which attempt to incorporate gravity with the other forces. The action used here may be just an approximation of the full theory, whose other terms stabilize the evolution. Or, on a classical level, a mass or potential term for the scalars could freeze the ϕ at some minimum, thus preventing the runaway growth of small perturbations. Any such modification would have to be examined in more detail to see if stability arises. However, given the problems associated with negative kinetic terms, multiple ST theories that are consistent with GR in the weak-field limit seem to be unstable.

V. CONCLUSION

Multiple scalar ST theories offer the possibility of providing a new theoretical framework in which to test GR. Because such theories can mimic GR in the solar system, where regular BD theories are severely limited, but yield substantially different results in other regimes, they are especially valuable. Our results complement previous work [3], which used values constant in time for the scalar field far from massive compact objects. The authors of this previous work also note the need for negative kinetic terms, and point out the potential for problems with quantum and stellar stability. We have found still another problem. In theories like this, with negative kinetic energy terms, the cosmological solutions which meet the solar system constraints are unstable as they evolve in time, and thus cannot be maintained.

Our time evolving solutions for the scalars are consistent with both solar system tests of GR and the cosmological field equations. If these solutions can be stabilized by some means such as an explicit potential term or a more complete theory of gravity, then they will be a useful probe of GR. However, without such stabilization, the only way to make these theories agree with GR in the solar system is to choose constants that make the theory approximate GR not only in the post-Newtonian approximation, but also in other regimes. This situation is in analogy with the single scalar JBD theory, and going to multiple scalars would have no advantage.

Finally, we note that our solution in the cosmological constant dominated regime exhibits power-law growth when the fields dominate, as seen in Eq. (2.43). For power greater than 1, this yields inflation. The extended inflation scenario [7] used the JBD theory to create power-law inflation, in which case the first order phase driving inflation would eventually complete, in contrast with the GR case. However, the JBD theory was inadequate, because values of the JBD constant which met the solar system constraint produced too many big bubbles, and thus an inhomogeneous Universe [10]. One might hope to use multiple scalar ST theories to give both power-law inflation as well as meeting both the solar system constraint and the big bubble constraint. However, this scenario will not likely occur, as the dust values of the fields are unstable. Should a stabilizing mechanism be found, then further investigation of the inflationary scenario in these models would be warranted.

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