False vacuum inflation with Einstein gravity

Edmund J. Copeland and Andrew R. Liddle

School of Mathematical and Physical Sciences, University of Sussex, Falmer, Brighton BN1 9QH, United Kingdom

David H. Lyth

School of Physics and Materials, Lancaster University, Lancaster LA1 4YB, United Kingdom

Ewan D. Stewart

Department of Physics, Kyoto University, Kyoto 606, Japan

David Wands

School of Mathematical and Physical Sciences, University of Sussex, Falmer, Brighton BN1 9QH, United Kingdom

We present a detailed investigation of chaotic inflation models which feature two scalar fields such that one field (the inflaton) rolls while the other is trapped in a false vacuum state. The false vacuum becomes unstable when the magnitude of the inflaton field falls below some critical value, and a first- or second-order transition to the true vacuum ensues. Particular attention is paid to the case termed "hybrid inflation" by Linde, where the false vacuum energy density dominates so that the phase transition signals the end of inflation. We focus mostly on the case of a second-order transition, but treat also the first-order case and discuss bubble production in that context for the first time. False-vacuum-dominated inflation is dramatically different from the usual true vacuum case, both in its cosmology and in its relation to particle physics. The spectral index of the adiabatic density perturbation originating during inflation can be indistinguishable from 1, or it can be up to ten percent or so higher. The energy scale at the end of inflation can be many orders of magnitude less than the value 10^{16} GeV, which is usual in the true vacuum case. Reheating occurs promptly at the end of inflation. Cosmic strings or other topological defects are almost inevitably produced at the end of inflation, and if the inflationary energy scale is near its upper limit they contribute significantly to large scale structure formation and the cosmic microwave background anisotropy. Turning to particle physics, false vacuum inflation occurs with the inflaton field far below the Planck scale and is therefore somewhat easier to implement in the context of supergravity than true vacuum chaotic inflation. The smallness of the inflaton mass compared with the inflationary Hubble parameter still presents a difficulty for generic supergravity theories. Remarkably, however, the difficulty can be avoided in a natural way for a class of supergravity models that follow from orbifold compactification of superstrings. This opens up the prospect of a truly realistic superstring-derived theory of inflation. One possibility, which we show to be viable at least in the context of global supersymmetry, is that the Peccei-Quinn symmetry is responsible for the false vacuum.

PACS number(s): 98.80.Cq, 04.50.+h

I. INTRODUCTION

An attractive proposal concerning the first moments of the observable Universe is that of chaotic inflation [1]. At some initial epoch, presumably the Planck scale, the various scalar fields existing in nature are roughly homogeneous and dominate the energy density. Their initial values are random, subject to the constraint that the energy density is at the Planck scale. Among them is the inflaton field ϕ, which is distinguished from the noninflaton fields by the fact that the potential is relatively flat in its direction. Before the inflaton field ϕ has had time to change much, the noninflaton fields quickly settle down to their minimum at fixed ϕ, after which inflation occurs as ϕ rolls slowly down the potential.

Two possibilities exist concerning the minimum into which the noninflaton fields fall. The simplest possibility is that it corresponds to the true vacuum; that is, the noninflaton fields have the same values as in the present Universe. Inflation then ends when the inflaton field starts to execute decaying oscillations around its own vacuum value, and the hot big bang ("reheating") ensues when the vacuum value has been achieved and the decay products have thermalized. This is the usually considered case, which has been widely explored. The other possibility is that the minimum corresponds to a false vacuum, with nonzero energy density. This case may be called *false vacuum inflation*, and is the subject of the present paper.

There are two fundamentally different kinds of false vacuum inflation, according to whether the energy density is dominated by the false vacuum energy density or by the potential energy of the inflaton field. (For simplicity we discount for the moment the intermediate possibility that the two contributions are comparable, though it will be dealt with in the body of the paper.) In all cases the false vacuum exists only when the value of the inflaton field is above some critical value. If the false vacuum

energy dominates, a phase transition occurs promptly when the inflaton field falls below the critical value, causing the end of inflation and prompt reheating. The result is a new model of inflation which is dramatically different from the usual one, and at least as attractive. It was first studied by Linde who termed it "hybrid inflation," and it is the main focus of the present paper. The phase transition may be of either first or second order. A first-order model of false-vacuum-dominated inflation has been considered by Linde [2] and (with minor differences but more thoroughly) by Adams and Freese [3]. A second-order model has been discussed by Linde [4,5] and explored in a preliminary way by Liddle and Lyth [6] and by Mollerach, Matarrese, and Lucchin [7]. As far as we know, these are the only references in the literature to false-vacuum-dominated inflation with Einstein gravity. Related models have been considered at some length in the context of extended gravity theories [8–10]; although such theories can be recast as Einstein gravity theories by a conformal transformation, the resulting potentials are of a different type and this case is excluded from the present paper.

The opposite case where the false vacuum energy is negligible (inflaton domination) is indistiguishable from the true vacuum case for couplings of order unity, though a variety of exotic effects can occur for small couplings. This case has been studied by several authors [11–19], and in the present paper it is treated fairly briefly.

From the viewpoint of cosmology, false-vacuum-dominated inflation differs from the usual true vacuum case in three important respects.

(1) The spectral index n of the adiabatic density perturbation is typically very close to the scale invariant value 1, and is in any case greater than 1. This is in contrast with other working models of inflation, where one typically finds $n < 1$, viable models covering a range from perhaps $n \simeq 0.7$ up to $n \simeq 1$ [6]. We shall, however, note that the extent to which n can exceed unity is quite limited, contrary to the claim in Ref. [7].

(2) Topological defects generally form at the *end* of inflation, in accordance with the homotopy groups of the breaking of the false vacuum to degenerate states, provided that these groups exist. The defects may be of any type (domain walls, gauge or global strings, gauge or global monopoles, textures or nontopological textures).

(3) The energy scale at the end of inflation can be far below the value $\sim 10^{16}$ GeV which is usual in true vacuum inflation. On the other hand, reheating occurs promptly at the end of inflation so that the reheat temperature T_R is of order the inflationary energy scale, again in contrast with the true vacuum case. The well known bound $T_R \lesssim 10^{11}$ GeV following from the existence of a gravitino with standard properties [20] would therefore require inflation to end below this scale, unless the gravitinos are diluted later by a decaying particle dumping entropy.[1]

---

[1]We are indebted to Subir Sarkar for valuable correspondence about the gravitino problem.

False-vacuum-dominated inflation is also very different from the true vacuum case from the viewpoint of particle physics. Sticking to the chaotic inflation scenario already described, let us consider as a specific example the inflationary potential

$$V(\phi) = V_0 + \tfrac{1}{2}m^2\phi^2 + \tfrac{1}{4}\lambda\phi^4, \qquad (1.1)$$

where V_0 is the false vacuum energy density. Consider first the true vacuum case, where V_0 vanishes. Inflation occurs while ϕ rolls slowly towards zero, and it ends when ϕ begins to oscillate, which occurs when ϕ is of order the Planck mass. In order to have sufficiently small cosmic microwave background (CMB) anisotropy, one needs $m \lesssim 10^{13}$ GeV and $\lambda \lesssim 10^{-12}$, with one or conceivably both of these limits saturated if inflation is to actually generate the observed anisotropy (and a primeval density perturbation leading to structure formation). To achieve the small λ in a natural way one should invoke supersymmetry. As long as one sticks to global supersymmetry this presents no problem, but there are sound particle physics reasons for invoking instead local supersymmetry, which is termed supergravity because it automatically includes gravity. In the context of supergravity, the fact that ϕ is of order the Planck mass during inflation is problematical, because in this regime it is difficult to arrange for a sufficiently flat potential.

As will become clear, things are very different in the false vacuum case. One still needs to have λ very small, and will still therefore wish to implement inflation in the context of supergravity. But now ϕ is far below the Planck scale during inflation (after the observable Universe leaves the horizon which is the cosmologically interesting era). As a result it becomes easier to construct a viable model of inflation, though the smallness of m in relation to the inflationary Hubble scale H still presents a severe problem for generic supergravity theories. Remarkably, though, it turns out that among the class of supergravity models emerging from orbifold compactifications of superstring theory, one can find a large subset for which this problem disappears. As a toy model, we will see how things work out with a specific choice for the perturbative part of the superpotential.

Another crucial difference concerns the mass m. The CMB anisotropy now determines not m, but a relation between m and M which is very roughly $M/10^{12}$ GeV $\sim$ $(m/1\,\text{TeV})^{2/5}$. By making m very low one can make M and therefore the inflationary energy scale very low. It is usually assumed that the lowest reasonable scale for the mass of a scalar field in our vacuum is $m \sim 100$ GeV, because this is the soft supersymmetry-breaking scale. However, it is not clear that this constraint should also be applied to scalar fields during inflation which corresponds to a different vacuum state.

In the superstring motivated models, we find that a high value of m probably obtains if the slope of the inflationary potential is dominated by one-loop corrections coming from the Green-Schwarz mechanism, whereas a low value can occur if these corrections are negligible. In the former case the spectral index n of the primeval curvature perturbation is typically around 1.05, the pre-

cise value being determined by the orbifold. A measured value in this range might therefore provide a window onto superstring physics.

A different possibility is that ϕ acquires its mass during inflation in the same way and at the same scale ($m \sim 100\,\text{GeV}$) as the superpartners of the standard model fermions do in our vacuum. In that case the corresponding false vacuum energy scale $V_0^{1/4} \sim 10^{11}\,\text{GeV}$ might be the one associated with Peccei-Quinn symmetry, a global U(1) symmetry which is perhaps the most promising explanation for the observed CP invariance of the strong interaction. This same symmetry provides the axion, which is one of the leading dark matter candidates, and the possibility that it might in addition provide the false vacuum for inflation is to say the least interesting. We explore this possibility in the context of global supersymmetry and find that it can easily be realized there. We have not gone on to explore it in the context of supergravity, but there seems to be no reason why it should not be realized within the context of the superstring-derived models considered earlier.

As will be clear from this Introduction, the present work is expected to be of interest to a very wide audience, ranging from observational astronomers to superstring theorists. With this in mind we have tried to keep separate the part of the paper that discusses the phenomenology of the false vacuum inflation models, and the part that relates these models to particle physics.

The outline of the paper is as follows. Section II introduces the specific second-order model upon which most of our discussion will be focused. We analyze the inflationary dynamics and density perturbation constraints by a combination of analytic and numerical methods to delineate the observationally viable models. Section III then takes our attention onto the formation of topological defects, which (almost) inevitably form at the end of inflation. Their possible existence constrains the models, and there is the further opportunity of a reconciliation of structure-forming defects with inflation. In Sec. IV we try to realize the model in the contexts of global supersymmetry, supergravity, and superstring-derived supergravity. In Sec. V we consider the related first-order model which also indicates the link with extended inflation models. Section VI summarizes the paper.

II. INFLATIONARY PHENOMENOLOGY

A. The model

Throughout this paper we assume Einstein gravity. During inflation the energy density is supposed to be dominated by the potential of two scalar fields, which is taken to be of the form

$$V(\phi, \psi) = \tfrac{1}{4}\lambda \left(\psi^2 - M^2\right)^2 + \tfrac{1}{2}m^2\phi^2 + \tfrac{1}{2}\lambda'\phi^2\psi^2 . \quad (2.1)$$

This potential possesses the symmetries $\phi \leftrightarrow -\phi$ and $\psi \leftrightarrow -\psi$, and is the most general renormalizable potential with this property except for a quartic term

$\lambda''\phi^4$.[2] We make the restrictions $0 < \lambda,\ \lambda' \lesssim 1$, and $M \lesssim m_{\text{Pl}}/\sqrt{8\pi} = 2.4 \times 10^{18}\,\text{GeV}$. Depending on the particle physics context of the model, there might also be the restriction $m \gtrsim 100\,\text{GeV}$ corresponding to the soft supersymmetry-breaking scale of our vacuum.

Provided that $\phi^2 > \phi_{\text{inst}}^2$, where

$$\phi_{\text{inst}}^2 = \lambda M^2/\lambda' , \quad (2.2)$$

there is a local minimum at $\psi = 0$ on the constant ϕ slices, corresponding to a false vacuum. Our assumption is that inflation occurs with the ψ field located in this false vacuum, so that the potential is

$$V(\phi) = \tfrac{1}{4}\lambda M^4 + \tfrac{1}{2}m^2\phi^2 . \quad (2.3)$$

If the false vacuum dominates, inflation ends when ϕ falls below ϕ_{inst}, the fields rapidly adjusting to their true vacuum values $\psi = M$ and $\phi = 0$.

This model was first considered by Kofman and Linde [11], who pointed out that it might produce cosmic strings with enough energy per unit length to form structure. They considered only what we shall term the inflaton-dominated regime (small false vacuum energy), as did several subsequent authors studying this and related models [12–19]. In order to obtain interesting effects, these authors had to assume (at least) that the coupling λ' was many orders of magnitude less than unity. The case of false vacuum domination, which is our main focus, was proposed by Linde who termed it "hybrid inflation," [4] and has received further attention from Liddle and Lyth [6], Linde [5], and Mollerach, Matarrese, and Lucchin [7]. In this case the couplings can be of order unity, but for completeness we explore also the regime of parameter space where they are very different from 1.

B. Inflationary dynamics

As usual, the inflationary dynamics are governed by the equations

$$H^2 = \frac{8\pi}{3m_{\text{Pl}}^2}\left[\tfrac{1}{2}\dot{\phi}^2 + \tfrac{1}{2}\dot{\psi}^2 + V(\phi,\psi)\right] , \quad (2.4)$$

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{\partial V(\phi,\psi)}{\partial\phi} , \quad (2.5)$$

$$\ddot{\psi} + 3H\dot{\psi} = -\frac{\partial V(\phi,\psi)}{\partial\psi} , \quad (2.6)$$

for two isotropic scalar fields in an expanding universe, with $H = \dot{a}/a$ the Hubble parameter, a the scale factor, m_{Pl} the Planck mass, and dots derivatives with respect to

---

[2]The pure quadratic term is the simplest possibility, and is also the one favored by particle physics considerations (Sec. IV). Nonrenormalizable potentials, involving higher powers of the fields, arise naturally in the context of supergravity, but for simplicity we ignore them here. As we discuss later the ψ field can have several components, but they do not affect the issues we discuss in the present section.

time. Our assumption is that there is a transitory regime during which the ψ field rolls to $\psi = 0$ from whatever its initial value may have been, and is followed by sufficient inflation on the $\psi = 0$ trajectory to erase any evidence of such a transient. Inflation then proceeds according to the usual single field equation for ϕ in the potential of Eq. (2.3). Without loss of generality, we shall assume that ϕ is initially positive.

We shall utilize the slow-roll approximation throughout. It is characterized by the conditions

$$\epsilon \ll 1, \quad |\eta| \ll 1, \qquad (2.7)$$

where the two dimensionless functions $\epsilon(\phi)$ and $\eta(\phi)$ are defined by

$$\epsilon(\phi) \equiv \frac{m_{\rm Pl}^2}{16\pi} \left(\frac{V'(\phi)}{V(\phi)} \right)^2, \qquad (2.8)$$

$$\eta(\phi) \equiv \frac{m_{\rm Pl}^2}{8\pi} \frac{V''(\phi)}{V(\phi)}. \qquad (2.9)$$

Here and throughout primes indicate derivatives with respect to the field ϕ. With justification from numerical results, it is standard to assume that if the potential satisfies these conditions, then the solutions for a broad range of initial conditions rapidly approach the attractor

$$3H\dot{\phi} \simeq -V'. \qquad (2.10)$$

When this is satisfied, there exists a simple expression for the number N of e-foldings of expansion which occur between two scalar field values ϕ_1 and ϕ_2:

$$N(\phi_1, \phi_2) \equiv \ln \frac{a_2}{a_1} \simeq -\frac{8\pi}{m_{\rm Pl}^2} \int_{\phi_1}^{\phi_2} \frac{V}{V'} \, d\phi. \qquad (2.11)$$

For our specific potential we have

$$\eta = \frac{m^2 m_{\rm Pl}^2}{2\pi \left(\lambda M^4 + 2m^2\phi^2 \right)}, \qquad (2.12)$$

$$\epsilon = \frac{m_{\rm Pl}^2 m^4 \phi^2}{\pi \left(\lambda M^4 + 2m^2\phi^2 \right)^2} = \frac{1}{2} \frac{8\pi}{m_{\rm Pl}^2} \eta^2 \phi^2, \quad (2.13)$$

$$N(\phi_1, \phi_2) = \frac{2\pi \lambda M^4}{m^2 m_{\rm Pl}^2} \ln \frac{\phi_1}{\phi_2} + \frac{2\pi}{m_{\rm Pl}^2} \left(\phi_1^2 - \phi_2^2 \right). \qquad (2.14)$$

Within the slow-roll approximation, the condition for inflation to occur is simply that ϵ be less than 1. However, slow roll is automatically a poor approximation should ϵ reach this value, though the amount of inflation that occurs as ϵ becomes large is always small. Numerical simulation indicates that for this potential, if ϵ and η grow to unity, shortly thereafter the inflationary condition $\ddot{a} > 0$ is violated and inflation ends. The number of e-foldings that occurs between these events is a tiny fraction of unity, and can be ignored. It is therefore sensible operationally to identify the end of inflation in this case with the precise condition that $\epsilon = 1$, should this occur, and we shall assume this subsequently.

There are therefore two separate ways in which inflation may end in this model, the one which is applicable

depending on the parameter values. These are as follows.

(1) If ϕ reaches $\phi_{\rm inst}$ while inflation is occurring, then inflation may end through the instability of the ψ field to roll to its global minimum. As noted by Linde [5] one expects this to happen, at least for λ and λ' not too small, if the false vacuum term $\lambda M^4/4$ dominates the potential. We look in some detail at this possibility in Sec. III, confirming the picture of rapid instability.

(2) If the logarithmic slope of the potential becomes too large *on the $\psi = 0$ trajectory*, then inflation can end while the ϕ field is still rolling down that trajectory. This is symptomized by ϵ growing to exceed unity. Some time later, ϕ will pass $\phi_{\rm inst}$ and the ψ field may roll away from $\psi = 0$.

The value of ϕ at which ϵ becomes equal to unity is[3]

$$\phi_\epsilon = \frac{m_{\rm Pl}}{\sqrt{16\pi}} \left(1 + \sqrt{1 - \frac{8\pi}{m_{\rm Pl}^2} \frac{\lambda M^4}{m^2}} \right). \qquad (2.15)$$

If $8\pi \lambda M^4 / m_{\rm Pl}^2 m^2 > 1$, then ϕ_ϵ does not exist at all. In that case inflation must end by instability. In the opposite limit, the position $\phi_\epsilon \to m_{\rm Pl}/\sqrt{4\pi}$ is familiar from chaotic inflation with a single field, and of course the standard results will be recovered in that limit with the ψ field playing no significant role.

We need to know the number $N(k)$ of Hubble times of inflation which occur after a given scale leaves the horizon.[4] With the assumptions (valid in our model) that H does not vary significantly and that reheating is prompt, it is given by [6]

$$N(k) = 62 - \ln \frac{10^{16} \, {\rm GeV}}{V_1^{1/4}} - \ln \frac{k}{a_0 H_0}, \qquad (2.16)$$

where subscript 0 indicates present value. The largest cosmologically interesting scale is of order the present Hubble distance (roughly the size of the observable universe), $k = a_0 H_0$, and other scales of cosmological interest leave the horizon at most a few Hubble times after this one. As for the inflationary energy scale, true vacuum inflation typically gives $V_1^{1/4} \sim 10^{16}$ GeV, which makes the observable universe leave the horizon about 60 e-folds before the end of inflation (the fact that reheating may be very inefficient in this model may reduce this number somewhat). As we shall see, false-vacuum-dominated inflation can give values as low as $V_1^{1/4} \sim 10^{11}$ GeV, which reduces the figure 60 to about 50. However, one only needs a rough estimate of N for most purposes because the potential is slowly varying, and for simplicity we suppose from now on that cosmologically interesting scales leave the horizon 60 e-folds before the end of inflation.

----

[3]There is also a second root at smaller ϕ, where ϵ drops back below unity. However, for second-order models it is easy to show that the attractor solution Eq. (2.10) cannot be attained for ϕ below this root, allowing inflation to restart, before the instability sets in.

[4]As usual we say that a comoving scale a/k leaves the horizon when $aH/k = 1$.

Provided the parameters are chosen in such a way as to produce the correct level of density perturbations to explain the Cosmic Background Explorer (COBE) satellite observations [21] of the CMB anisotropies, there is no problem in obtaining sufficient inflation to resolve the horizon and flatness problems or with ensuring that a classical description of the evolution is adequate. We thus need only investigate the density perturbation constraint in order to completely fix the model.

C. Density perturbations

The adiabatic density perturbation, which is generally thought to be responsible for large scale structure, originates as a vacuum fluctuation during inflation. Its spectrum is determined by a quantity δ_H, which loosely speaking gives the density contrast at horizon crossing and is defined formally in [6]. The inflationary prediction for the spectrum is

$$\delta_H^2(k) = \frac{32}{75} \frac{V_*}{m_{\text{Pl}}^4} \frac{1}{\epsilon_*} , \qquad (2.17)$$

where ϵ is the slow-roll parameter defined earlier and the subscript $*$ indicates that the right hand side is to be evaluated as the comoving scale k equals the Hubble radius ($k = aH$) during inflation.

By virtue of the slow-roll conditions Eqs. (2.7)–(2.10), this formula gives a value of $\delta_H(k)$ which is nearly independent of k on scales of cosmological interest, in agreement with observation. For a sufficiently flat spectrum, and provided that no significant generation of long wavelength gravitational wave modes occurs, the central value of the COBE $10°$ anisotropy, $30\ \mu K$, is reproduced provided one has $\delta_H = 1.7 \times 10^{-5}$ [6].[5] Thus the inflationary energy scale when cosmologically interesting scales leave the horizon is given by

$$V_{60}^{1/4} = 6\epsilon_{60}^{1/4} \times 10^{16}\ \text{GeV} , \qquad (2.18)$$

where a subscript 60 denotes 60 e-folds before the end of inflation.

The most efficient way to proceed is as follows. First, fix the couplings λ and λ'. Then, having chosen a value for the mass scale M, find the value(s) of m such that the density perturbation constraint is satisfied. Assuming that inflation ends promptly if ϕ falls below ϕ_{inst}, we can determine the means by which inflation ends and the

---

[5]This figure assumes a Gaussian beam profile, and is raised by 16% if the precise profile of the experiment is used and a correction applied for the incomplete sky coverage inducing errors in the monopole and dipole subtractions [22]. For an accurate analysis one has to include (here and elsewhere) the effect of spectral tilt and gravitational waves on the COBE normalization [23]. Such changes are not significant in the present context except for extreme parameter values, and for simplicity we shall not include them in the normalization though we shall discuss tilt and gravitational waves later.

corresponding value of ϕ:

$$\phi_{\text{end}} = \max\{\phi_\epsilon, \phi_{\text{inst}}\} . \qquad (2.19)$$

We then use Eq. (2.14) to determine the value of ϕ 60 e-foldings from the end of inflation, ϕ_{60}, and evaluate δ_H as

$$\delta_H^2 = \frac{8\pi}{75} \frac{\left(\lambda M^4 + 2m^2\phi_{60}^2\right)^3}{m_{\text{Pl}}^6 m^4 \phi_{60}^2} . \qquad (2.20)$$

To find the value(s) of m which satisfy the COBE normalization, remember that ϕ_{end}, and hence ϕ_{60}, is a function of m. In general this procedure cannot be carried out analytically, and we compute using an iterative numerical method. However, the problem can be solved analytically and self-consistently in two regimes. As we shall see, provided M is not too large then for each M there are *two* possible choices of m which give the right perturbation amplitude. One corresponds to the traditional polynomial chaotic inflation scenario, where the first term in Eq. (2.3) plays a negligible role [and by implication the first term in the numerator of Eq. (2.20) likewise]. (There is a variant on this regime, also discussed below, where the instability sets in while the false vacuum energy is still negligible.) The second, and for our purposes more interesting, possibility involves a value of $m \ll M$, and corresponds to domination by the first term in Eq. (2.3).

D. Delineating parameter space

We shall now examine different analytic and numerical regimes. The results are concisely summarized in Fig. 1.

The inflaton-dominated regime

The simplest scenario of all is one in which the ψ field plays no role whatsoever, leaving just the ϕ field to govern inflation. The potential $V = m^2\phi^2/2$ was proposed by Linde [1] as a simple realization of chaotic inflation. With this potential, inflation ends when ϕ starts to oscillate around its minimum, which occurs when $\epsilon \simeq 1$ corresponding to

$$\phi_{\text{end}} \simeq m_{\text{Pl}}/\sqrt{4\pi} . \qquad (2.21)$$

The condition that our potential Eq. (2.3) be a good approximation to this one is therefore

$$\frac{8\pi}{m_{\text{Pl}}^2} \frac{\lambda M^4}{4m^2} \ll 1 . \qquad (2.22)$$

In Eq. (2.14) the second term dominates, giving

$$\phi_{60} \simeq \sqrt{\frac{60}{2\pi}}\ m_{\text{Pl}} . \qquad (2.23)$$

Note that in this regime the characteristic scale of ϕ_{60} is the Planck scale. The density perturbation amplitude is independent of M, λ, and λ' in this limit, and the correct
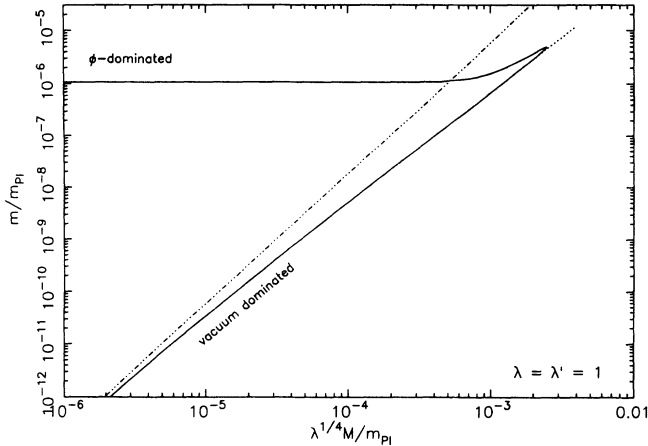
FIG. 1. The solid line shows the loci of M and m which satisfy the COBE normalization for $\lambda = \lambda' = 1$. The two analytic branches are clearly seen. The dot-dashed line indicates the analytic solution for the extreme vacuum dominated branch, as utilized by Linde [5]; the deviation of the exact solution from it is caused by the increasing significance of the exponential term in Eq. (2.37), which is included in the parametric analytic solution Eq. (2.36), as used in [7] and indicated here by the dotted line. The dotted line (hidden under the solid for most of its length) terminates when the regime becomes invalid, though it actually extends somewhat beyond the exact solution because we have interpreted $\ll$ as $\lesssim$ in places.

value is obtained with

$$\frac{\sqrt{8\pi}}{m_{\rm Pl}}m = \frac{\pi}{4\sqrt{6}}\delta_H = 5.5 \times 10^{-6}. \tag{2.24}$$

The condition for the validity of this approximation is therefore

$$\frac{\sqrt{8\pi}}{m_{\rm Pl}}\lambda^{1/4}M \ll 3 \times 10^{-3}. \tag{2.25}$$

The above analysis assumes that $\phi > \phi_{\rm inst}$ throughout inflation, which from Eq. (2.21) fails if $\phi_{\rm inst} \gtrsim m_{\rm Pl}/\sqrt{4\pi}$, or equivalently

$$\frac{\lambda'^2}{\lambda} \lesssim \frac{1}{4}\lambda M^4 \left(\frac{8\pi}{m_{\rm Pl}^2}\right)^2 \lesssim 10^{-11}. \tag{2.26}$$

[The final inequality is Eq. (2.25).] If ϕ falls below $\phi_{\rm inst}$, then as discussed in Sec. III A ψ may roll towards its minimum at fixed ϕ,

$$\psi_{\rm vac}^2(\phi) = M^2 \left(1 - \frac{\phi^2}{\phi_{\rm inst}^2}\right). \tag{2.27}$$

It oscillates around the minimum, losing energy through the expansion of the universe so that after a few Hubble times $\psi \simeq \psi_{\rm vac}$ (if its spatial gradient is not negligible it may settle down more quickly through thermalization.) Inserting $\psi = \psi_{\rm vac}$ into Eq. (2.1) gives [24]

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4}M^4 \left[1 - \left(1 - \frac{\phi^2}{\phi_{\rm inst}^2}\right)^2\right], \tag{2.28}$$

$$V'(\phi) = m^2\phi \left[1 + \frac{\lambda M^4}{m^2\phi_{\rm inst}^2}\left(1 - \frac{\phi^2}{\phi_{\rm inst}^2}\right)\right], \tag{2.29}$$

$$V''(\phi) = m^2 \left[1 + \frac{\lambda M^4}{m^2\phi_{\rm inst}^2}\left(1 - 3\frac{\phi^2}{\phi_{\rm inst}^2}\right)\right]. \tag{2.30}$$

Since we are in the regime $\phi_{\rm inst} \gtrsim m_{\rm Pl}/\sqrt{8\pi}$, the condition Eq. (2.22) written down earlier guarantees again that the modification to V will be negligible.

It therefore appears that, when Eq. (2.22) is well satisfied, the evolution of ϕ will not be significantly affected even if ϕ falls below $\phi_{\rm inst}$. If Eq. (2.22) is only marginally satisfied, the evolution of ϕ might be substantially altered, leading to a significant change in the predicted adiabatic density perturbation. The simplest assumption is that the potential is given by Eq. (2.28). In that case, if one ignores the inhomogeneity of ϕ caused by the phase transition, the perturbation is still given by the usual formula Eq. (2.17), with the new potential [24]. However, this formula depends crucially on the assumption that each Fourier mode of ϕ is in the vacuum state before leaving the horizon, whereas the phase transition will inevitably populate some of the modes with nonzero particle number. Taking this into account, the adiabatic perturbation on scales leaving the horizon after $\phi = \phi_{\rm inst}$ might be quite different (non-Gaussian, with a nonflat spectrum and a different normalization). An additional adiabatic perturbation might also be generated by the perturbation in ψ [13,15,16], as discussed in Sec. III A.

The vacuum-dominated regime

We now explore the opposite regime, where the vacuum energy density notionally associated with the ψ field dominates the potential. Special cases of this regime of parameter space have already been considered in [4]– [7].

As noted earlier, inflation is expected to occur only if η and ϵ are small compared with unity. The first of these parameters is independent of ϕ in the limit of vacuum domination, with the value

$$\eta = \frac{m_{\rm Pl}^2}{8\pi}\frac{4m^2}{\lambda M^4}. \tag{2.31}$$

Thus, the requirement that $\eta \lesssim 1$ in this regime is precisely the opposite of the condition Eq. (2.22) which characterizes the regime in which the vacuum does not dominate. The parameter ϵ decreases as inflation proceeds, and during the era $\phi < \phi_{60}$ that we are interested in we have

$$\epsilon < \epsilon_{60} = \frac{1}{2}\frac{8\pi}{m_{\rm Pl}^2}\eta^2\phi_{60}^2. \tag{2.32}$$

The condition for vacuum domination is

$$\frac{1}{2}\frac{8\pi}{m_{\rm Pl}^2}\phi_{60}^2\eta \ll 1, \tag{2.33}$$

or equivalently

$$\epsilon_{60} \ll \eta \, . \tag{2.34}$$

The first term of Eq. (2.14) dominates the formula for ϕ_{60}, giving

$$\phi_{60}^2 = \frac{\lambda}{\lambda'} M^2 e^{120\eta} \, . \tag{2.35}$$

The COBE normalization Eq. (2.20) is therefore

$$\frac{\sqrt{8\pi}}{m_{\text{Pl}}} \sqrt{\lambda'} M = 10\sqrt{3}\pi \delta_H \eta e^{60\eta} \, , \tag{2.36}$$

$$= 9.3 \times 10^{-4} \eta e^{60\eta} \, . \tag{2.37}$$

It involves the two masses and the two coupling constants only in the dimensionless combinations

$$\hat{M} \equiv \frac{\sqrt{8\pi}}{m_{\text{Pl}}} \sqrt{\lambda'} M \, , \tag{2.38}$$

$$\hat{m} \equiv \frac{\sqrt{8\pi}}{m_{\text{Pl}}} \sqrt{\frac{\lambda'^2}{\lambda}} \, m \, , \tag{2.39}$$

since $\eta = 4\hat{m}^2/\hat{M}^4$. The restrictions $M \lesssim m_{\text{Pl}}/\sqrt{8\pi}$ and $\lambda' \lesssim 1$ that we have agreed to impose because of particle physics considerations mean that we are in the regime $\hat{M} \lesssim 1$, which with the COBE constraint corresponds to $\eta \lesssim 0.15$.

The situation becomes especially simple in the regime $60\eta \ll 1$, which we call the *extreme vacuum-dominated regime*. It corresponds to $\hat{M} \ll 10^{-4}$, and the quantity η varies linearly with $\hat{M}$ leading to [4,5]

$$\frac{\hat{M}^5}{\hat{m}^2} = 40\sqrt{3}\pi \delta_H = 3.7 \times 10^{-3} \, . \tag{2.40}$$

Inserting the Planck mass and working with the masses themselves this formula becomes

$$\frac{M}{5.5 \times 10^{11} \, \text{GeV}} = \lambda'^{-1/10} \lambda^{-1/5} \left(\frac{m}{1 \, \text{TeV}} \right)^{\frac{2}{5}} \, . \tag{2.41}$$

In the other regime $60\eta \gtrsim 1$ [6,7], η varies only logarithmically with M, and the power $m^{2/5}$ gradually changes to $m^{1/2}$.

Although the CMB constraint can be expressed in terms of just the two quantities $\hat{m}$ and $\hat{M}$, the vacuum domination condition involves three quantities which are conveniently chosen to be m, $\lambda^{1/4} M$, and λ'^2/λ. It is therefore useful to express the CMB constraint as a constraint on m and $\lambda^{1/4} M$ at fixed λ'^2/λ. (Another good reason for doing this is that λM^4 is the false vacuum energy density.) The extreme vacuum-dominated regime $60\eta \ll 1$ corresponds to

$$\lambda^{1/4} M \ll 4 \times 10^{-5} \left(\frac{\lambda'^2}{\lambda} \right)^{-1/4} \frac{m_{\text{Pl}}}{\sqrt{8\pi}} \, . \tag{2.42}$$

In this regime η increases linearly with $\lambda^{1/4} M$,

$$\frac{\eta}{4} \equiv \frac{m_{\text{Pl}}^2}{8\pi} \frac{m^2}{\lambda M^4} \simeq 270 \left(\frac{\lambda'^2}{\lambda} \right)^{1/4} \frac{\sqrt{8\pi}}{m_{\text{Pl}}} \lambda^{1/4} M \, , \tag{2.43}$$

while for larger values it increases only logarithmically giving the normalization

$$\frac{m_{\text{Pl}}^2}{8\pi} \frac{m^2}{\lambda M^4} \simeq 0.004 - 0.04 \tag{2.44}$$

with the upper limit corresponding to $\eta = 0.15$.

We have yet to invoke the false-vacuum-domination condition Eq. (2.33). Using Eq. (2.37), it becomes

$$\eta^3 e^{240\eta} \ll 2 \times 10^6 \frac{\lambda'^2}{\lambda} \, . \tag{2.45}$$

With $\lambda'^2/\lambda = 1$, this bound is saturated for $\eta = 0.09$, and a similar limit is obtained for any value of the ratio within a few orders of magnitude of unity. Setting $\eta = 0.09$, one learns that the false-vacuum-dominated regime is restricted to

$$\frac{\sqrt{8\pi}}{m_{\text{Pl}}} \lambda^{1/4} M \lesssim 2 \times 10^{-2} \left(\frac{\lambda'^2}{\lambda} \right)^{-1/4} \, , \tag{2.46}$$

$$\frac{\sqrt{8\pi}}{m_{\text{Pl}}} m \lesssim 8 \times 10^{-5} \left(\frac{\lambda'^2}{\lambda} \right)^{-1/2} \, . \tag{2.47}$$

The upper limit on $\lambda^{1/4} M$ is not far below the one following just from the fact that $\epsilon_* < 1$ in the CMB constraint Eq. (2.17), which using $\frac{1}{4}\lambda M^4 < V$ is[6]

$$\frac{\sqrt{8\pi}}{m_{\text{Pl}}} \lambda^{1/4} M < 5 \times 10^{-2} \, . \tag{2.48}$$

The intermediate regime

We have now investigated the extreme cases, first the one in which the false vacuum energy is negligible, and second the one in which it dominates. There remains the intermediate case where both are comparable, during at least part of the cosmologically significant era $\phi < \phi_{60}$.

Plotted with m the vertical axis and $\lambda^{1/4} M$ the horizontal axis, we have learned that the first regime corresponds to a straight horizontal line, whereas the second one corresponds to a line with positive slope. Unless λ'^2/λ is several orders of magnitude away from unity, comparison of Eqs. (2.24) and (2.25) with Eqs. (2.47) and (2.46) shows that the right hand ends of these two lines are separated by at most an order of magnitude or so in the m and $\lambda^{1/4} M$ variables. Therefore the intermediate regime is not very extensive, but it is still important to investigate it in order to see if new physics occurs.

Even in the intermediate regime the upper bound Eq. (2.48) holds. Apart from this fact, numerical techniques are required to solve the density perturbation constraint. The solution, as might be expected, is that as $\lambda^{1/4} M$ is increased the two solution branches approach

---

[6]To understand the relation between the two limits, note that, when the vacuum domination condition Eq. (2.33) is saturated, $V = \frac{1}{2}\lambda M^4$ and $\epsilon_{60} = \eta$.

each other and merge continuously. This merger specifies the maximum allowed value of $\lambda^{1/4}M$; for higher values it becomes impossible to obtain a sufficiently low perturbation amplitude regardless of the choice of m. (The maximum value does depend on λ and λ', of course.) Figure 1 illustrates the complete set of viable models for the couplings both set to unity, showing both the asymptotic regimes and the merger region.

E. Tilt and gravitational waves

Although the inflationary prediction in Eq. (2.17) for the spectrum $\delta_H(k)$ is almost flat, there is always some k dependence, usually referred to in the literature as *tilt*. On cosmologically interesting scales the tilt can usually be well characterized by a constant spectral index n, such that $\delta_H^2 \propto k^{n-1}$, and in that case one learns from the slow roll conditions Eqs. (2.7)–(2.10) that [25,6]

$$n - 1 = 2\eta_{60} - 6\epsilon_{60} . \qquad (2.49)$$

As always, we take "cosmologically interesting" to mean scales that leave the horizon 60 e-folds before the end of inflation.

In addition to the adiabatic density (scalar) perturbation, inflation also generates gravitational waves, whose contribution R to the CMB anisotropy $(\Delta T/T)^2$ relative to that of the scalar modes is [26,25,6]

$$R \simeq 12\epsilon_{60} . \qquad (2.50)$$

For true vacuum inflation with a potential $V \propto e^{A\phi}$, $\eta = 2\epsilon$ so that n is less than 1 and $R \simeq 6(1-n)$. Replacing the exponential by a power ϕ^α gives tilt $n - 1 = -(2 + \alpha)/120$, still negative, and for $\alpha \geq 2$ it gives $3(1 - n) \lesssim R \lesssim 6(1 - n)$. Thus, true vacuum inflation with a ϕ^α potential typically makes n a few percent below unity and it makes R tens of percent. Both of these predictions are big enough to be cosmologically significant.

The case of false-vacuum-dominated inflation is dramatically different. The condition Eq. (2.33) for false vacuum domination is $\epsilon \ll \eta$, and unless it is almost saturated η is very small. As a result, the tilt and gravitational wave contribution are both indistinguishable from zero, for generic choices of the parameters. For fixed couplings, they become significant only at the upper end of the mass range allowed by Eq. (2.33), and in contrast with the true vacuum case n is *greater* than 1 until Eq. (2.33) is almost saturated.

The value of n for a given parameter choice is obtained by substituting Eq. (2.35) into Eq. (2.49). With $\lambda'^2/\lambda = 1$, n is equal to 1.0001 for the minimum value $m \sim 100$ GeV at the lower end, and rises to a maximum value $n = 1.14$ near the upper end of the allowed range [6]. The biggest possible value of n, corresponding to $\eta = 0.15$ and $\epsilon \ll 1$, is $n = 1.30$, but this is only achieved with extreme values of the parameters $M \sim m_{\rm Pl}/\sqrt{8\pi}$ and $\lambda'^2/\lambda \gtrsim 10^9$.

We have extended these results numerically to the intermediate regime for the case where the couplings are unity. The result for n is shown in Fig. 2 as a function of
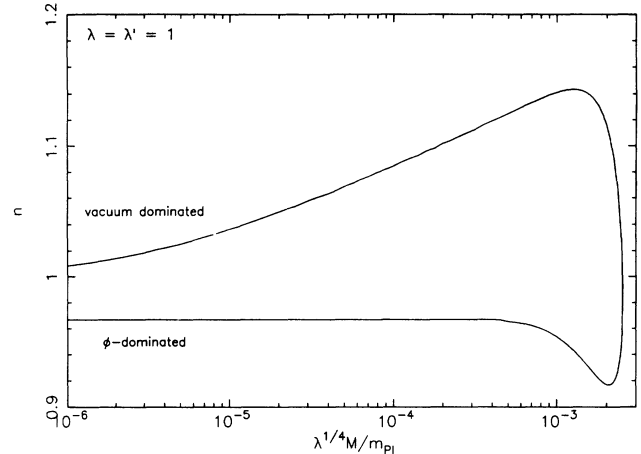

FIG. 2. The spectral index n is shown for the exact COBE normalized models of Fig. 1.

$\lambda^{1/4}M$. The two solution branches are as in Fig. 1. The analytic vacuum-dominated result agrees well with the exact one until well beyond the maximum, and in particular the maximum value $n \simeq 1.14$ is essentially the same. This number is considerably less than that which was suggested could be obtained in this model by Mollerach *et al.* [7]; they extrapolated the vacuum-dominated case beyond its regime of validity and neglected ϵ to obtain their larger values. As we commented above, this conclusion is not altered unless the couplings are changed (and separated) by orders of magnitude.

Another interesting feature is the dip in n to around 0.92 as one exits from the inflaton-dominated regime into the intermediate region, indicating that these models are also capable of providing a significant (though not startling) tilt in the opposite direction. In the limit of small M, the vacuum-dominated case asymptotes to unity and the inflaton-dominated case to the standard value 0.97.

We have also calculated the gravitational wave component, though we have not attempted to include it (or the tilt) into the COBE normalization. Gravitational waves make only a small contribution to COBE except in the intermediate regime where they can reach a peak of tens of percent (though as expected when n exceeds unity the gravitational wave component is suppressed by the small ϵ required), indicating that a proper treatment of them is required to develop the precise phenomenology of the intermediate regime.

III. THE SECOND-ORDER PHASE TRANSITION

If ϕ falls below $\phi_{\rm inst}$ before the end of inflation, the false vacuum is destabilized and there is a possibility of a second-order phase transition, of a kind quite different from the usual thermal phase transition. In this section we consider the nature of this transition, treating separately the very different regimes of inflaton domination and vacuum domination.

For simplicity we continue to suppose that ψ is a single

real field with the potential Eq. (2.1). This potential has the discrete symmetry $\psi \leftrightarrow -\psi$, which of course implies that the phase transition creates domain walls located at surfaces in space where ψ vanishes. One can instead have N real fields, and replace ψ^2 by $\sum |\psi_i|^2$ in the potential Eq. (2.3), which has $O(N)$ symmetry. This will give global strings ($N = 2$), global monopoles ($N = 3$), or textures ($N \geq 4$) if the symmetry is global, or gauge strings ($N = 2$) or gauge monopoles ($N = 3$) if it is local. We expect that in all cases our discussion of the evolution of ϕ and ψ should be roughly correct, provided that ψ is taken to represent the "radial" degree of freedom with respect to which the potential has a maximum, as opposed to the "angular" degrees of freedom with respect to which it is constant.

One other significance of having a local symmetry rather than a global one, emphasized by Linde [5], is that one might have no defects forming simply because the lowest homotopy groups all vanish. This takes advantage of there being no such thing as local texture or nontopological texture, due to the gauge degrees of freedom canceling the scalar gradients. For global symmetries, however, scalar gradients can still play a harmful role even in the absence of topological constraints.

We will see that the formation of topological defects at the end of a period of inflation offers the intriguing possibility that we could make use of the inflationary epoch to solve the flatness issues of our universe, and yet retain the possibility of utilizing defects as the source of the density fluctuations to seed large scale structure.

A. The inflaton-dominated regime

In Sec. II D, we saw that in the inflaton-dominated regime the false vacuum is maintained right up to the end of inflation unless λ' is very small. In that case the phase transition to the true vacuum will take place after inflation, and will presumably be of the usual thermal type, any topological defects forming by the usual Kibble mechanism [27].

If λ' is sufficiently small, ϕ can fall below ϕ_{inst} before the end of inflation. Depending on the regime of parameter space, the transition to the false vacuum and the formation of defects may then occur before the end of inflation. This case has been treated by several authors[7] [11–19], and we look briefly at the results of these authors because they provide a starting point for our discussion of the vacuum-dominated case, which is our main focus. As we noted in Sec. II D, the back reaction of ψ on ϕ is negligible in the vacuum-dominated regime. As a result we can in principle follow the evolution of ψ explicitly, using quantum field theory in curved spacetime [12–19]. Depending on the values of the couplings λ and λ' and the mass scale M, we can have very different situations; in-

deed even when considering the same regime the authors cited above are not always in agreement. Still, a reasonably definite picture emerges provided that the values of the parameters are not too extreme (discounting the necessarily small λ' of course), which we now summarize before mentioning more exotic possibilities.

For a rough description of what is going on, we can ignore the spatial gradient of ψ, and treat it classically. As long as it is small in the sense that

$$|\psi| \ll \psi_{\text{vac}}, \tag{3.1}$$

its equation of motion is

$$\ddot{\psi} + 3H\dot{\psi} + M_\psi^2(\phi)\psi = 0, \tag{3.2}$$

with

$$M_\psi^2(\phi) = \lambda'(\phi^2 - \phi_{\text{inst}}^2). \tag{3.3}$$

As long as $\phi > \phi_{\text{inst}}$, the effective mass M_ψ^2 is positive and ψ is equal to zero apart from its quantum fluctuation. When ϕ first falls below ϕ_{inst}, $|M_\psi|$ is negligible compared with H, and ϕ remains almost constant. After some time, which may be either small or large on the Hubble scale depending on the regime of parameter space, $|M_\psi|$ grows to exceed H, and one can start to use the opposite approximation of ignoring H:

$$\ddot{\psi} + M_\psi^2(\phi)\psi = 0. \tag{3.4}$$

There are now two possibilities, according to whether or not the adiabatic condition $|\dot{M}_\psi| \ll |M_\psi|^2$ is satisfied. If it is, the solution of Eq. (3.2) is

$$\psi \simeq \text{const} \times |M_\psi|^{-1/2} \exp\left(\int_0^t |M_\psi(t)|dt\right). \tag{3.5}$$

Taking $t = 0$ to be the epoch when $|M_\psi| = H$, the exponential becomes large within a Hubble time, and Eq. (3.1) will be violated more or less independently of the initial value of ψ. When that happens ψ will quickly roll down to its minimum ψ_{vac}. If on the other hand the adiabatic condition is not satisfied when $|M_\psi|$ first grows to be of order H, there will typically be little change in ψ until it is satisfied, after which Eq. (3.5) will again hold. Thus the conclusion is that ψ rolls down rapidly towards its vacuum value at the epoch $|M_\psi| \sim H$ or the epoch $|\dot{M}_\psi|/|M_\psi|^2 = 1$, whichever is later. (The insufficiency of just the former condition was pointed out in [19].)

Though the spatial gradient of ψ is not crucial initially, it becomes so after roll down, because domain walls form at the places in space where ψ is trapped with its false vacuum value $\psi = 0$. As we already noted, more general defects can form if ψ is replaced by an N-component object. To determine the stochastic properties of the spatial distribution of the defects, one needs to consider the spatial variation of ψ. The basic assumption is that during inflation ψ vanishes except for its quantum fluctuation. Once ϕ falls below ϕ_{inst}, the fluctuation in ψ can be easily evaluated, because its Fourier modes decouple, until its rms has grown to be of order ψ_{vac}. The classical equation for each mode is the generalization of Eq. (3.2) including

---

[7]Some of these authors consider a coupling of ψ to the spacetime curvature R rather than to the inflaton field, but this is equivalent for the present purpose.

the spatial gradient

$$\ddot{\psi}_k + 3H\dot{\psi}_k + \left[\left(\frac{k}{a}\right)^2 + M_\psi^2(\phi)\right]\psi_k = 0\,, \qquad (3.6)$$

where k is the comoving wave number of the mode under examination. Now cosmologically interesting (and smaller) comoving scales presumably leave the horizon many Hubble times after inflation begins, with the corresponding Fourier modes initially in the vacuum, i.e., containing no ψ particles ([6], p. 46). As a result the initial value of the quantum expectation $\langle|\psi_k|^2\rangle$ is known, and so is its time dependence, which is given simply by the modulus squared of the solution of the classical field equation. For the nonrelativistic modes $k/a \ll |M_\psi|$, Eq. (3.6) reduces to Eq. (3.2), and $\langle|\psi_k|^2\rangle$ begins to grow as the solution Eq. (3.5) becomes valid. At this point, but not earlier, ψ_k can be regarded as a classical quantity in the sense that its quantum state corresponds to a superposition of states with almost well defined values $\psi_k(t)$ [28,29]. Thus after smearing the field over a distance $1/M_\psi$ (i.e., dropping the relativistic modes),[8] one has a classical field $\psi(\mathbf{x})$ which has a Gaussian inhomogeneity whose stochastic properties are specified entirely by the spectrum $\langle|\psi_k|^2\rangle$. Once the behavior Eq. (3.5) sets in the spectrum grows rapidly, and the rms of the smeared field rolls down to ψ_{vac} in accordance with the earlier conclusion.

In order for this simple picture to be self-consistent, the smeared field ψ must still be small enough to satisfy Eq. (3.1), at the epoch when the nonrelativistic modes ψ_k begin to grow according to Eq. (3.5). This can fail to be true if the parameters are far from their natural values, for instance if λ is very small, and then one has a more complicated situation which has been looked at by various authors [13,15,16,18,19]. In particular, the inhomogeneity in ψ might generate an adiabatic density perturbation on scales far outside the horizon, which would then survive the subsequent phase transition.[9]

Topological defect production in the inflaton-dominated case

From the stochastic properties of $\psi(\mathbf{x})$ just before roll down, one can in principle calculate the stochastic properties of the initial configuration of the defects, since they will form at the places in space where $\psi(\mathbf{x}) = 0$. In particular one can estimate the typical spacing of the defects. The smallest possible spacing corresponds to the defect size,[10] which at least for couplings of order unity is of

order M^{-1}. For a thermal phase transition the typical spacing at formation is $(\lambda M)^{-1}$ [27]. We would like to know if the same is true in the inflationary case. The different estimates [12,14,18,19] do not entirely agree but they do seem to indicate that the spacing is still very roughly M^{-1}, at least to within a few orders of magnitude.

It does not, however, follow that the cosmological effects of the defects are the same in the two cases, their subsequent evolution being quite different. In the thermal case, where the defects are created during a noninflationary (typically radiation-dominated) era, the Hubble distance H^{-1} increases steadily in comoving distance units. Except in the case of gauge monopoles, the spatial distribution of the defects typically loses all memory of the initial conditions on scales smaller than the Hubble distance, exhibiting scaling behavior whereby the stochastic properties become more or less fixed in units of the Hubble distance. In particular the typical spacing becomes of order the Hubble radius H^{-1}. Only on scales much larger than H^{-1} does the initial distribution expand with the Universe, remaining fixed in comoving distance units. (The case of gauge monopoles, which do not scale, is considered in greater detail in Sec. III C.)

The case where the defects are created during inflation is quite different. During inflation, H^{-1} decreases, typically dramatically, in comoving distance units. As a result the distribution of the defects is frozen in comoving distance units, and in particular the typical spacing remains roughly of order the comoving distance scale which left the horizon (became bigger than H^{-1}) at the epoch when the defects form. This remains true until the era, long after inflation ends, when that scale reenters the horizon. Only then will the defect distribution become the same as in the thermal case, as the "scaling" solution is established.

The cosmological significance of this different evolution depends on when the defects form. If they form *after* cosmologically interesting scales leave the horizon (50 or 60 Hubble times before the end of inflation), the scaling solution has been established by the time that these scales enter the horizon and there should be no significant difference from the thermal case. If they form *before*, their typical spacing is still much bigger than the horizon size and we presumably see no defects (unless of course we are in an atypical region of the universe [12,18]). Finally, if they form at *about the same time*, the configuration of the defects will differ from the scaling solution, as has been discussed at some length in the case of structure-forming gauge strings [12,14,18].

B. The vacuum-dominated regime

Coming now to the regime of vacuum domination, Linde [4,5] has argued that at least for couplings of order unity inflation will end promptly (within less than a Hubble time) after $\phi = \phi_{\mathrm{inst}}$. He demonstrated this by assuming that ϕ continues to slow roll down the potential Eq. (2.3) for a Hubble time, and showing that this inevitably leads to a contradiction. We present below

---

[8]Since $|M_\psi| \gg H$, these modes have yet to leave the horizon and so are still in the vacuum state.

[9]As has already been pointed out [30–32] in the somewhat different context of axions, failure to take proper account of the phase transition has led some authors to draw incorrect conclusions from this type of calculation.

[10]The thickness of a domain wall or string, or the radius of a monopole, outside which ψ has its vacuum value.

a more detailed version of this argument, repairing an omission in the original and examining also the case of small couplings.

To proceed, one has to make the technical assumption that the spatial gradients of both ϕ and ψ are negligible. It is hard to see how they could be crucial in maintaining slow roll, so the contradiction which we shall establish presumably indicates failure of slow roll rather than significant gradients in the presence of slow roll.

The slow-roll expression for ϕ is

$$\dot{\phi} = -\frac{V'}{3H} = -\frac{2}{\sqrt{3\lambda}} \frac{m_{\rm Pl}}{\sqrt{8\pi}} \frac{m^2}{M^2} \phi \,, \tag{3.7}$$

where we have used the vacuum-dominated value for H:

$$H^2 = \frac{2\pi}{3m_{\rm Pl}^2} \lambda M^4 \,. \tag{3.8}$$

During one Hubble time the change in ϕ is given by

$$\frac{\Delta\phi}{\phi} = \frac{\dot{\phi}}{H\phi} = \frac{-4m^2}{\lambda M^4} \frac{m_{\rm Pl}^2}{8\pi} \ll 1 \,. \tag{3.9}$$

It follows that after one Hubble time

$$|M_\psi^2| = \frac{8m^2}{\lambda M^4} \lambda M^2 \frac{m_{\rm Pl}^2}{8\pi} \ll M^2 \tag{3.10}$$

and

$$\frac{|M_\psi|^2}{H^2} = \frac{96m^2}{\lambda M^6} \left(\frac{m_{\rm Pl}^2}{8\pi}\right)^2 \,. \tag{3.11}$$

Using Eqs. (2.43) and (2.44), we see that this is much bigger than unity unless we are in the regime of Eq. (2.43), which then requires

$$\lambda' \lesssim 10^{-9} \frac{8\pi}{m_{\rm Pl}^2} M^2 \lesssim 10^{-9} \,. \tag{3.12}$$

We shall not consider these very small values of λ'. One also finds after one Hubble time

$$\frac{|\dot{M}_\psi|}{|M_\psi|^2} = \frac{1}{2} \left(\frac{\lambda M^6}{96m^2}\right)^{1/2} \frac{8\pi}{m_{\rm Pl}^2} \,. \tag{3.13}$$

Comparing with Eq. (3.11) one sees that except for the factor $1/2$ the right hand side is just $H/|M_\psi|$. Thus the adiabaticity condition is satisfied as well (it was not considered by Linde). As a result Eq. (3.5) shows that ψ will have rolled down to become of order $\psi_{\rm vac}$ given by Eq. (2.27).

The next step is to demonstrate that the roll down of ψ is actually inconsistent with slow-roll inflation. Since only one Hubble time has elapsed ψ will be oscillating around $\psi_{\rm vac}$ rather than located in it, but for a crude estimate we can ignore the oscillation, so that the potential is given by Eq. (2.28). Using Eqs. (2.29) and (2.30) we find

$$\epsilon \simeq 128 \frac{\lambda'}{\lambda^3} \frac{m^4}{M^{10}} \left(\frac{m_{\rm Pl}^2}{8\pi}\right)^3 \,, \tag{3.14}$$

$$\eta \simeq -\frac{8\lambda'}{\lambda M^2} \frac{m_{\rm Pl}^2}{8\pi} \,. \tag{3.15}$$

For couplings of order unity it is easy to check that the CMB constraint Eqs. (2.43) and (2.44) implies that ϵ and $|\eta|$ are both $\gg 1$. The slow roll solution we started with is therefore presumably invalid. (Even if valid, it is certainly not inflationary since $\epsilon \gg 1$.) More generally, it follows from Eq. (2.48) that $\eta > 1$ unless $\lambda' \lesssim 10^{-4}\lambda^{1/2}$, which again presumably means that the slow-roll solution is invalid. (The condition that $\epsilon > 1$ is too complicated to be worth discussing in the general case, but one expects slow roll only if both η and ϵ are less than unity.)

The conclusion is that (unless λ' is very small) slow-roll inflation ends within a Hubble time of the epoch $\phi = \phi_{\rm inst}$. Since the field equations contain a mass scale $M \gg H$, it is reasonable to suppose that in fact inflation ends altogether, giving way to an epoch when the energy density is dominated by the spacetime gradients of the fields. What happens next is associated with the question of defect production, to which we now turn.

Defect production in the vacuum-dominated case

In contrast with the inflaton-dominated case, defect production has not previously been considered for the vacuum-dominated regime. The following discussion assumes that the couplings are of order unity, or to be more precise that they are not extremely small for in that case a qualitatively different scenario could ensue. In particular, we assume that λ' is not small enough to satisfy Eq. (3.12), so that as argued above the phase transition marks the end of inflation.

Ideally one would like to follow the evolution of the fields using quantum field theory in curved spacetime, as in the inflaton-dominated case, and hence calculate explicitly the typical spacing and other stochastic properties of the initial distribution of the defects. However, that calculation relies crucially on the fact that the back reaction of ψ on ϕ is negligible which one easily checks is not the case in the vacuum-dominated regime,[11] and indeed an estimate using the techniques described in Sec. III A indicates rather that the back reaction hits ϕ long before ψ has had a chance to roll down. In the absence of this simplifying feature, it is not even possible to give a qualitative account of the evolution, let alone follow it in detail. When ϕ is hit by the back reaction from ψ, it will acquire a spatial gradient of order M_ψ, which will soon become much bigger than m. As a result of the

---

[11]It is important in this connection not to be misled by the evolution discussed in the previous subsection, which *was* similar to that seen in the inflaton-dominated case. That discussion was a purely hypothetical one, used to establish a contradiction; the premise that slow roll continues for one Hubble time leads to that evolution, which in turn leads to the contradictory result that slow roll does *not* continue for one Hubble time. One does not expect anything like it to actually occur, and in particular there is no reason to suppose that ψ first rolls down to $\psi_{\rm vac}(\phi)$, after which ϕ falls down due to the destabilizing action of ψ. (This "waterfall" sequence of events was suggested in [5].)

back reaction, ϕ will look more like a collection of interacting plane waves than a homogeneous field, and cannot be said to "roll down" to its true vacuum. Moreover, ψ will in general still be an essentially quantum object at this stage (i.e., the state of the system is not a superposition of states in which it has an almost well defined value over an extended period of time), so ϕ will become one as well.

In the absence of an explicit calculation one must rely on order of magnitude arguments, which as we now see actually point to rather definite conclusions.[12] The crucial point is that the ϕ and ψ fields are coupled to each other, and also in general to the quark, lepton, etc., fields. Since at least the scale M is much bigger than H (and even m is not many orders of magnitude less), one expects the fields to thermalize quickly on the Hubble time scale; reheating in the vacuum-dominated case will occur promptly at the end of inflation. The reheat temperature T_{reh} is therefore given by the familiar formula $T_{\text{reh}} = (30/\pi^2 g_*)^{1/4} \rho^{1/4}$, or

$$T_{\text{reh}} = (30\lambda/4\pi^2 g_*)^{1/4} M , \qquad (3.16)$$

where g_* is the effective number of degrees of freedom at that temperature, presumably at least of order 10^2 (e.g., in the minimal supersymmetric standard model $g_* = 229$). Thus the reheat temperature is of order M. The defects that have been formed find themselves effectively in a thermal bath at a temperature T_{reh}, and may be in thermal equilibrium. If so, then for a string network, for example, the most likely configuration will be the one which maximizes the allowed density of states. For the case of cosmic strings, such a configuration below the Hagedorn transition consists of maximizing the number of possible loops that can form, with the long strings being exponentially suppressed [34]. Since this distribution is similiar to that found soon after a thermal phase transition has produced strings, it is possible that the effect of such a rapid reheating could lead to a configuration of defects much the same as if there had been a thermal phase transition. However, we have not explicitly demonstrated this to be the case here; it would require a detailed numerical simulation of the reheating to rigorously establish how the network behaves.

Local cosmic strings are perhaps the most interesting defect for cosmology. The primary motivation for employing them here would be so they could contribute as seeds for the observed large scale structure and the anisotropies in the microwave background. However, if we do try to make use of them in the context of this inflation model, we must be cautious; it is expected that strings would have a very important influence on the CMB if they are to be massive enough to affect struc-

ture formation [35]. Therefore we must reassess the estimates of the allowed model parameters determined in the previous section, for these were obtained assuming that the inflaton field alone was responsible for the CMB anisotropies. With two sources (assumed uncorrelated) of anisotropies, the contributions add in quadrature. As a benchmark figure we reduce the inflation contribution to 10% of the total anisotropy, corresponding to dropping δ_H by $\sqrt{10}$. Numerical calculation shows that for unit couplings, this only reduces M_{max} from $2.4 \times 10^{-3} m_{\text{Pl}}$ to $1.3 \times 10^{-3} m_{\text{Pl}}$.

Recent simulations of cosmic string networks have shown the importance of the small scale structure on the network. An important effect of this structure is to renormalize the string mass per unit length μ, relating it to the original mass per unit length μ_0 by $\mu \sim 1.4\mu_0$ [36]. Recalling that $\mu_0 \simeq \pi M^2$ (provided scalar and vector masses are not too disparate), we are therefore allowed a mass per unit length of up to $9 \times 10^{-6} m_{\text{Pl}}$, which is comfortably high enough to allow μ to fall in the favored range of values for structure formation $\mu \sim (2-4) \times 10^{-6} m_{\text{Pl}}^2$ [37]. An equivalent calculation indicates that global textures can be similarly reconciled, though more marginally. Indeed, for the highest values we can obtain, the strings would create excessive microwave fluctuations; the best current bound on μ arises from CMB anisotropies on scales less than 10 arc min, and yields $\mu < 3 \times 10^{-6} m_{\text{Pl}}^2$ [38]. Defect production can therefore provide an additional constraint on the viable parameters of the inflationary theory.

Finally, recall that the above is with the unfavorable assumption of unit couplings; the analysis of Sec. II indicates that the upper limit on M is yet higher if the couplings are reduced, which to lowest order does not alter the string tension.

C. Nonthermal monopole production

We now consider gauge monopole production in a nonthermal phase transition. Some aspects of such production have already been discussed in [39]. The case of gauge monopoles is particularly interesting because they do not reach a scaling regime. Let the initial correlation length be some fraction ζ of the Hubble radius, $\xi \equiv \zeta H^{-1}$. For the thermal case, it is easy to show that $\zeta_{\text{th}} \sim g_*^{1/2} M/\lambda m_{\text{Pl}}$. In the vacuum-dominated case all we can be sure of (for fast roll down) is that $\zeta_{\text{vac}} \leq 1$. Of course the uncertainties in the initial distribution will be reflected in our lack of knowledge of the form ζ_{vac} should take.

Now in general we can write the initial number density of monopoles and temperature as

$$\frac{n_i}{T_i^3} \sim \frac{1}{\xi^3 T_i^3} \sim \frac{1}{\zeta^3} \frac{H^3}{T_i^3}. \qquad (3.17)$$

Once we know T we can determine H, and hence the future evolution of n/T^3 for a given value of ζ. We have demonstrated that, in the false-vacuum-dominated case, reheating is prompt, leading to a temperature af-

---

[12]Both here and in the earlier discussion, we are assuming that the defects are stable structures that do not "know" about the expansion of the Universe. This assumption has recently been questioned [33], and the subject deserves more attention, particularly in the case of false vacuum inflation.

ter inflation given by Eq. (3.16). Now this means that in Eq. (3.17), for a given reheat temperature and assuming $\zeta \leq 1$, we obtain a similar scenario to the thermal case [40,41]. Neglecting the effects of annihilation, which should be valid for monopole masses, $M_{\text{mon}} \leq 10^{17}$ GeV and $\zeta \sim 1$, we obtain

$$\Omega_{\text{mon}} h^2 \simeq \frac{10^{11}}{\zeta^3} \left(\frac{T_{\text{reh}}}{10^{14} \, \text{GeV}} \right)^3 \left(\frac{M_{\text{mon}}}{10^{16} \, \text{GeV}} \right), \quad (3.18)$$

where h is the Hubble parameter today in units of 100 km s^{-1} Mpc^{-1} and $0.4 < h < 1$ [41]. In other words, we are unable to differentiate between the thermal production of monopoles and this particular nonthermal case, for a given initial temperature. The original grand unified theory (GUT) scale monopole problem still exists in the nonthermal case.

Demanding $\Omega_{\text{mon}} h^2 < 1$ constrains us to a region $M_{\text{mon}} < 10^{13}$ GeV. This is not the strongest bound though, especially for the case of light monopoles. The Parker limit [42] (see [41] for details), places a constraint on the allowed flux in monopoles of mass below 10^{17} GeV. For consistency we find that $M_{\text{mon}} < 10^{12}$ GeV, slightly tighter than the density bound. Thus it appears that, at least under the assumption that $\zeta \sim 1$, the monopole problem is still very much present in the vacuum-dominated region.

IV. PARTICLE PHYSICS MODELS

No matter how simple it might be, and no matter how well its predictions agree with observation, no model of inflation can be regarded as satisfactory unless it emerges from a sensible theory of particle physics. In the present context [Eq. (2.1)] this means that we want to identify ϕ and ψ with fields belonging to such a theory, and to show that ϕ can have a sufficiently flat potential without fine-tuning, in particular to show that the mass m of the ϕ field can be sufficiently small ($m \ll H$) and that there is no ϕ^4 term.

We start by considering the case of global supersymmetry [43]. Here it is natural to focus on the regime 100 GeV to 1 TeV for m, which is the smallest one commonly considered for scalar fields. The requirement of having no ϕ^4 term means that one cannot identify ϕ with a Higgs field, but it might be one of the scalar fields suggested by supersymmetric theories. With m in this range, the COBE normalization requires that M is of order 10^{11} GeV which, as noted earlier [6], suggests the possibility that the false vacuum is that of Peccei-Quinn symmetry. One is therefore led to ask whether, by considering Peccei-Quinn symmetry in the context of supersymmetry, there emerges a field ϕ with a quadratic coupling to the Peccei-Quinn field and a mass of order 1 TeV, but no ϕ^4 coupling. For global supersymmetry the answer to our question is remarkable; the very first model of supersymmetric Peccei-Quinn symmetry, proposed by Kim in 1984 [44], indeed has a suitable field ϕ. If we could stop at this point, we would have satisfied the wildest dreams of particle cosmologists. A model moti-

vated purely by particle physics would subsequently be seen (in this case nine years later) to lead to an observationally viable epoch of inflation without any fine-tuning of its parameters.

Unfortunately, there are sound particle physics reasons for rejecting global supersymmetry and replacing it by supergravity [43]. In a general supergravity theory it is difficult to construct a model of inflation without fine-tuning, because in addition to the global supersymmetric type terms there is an infinite series of higher order non-renormalizable terms, the first of which usually gives any would-be inflaton an effective mass of order H.

However, supergravity can only be regarded as an effective theory with a cutoff at the Planck scale. So in order to get a better handle on the crucial nonrenormalizable terms we should consider a theory of everything. Superstrings provide one possible candidate theory. Here we find another superstring miracle. The class of supergravity theories derived as the low-energy limit of orbifold compactifications of superstrings can automatically cancel the harmful nonrenormalizable corrections to the inflaton's potential. We go on to make the first steps towards constructing a truly realistic, superstring-derived model of inflation.

A. A simple supersymmetric model

The simplest superpotential [43] that spontaneously breaks a U(1) symmetry is

$$W = \sigma(\Psi_1 \Psi_2 + \Lambda^2)\Phi, \quad (4.1)$$

where Φ, Ψ_1, and Ψ_2 are chiral superfields which we take to have canonical kinetic terms, Λ is a mass which sets the scale of the spontaneous symmetry breaking, σ is a coupling constant, and the U(1) symmetry is $\Psi_1 \to e^{i\theta} \Psi_1$, $\Psi_2 \to e^{-i\theta} \Psi_2$. This superpotential is often used in supersymmetric model building [43,44], and in particular was used by Kim [44] to construct the first supersymmetric realization of Peccei-Quinn symmetry. We shall now show that, for fairly generic initial conditions, it leads to the false vacuum inflation model of the previous sections with the identifications $\lambda' = 2\lambda = \sigma^2/2$ and $M = 2\Lambda$.

The scalar potential derived from this superpotential is

$$V = \left| \frac{\partial W}{\partial \Phi} \right|^2 + \left| \frac{\partial W}{\partial \Psi_1} \right|^2 + \left| \frac{\partial W}{\partial \Psi_2} \right|^2 \quad (4.2)$$

$$= \sigma^2 \left| \Psi_1 \Psi_2 + \Lambda^2 \right|^2 + \sigma^2 \left(|\Psi_1|^2 + |\Psi_2|^2 \right) |\Phi|^2, \quad (4.3)$$

where Φ, Ψ_1, and Ψ_2 now represent just the (complex) scalar component of the respective chiral superfields. Adding a soft supersymmetry-breaking mass m of order 1 TeV, for Φ, we obtain

$$V = \sigma^2 \left| \Psi_1 \Psi_2 + \Lambda^2 \right|^2 + \sigma^2 \left(|\Psi_1|^2 + |\Psi_2|^2 \right) |\Phi|^2 + m^2 |\Phi|^2. \quad (4.4)$$

We want to show that Φ can be the inflaton, and so to obtain the effective potential during inflation we will

minimize this potential for fixed Φ. The potential is minimized at $\arg \Psi_1 + \arg \Psi_2 = \pi$, and the canonically normalized field corresponding to the phase of the Ψ field with smaller magnitude has an effective mass $\geq \sigma\Lambda$ there, and so $\arg \Psi_1 + \arg \Psi_2$ will be anchored at the value π. The potential is independent of the other two angular degrees of freedom, namely, $\arg \Psi_1 - \arg \Psi_2$ (which corresponds to the axion field) and $\arg \Phi$, and Hubble damping will make them practically time independent more or less independently of the initial conditions. This leaves only the radial degrees of freedom, corresponding to the three canonically normalized real fields $\phi = \sqrt{2}\,|\Phi|$, $\psi_1 = \sqrt{2}\,|\Psi_1|$, and $\psi_2 = \sqrt{2}\,|\Psi_2|$. In terms of them, the potential is

$$V(\phi, \psi_1, \psi_2) = \frac{\sigma^2}{4}\left(\psi_1\psi_2 - 2\Lambda^2\right)^2 + \frac{\sigma^2}{4}\left(\psi_1^2 + \psi_2^2\right)\phi^2$$
$$+ \frac{1}{2}m^2\phi^2 \,. \qquad (4.5)$$

For $\phi > 0$, the degree of freedom orthogonal to $\psi_1\psi_2$ (which corresponds to the saxino) has its minimum at $\psi_1 = \psi_2$, and it is straightforward to show that the effective mass of the canonically normalized saxino field is everywhere $\geq \sigma\phi/\sqrt{2}$ and so the saxino will be firmly fixed at its minimum during inflation.[13] Then in terms of ϕ and the canonically normalized field $\psi = \sqrt{2\psi_1\psi_2}$,

the potential is

$$V(\phi, \psi) = \frac{\sigma^2}{16}\left(\psi^2 - 4\Lambda^2\right)^2 + \frac{\sigma^2}{4}\phi^2\psi^2 + \frac{1}{2}m^2\phi^2 \,. \qquad (4.6)$$

Thus we have the model of Secs. II and III, with $\lambda' = 2\lambda = \sigma^2/2$ and $M = 2\Lambda$. From Eq. (2.41) it follows that the usual axion parameter f_a is given by

$$f_a = 2\Lambda = (8 \times 10^{11}\ \text{GeV})\ \sigma^{-\frac{3}{5}}\left(\frac{m}{\text{TeV}}\right)^{\frac{2}{5}} \,, \qquad (4.7)$$

which is at the right scale for the axion.

Note that for chaotic initial conditions Λ and m will initially be negligible and so the potential Eq. (4.4) will initially have the simple form

$$V = \sigma^2\left(|\Psi_1\Psi_2|^2 + |\Psi_2\Phi|^2 + |\Phi\Psi_1|^2\right) \,. \qquad (4.8)$$

Thus if initially $|\Phi| > |\Psi_1|, |\Psi_2|$, i.e., one-third of the initial condition space, the fields will rapidly approach the inflating trajectory $|\Psi_1| = |\Psi_2| = 0$ and $|\Phi| \gg \Lambda$ given above.

B. Supergravity

The scalar potential in supergravity [43] has the general form

$$V = \exp\left(\frac{8\pi}{m_{\text{Pl}}^2}K\right)\left[\sum_{\alpha,\beta}\left(\frac{\partial^2 K}{\partial\bar{\phi}_\alpha\partial\phi_\beta}\right)^{-1}\left(\frac{\partial W}{\partial\phi_\alpha} + \frac{8\pi}{m_{\text{Pl}}^2}W\frac{\partial K}{\partial\phi_\alpha}\right)\left(\frac{\partial\bar{W}}{\partial\bar{\phi}_\beta} + \frac{8\pi}{m_{\text{Pl}}^2}\bar{W}\frac{\partial K}{\partial\bar{\phi}_\beta}\right) - 3\frac{8\pi}{m_{\text{Pl}}^2}|W|^2\right] + D\ \text{term}\,,$$

$$(4.9)$$

where the Kähler potential $K(\phi, \bar{\phi})$ is a real function of the complex scalar fields ϕ_α and their Hermitian conjugates $\bar{\phi}_\alpha$, and the superpotential $W(\phi)$ is an analytic function of ϕ. The D term is quartic in the charged fields, and we will assume that it is flat along the inflationary trajectory so that it can be ignored during inflation. It may, however, play a vital role in determining the trajectory and in stabilizing the noninflaton fields. The term given explicitly is called the F term.

The kinetic terms are

$$\sum_{\alpha,\beta}\frac{\partial^2 K}{\partial\phi_\alpha\partial\bar{\phi}_\beta}\partial_\mu\phi_\alpha\partial^\mu\bar{\phi}_\beta \,, \qquad (4.10)$$

where μ is a spacetime index. It follows that for canonically normalized fields

$$K = \sum_\alpha|\phi_\alpha|^2 + \cdots \,, \qquad (4.11)$$

where the ellipsis stands for higher order terms.[14] Then the F-term part of the scalar potential becomes

$$V = \exp\left(\frac{8\pi}{m_{\text{Pl}}^2}\sum_\gamma|\phi_\gamma|^2 + \cdots\right)$$

$$\times \left\{\sum_{\alpha,\beta}(\delta_{\alpha\beta} + \cdots)\left[\frac{\partial W}{\partial\phi_\alpha} + \frac{8\pi}{m_{\text{Pl}}^2}(\bar{\phi}_\alpha + \cdots)W\right]\right.$$

$$\times \left.\left[\frac{\partial\bar{W}}{\partial\bar{\phi}_\beta} + \frac{8\pi}{m_{\text{Pl}}^2}(\phi_\beta + \cdots)\bar{W}\right] - 3\frac{8\pi}{m_{\text{Pl}}^2}|W|^2\right\} \,.$$

$$(4.12)$$

Thus, for *any* model of inflation, the lowest order (i.e., global supersymmetric) inflationary potential

---

[13] $\phi > \phi_{\text{inst}} = \sqrt{2}\,\Lambda$ during inflation [see Eq. (4.6) and Secs. II and III].

[14] Global supersymmetry corresponds to the case where these terms are absent, and one has taken the limit $m_{\text{Pl}} \to \infty$ to obtain the potential $V = \sum_\alpha |\partial W/\partial\phi_\alpha|^2$ that we used earlier. Supergravity corresponds to keeping m_{Pl} finite.

$$V_{\text{global}} \equiv \sum_{\alpha} \left| \frac{\partial W}{\partial \phi_{\alpha}} \right|^2 , \qquad (4.13)$$

will receive corrections[15] giving

$$V = V_{\text{global}} \left(1 + \frac{8\pi}{m_{\text{Pl}}^2} \sum_{\alpha} |\phi_{\alpha}|^2 + \text{other terms} \right)$$
$$+ \text{other terms} . \quad (4.14)$$

The $|\phi_{\alpha}|^2$ term in this equation gives a contribution $8\pi V_{\text{global}}/m_{\text{Pl}}^2 \simeq 3H^2$ to the effective mass squared of *all* scalar fields; therefore, assuming the inflaton is the modulus of a scalar field,[16] it gives a contribution of order unity to $\eta \equiv m_{\text{Pl}}^2 V''/8\pi V$ (see Sec. II B). But $|\eta| \ll 1$ is necessary for inflation to work (at least in the usual slow-roll form). As a result practically all[17] of the supergravity models of inflation proposed so far [46] have involved unmotivated fine-tuning of the Kähler potential and/or the superpotential in order to cancel the harmful nonrenormalizable corrections [i.e., to get the "other terms" in Eq. (4.14) to cancel the $|\phi_{\alpha}|^2$ term]. However, supergravity can only be regarded as an effective theory with a cutoff at the Planck scale. So in order to get a better handle on the crucial nonrenormalizable terms we should consider a theory of everything. Superstrings provide the most promising candidate, and in the next section we will find that for the Kähler potential derived from orbifold compactification of superstrings the cancellation can occur without any fine-tuning.

To end this section we just note that for the special choice of supergravity with minimal kinetic terms[18] (i.e., Eq. (4.11) and Eq. (4.12) without the higher order corrections ...] and the superpotential of the previous section, the "other terms" in Eq. (4.14) cancel the $|\phi_{\alpha}|^2$ term for the inflaton, as we will now show.

Substituting $K = |\Phi|^2 + |\Psi_1|^2 + |\Psi_2|^2$ and the superpotential of the previous section, Eq. (4.1), into Eq. (4.9) gives

$$V(\Phi, \Psi_1, \Psi_2) \simeq \sigma^2 \exp \left(\frac{8\pi}{m_{\text{Pl}}^2} |\Phi|^2 \right) \left[|\Psi_1 \Psi_2 - \Lambda^2|^2 \right.$$
$$\times \left(1 - \frac{8\pi}{m_{\text{Pl}}^2} |\Phi|^2 + \frac{(8\pi)^2}{m_{\text{Pl}}^4} |\Phi|^4 \right)$$
$$\left. + \left(|\Psi_1|^2 + |\Psi_2|^2 \right) |\Phi|^2 \right] . \quad (4.15)$$

---

[15]We assume that the corrections are small as will be the case if $|\phi_{\alpha}| \ll m_{\text{Pl}}/\sqrt{8\pi}$. If $|\phi_{\alpha}| \gtrsim m_{\text{Pl}}/\sqrt{8\pi}$ then a glance at the exponential factor in Eq. (4.12) shows that the problems will then be even more severe.

[16]Natural inflation [45] avoids this problem because its inflaton is the phase of a complex scalar field.

[17]The only exception known to us is "natural inflation" [45], as mentioned above.

[18]Although this is in some sense the simplest supergravity theory, it is not well motivated physically and its adoption must be regarded as fine-tuning to some extent.

Minimizing with respect to Ψ_1 and Ψ_2 for $|\Phi| > \Lambda$, as in the previous section, gives

$$V(\phi) = \sigma^2 \Lambda^4 \exp \left(\frac{1}{2} \frac{8\pi}{m_{\text{Pl}}^2} \phi^2 \right)$$
$$\times \left(1 - \frac{1}{2} \frac{8\pi}{m_{\text{Pl}}^2} \phi^2 + \frac{1}{4} \frac{(8\pi)^2}{m_{\text{Pl}}^4} \phi^4 \right)$$
$$= \sigma^2 \Lambda^4 \left(1 + \frac{1}{8} \frac{(8\pi)^2}{m_{\text{Pl}}^4} \phi^4 + \cdots \right) . \quad (4.16)$$

Thus the problematic mass term cancels out. However, we do not regard this as a realistic model and so will not pursue it further.

C. Superstrings

The inflation that inflated the observable Universe beyond the Hubble radius, and could have produced the seed inhomogeneities necessary for galaxy formation and the anisotropies recently observed by COBE, must occur at an energy scale $V^{1/4} \leq 4 \times 10^{16}$ GeV [23], well below the Planck scale. At these relatively low energies, superstrings are described by an effective $N = 1$ supergravity theory [43]. The properties of that supergravity theory are known in most detail for orbifold compactification schemes, and so we will restrict ourselves to such compactifications, although our results may be more general. Also, for simplicity, we will ignore the twisted sector of the theory. For the remainder of this section we set $m_{\text{Pl}}/\sqrt{8\pi} = 1$.

Following [47], we will assume the following form for the one-loop corrected Kähler potential K of the supergravity theory derived from orbifold compactification of superstrings [48,49,47,50].

$$K = -\ln Y - \sum_{i=1}^{3} \ln X_i , \qquad (4.17)$$

with

$$Y = S + \bar{S} + \frac{1}{4\pi^2} \sum_{i=1}^{3} \delta_i^{\text{GS}} \ln X_i , \qquad (4.18)$$

and

$$X_i = T_i + \bar{T}_i - \sum_{\alpha} |\phi_i^{\alpha}|^2 , \qquad (4.19)$$

where S is the dilaton whose real part gives the tree-level gauge coupling constant (Re $S \sim g_{\text{GUT}}^{-2}$), T_i are untwisted moduli whose real parts give the radii of the three compact complex dimensions of the orbifold, and ϕ_i^{α} are the untwisted matter fields associated with T_i. The terms with coefficients δ_i^{GS} are one-loop corrections coming from the Green-Schwarz mechanism, but their matter field dependence is speculative [47] (note that our convention for the sign and magnitude of these coefficients follows [49,50], not [47]).

For initial orientation we make the standard assump-

tion that the dilaton and moduli have expectation values of order 1, showing later how this may be achieved. (In our vacuum, $\langle \mathrm{Re}\, S \rangle \sim 2$ [50], but as we note later it may have a different value during inflation.) We will also make the standard assumption that the matter fields have expectation values much less than 1. The fact that this can and does include the inflaton is an important advantage of this model of inflation. The values of the dimensionless coefficients δ_i^{GS} depend on the orbifold assumed. Some values for δ_i^{GS} that have been calculated in [49] are 0, 5, and 15. We will assume $\delta_i^{\mathrm{GS}} \geq 0$ as is the case for all orbifolds considered up to now [49,51], and that we are in the perturbative regime $\delta_i^{\mathrm{GS}}/4\pi^2 \ll S + \bar{S}$ so that the loop corrections are small.

The F-term part of the scalar potential corresponding to this Kähler potential is [47]

$$V = \frac{1}{Y \prod_{i=1}^{3} X_i} \left\{ \left| W - Y \frac{\partial W}{\partial S} \right|^2 - 3|W|^2 \right.$$

$$+ \sum_{i=1}^{3} \frac{Y}{Y + \frac{1}{4\pi^2}\delta_i^{\mathrm{GS}}} \left(\left| W + \frac{1}{4\pi^2}\delta_i^{\mathrm{GS}}\frac{\partial W}{\partial S} - X_i \frac{\partial W}{\partial T_i} \right|^2 \right.$$

$$\left. \left. + X_i \sum_{\alpha} \left| \frac{\partial W}{\partial \phi_i^{\alpha}} + \bar{\phi}_i^{\alpha}\frac{\partial W}{\partial T_i} \right|^2 \right) \right\} . \qquad (4.20)$$

To lowest order in δ^{GS} and the matter fields, the kinetic terms given by Eq. (4.10) are

$$\frac{1}{(S + \bar{S})^2} \partial_\mu S \partial^\mu \bar{S} + \sum_i \frac{1}{(T_i + \bar{T}_i)^2} \partial_\mu T_i \partial^\mu \bar{T}_i$$

$$+ \sum_{i,\alpha} \frac{1}{T_i + \bar{T}_i} \partial_\mu \phi_i^{\alpha} \partial^\mu \bar{\phi}_i^{\alpha} . \qquad (4.21)$$

The superpotential W is composed of a perturbative part $W_{\mathrm{pert}}(\phi, T)$, and a nonperturbative part $W_{\mathrm{np}}(\phi, S, T)$. To lowest order in the matter fields, W_{pert} has the general form [48,47,50]

$$W_{\mathrm{pert}} = \sum_{\alpha,\beta,\gamma} w_{\alpha\beta\gamma} \phi_1^{\alpha} \phi_2^{\beta} \phi_3^{\gamma} , \qquad (4.22)$$

where $w_{\alpha\beta\gamma} = 0$ or 1. W_{np} is not very well understood. However, it should have an expansion in powers of e^S to reflect its nonperturbative nature [52,50]. Also, orbifold compactifications of superstrings are invariant under target-space duality symmetries to all orders of string perturbation theory and, it is thought, nonperturbatively as well [53,52,54]. These duality transformations act on the moduli as

$$T_i \to \frac{a_i T_i - ib_i}{ic_i T_i + d_i} , \quad a_i d_i - b_i c_i = 1 . \qquad (4.23)$$

The parameters a_i, b_i, c_i, d_i are in general a discrete set of real numbers. In many cases the duality group is given by the product of three modular groups, i.e., $a_i, b_i, c_i, d_i \in \mathbf{Z}$, and for simplicity we will assume this to be the case here. Then the matter fields ϕ_i^{α} transform in the same way as $1/[\eta(T_i)]^2$ where

$$\eta(T_i) = e^{-\frac{\pi T_i}{12}} \prod_{n=1}^{\infty} \left(1 - e^{-2n\pi T_i} \right) \qquad (4.24)$$

is the Dedekind function. It will also be useful to define the modular-invariant dilaton field [49,47]

$$S' = S - \frac{1}{4\pi^2} \sum_{i=1}^{3} \delta_i^{\mathrm{GS}} \ln \left[\eta(T_i) \right]^2 \qquad (4.25)$$

from which the transformation properties of the dilaton can be deduced. Requiring modular invariance then puts strong constraints on the form of the low-energy supergravity theory and in particular on W_{np} [53,52,47,50].

As we shall see, the Kähler potential of Eq. (4.17) has some very special properties as far as inflation is concerned. The crucial point is the cancellation of the X_i factor in front of the global supersymmetric $\partial W/\partial \phi_i^{\alpha}$ term in Eq. (4.20). This has the consequence that if the $\partial W/\partial \phi_i^{\alpha}$ terms for one value of i, say $i = 3$, dominate the inflationary potential energy then the canonically normalized T_3 and ϕ_3^{α} fields do not acquire corrections of order H to their effective masses[19] as would be expected in supergravity in general (see the previous section). This opens up a path to inflation without fine tuning. Note that the above conditions for inflation are asymmetric with respect to the i subscripts which means that the Kähler potential $K = -\ln(S + \bar{S}) - 3\ln\left(T + \bar{T} - \sum_\alpha |\phi^\alpha|^2\right)$ [56] cannot be regarded as equivalent to the Kähler potential of Eq. (4.17) and does not share its inflationary properties. We will now go on to chart this path to inflation for the case of false vacuum inflation.[20]

Since we are assuming $|\phi_i^{\alpha}| \ll 1$, it is not unreasonable to assume that the $\partial W/\partial \phi_i^{\alpha}$ terms dominate the potential during inflation.[21] Then

$$V = \sum_{i=1}^{3} \frac{\sum_\alpha \left| \frac{\partial W}{\partial \phi_i^{\alpha}} \right|^2}{\left(Y + \frac{1}{4\pi^2}\delta_i^{\mathrm{GS}}\right) \prod_{j \neq i} X_j} . \qquad (4.26)$$

Next we minimize the potential, with respect to the matter field dependence of the $\partial W/\partial \phi_i^{\alpha}$ terms,[22] for a fixed inflationary value (i.e., $\phi > \phi_{\mathrm{inst}}$, see Sec. II) of the inflaton (whatever it may turn out to be). We assume that the $\partial W/\partial \phi_i^{\alpha}$ terms will then be independent of the inflaton,

---

[19] An important exception to this statement is a ϕ_3^{α} field whose $\partial W/\partial \phi_3^{\alpha}$ term contributes to the inflationary potential energy. Such fields are likely to pick up masses of order H from the $|W|^2$ terms in Eq. (4.20).

[20] One of us will consider the case of true vacuum inflation elsewhere [55].

[21] We will consider what effect the other terms might have on the inflaton's potential later.

[22] These matter fields are likely to have masses of order $V^{1/4}$ much greater than the other fields (such as the dilaton, etc., which are likely to have masses of order H) and so they will settle to their values on the inflationary trajectory before the other fields have moved significantly.

as will be the case for false vacuum inflation. We also assume that some of them will be nonzero. Dropping temporarily the superscripts on the matter fields, W transforms under modular transformations like $\phi_1\phi_2\phi_3$, and therefore $\partial W/\partial\phi_i$ transforms like $\prod_{j\neq i}\phi_j$ which, as we noted earlier, transforms like $\prod_{j\neq i}[1/\eta(T_j)]^2$. Since the S dependence arises entirely from nonperturbative effects, one therefore expects the functional form [53,52,47,50]

$$\frac{\partial W}{\partial\phi_i^\alpha} = \frac{\sum_n a_{in}^\alpha(T)e^{-b_nS'}}{\prod_{j\neq i}[\eta(T_j)]^2}, \qquad (4.27)$$

where the a_{in}^α's will in general be arbitrary modular invariant functions of the T_i, but for the most part we will assume that they are constants as is the case in gaugino condensation scenarios [50]. Also, we will assume that the b_n's are positive as in such scenarios.

Now, as will soon become clear, in order to get inflation we need the false vacuum energy density V_0 to be dominated by one or more $\partial W/\partial\phi_i^\alpha$ terms with the same value of i, say $i=3$. For the moment let us ignore the other i values altogether. Then the potential during inflation is given by

$$V_{\text{infl}} = \frac{\sum_\alpha\left|\frac{\partial W}{\partial\phi_3^\alpha}\right|^2}{\left(Y + \frac{1}{4\pi^2}\delta_3^{\text{GS}}\right)\prod_{i\neq 3}X_i}, \qquad (4.28)$$

$$= \frac{\sum_\alpha\left|\sum_n a_{3n}^\alpha e^{-b_nS'}\right|^2}{S' + \bar{S}' + \frac{1}{4\pi^2}\delta_3^{\text{GS}} + \frac{1}{4\pi^2}\sum_{i=1}^3\delta_i^{\text{GS}}\ln\left[\left(T_i + \bar{T}_i - \sum_\beta\left|\phi_i^\beta\right|^2\right)|\eta(T_i)|^4\right]}\frac{1}{\prod_{i\neq 3}\left(T_i + \bar{T}_i - \sum_\beta\left|\phi_i^\beta\right|^2\right)|\eta(T_i)|^4}, \qquad (4.29)$$

$$= V_0\left\{1 + \sum_{i\neq 3}\sum_\beta\frac{\left|\phi_i^\beta\right|^2}{T_i + \bar{T}_i} + \frac{\delta_3^{\text{GS}}}{4\pi^2\left(S + \bar{S}\right)}\sum_\beta\frac{\left|\phi_3^\beta\right|^2}{T_3 + \bar{T}_3} - \frac{\delta_3^{\text{GS}}}{4\pi^2\left(S + \bar{S}\right)}\ln\left[\left(T_3 + \bar{T}_3\right)|\eta(T_3)|^4\right] + \cdots\right\}, \qquad (4.30)$$

where

$$V_0(S', T_1, T_2) = \frac{\sum_\alpha\left|\sum_n a_{3n}^\alpha e^{-b_nS'}\right|^2}{\left(S' + \bar{S}'\right)\prod_{i\neq 3}\left(T_i + \bar{T}_i\right)|\eta(T_i)|^4}. \qquad (4.31)$$

As pointed out by Brustein and Steinhardt [57] and Carlos et al. [58], the dilaton provides the biggest obstacle to constructing a model of inflation in superstrings.[23] Our model helps with the difficulty pointed out by Brustein and Steinhardt, because V_0 can give S' a suitable minimum during inflation in much the same way as double gaugino condensation scenarios do in the true vacuum [50]. Since we are supposing that the b_n's are positive, we need for this purpose at least two distinct values of n for some α so as to obtain a minimum at a finite value of S', and then at least one more term with a different value of α to make V_0 nonzero. A minimum with $V_0 > 0$ and mass greater than H can then be obtained for reasonable, but significantly constrained, values of the a's and b's. There is, though, still the problem pointed out by Brustein and Steinhardt that for a potential of the form of V_0, and for generic initial values, S' will tend to roll past the desired minimum and on to the minimum at $S' = \infty$. As this is also a problem for the true vacuum it should be regarded as a problem for the

assumption of all positive b_n's rather than of the model of inflation. It might be solved by anthropic arguments, which in any case seem likely to be needed because of the huge degeneracy in the superstring vacuum.

There remains the problem that the V_0-induced expectation value for S' is likely to be different from its vacuum expectation value after inflation because V_0 disappears at the end of inflation. As pointed out by Carlos et al. [58], this might lead to cosmological problems because S' will in general be left far from its minimum at the end of inflation. We do not address that difficulty here.

Next consider the moduli T_i for $i \neq 3$. The function $(T_i + \bar{T}_i)|\eta(T_i)|^4$ has its maxima at $T_i = e^{i\pi/6}$ and points equivalent under modular transformations, and

$$(T_i + \bar{T}_i)|\eta(T_i)|^4\big|_{T_i = e^{i\pi/6} + \sqrt{3}\,t_i}$$

$$= \sqrt{3}\left|\eta\left(e^{i\pi/6}\right)\right|^4\left[1 - |t_i|^2 + O\left(t_i^3\right)\right], \qquad (4.32)$$

where $\left|\eta\left(e^{i\pi/6}\right)\right| \simeq 0.8006$. Therefore V_0 is minimized for $T_i = e^{i\pi/6}, i \neq 3$, and since, to lowest order in δ^{GS} and the matter fields, the canonically normalized T_i fields [see Eq. (4.21)] have masses $\sqrt{V_0}$ there, they will be firmly anchored during inflation. However, after inflation they will be left far from their true vacuum minima (which are close to $T=1.23$ in some models [50]) potentially giving cosmological problems [58]. Note that, if the a_{3n}^α's were functions of the T_i for $i \neq 3$, then they would merely shift the T_i's expectation values during inflation, while if they depended on T_3 they would fix T_3 during inflation, simplifying the following discussion.

---

[23]At least if we do not assume something like S duality [59], which requires some of the b_n to be negative.

Next, the canonically normalized ϕ_i^β matter fields [see Eq. (4.21)], for $i \neq 3$, acquire masses $\sqrt{V_0} \simeq \sqrt{3} H$ [see Eq. (4.30)], making them unviable as inflatons and firmly fixing them during inflation.

Having argued for the stability of V_0 during inflation, let us consider the possible inflatons. The canonically normalized ϕ_3^β fields acquire a mass squared (to lowest order in δ^{GS} and the matter fields)

$$m_{\mathrm{GS}}^2 = \frac{\delta_3^{\mathrm{GS}} V_0}{4\pi^2 \langle S + \bar{S} \rangle} , \qquad (4.33)$$

much less than V_0 (assuming that we are in the perturbative regime so that the loop corrections are small). Note that here $\langle S + \bar{S} \rangle$ is the expectation value of the dilaton during inflation, which is probably different from its value in our vacuum. However, it may be reasonable to assume that it has a similar value during inflation as now because, in both cases, we need to be in the perturbative regime, $\mathrm{Re}\, S \gtrsim 1$, but avoid the potentially runaway behaviour at large $\mathrm{Re}\, S$ [57]. Also, V_{infl} is minimized for $T_3 = e^{i\pi/6}$. Defining $T_3 = e^{i\pi/6} + \sqrt{3}\, t_3$, and assuming $|t_3| \ll 1$ (and $\left|\phi_3^\beta\right| \ll 1$, which we have been assuming all along), the ϕ_3^β and T_3 dependence of the inflationary potential energy is given by

$$V_{\mathrm{infl}} = V_0 \left[1 + \frac{\delta_3^{\mathrm{GS}}}{4\pi^2 \langle S + \bar{S} \rangle} \left(\sum_\beta \left| \Phi_3^\beta \right|^2 + |t_3|^2 \right) + \cdots \right] , \qquad (4.34)$$

where Φ_3^β and t_3 are the canonically normalized ϕ_3^β and T_3 fields. Then defining the canonically normalized (inflaton) field $\phi = \sqrt{2} \sqrt{\sum_\beta \left| \Phi_3^\beta \right|^2 + |t_3|^2}$, and making the reasonable assumption that the orthogonal degrees of freedom are time independent, we get the potential during inflation,

$$V_{\mathrm{infl}} = V_0 \left[1 + \frac{1}{2} \frac{\delta_3^{\mathrm{GS}}}{4\pi^2 \langle S + \bar{S} \rangle} \phi^2 \right] . \qquad (4.35)$$

Therefore, from Sec. II E, the density perturbations produced during inflation will have a spectral index

$$n = 1 + \frac{\delta_3^{\mathrm{GS}}}{2\pi^2 \langle S + \bar{S} \rangle} , \qquad (4.36)$$

directly related to fundamental superstring parameters. For example, taking $\langle S + \bar{S} \rangle = 4$ [50] and $\delta_3^{\mathrm{GS}} = 5$ [49] gives $n = 1.06$.

For $\delta_3^{\mathrm{GS}} = 0$ [49], T_3 and ϕ_3^β receive no contribution to their potential from V_{infl} and so either the terms in Eq. (4.26) neglected in Eq. (4.28) or the terms in Eq. (4.20) neglected in Eq. (4.26) will dominate. In the first case, if the $\partial W / \partial \phi_i^\alpha$ terms for $i \neq 3$ are nonzero but still much smaller than the $i = 3$ terms, then they could provide T_3 and the ϕ_3^β with masses $\ll H$ in the same way as the $i = 3$ terms provide the T_i and ϕ_i^β for $i \neq 3$ with masses of order H. Note that if the $\partial W / \partial \phi_i^\alpha$ terms for $i \neq 3$ are of the same order as the $i = 3$ terms

then all the fields will have masses of order H and inflation will not be possible. In the latter case, the neglected terms are the terms that are thought to provide the soft supersymmetry-breaking terms in our vacuum and so, during inflation, they might also provide the ϕ_3^β with soft supersymmetry-breaking mass terms and give T_3 a minimum with a mass of the same order. However, as the expectation value of the dilaton during inflation may be different from its value in our vacuum, the soft supersymmetry-breaking scale may also be different. Thus for $\delta_3^{\mathrm{GS}} = 0$, the results will depend on the specific superpotential.

Finally let us consider briefly the possibility that $\phi \gtrsim 1$. Then Eq. (4.30) will no longer be a good approximation. For example, consider the simplest case of constant T_3 and, without loss of generality, one ϕ_3^β field which we will call ϕ_3. Then, to lowest order in δ^{GS} and the other matter fields, ϕ_3's kinetic term is

$$\frac{\partial^2 K}{\partial \phi_3 \partial \bar{\phi}_3} |\partial_\mu \phi_3|^2 = \frac{T_3 + \bar{T}_3}{\left(T_3 + \bar{T}_3 - |\phi_3|^2 \right)^2} |\partial_\mu \phi_3|^2 . \qquad (4.37)$$

Therefore the canonically normalized real field ϕ corresponding to $|\phi_3|$ is given by

$$|\phi_3| = \sqrt{T_3 + \bar{T}_3} \tanh \frac{\phi}{\sqrt{2}} . \qquad (4.38)$$

Note that $\phi \gtrsim 1$ corresponds to $|\phi_3| \sim \sqrt{T_3 + \bar{T}_3}$ but still $|\phi_3| < \sqrt{T_3 + \bar{T}_3}$. This is the only place where we will relax the assumption of $|\phi_3| \ll \sqrt{T_3 + \bar{T}_3}$. One of the problems of relaxing this assumption is that we neglected the terms in Eq. (4.20) proportional to $1/X_3 = \cosh^2(\phi/\sqrt{2})/(T_3 + \bar{T}_3)$.[24] This will be reasonable for $\phi \ll 1$ but is unlikely to be so for $\phi \gg 1$. However, it may just be acceptable for $\phi \sim 1$, and so, making the big assumption that the derivation of Eq. (4.30) is still valid for $\phi \gtrsim 1$, we get the inflationary potential

$$V_{\mathrm{infl}} = V_0 \left[1 + 2 \frac{\delta_3^{\mathrm{GS}}}{4\pi^2 \langle S + \bar{S} \rangle} \ln \cosh \frac{\phi}{\sqrt{2}} + \cdots \right] . \qquad (4.39)$$

For example, taking $\langle S + \bar{S} \rangle = 4$ [50], $\delta_3^{\mathrm{GS}} = 15$ [49], and $\phi_{\mathrm{inst}} \sim V_0^{1/4}$ could give an observable signature of superstrings in the varying spectral index of the density perturbations produced during inflation, although one would need the two-loop result to quantify this. Note that we must be in the vacuum-dominated regime for the loop expansion (expansion in $\delta^{\mathrm{GS}}/4\pi^2 \langle S + \bar{S} \rangle$) to be reliable.

A specific model

The arguments that we have given suggest that false vacuum inflation can be achieved, provided that the su-

---

[24]Note that these terms, although not inflationary, could lead to a scaling $a \propto t$ of the scale factor of the Universe, taking us down from the Planck scale to the energy scale at which inflation proper starts.

perpotential satisfies certain conditions. We have not, however, demonstrated that such a superpotential exists, nor have we discussed the instability mechanism which ends inflation (see Secs. II and III). We therefore end by showing how things work out with a specific choice for the perturbative part of the superpotential. The form we choose is

$$W_{\rm pert} = \left(\hat{\phi}_1^{(1)} \hat{\phi}_2^{(1)} + \hat{\phi}_1^{(2)} \hat{\phi}_2^{(2)} + \hat{\phi}_1^{(3)} \hat{\phi}_2^{(3)} \right) \hat{\phi}_3$$

$$+ \left(\hat{\phi}_1^{(1)} \hat{\phi}_2^{(4)} + \hat{\phi}_1^{(4)} \hat{\phi}_2^{(1)} \right) \tilde{\phi}_3 + \left(\hat{\phi} \to \check{\phi} \right) , \quad (4.40)$$

where $\hat{\ }$, $\check{\ }$, $\tilde{\ }$, and $^{(\alpha)}$ correspond to the generic α label

used previously. We also assume that $\hat{\phi}_i^{(\alpha)}$ and $\check{\phi}_i^{(\alpha)}$ for $i = 1, 2$ and $\alpha = 2, 3$ acquire the expectation values

$$\hat{\phi}_i^{(\alpha)} = \hat{\Lambda}_i^{(\alpha)} (S, T) = \frac{\hat{a}_i^{(\alpha)} e^{-\hat{b}_i^{(\alpha)} S'}}{[\eta (T_i)]^2} , \quad (4.41)$$

and similarly for $\check{\phi}$. The form of these expectation values is motivated by gaugino condensation scenarios (see for example Eq. (36) of [50]). These expectation values might be induced by the D-term part of the scalar potential or, more directly, by a nonperturbative part of the superpotential. We will assume $\tilde{\phi}_3$ is D flat because it will become (part of) the inflaton.

Substituting into Eq. (4.26) we get

$$V = \frac{1}{X_2 X_3 \left(Y + \frac{1}{4\pi^2} \delta_1^{\rm GS} \right)} \left[\left| \hat{\phi}_2^{(1)} \hat{\phi}_3 + \hat{\phi}_2^{(4)} \tilde{\phi}_3 \right|^2 + \left| \hat{\Lambda}_2^{(2)} \hat{\phi}_3 \right|^2 + \left| \hat{\Lambda}_2^{(3)} \hat{\phi}_3 \right|^2 + \left| \hat{\phi}_2^{(1)} \tilde{\phi}_3 \right|^2 + (\hat{\ } \to \check{\ }) \right]$$

$$+ (\text{subscript } 1 \leftrightarrow \text{subscript } 2) + \frac{1}{X_1 X_2 \left(Y + \frac{1}{4\pi^2} \delta_3^{\rm GS} \right)}$$

$$\times \left[\left| \hat{\phi}_1^{(1)} \hat{\phi}_2^{(1)} + \hat{\Lambda}_1^{(2)} \hat{\Lambda}_2^{(2)} + \hat{\Lambda}_1^{(3)} \hat{\Lambda}_2^{(3)} \right|^2 + (\hat{\ } \to \check{\ }) + \left| \hat{\phi}_1^{(1)} \hat{\phi}_2^{(4)} + \hat{\phi}_1^{(4)} \hat{\phi}_2^{(1)} + \left(\hat{\phi} \to \check{\phi} \right) \right|^2 \right] . \quad (4.42)$$

For $|\tilde{\phi}_3| > 0$ this potential is minimized for $\hat{\phi}_3 = \check{\phi}_3 = \hat{\phi}_1^{(4)} = \check{\phi}_1^{(4)} = \hat{\phi}_2^{(4)} = \check{\phi}_2^{(4)} = 0$. Then

$$V = \frac{\left(\left| \hat{\phi}_2^{(1)} \right|^2 + \left| \check{\phi}_2^{(1)} \right|^2 \right) \left| \tilde{\phi}_3 \right|^2}{X_2 X_3 \left(Y + \frac{1}{4\pi^2} \delta_1^{\rm GS} \right)}$$

$$+ (\text{subscript } 1 \leftrightarrow \text{subscript } 2)$$

$$+ \frac{\left| \hat{\phi}_1^{(1)} \hat{\phi}_2^{(1)} + \hat{\Lambda}_1^{(2)} \hat{\Lambda}_2^{(2)} + \hat{\Lambda}_1^{(3)} \hat{\Lambda}_2^{(3)} \right|^2 + (\hat{\ } \to \check{\ })}{X_1 X_2 \left(Y + \frac{1}{4\pi^2} \delta_3^{\rm GS} \right)} . \quad (4.43)$$

Defining the canonically normalized fields $\tilde{\Phi} \propto \tilde{\phi}_3$, $\hat{\Psi}_1 \propto \hat{\phi}_1^{(1)}$, $\hat{\Psi}_2 \propto \hat{\phi}_2^{(1)}$, $\hat{\Lambda}^2 \propto \hat{\Lambda}_1^{(2)} \hat{\Lambda}_2^{(2)} + \hat{\Lambda}_1^{(3)} \hat{\Lambda}_2^{(3)}$, and similarly for the checked (i.e., $\check{\ }$) symbols, then working to lowest order in $\delta^{\rm GS}$ and the matter fields, we obtain

$$V = \frac{1}{S + \bar{S}} \left[\left| \hat{\Psi}_1 \hat{\Psi}_2 + \hat{\Lambda}^2 \right|^2 + \left(\left| \hat{\Psi}_1 \right|^2 + \left| \hat{\Psi}_2 \right|^2 \right) \left| \tilde{\Phi} \right|^2 \right.$$

$$\left. + (\hat{\ } \to \check{\ }) \right] . \quad (4.44)$$

Proceeding as in Sec. IV A then gives

$$V = \frac{1}{S + \bar{S}} \left[\frac{1}{16} \left(\hat{\psi}^2 - 4|\hat{\Lambda}|^2 \right)^2 + \frac{1}{4} \tilde{\phi}^2 \hat{\psi}^2 + (\hat{\ } \to \check{\ }) \right] . \quad (4.45)$$

For $\tilde{\phi} > \sqrt{2} \max\{\hat{\Lambda}, \check{\Lambda}\}$, this is minimized for $\hat{\psi} = \check{\psi} = 0$, giving the false vacuum energy density

$$V = V_0 = \frac{|\hat{\Lambda}|^4 + |\check{\Lambda}|^4}{S + \bar{S}} , \quad (4.46)$$

as discussed above. The higher order terms give the in-

flaton a potential, also as discussed above. However, at $\tilde{\phi} = \tilde{\phi}_{\rm inst} = \sqrt{2} \max\{\hat{\Lambda}, \check{\Lambda}\}$ an instability sets in, ending inflation in a manner similar to that described in Secs. II and III.

V. A FIRST-ORDER MODEL

The one context in which the dynamical effect of more than one scalar field during inflation has been considered in some detail in the literature is in models of inflation ended by a first-order phase transition, where a field must tunnel from the metastable false vacuum, through a classically forbidden region, to the true vacuum. In the case of a single scalar field (Guth's old inflation model [60]) the metric rapidly reaches the static de Sitter metric with a fixed nucleation rate to the true vacuum and the transition must either complete at once (without sufficient inflation) or not at all. The critical parameter here is the percolation parameter p, the average number of bubbles nucleated per Hubble volume per Hubble time. To complete the transition p must exceed some critical value, $p_{\rm cr} \sim 1$ [61]. By introducing a second scalar field which can evolve with time, p can grow, allowing sufficient inflation before the transition completes.

To incorporate such a first-order transition into our model we must extend our basic potential Eq. (2.1), to include asymmetric terms which can break the degeneracy of the two vacuum states at low energies. Thus we will consider the more general potential

$$V(\phi, \psi) = \frac{1}{4} \lambda \left(M^4 + \psi^4 \right) + \frac{1}{2} \alpha M^2 \psi^2 - \frac{1}{3} \gamma M \psi^3$$

$$+ \frac{1}{2} m^2 \phi^2 + \frac{1}{2} \lambda' \phi^2 \psi^2 . \quad (5.1)$$

The cubic term spoils the degeneracy, and choosing α greater or less than zero determines whether $\psi = 0$, $\phi = 0$

is a local minimum or saddle point, respectively. Thus in addition to the two mass scales M and m we now have four dimensionless coupling constants λ, λ', α, and γ. Requiring the energy density of the true vacuum to be zero [$V(0, \psi_{\rm true}) = 0$] can be used to specify γ, say, in terms of α and λ. Thus we have one more free parameter, α, than in our second-order model (which corresponds to the particular case $\alpha = -\lambda$, $\gamma = 0$).

At large values of $|\phi|$ [where $\lambda'(\phi/M)^2 > (\gamma^2/4\lambda) - \alpha$] the potential has only one turning point with respect to ψ, a minimum at $\psi = 0$, while for smaller values of $|\phi|$ a second minimum appears, initially as a point of inflection at $\psi = \gamma/2\lambda$. Although it is this second minimum that develops into the true vacuum with $V = 0$ when $\phi = 0$, for $\gamma \neq 0$ it initially has an energy density greater than that of the false vacuum so that if the fields follow the "path of least resistance" they will remain in the false vacuum for $\alpha + \lambda'(\phi/M)^2 > 0$.

A. Inflationary dynamics

While ψ is restricted to the false vacuum ($\psi = 0$) the potential for ϕ remains that given in Eq. (2.3), and the dynamics are the same as considered in Sec. II, except that the effective mass of the ψ field is now

$$M_\psi^2 = \alpha M^2 + \lambda' \phi^2 \equiv \tilde{\alpha}(\phi) M^2 , \qquad (5.2)$$

and a second-order transition is not possible for $\alpha > 0$. Instead the transition must proceed by nucleating bubbles of the true vacuum and the end point $\phi_{\rm inst}$ is replaced by the critical value $\phi_{\rm cr}$ where the percolation parameter reaches $p_{\rm cr}$.

Notice then that inflation ends at

$$\phi_{\rm end} = \max\{\phi_\epsilon, \phi_{\rm cr}\} , \qquad (5.3)$$

where the slow-roll condition may break down at ϕ_ϵ (defined as in Sec. II) before the true vacuum percolates. If this is the case then we are again in the inflaton-dominated limit, the false vacuum energy density is negligible ($\lambda M^4 \ll m^2 \phi_{60}^2$), and the constraints are exactly the same as when the eventual transition to true vacuum is second order. The precise mechanism of the phase transition becomes irrelevant as this now occurs after inflation has ended. After passing ϕ_ϵ the field reaches $\phi = 0$ within one Hubble time. Unlike in the second-order model this does not immediately cause an instability, and oscillations about $\phi = 0$ could be sufficiently damped to restart inflation if $\phi_{\rm cr}$ lies very close to zero. However, even in this case we can show that the number of e-foldings, given by Eq. (2.14), during any subsequent stage of inflation,

$$N < \frac{1}{8} + \frac{1}{4} \ln \left(\sqrt{\frac{\lambda'}{16\pi\lambda}} \frac{m_{\rm Pl}}{M} \right) , \qquad (5.4)$$

must be very small.

Thus we will consider only the vacuum-dominated branch in what follows, where we may take $\lambda M^4 \gg m^2 \phi_{\rm cr}^2$. Another reason for doing this is that we will have to ignore the evolution of the ϕ field while calculat-

ing the nucleation rate for ψ from the false to the true vacuum. The correct two-field result is not known so, in common with all other models of first-order inflation, we will calculate instead the tunneling rate for the quasi-static potential $V(\psi)$. We would only expect this to be valid if $m \lesssim H$, which is indeed guaranteed if we are in the vacuum-dominated regime.

The percolation parameter is then given by

$$p \simeq \frac{\lambda M^4}{4H^4} \exp(-S_E) , \qquad (5.5)$$

where the term in the exponential is the Euclidean action of the tunneling configuration [62], recently given for first-order quartic potentials $V(\psi)$ by Adams [63] as $S_E = 2\pi^2 B_4/\lambda$, with B_4 a numerically calculated monotonically increasing fitting function of the parameter

$$\delta(\phi) \equiv \frac{9\lambda\tilde{\alpha}}{\gamma^2} . \qquad (5.6)$$

In our model $\delta(\phi)$ decreases as ϕ rolls down its potential during inflation, until S_E is sufficiently small for the percolation parameter to reach unity, allowing the first-order transition to complete. This corresponds to

$$S_{\rm cr} = \ln \frac{\lambda M^4}{4p_{\rm cr} H^4} \simeq 4 \ln \frac{m_{\rm Pl}}{M} . \qquad (5.7)$$

Figure 3 shows the corresponding value of $\delta_{\rm cr}$ required for the transition to complete at different values of the false vacuum energy density. Clearly for a given α there is a lower bound, $\delta > \delta_0 = 9\lambda\alpha/\gamma^2$, and a corresponding bound on the nucleation rate, so the first-order transition cannot complete when the energy density of the false vacuum is above a given value. For instance, if $\lambda = 1$ and $\alpha \gtrsim 1$ then the transition will never complete for $M \gtrsim 10^{14}$ GeV.

Assuming then that α is sufficiently small [i.e., $\delta_0 < \delta_{\rm cr}(M)$], bubbles of the true vacuum will percolate at $\phi_{\rm cr} = \sqrt{\lambda/\lambda'_{\rm eff}}\, M$, where, to utilize the results of Sec.
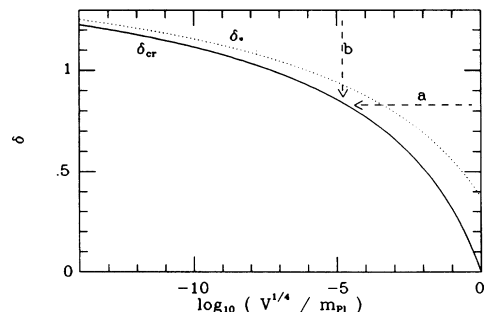

FIG. 3. $\delta_{\rm cr}$ and δ_* plotted as functions of $V^{1/4}/m_{\rm Pl}$ for first-order inflation. 55 e-foldings from the end of inflation δ_{55} must lie above the dotted line δ_*, but then reach the solid line $\delta_{\rm cr}$, to bring inflation to an end. The two trajectories, plotted as dashed lines, represent the typical evolution of δ and $M/m_{\rm PL}$ for (a) extended inflation where δ =const, and (b) false vacuum inflation in Einstein gravity where $V^{1/4}/m_{\rm Pl} \simeq$ const.

II, we write

$$\lambda'_{\rm eff} = \frac{9\lambda^2}{\gamma^2(\delta_{\rm cr} - \delta_0)}\lambda' \, . \qquad (5.8)$$

To complicate the matter somewhat, $\lambda'_{\rm eff}$ is now a function of M through the dependence of $\delta_{\rm cr}$ on the energy density, but this dependence is very weak for the energy scales significantly below the Planck scale which we are interested in. With this proviso then the results for the vacuum-dominated branch of Sec. II in the second-order model may be carried through to the first-order model by replacing λ' by $\lambda'_{\rm eff}$.

B. Big-bubble constraints

The production of large true vacuum voids, nucleated early on during inflation and swept up to astrophysical sizes by the subsequent expansion, can severely constrain some models of first-order inflation [64,65]. The isotropy of the microwave background can be used to rule out the possibility that there are any voids with a comoving size greater than about $20h^{-1}$ Mpc on the last scattering surface [65], which corresponds to a filling fraction of less than about 10^{-5} for bubbles nucleated around 55 e-foldings before the end of inflation. This means that the percolation parameter at this point during inflation must be less than 10^{-5}, requiring

$$S_{55} \gtrsim 4 \ln \frac{m_{\rm Pl}}{M} + 11.5 \, . \qquad (5.9)$$

This gives the second line in Fig. 3 showing the minimum permissible value of δ (denoted by δ_*) at 55 e-foldings before the end of inflation at different false vacuum energy densities.

Obeying this extra constraint, $\delta_{55} \gtrsim \delta_*$, requires $\tilde{\alpha}$ to be greater than a minimum value at this point and thus

$$\phi_{55} \gtrsim \gamma\sqrt{\frac{\delta_* - \delta_0}{9\lambda\lambda'}}M \, . \qquad (5.10)$$

In the vacuum-dominated regime the value of ϕ can be given as a function of the number of e-foldings before the end of inflation from Eq. (2.14)

$$\phi \simeq \phi_{\rm cr} \exp\left(\frac{Nm^2 m_{\rm Pl}^2}{2\pi\lambda M^4}\right) \, , \qquad (5.11)$$

which gives the constraint in Eq. (5.10) as a constraint on the mass scales

$$\frac{m^2 m_{\rm Pl}^2}{M^4} \gtrsim \frac{\pi\lambda}{55} \ln\left(\frac{\delta_* - \delta_0}{\delta_{\rm cr} - \delta_0}\right) \, . \qquad (5.12)$$

In other words, the mass of the ϕ field, m, must be large enough for the decrease in the effective mass of the ψ field during the last 55 e-foldings of inflation to raise the percolation parameter from 10^{-5} to unity. The numerical factor on the right hand side of this equation is fairly small, typically about 10^{-2} for $\lambda \sim 1$, so this does not threaten to force us out of the small m limit. Clearly it is

minimized for small α, as $\delta_0 \to 0$, but can become large if $\delta_{\rm cr}$ is too close to δ_0.

Normalizing the parameters of the model by the observed density perturbations, as described in Sec. II, gives another relation between m and M which, combined with the big-bubble constraint, provides limits on either m or M alone:

$$\frac{M}{m_{\rm Pl}} \gtrsim (3 \times 10^{-5})\frac{\pi\gamma}{55\lambda}\sqrt{\frac{\delta_{\rm cr} - \delta_0}{9\lambda'}} \ln\left(\frac{\delta_* - \delta_0}{\delta_{\rm cr} - \delta_0}\right) \, , \qquad (5.13)$$

$$\frac{m}{m_{\rm Pl}} \gtrsim (3 \times 10^{-5})^2 \frac{\gamma^2}{\lambda^{3/2}}\left(\frac{\delta_{\rm cr} - \delta_0}{9\lambda'}\right)\left(\frac{\pi}{55}\right)^{5/2}$$

$$\times \left(\ln\frac{\delta_* - \delta_0}{\delta_{\rm cr} - \delta_0}\right)^{5/2} \, . \qquad (5.14)$$

For reasonable values of the coupling parameters, of order unity, we would expect the right hand side of Eq. (5.13) to be $\sim 10^{-6}$, placing a lower limit on M of around 10^{13} GeV in a first-order model, unless we have $\delta_{\rm cr}$ very close to δ_0. The other way to allow first-order models at lower energy scales would be to introduce a strong coupling λ', much larger than unity, between the two fields, which is clearly always possible, as this enables only a small change in ϕ to effect a large change in the bubble nucleation rate.

C. Other first-order models

The ability of a second scalar field to allow a first-order inflationary phase transition to complete was first emphasized by La and Steinhardt [8]. This is the basis of models of extended inflation based on extensions to the gravitational Lagrangian beyond the Einstein-Hilbert action of general relativity [8–10]. In Brans-Dicke gravity, for instance, the Ricci scalar appears in the action coupled to a scalar field rather than Newton's constant, and it is this growing Brans-Dicke field, $\Phi \equiv m_{\rm Pl}^2$, which triggers the completion of the phase transition in the ψ field. However, Linde [2] and Adams and Freese [3] pointed out that this basic scenario can also be realized in general relativity by coupling the inflaton to a second scalar field. Linde used the same basic first-order potential $V(\psi)$ as we have, although he used a Coleman-Weinberg-type potential for ϕ rolling down from $+\infty$, introducing a minimum at a nonzero value. This would have to be included in the minimum value of $\tilde{\alpha}$ and thus δ. Adams and Freese considered a specific interaction rather different to ours where, as ϕ rolled down its potential, the energy of the false vacuum state actually increased relative to the true vacuum, but their more general discussion was clearly intended to include models such as the one we have examined here.

The bubble nucleation constraints in terms of $\delta_{\rm cr}$ and δ_{55} are independent of the type of first-order inflation being considered. Extended inflation models consider a first-order potential for the inflaton which does not change during inflation. Thus δ remains a constant,

as does the false vacuum energy density. The time-varying quantity here is the Planck mass which grows during inflation. Thus extended inflation models proceed horizontally, from right to left across the parameter space in Fig. 3, completing the phase transition when $\delta_{\mathrm{cr}}(M/m_{\mathrm{Pl}}) = \delta$. The general relativistic models considered here, and those considered by Linde and by Adams and Freese, proceed almost vertically as the false vacuum energy density remains approximately constant as δ decreases with ϕ. Because the percolation parameter p is exponentially dependent on the Euclidean action S_E, it is relatively easy to evade the big-bubble constraints in the general relativistic models varying δ. In models where only M or m_{Pl} varies, the percolation parameter tends to grow comparatively slowly, making the big-bubble contraint much more severe, especially as V/m_{Pl}^4 and V'/V are already constrained by density pertubations at 60 e-foldings [25].

This has led other authors [5,39,10] recently to consider models of extended inflation where nonminimal coupling can also change the shape of the "effective potential," making $M_{\psi}^2 = \xi R - \lambda M^2$ for instance. In such cases inflation could again end by a first- or second-order transition. In a de Sitter metric the Ricci scalar R is a constant ($R = 12H^2$) so a false-vacuum-dominated universe in general relativity does not yield a time-varying mass. But in Brans-Dicke gravity, for instance, where the dominant coupling to the Ricci scalar is via the Brans-Dicke field (rather than a constant), the expansion is power law [66] rather than exponential and $R \propto t^{-2}$, triggering an instability when $R \leq \sqrt{\lambda/\xi}\, m$. Similar models have been proposed in higher order gravity theories, coupling the ψ field to R^2 terms [10]. These models extending the gravity Lagrangian can be rewritten in terms of a general relativistic model with two interacting scalar fields (the defect field and a dilaton field that acts as the inflaton) using a conformally rescaled metric [67]. But the scalar field Lagrangian in this case is rather different from our model as not only M_{ψ} but all the mass scales are changed by the dilaton field. These first-order models thus correspond to a more complicated path on Fig. 3, and by making δ a function of time can also evade the big-bubble constraint.

VI. DISCUSSION AND CONCLUSIONS

In conclusion, models of inflation based on Einstein gravity, but driven by a false vacuum, offer a range of new possibilities for both theory and phenomenology.

On the particle physics side, we have shown how false vacuum inflation points to new possibiities for model building. In particular, we have shown that it can occur in a class of supergravity models implied by orbifold compactification of superstrings. One outcome of that discussion was the intriguing possibility of obtaining a handle on the superstring orbifold, through the fact that one-loop corrections might be the dominant effect determining the spectral index. Much remains to be done of course. For instance, although we have exhibited a toy model for the scalar field sector of the string-derived su-

pergravity theory, we have made no attempt to put it in the context of a realistic model involving other fields as well. In particular, we have not tried to extend to supergravity the identification of the false vacuum with that of Peccei-Quinn symmetry, which we found was both viable and attractive in the context of global supersymmetry.

In terms of direct cosmological phenomenology, the most crucial feature of false-vacuum-dominated inflation is that the energy scale at the end of inflation can be almost arbitrarily low, the value $\sim 10^{16}$ GeV which is usual in the true vacuum case appearing only as an upper limit. If the upper limit is approached one has the unusual feature of a spectral index for the density perturbations exceeding unity, though we have demonstrated that with the COBE normalization the deviation can only be rather modest with a plausible maximum of around $n = 1.14$.

An additional feature of interest is that one expects topological defects to form as the false vacuum decays; because essentially all the energy density is available to go into the defect fields, the energy available is much greater than in the usual models where reheating is required first, redistributing the energy into a large number of fields. Because of this, structure-forming defects are comfortably compatible with our inflation model when the masses are towards the top of their allowed ranges. We have made a preliminary investigation of the details of the phase transition in different regimes, though much remains to be done. For a second-order phase transition, results already exist in the literature describing the inflaton-dominated regime. We have demonstrated that, barring very weak couplings, the phase transition proceeds very rapidly in the vacuum-dominated regime, but have been unable to develop a solid understanding of the statistics of the defects produced in such a transition. In the first-order case, where the transition completes via bubble nucleation, we have gone on to calculate the bubble distribution and the constraints upon it. We note that first-order inflation models based on Einstein gravity are generally easier to implement than those of the extended inflation type.

That one can have both structure-forming topological defects and inflation raises a host of possible structure formation scenarios, as one could choose to utilize only one of these two or a combination of the two. It is believed [35] that, for a given size of density perturbation (i.e., perturbation in the gravitational potential), defects give a larger microwave background temperature anisotropy, by a factor of a few. One could therefore arrange for defects to be the source of a component of the COBE signal while having only a modest effect on structure formation; alternatively one could aim to have inflation and defects contributing roughly equally to structure formation, in which case the defects would be predominant in the microwave background. It is conceptually (and calculationally) preferable to take the option of using only one source, lowering the energy scale of the other to make its effects negligible, but one should be aware that the required scales of the two are similar, and should a realistic model along our suggested lines be devised it would not be a particular surprise should both contributions have a role to play.

[1] A. D. Linde, Phys. Lett. **129B**, 177 (1983); *Particle Physics and Inflationary Cosmology* (Harwood Academic, Chur, Switzerland, 1990).

[2] A. Linde, Phys. Lett. B **249**, 18 (1990).

[3] F. C. Adams and K. Freese, Phys. Rev. D **43**, 353 (1991).

[4] A. D. Linde, Phys. Lett. B **259**, 38 (1991).

[5] A. D. Linde, Phys. Rev. D **49**, 748 (1994).

[6] A. R. Liddle and D. H. Lyth, Phys. Rep. **231**, 1 (1993).

[7] S. Mollerach, S. Matarrese, and F. Lucchin, "Blue Perturbation Spectra from Inflation," CERN Report No. astro-ph/9309054, 1993 (unpublished).

[8] D. La and P. J. Steinhardt, Phys. Rev. Lett. **62**, 376 (1989).

[9] P. J. Steinhardt and F. S. Accetta, Phys. Rev. Lett. **64**, 2740 (1990); J. D. Barrow and K. Maeda, Nucl. Phys. **B341**, 294 (1990).

[10] L. Amendola, S. Capozziello, M. Litterio, and F. Occhionero, Phys. Rev. D **45**, 417 (1992).

[11] L. A. Kofman and A. D. Linde, Nucl. Phys. **B282**, 555 (1987).

[12] E. T. Vishniac, K. A. Olive, and D. Seckel, Nucl. Phys. **B289**, 717 (1987).

[13] L. A. Kofman and D. Yu. Pogosyan, Phys. Lett. B **214**, 508 (1988).

[14] J. Yokoyama, Phys. Lett. B **212**, 273 (1988); Phys. Rev. Lett. **63**, 712 (1989).

[15] L. Kofman, G. R. Blumenthal, H. Hodges, and J. R. Primack, in Proceedings of the Workshop on Large Scale Structure, Rio, 1989, edited by D. W. Latham and L. N. da Costa (unpublished).

[16] D. S. Salopek, J. R. Bond, and J. M. Bardeen, Phys. Rev. D **40**, 1753 (1989).

[17] D. H. Lyth, Phys. Lett. B **246**, 359 (1990).

[18] H. M. Hodges and J. R. Primack, Phys. Rev. D **43**, 3155 (1991).

[19] M. Nagasawa and J. Yokoyama, Nucl. Phys. **B370**, 472 (1992).

[20] J. Ellis, D. V. Nanopoulos, and S. Sarkar, Nucl. Phys. **B259**, 175 (1985); M. Kawashi and K. Sato, Phys. Lett. B **189**, 23 (1987); M. H. Reno and D. Seckel, Phys. Rev. D **37**, 3441 (1988); J. Ellis *et al.*, Nucl. Phys. **B373**, 399 (1992).

[21] G. F. Smoot *et al.*, Astrophys. J. Lett. **396**, L1 (1992).

[22] E. L. Wright *et al.*, Astrophys. J. (to be published).

[23] A. R. Liddle, Phys. Rev. D **49**, 739 (1994).

[24] A. Linde, Phys. Lett. B **284**, 215 (1992).

[25] A. R. Liddle and D. H. Lyth, Phys. Lett. B **291**, 391 (1992).

[26] A. A. Starobinsky, Pis'ma Astron. Zh. **11**, 323 (1985) [Sov. Astron. Lett. **11**, 133 (1985)].

[27] T. W. B. Kibble, J. Phys. A **9**, 1387 (1976).

[28] D. H. Lyth, Phys. Lett. **147B**, 403 (1984); **150B**, 465(E) (1985).

[29] A. H. Guth and S.-Y. Pi, Phys. Rev. D **32**, 1899 (1985).

[30] A. D. Linde and D. H. Lyth, Phys. Lett. B **246**, 353 (1990).

[31] D. H. Lyth, Phys. Rev. D **45**, 3394 (1992).

[32] D. H. Lyth and E. D. Stewart, Phys. Rev. D **46**, 532 (1992).

[33] A. D. Linde, "Monopoles as Big as the Universe," Stanford Report No. astro-ph/9402031, 1994 (unpublished); A. Vilenkin, "Topological Inflation," Tufts Report No. hep-th/9402031, 1994 (unpublished); R. Basu and A. Vilenkin, "Evolution of Topological Defects during Inflation," Tufts report, 1994 (unpublished); A. Linde and D. Linde, "Topological Defects as Seeds for Eternal Inflation," Stanford Report No. hep-th/9402115, 1994 (unpublished).

[34] E. Copeland, D. Haws, S. Holbraad, and R. Rivers, in *The Formation and Evolution of Cosmic Strings*, edited by G. Gibbons, S. Hawking, and T. Vachaspati (Cambridge University Press, Cambridge, England, 1990).

[35] A. Stebbins, Ann. N.Y. Acad. Sci. **688**, 824 (1993).

[36] T. Vachaspati and A. Vilenkin, Phys. Rev. Lett. **67**, 1057 (1991).

[37] A. Albrecht and A. Stebbins, Phys. Rev. Lett. **69**, 2615 (1992).

[38] M. Hindmarsh, Astrophys. J. (to be published).

[39] A. M. Laycock and A. R. Liddle, Phys. Rev. D **49**, 1827 (1994).

[40] J. Preskill, Phys. Rev. Lett. **43**, 1365 (1979).

[41] E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley, Redwood City, CA, 1990).

[42] E. N. Parker, Astrophys. J. **160**, 383 (1970); M. S. Turner, E. N. Parker, and T. Bogdan, Phys. Rev. D **26**, 1296 (1982).

[43] For a review of supersymmetry and supergravity, see H. P. Nilles, Phys. Rep. **110**, 1 (1984).

[44] J. E. Kim, Phys. Lett. **136B**, 378 (1984); J. E. Kim and H. P. Nilles, *ibid.* **138B**, 150 (1984); K. Rajagopal, M. S. Turner, and F. Wilczek, Nucl. Phys. **B358**, 447 (1991); T. Goto and M. Yamaguchi, Phys. Lett. B **276**, 103 (1992).

[45] K. Freese, J. A. Frieman, and A. V. Olinto, Phys. Rev. Lett. **65**, 3233 (1990); F. C. Adams, J. R. Bond, K. Freese, J. A. Frieman, and A. V. Olinto, Phys. Rev. D **47**, 426 (1993).

[46] K. Olive, Phys. Rep. **190**, 307 (1990).

[47] D. Lüst and C. Muñoz, Phys. Lett. B **279**, 272 (1992).

[48] S. Ferrara, C. Kounnas, and M. Porrati, Phys. Lett. B **181**, 263 (1986).

[49] J.-P. Derendinger, S. Ferrara, C. Kounnas, and F. Zwirner, Nucl. Phys. **B372**, 145 (1992).

[50] B. de Carlos, J. A. Casas, and C. Muñoz, Nucl. Phys. B

399, 623 (1993).

[51] A. Brignole, L. E. Ibáñez, and C. Muñoz, "Towards a Theory of Soft Terms for the Supersymmetric Standard Model," Universidad Autónoma de Madrid Report No. FTUAM-26/93, hep-ph/9308271, 1993 (unpublished).

[52] M. Cvetič, A. Font, L. E. Ibáñez, D. Lüst, and F. Quevedo, Nucl. Phys. **B361**, 194 (1991).

[53] S. Ferrara, D. Lüst, A. Shapere, and S. Theisen, Phys. Lett. B **225**, 363 (1989); S. Ferrara, D. Lüst, and S. Theisen, Phys. Lett. B **233**, 147 (1989).

[54] L. E. Ibáñez and D. Lüst, Nucl. Phys. **B382**, 305 (1992).

[55] E. D. Stewart, "Inflation, Supergravity and Superstrings," Kyoto report, 1994 (unpublished).

[56] E. Witten, Phys. Lett. **155B**, 151 (1985).

[57] R. Brustein and P. J. Steinhardt, Phys. Lett. B **302**, 196 (1993).

[58] B. de Carlos, J. A. Casas, F. Quevedo, and E. Roulet, Phys. Lett. B **318**, 447 (1993).

[59] A. Font, L. E. Ibáñez, D. Lüst, and F. Quevedo, Phys. Lett. B **249**, 35 (1990).

[60] A. H. Guth, Phys. Rev. D **23**, 347 (1981).

[61] A. H. Guth and E. J. Weinberg, Nucl. Phys. **B212**, 321 (1983).

[62] S. Coleman, Phys. Rev. D **15**, 2929 (1977).

[63] F. C. Adams, Phys. Rev. D **48**, 2800 (1993).

[64] E. J. Weinberg, Phys. Rev. D **40**, 3950 (1989); A. R. Liddle and D. Wands, *ibid.* **45**, 2665 (1992); M. S. Turner, E. J. Weinberg, and L. M. Widrow, *ibid.* **46**, 2384 (1992).

[65] A. R. Liddle and D. Wands, Mon. Not. R. Astron. Soc. **253**, 637 (1991).

[66] H. Nariai, Prog. Theor. Phys. **42**, 544 (1969).

[67] K. Maeda, Phys. Rev. D **39**, 3159 (1989).