# Neutralino annihilation into gluons

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We present a complete calculation of the cross section for neutralino annihilation into the two-gluon final state. This channel can be quite important for the phenomenology of neutralino annihilation due to the well-known helicity suppression of neutralino annihilation into light quarks and leptons. In addition, we calculate the cross section for annihilation of neutralinos into a gluon and quark-antiquark pair, and discuss QCD corrections to the tree-level cross sections for neutralino annihilation into quarks. If the neutralino is lighter than the top quark, the effect of these results on high-energy neutrino signals from neutralino annihilation in the Sun and in the Earth can be significant, especially if the neutralino is primarily gaugino. On the other hand, our results should have little effect on calculations of the cosmological abundance of neutralinos. We also briefly discuss implications for cosmic-ray antiprotons from neutralino annihilation in the galactic halo.

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## I. INTRODUCTION

Luminous matter almost certainly does not account for all matter in the Universe [1], and this matter deficit inspires both particles-physics and astrophysics speculation. Among the particle-physics solutions discussed, perhaps the most attractive idea is that stable weakly interacting massive particles (WIMP's) may make up the dark matter. Currently, the most promising candidate WIMP is the neutralino [2], a linear combination of the supersymmetric partners of the photon, $Z^0$, and Higgs bosons.

It is well known that the cosmological abundance of a WIMP is inversely proportional to its annihilation cross section. In the earliest calculations, annihilation of neutralinos into light fermions was considered [3,4]; subsequently, annihilation into gauge and Higgs bosons [5–7] as well as some three-body final states in certain models [8] was taken into account. At this point, the cross sections for annihilation into all two-body final states which occur at the tree level had been calculated. The basic conclusion of all these papers is that, in almost all regions of supersymmetric parameter space, the neutralino makes an excellent candidate for the dark matter in galactic halos and, in some cases, can provide an abundance suitable to account for a flat universe.

The neutralino annihilation cross section is also needed to determine event rates in numerous schemes for indirect detection of WIMP's in the galactic halo. The existence of WIMP's in the halo may be inferred through observation of distinctive spectra of cosmic-ray antiprotons, $\gamma$ rays, or positrons produced by annihilation of WIMP's in the halo [9]. Perhaps a more promising method of detection involves observation of energetic neutrinos from WIMP annihilation in the Sun and/or Earth [10,11]. If neutralinos reside in the galactic halo, then they will be captured in the Sun and Earth [12]. Annihilation of neutralinos therein would produce, among other things, neutrinos whose energy is some fraction of the neutralino mass, which could be detected in current [e.g., IMB, Kamiokande II, the Monopole, Astrophysics and Cosmic Ray Observatory (MACRO)] or next-generation [e.g., Deep Underground Muon and Neutrino Detector (DUMAND), Antarctic Muon and Neutrino Detector Array (AMANDA), super-Kamiokande] experiments [13].

In this paper we calculate the cross section for annihilation of neutralinos into two gluons ($\tilde{\chi}\tilde{\chi}\rightarrow gg$) and discuss the implications for indirect-detection searches, especially those involving observation of energetic neutrinos from neutralino annihilation in the Sun and Earth. We also calculate the cross section for annihilation into the quark- antiquark-gluon final state ($\tilde{\chi}\tilde{\chi}\rightarrow q\bar{q}g$), and we improve the tree-level cross sections for annihilation of neutralinos into quarks by including leading-logarithmic QCD corrections.

There are a few simple reasons to believe that the two-gluon final state may be important for indirect-detection experiments, although we do not expect this final state to have much impact on cosmological relic-abundance calculations. Neutralinos in the halo, Sun, and Earth move with velocities $v \lesssim 10^{-3}$, and so annihilation through $P$ waves (i.e., angular momentum $l=1$) is suppressed by roughly $v^2 \lesssim 10^{-6}$; in other words, annihilation can occur

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only through an $S$ wave. Neutralinos are Majorana particles, and so the $S$-wave cross section for annihilation into light quarks and leptons is suppressed by $(m_f/m_{\tilde{\chi}})^2$ [14], where $m_f$ is the quark or lepton mass and $m_{\tilde{\chi}}$ is the neutralino mass. On the other hand, there is no such suppression of $S$-wave annihilation into gluons. Therefore, even though annihilation into gluons is formally suppressed relative to that into quarks by $\alpha_s^2$, the square of the strong coupling constant, in practice, $S$-wave annihilation of neutralinos into gluons may be comparable to or even stronger than annihilation into quarks, as first noted by Rudaz, Bergstrom, and others [9]. For given supersymmetric (SUSY) parameters, if the neutralino annihilates primarily into light fermions at the tree level, we expect our results to be important [11]. This will be the case if the neutralino is lighter than the $W$ boson. It may also be the case if the neutralino is primarily gaugino and heavier than the $W$ boson but lighter than the top quark [5]. In addition, since $S$-wave annihilation into light fermions is additionally suppressed in the limit of large neutralino mass, the $gg$ annihilation channel should become increasingly important as the neutralino mass is increased.

When neutralinos freeze out in the early Universe, they are moving with velocities of order 0.5, and so the cross section for annihilation contains significant $P$-wave as well as $S$-wave contributions. Since $P$-wave annihilation into light fermions is *not* suppressed in this case, annihilation into gluons is suppressed by $\alpha_s^2$ relative to that into light fermions. Therefore inclusion of the $gg$ (and $q\bar{q}g$ [8]) final state should have no more than a small effect on the cosmological abundance, and thus we do not consider these abundance calculations further.

In the following section we describe the calculation and present results of the cross sections for $\tilde{\chi}\tilde{\chi}\to gg$ and $\tilde{\chi}\tilde{\chi}\to q\bar{q}g$. We also discuss the leading-logarithmic QCD correction to the annihilation cross section for $\tilde{\chi}\tilde{\chi}\to q\bar{q}$. We present our results for a neutralino of arbitrary mass and mixing and for arbitrary squark masses and mixings. From these, the cross sections for any given supersymmetric parameters can be obtained. In Sec. III, we illustrate our results for a few simple examples and discuss the importance of the new channels in various regions of parameter space. In Sec. IV we review production of energetic neutrinos from annihilation of neutralinos in the Earth, and we discuss the effect of the new cross sections on the predicted event rates. In the final section we briefly summarize and comment on the implications of our results from predicted fluxes of cosmic-ray antiprotons produced by annihilation of neutralinos in the halo.

## II. ANNIHILATION CROSS SECTIONS

In this section we review some relevant supersymmetry (SUSY) formalism and present results for the cross sections. We use the conventions and notation of Ref. [7]. There are four neutralinos which are linear combinations of $\tilde{B}$, $\tilde{W}_3$, $\tilde{h}_1^0$, and $\tilde{h}_2^0$, the supersymmetric partners of the U(1) gauge field, the third component of the SU(2) gauge field, and Higgs fields, respectively. In the $(\tilde{B},\tilde{W}_3,\tilde{h}_1^0,\tilde{h}_2^0)$ basis, the neutralino mass matrix is

$$\begin{pmatrix} M_1 & 0 & -m_Z s_W\cos\beta & m_Z s_W\sin\beta \\ 0 & M_2 & m_Z c_W\cos\beta & -m_Z c_W\sin\beta \\ -m_Z s_W\cos\beta & m_Z c_W\cos\beta & 0 & -\mu \\ m_Z s_W\sin\beta & -m_Z c_W\sin\beta & -\mu & 0 \end{pmatrix}, \qquad (2.1)$$

where $\mu$ is the Higgsino mass parameter, $M_1$ and $M_2$ are the gaugino mass parameters, $s_W=\sin\theta_W$, and $c_W=\cos\theta_W$, and we adopt the grand unified theory (GUT) relation $M_2=\frac{5}{3}M_1\tan^2\theta_W$. The ratio of Higgs vacuum expectation values is $\tan\beta=\langle H_2^0\rangle/\langle H_1^0\rangle$. The elements of the matrix that diagonalize the mass matrix are $N_{ij}$. Unlike in Ref. [2] we allow the neutralino mass eigenvalues to take on negative values, and so the $N_{ij}$ are always real. We denote the lightest neutralino by the subscript 0; specifically, the lightest neutralino is

$$\tilde{\chi}=N_{01}\tilde{B}+N_{02}\tilde{W}_3+N_{03}\tilde{h}_1^0+N_{04}\tilde{h}_2^0 . \qquad (2.2)$$

### A. Cross section for $\tilde{\chi}\tilde{\chi}\to gg$

In this subsection we present the results of our calculation of the cross section for annihilation of nonrelativistic neutralinos into two gluons. Our results are only for the limit of vanishing relative incident velocity, $v\to 0$, and are therefore suitable for use in indirect-detection calculations. Although they are not valid for relic-abundance calculations, they could be used to estimate the magnitude of the annihilation cross section at $v\sim 0.5$.

In Fig. 1 the one-loop diagrams for annihilation of neutralinos into two gluons are displayed, and antisymmetrization (symmetrization) on the identical particles in the initial (final) state is understood. Because of the helicity structure in the $v\to 0$ limit, only diagrams (a), (b), (e), and (f) contribute. This is because the $CP$ eigenvalue of the initial two-neutralino state in this limit is $-1$. The two gluons in the final state could therefore be produced only by an operator of the form $G_{\mu\nu}^a\tilde{G}^{\mu\nu b}$, where $G_{\mu\nu}^a$ is the gluon field-strength tensor. Inspection of the Dirac structure of diagrams (c) and (d) shows that they cannot give rise to such a term. Furthermore, the Dirac structure of the remaining diagrams also simplifies considerably in the $v\to 0$ limit. This allows us to write a relatively compact form for the final result.

The matrix element for $\tilde{\chi}\tilde{\chi}\to qq$ is

$$\mathcal{M}=\frac{1}{4\pi^2}\epsilon(k_1,k_2,\epsilon_1,\epsilon_2)(-1)^{\bar{h}+1/2}\delta_{h,\bar{h}}\tilde{\mathcal{M}}\frac{g_s^2}{2}\delta_{ab} . \qquad (2.3)$$

The imaginary part of $\tilde{\mathcal{M}}$ is

$$\mathrm{Im}\tilde{\mathcal{M}} = -\pi \sum_q \theta(m_{\tilde\chi}^2 - m_q^2) \ln\frac{1+\beta_q}{1-\beta_q} \left\{ \sum_{\tilde q} \left[ \frac{1}{m_{\tilde q}^2 + m_{\tilde\chi}^2 - m_q^2} \left[\frac{1}{2}\right] \left[ S_q \frac{m_q^2}{m_{\tilde\chi}^2} + D_q \frac{m_q}{m_{\tilde\chi}} \right]\right] - \frac{2m_q}{m_{\tilde\chi}} \frac{g_{Aqq} g_{A\tilde\chi\tilde\chi}}{4m_{\tilde\chi}^2 - m_A^2} - \frac{m_q^2}{m_{\tilde\chi}^2 m_Z^2} I_{3q} \frac{g_{Z\tilde\chi\tilde\chi} g}{\cos\theta_W} \right\}, \tag{2.4}$$

where $\beta_q = \sqrt{1 - m_q^2/m_{\tilde\chi}^2}$. The real part of $\tilde{\mathcal{M}}$ is

$$\begin{aligned}
\mathrm{Re}\tilde{\mathcal{M}} = \sum_q \Bigg[ \sum_{\tilde q} \Bigg\{ & -\frac{1}{2m_{\tilde\chi}^2} \int_0^1 dx \left[ \frac{S_q}{x} \ln \left| \frac{x^2 a + x(b-1-a)+1}{-x^2 a + x(b-1+a)+1} \right| \right. \\
& + \frac{S_q b + D_q\sqrt{ab}}{1+a-b} \left[ \frac{1}{1-x} + \frac{1}{1+x} \right] \ln \left| \frac{x^2 a + x(b-a-1)+1}{b + a(1-x^2)} \right| \\
& + \frac{1}{1-b+xa} \left[ S_q b \left[ \frac{1}{x} + \frac{1}{1-x} \right] + D_q \sqrt{ab} \frac{a}{1-x} \right] \ln \left| \frac{b}{x^2 a - x(a+b-1) - 1} \right| \\
& \left. + \frac{1}{b-1+ax} \left[ S_q b \left[ \frac{1}{x} - \frac{1}{1+x} \right] - D_q \sqrt{ab} \frac{1}{x+1} \right] \ln \left| \frac{b}{x^2 a + x(b-1-a)+1} \right| \right] \Bigg\} \\
& + 2I \left[ \frac{m_q^2}{m_{\tilde\chi}^2} \right] \frac{m_q}{m_{\tilde\chi}} \left[ 2 \frac{g_{Aqq} g_{A\tilde\chi\tilde\chi}}{4m_{\tilde\chi}^2 - m_A^2} + \frac{m_q}{m_{\tilde\chi}} I_{3q} \frac{g_{Z\tilde\chi\tilde\chi} g}{m_Z^2 \cos\theta_W} \right] \Bigg] .
\end{aligned} \tag{2.5}$$

Note that there is a sum over quarks, and for each quark, there is an additional sum over the two squarks in the term containing the parametric integral. We have defined $S_q = A_q^2 + B_q^2$, and $D_q = A_q^2 - B_q^2$, and $A_q$ and $B_q$ are quark-squark-neutralino couplings defined below. We have also defined $a \equiv m_{\tilde\chi}^2/m_{\tilde q}^2$ and $b \equiv m_q^2/m_{\tilde q}^2$, and it is important to note that the sign of $\sqrt{ab}$ is the sign of $m_{\tilde\chi}$.

Generally, left and right squarks ($\tilde q_L$ and $\tilde q_R$) may mix [15], and the squark mass eigenstates are then (see, e.g., Ref. [7])

$$\begin{aligned}
\tilde q_1 &= \tilde q_L \cos\theta_q + \tilde q_R \sin\theta_q , \\
\tilde q_2 &= -\tilde q_L \sin\theta_q + \tilde q_R \cos\theta_q ,
\end{aligned} \tag{2.6}$$

where $\theta_q$ are the squark mixing angles. The squark-quark-neutralino couplings for the lighter squark eigenstates, denoted by the subscript 1, are

$$\begin{aligned}
A_q &= \tfrac{1}{2}[\cos\theta_q (X_q + Z_q) + \sin\theta_q (Y_q + Z_q)] , \\
B_q &= \tfrac{1}{2}[\cos\theta_q (X_q - Z_q) + \sin\theta_q (Z_q - Y_q)] .
\end{aligned} \tag{2.7}$$

The squark-quark-neutralino couplings for the heavier squark eigenstate, denoted by a subscript 2, are obtained by making the substitutions $\sin\theta_q \rightarrow \cos\theta_q$ and $\cos\theta_q \rightarrow -\sin\theta_q$. Here we have defined

$$\begin{aligned}
X_q &= -\sqrt{2}[g T_{3q} N_{02} - g'(T_{3q} - e_q) N_{01}] , \\
Y_q &= \sqrt{2} g' N_{01} e_q .
\end{aligned} \tag{2.8}$$

For up-type quarks we have

$$Z_q = -\frac{g m_q N_{04}}{\sqrt{2} m_W \sin\beta} , \tag{2.9}$$

and for down-type quarks we have

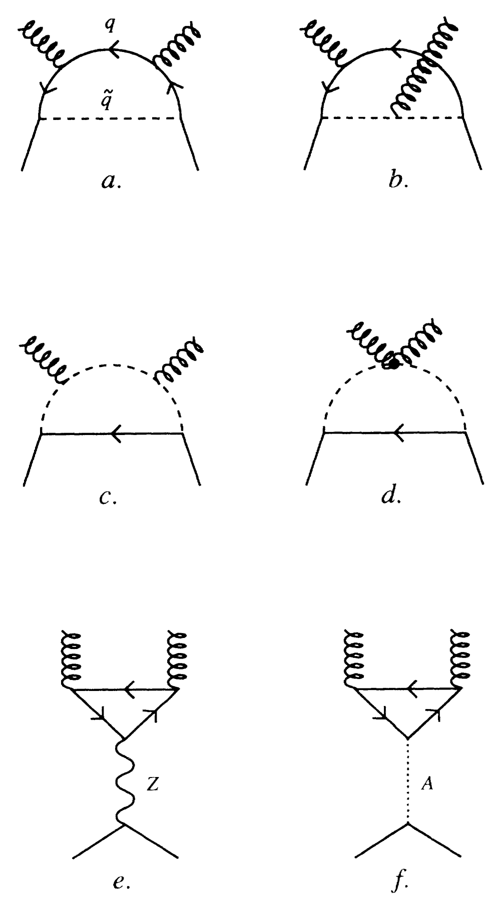


FIG. 1. Feynman diagrams for the one-loop annihilation of neutralinos into two gluons: (a)–(d) Quark-squark loops and (e) and (f) the exchange of the $Z^0$ boson or the pseudoscalar Higgs boson $A^0$.

$$Z_q = -\frac{g m_q N_{03}}{\sqrt{2} m_W \cos\beta} \ . \tag{2.10}$$

In the above, $T_{3q} = \pm\frac{1}{2}$ is the weak isospin of the quark, $e_q$ is its electric charge, $g$ is the $SU(2)_L$ gauge coupling, $g'$ is the $U(1)_Y$ gauge coupling, and $m_W$ is the $W$-boson mass.

The mass of the pseudoscalar Higgs boson $A^0$ (sometimes referred to in the literature as $H_3^0$) is $m_A$; the couplings of the pseudoscalar Higgs boson to up-type quarks are

$$g_{Aqq} = -\frac{g m_q \cot\beta}{2 m_W} \ , \tag{2.11}$$

and the couplings to down-type quarks are obtained by making the substitution $\cot\beta \to \tan\beta$. The coupling of the pseudoscalar Higgs boson to the neutralinos is

$$g_{A\tilde{\chi}\tilde{\chi}} = \frac{g}{2}(N_{02} - \tan\theta_W N_{01})(N_{03}\sin\beta - N_{04}\cos\beta) \ , \tag{2.12}$$

and the coupling of the $Z^0$ boson to the neutralinos is

$$g_{Z\tilde{\chi}\tilde{\chi}} = \frac{g(N_{03}^2 - N_{04}^2)}{4\cos\theta_W} \ . \tag{2.13}$$

The function $I(x)$, which arises from the three-point function in diagrams (e) and (f) of Fig. 1, is given by

$$I(x) = \begin{cases} -\frac{1}{4}\left[\ln^2\frac{1+\beta(x)}{1-\beta(x)} - \pi^2\right] & \text{for } x \le 1 \ , \\ \left[\arctan\frac{1}{\sqrt{x-1}}\right]^2 & \text{for } x > 1 \ , \end{cases} \tag{2.14}$$

where $\beta(x) = \sqrt{1-x}$.

Note that the integration over the Feynman parameter $x$ can be transformed in many ways. In particular, by judicious use of the transformation $x \to 1-x$ in some of the terms of the integrand, it is possible to improve the convergence of numerical integration. A poor transformation of this type would yield singularities at the end points which would, in principle, cancel when integrated. As we have written it above, the integrand has been transformed so that it has no singularities at the end points, and this is the form most suitable for numerical integration.

Given the matrix element above, the cross section times relative velocity is

$$\sigma_{gg} v = |\tilde{\mathcal{M}}|^2 \frac{\alpha_s^2 m_{\tilde{\chi}}{}^2}{8\pi^3} \ . \tag{2.15}$$

Although the calculation is lengthy and complicated, our result for the cross section was obtained independently by several of the authors. In addition, the diagrams for neutralino annihilation into two gluons are similar to some that appear in the calculation for annihilation into two photons. Rudaz performed the calculation for pure photinos and Higgsinos in the limit of large squark masses; Giudice and Griest obtained similar results and generalized to neutralinos of arbitrary mixing, and Bergstrom performed the calculation for photinos for arbitrary squark and photino masses [9]. For large squark masses ($a, b \ll 1$), the expression for the real part of the matrix element [Eq. (2.5)] simplifies to

$$\mathrm{Re}\tilde{\mathcal{M}} \simeq \sum_q \left\{ -\frac{1}{m_{\tilde{q}}^2}\sum_{\tilde{q}}\left[ -S_q + \left[\frac{m_q^2}{m_{\tilde{\chi}}{}^2}S_q + \frac{m_q}{m_{\tilde{\chi}}}D_q\right] I\left[\frac{m_q^2}{m_{\tilde{\chi}}^2}\right]\right] + 2I\left[\frac{m_q^2}{m_{\tilde{\chi}}{}^2}\right]\left[\frac{2m_q}{m_{\tilde{\chi}}}\frac{g_{Aqq}g_{A\tilde{\chi}\tilde{\chi}}}{4m_{\tilde{\chi}}{}^2 - m_A^2} + \frac{m_q^2}{m_{\tilde{\chi}}{}^2 m_Z^2} I_{3q} g_{Z\tilde{\chi}\tilde{\chi}} \frac{g}{\cos\theta_W}\right]\right\} \ . \tag{2.16}$$

If the neutralino is a pure photino, the terms due to $Z^0$ and $A^0$ exchange disappear, and $D_q \to 0$. In this limit we reproduce Bergstrom's result [9] for the box diagram numerically, but our expression (2.5) is significantly more compact, involving only a single (rather than double) integral over Feynman parameters. Note also that, even in the limit $m_q \to 0$, the cross section is nonzero, which reflects the fact that there is no chirality suppression of $S$-wave neutralino annihilation into gluons.

### B. Cross section for $\tilde{\chi}\tilde{\chi} \to g q \bar{q}$

In this subsection we give the cross section for the annihilation of neutralinos into a gluon and quark-antiquark pair. This cross section has been discussed when the neutralino is a pure gaugino [8]; here, we perform the calculation for a neutralino of arbitrary composition. This cross section also remains finite as $m_q \to 0$, as does the gluon-pair cross section discussed in the previous subsection. Moreover, it contains only a single strong-interaction vertex, i.e., only one factor of $\alpha_s$. If the neutralino is heavier than the top quark, annihilation into top quarks is unsuppressed, and the $g\bar{q}q$ final state will be negligible in comparison. Therefore we consider only the case that the neutralino is lighter than the top quark and calculate this cross section in the limit of zero mass for the final state quarks.

In Fig. 2 the diagrams for the cross section for $\tilde{\chi}\tilde{\chi} \to g\bar{q}q$ are displayed. It is worth noting that the expected infrared divergence from soft-gluon emission in these diagrams does not occur in the zero-quark-mass limit. Diagrams (a) and (b) do not vanish in this limit, but their contributions are infrared finite and combine with the infrared finite contribution of diagram (c). As in Fig. 1, the antisymmetrization on the identical initial particles in Fig. 2 is understood. Note that the contributions from $Z^0$ or $A^0$ exchange in the $s$ channel vanish in the massless-quark limit, so that the diagrams of Fig. 2 are the only ones to consider.

After some algebra, we reduce the calculation to the phase space integral

$$\sigma_{g\bar{q}q}v=\sum_{\tilde{q}}\frac{8A_q^4\alpha_s}{\pi^2 m_{\tilde{\chi}}^2}\int_0^1 dx_1\int_0^1 dx_2\Theta(x_1+x_2-1)\frac{(x_1+x_2-1)[x_1^2+x_2^2-2(x_1+x_2-1)]}{(1-2x_1-r_{\tilde{q}})^2(1-2x_2-r_{\tilde{q}})^2}\,, \tag{2.17}$$

where $x_i \equiv E_i/m_{\tilde{\chi}}$ are the normalized energies of the quark and antiquark, $r_{\tilde{q}} \equiv m_{\tilde{q}}^2/m_{\tilde{\chi}}^2$, and $A_q$ is the quark-squark-neutralino coupling defined above ($A_q=\pm B_q$ for massless quarks). The displayed form of the integrand is most convenient as a representation for the double-differential cross section, $d\sigma/dx_1dx_2$. One of the remaining integrals in (2.17) is elementary; integrating this gives

$$\sigma_{g\bar{q}q}v=\sum_{\tilde{q}}\frac{A_q^4\alpha_s}{2\pi^2 m_{\tilde{\chi}}^2}\times\int_0^1 dx\frac{f_1(x)+f_2(x)\ln\left[\dfrac{1+r_{\tilde{q}}}{1+r_{\tilde{q}}-2x}\right]}{(1+r_{\tilde{q}})(2x+r_{\tilde{q}}-1)^2}\,, \tag{2.18}$$

where

$$\begin{aligned} f_1(x)&=-14x-12r_{\tilde{q}}x-6r_{\tilde{q}}^2x+18x^2+2r_{\tilde{q}}x^2-8x^3\,,\\ f_2(x)&=7+13r_{\tilde{q}}+9r_{\tilde{q}}^2+3r_{\tilde{q}}^3-12x-16r_{\tilde{q}}x\\ &\quad-4r_{\tilde{q}}^2x+4x^2+4r_{\tilde{q}}x^2\,. \end{aligned} \tag{2.19}$$

The remaining integral can be treated numerically in direct fashion.

### C. Cross section for $\tilde{\chi}\tilde{\chi}\to f\bar{f}$

For clarity and comparison, we present the cross section for annihilation of neutralinos into fermions and quarks [4,5,7].

In the limit of zero relative velocity, the cross section is

$$\sigma_{\bar{f}f}v=\sum_f\frac{c_f\theta(m_{\tilde{\chi}}^2-m_f^2)}{8\pi}m_{\tilde{\chi}}^2\left[1-\frac{m_f^2}{m_{\tilde{\chi}}^2}\right]^{1/2}\times\left\{\sum_{\tilde{f}}\left[\frac{1}{m_{\tilde{f}}^2+m_{\tilde{\chi}}^2-m_f^2}\left[D_f+S_f\frac{m_f}{m_{\tilde{\chi}}}\right]\right]-\frac{4g_{A\tilde{\chi}\tilde{\chi}}g_{Aqq}}{4m_{\tilde{\chi}}^2-m_A^2}-2g_{Z\tilde{\chi}\tilde{\chi}}I_{3f}\frac{m_q}{m_{\tilde{\chi}}m_Z^2}\frac{g}{\cos\theta_W}\right\}^2\,, \tag{2.20}$$

where the sum on $f$ is over leptons as well as quarks, and the color factor $c_f$ is 3 for quarks and 1 for leptons. This cross section is proportional to the square of the quark mass. (Recall that $|A_f|=|B_f|$ for massless fermions, i.e., $D_f \propto m_f$.) Also note that the cross section is proportional to the square of the imaginary part of the amplitude for neutralino annihilation into gluons, as it should be.

Previously, the cross section for neutralino annihilation into quarks has been evaluated only at the tree level. In addition to the tree-level contributions, there will be QCD corrections to this cross section which can be important. These QCD corrections can broadly be grouped into three classes: emission of a hard gluon at large angle to both quark and antiquark, emission of a hard gluon (almost) collinear to either quark or antiquark, and virtual corrections plus soft-gluon emission (these two must be added to cancel infrared divergences). The first correction has been treated in the previous subsection, where we saw that it remains finite as $m_q \to 0$. The main effect of the second contribution is to change the energy spectrum of the produced quarks; this will be included using Monte Carlo results, as explained in Sec. IV. Finally, the third class of corrections only renormalizes the overall value of the cross section. As usual, the leading logarithms in this correction can be most easily taken into account by introducing "running parameters." This procedure, which is based on the renormalization group, also automatically resums all powers of these logarithms as they arise in higher orders of perturbation theory.

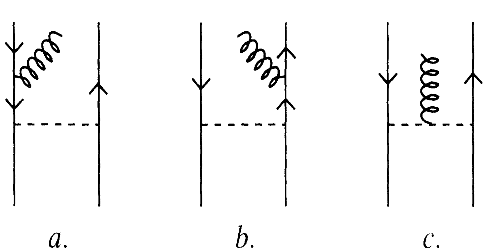


FIG. 2. Feynman diagrams for the tree-level annihilation of neutralinos into a gluon and a quark antiquark pair.

In the case at hand, there are two parameters that "run" due to the QCD corrections: the Yukawa contribution $Z_q$ to the lightest-supersymmetric-particle-(LSP-) quark–squark couplings and the quark mass $m_q$. Equations (2.9) and (2.10) show that Yukawa couplings and masses are proportional, and thus run in the same way. Running quark masses have previously been utilized in calculations of Higgs boson decay widths [16], but they are also useful for the treatment of QCD corrections to the production of massive quark pairs in $e^+e^-$ annihilation [17], which more closely resembles our case of LSP annihilation.

For momentum transfer $m_b < Q < m_t$, where $m_b$ and $m_t$ are the bottom- and top-quark masses, the running masses are given by [16]

$$m_b(Q)=m_b(m_b)\left[\frac{\alpha_s(Q)}{\alpha_s(m_b)}\right]^{12/23} \tag{2.21}$$

and

$$m_c(Q)=m_c(m_c)\left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{12/25}\left[\frac{\alpha_s(Q)}{\alpha_s(m_b)}\right]^{12/23}; \tag{2.22}$$

if $Q > m_t$, the last factor becomes

$$\left[\frac{\alpha_s(Q)}{\alpha_s(m_b)}\right]^{12/23} \rightarrow \left[\frac{\alpha_s(m_t)}{\alpha_s(m_b)}\right]^{12/23}\left[\frac{\alpha_s(Q)}{\alpha_s(m_t)}\right]^{12/21}, \tag{2.23}$$

where $m_b(m_b)=4.5$ GeV and $m_c(m_c)=1.35$ GeV are the on-shell quark masses, and $\alpha_s(Q)$ is the strong coupling constant at scale $Q$ [18]. We use $Q=m_{\tilde{\chi}}$ in our calculation.

For neutralino masses in the range of $\sim 10-100$ GeV, the running of the quark masses is significant, and since the tree-level cross section is proportional to the square of the quark mass, this affects the cross sections for annihilation into quarks significantly. For example, if the neutralino mass is 80 GeV, the running $c$-quark mass is about $\frac{3}{5}$ its tree-level value and the running $b$-quark mass is about $\frac{3}{4}$ its tree-level value; so the cross section for annihilation into quarks is roughly half that suggested by using the tree-level masses. The consequences for the rate of energetic-neutrino production from neutralino annihilation will be discussed below. The running of the quark masses should have little effect on cosmological relic-neutralino abundances because the cross section for neutralino annihilation through a $P$ wave is not governed by the quark mass.

## III. ILLUSTRATIVE EXAMPLES

The cross sections for the various annihilation channels are complicated and depend on many parameters, making it difficult to analytically assess the relative importance of each channel for arbitrary input parameters. Therefore, to illustrate the possible importance of the new channels, at least for a neutralino of some given simple composition, we consider several specific examples.

The first limit we consider is that where the neutralino is a pure photino: $N_{01}=\cos\theta_W$, $N_{02}=\sin\theta_W$, and $N_{03}=N_{04}=0$. In addition, we also assume that the light-fermion masses are small ($m_q \ll m_{\tilde{\chi}}$) and that the photino mass is negligible compared with the squark and top-quark masses ($m_{\tilde{\chi}} \ll m_{\tilde{q}}, m_t$). Furthermore, the pure photino has no coupling to the $Z^0$ or $A^0$ bosons. We also take degenerate squarks and assume no squark mixing. Then $D_q=0$ and $S_q=4\pi\alpha e_q^2$, where $\alpha$ is the electromagnetic fine-structure constant. We also note that $I(x)\simeq 1/x$ for $x \gg 1$. With these assumptions the cross section for photino annihilation into gluons is

$$\sigma_{gg} v \simeq 4\alpha^2\alpha_s^2 \frac{m_{\tilde{\chi}}^{\,2}}{m_{\tilde{q}}^4} . \tag{3.1}$$

With the same assumptions, annihilation into fermions occurs primarily into $\bar{\tau}\tau$ pairs and, to a smaller degree, into $\bar{c}c$ and $\bar{b}b$ pairs. The cross section for annihilation into these channels is

$$\sigma_{\bar{f}f} v \simeq \sum_f \frac{8\pi c_f \alpha^2 e_f^4 m_f^2}{m_{\tilde{f}}^4} ,$$
$$\simeq \frac{120\alpha^2}{m_{\tilde{f}}^2}\left[\frac{1\text{ GeV}}{m_{\tilde{f}}}\right]^2 . \tag{3.2}$$

We have not used running quark masses here. Use of running quark masses decreases this estimate only slightly due to the slow running of the $\tau$-lepton mass. With similar assumptions the cross sections for annihilation into the three-body final states are suppressed by larger inverse powers of the squark mass.

Comparing the annihilation cross sections for the pure photino, we find the ratio

$$\frac{\sigma_{gg}}{\sigma_{\bar{f}f}} \simeq \left[\frac{m_{\tilde{\chi}}}{45\text{ GeV}}\right]^2 , \tag{3.3}$$

valid for $m_b \ll m_{\tilde{\chi}} \ll m_t$ and $m_{\tilde{\chi}} \ll m_{\tilde{q}}$. Therefore the relative importance of the gluon annihilation channel increases roughly with the square of the photino mass and becomes dominant for photino masses greater than about 45 GeV.

We now consider the pure $B$-ino limit: $N_{01}=1$ and $N_{02}=N_{03}=N_{04}=0$. If we use the GUT relation $M_1=\frac{5}{3}M_2\tan^2\theta_W$ and if $M_2 \ll \mu$, then the neutralino is usually primarily $B$-ino. In fact, in most cases where the neutralino is primarily gaugino, it more closely resembles a $B$-ino than a photino. Once again, we take $m_b \ll m_{\tilde{\chi}} \ll m_t, m_{\tilde{q}}$. In this case the cross section for $B$-ino annihilation into gluons is

$$\sigma_{gg} v \simeq \left[\frac{49}{36}\right]^2 \frac{g'^4}{m_{\tilde{q}}^4}\frac{\alpha_s^2 m_{\tilde{\chi}}^2}{8\pi^3} , \tag{3.4}$$

while the cross section for annihilation into light fermions is

$$\sigma_{\bar{f}f} v \simeq \frac{1}{8\pi m_{\tilde{f}}^4}\sum_f c_f m_f^2 (S_f^L + S_f^R)^2$$
$$\simeq 0.3\frac{g'^4}{m_{\tilde{f}}^2}\left[\frac{1\text{ GeV}}{m_{\tilde{f}}}\right]^2 . \tag{3.5}$$

Again, using bare quark masses, we find the ratio

$$\frac{\sigma_{gg}}{\sigma_{\bar{f}f}} \simeq \left[\frac{m_{\tilde{\chi}}}{50\text{ GeV}}\right]^2 , \tag{3.6}$$

a result similar to that in the photino case.

The next limit we consider is the pure-Higgsino limit: $N_{01}=N_{02}=0$ and $N_{03}=N_{04}=1/\sqrt{2}$. We make the same assumptions about the masses. Again, in this limit, $D_q=0$. The main contribution to the $gg$ cross section comes from $t$-quark loops, unless $\tan\beta$ is very large, and the cross section is roughly

$$\sigma_{gg}v \simeq \frac{m_{\tilde{\chi}}{}^6 S_t^2 \alpha_s^2}{18\pi^3 m_{\tilde{q}}^4 m_t^4}\ , \tag{3.7}$$

where we have assumed $m_{\tilde{\chi}}{}^2 \ll m_t^2$. For the light-fermion final states, annihilation occurs primarily into $b$ quarks and

$$\sigma_{\bar{f}f} \simeq \frac{3S_b^2 m_b^2}{2\pi m_{\tilde{q}}^4}\ . \tag{3.8}$$

Using on-shell quark masses, we find that

$$\frac{\sigma_{gg}}{\sigma_{\bar{f}f}} \simeq \left[\frac{m_{\tilde{\chi}}}{20\ \text{GeV}}\right]^6 \cot^4\beta\ . \tag{3.9}$$

If we use running quark masses, the number is closer to 15 GeV. We caution, however, that the LSP is only very rarely sufficiently pure Higgsino for Eq. (3.9) to be applicable even approximately.

In the previous examples, we have considered neutralinos with no coupling to the $Z^0$ or $A^0$ bosons. Now let us consider what happens if the squarks are heavy enough that annihilation into gluons and fermions occurs primarily through, for example, the $A^0$-boson resonance. Since two neutralinos in an $S$ wave cannot produce a physical $Z^0$ boson, the contribution from $Z^0$ exchange is almost always subdominant. Again, we consider $m_b \ll m_{\tilde{\chi}} \ll m_t$. Then decay of the intermediate virtual $A^0$ occurs primarily through the top-quark loop. Of the final-state light fermions, decay occurs primarily into $\bar{b}b$ pairs. With this information we find the ratio

$$\frac{\sigma_{gg}}{\sigma_{\bar{b}b}} \simeq \frac{\alpha_s^2}{3\pi^2\tan^4\beta}\left[\frac{m_{\tilde{\chi}}}{m_b}\right]^2 \simeq \frac{0.15}{\tan^4\beta}\left[\frac{m_{\tilde{\chi}}}{100\ \text{GeV}}\right]^2 . \tag{3.10}$$

Therefore, if the neutralino annihilates primarily through the $A^0$, we do not expect the gluon final state to be significant. Note that this can be a somewhat artificial example since it requires that $2m_{\tilde{\chi}}$ be near $m_A$ for the contribution from the intermediate $A^0$ to be dominant.

In the limit of large squark masses, the cross section for neutralino annihilation into the $\bar{q}qg$ three-body final state to lowest order in $\alpha_s$ is suppressed relative to those for annihilation into the light-quark and two-gluon final states by a factor of $(m_{\tilde{\chi}}/m_{\tilde{q}})^4$ [8]. Although the matrix elements from diagrams (a) and (b) in Fig. 2 are each inversely proportional to $m_{\tilde{q}}^2$ in the large squark-mass limit, these leading-order contributions cancel and the contribution from diagrams (a) and (b), as well as that from diagram (c), to the matrix element is proportional to $m_{\tilde{q}}^{-4}$ in the large squark-mass limit. Therefore, if the squark masses are large, the cross section for annihilation into the three-body channels is smaller than those for annihilation into the $gg$ and $\bar{q}q$ channels. Flores, Olive, and Rudaz have pointed out that to higher order in $\alpha_s$, the cross section for annihilation into $\bar{q}qg$ is *not* suppressed by large inverse powers of the squark mass [8]; however, the cross section is still suppressed relative to the $\tilde{\chi}\tilde{\chi}\to gg$ cross section by additional powers of coupling constants.

We now illustrate the effect of using running quark masses in the cross section for annihilation of neutralinos into $\bar{\tau}\tau$, $\bar{c}c$, and $\bar{b}b$ pairs. If the neutralino is a pure photino, then $\sigma_{\bar{f}f} \propto c_f m_f^2 e_f^4$. Quarks have fractional charge, and so annihilation occurs primarily into $\bar{\tau}\tau$ pairs, and use of the running quark masses decreases the total cross section for annihilation into light fermions only slightly. The same is true if the neutralino is pure $B$-ino.

On the other hand, if the neutralino is a pure Higgsino, the squark-quark-neutralino coupling is a Yukawa coupling and is proportional to the quark mass. Therefore, the cross section for annihilation into $\bar{f}f$ is proportional to $m_f^6$, and so annihilation occurs primarily into $\bar{b}b$ pairs. If $m_b^{\text{run}}$ is the running quark mass and $m_b^{\text{on shell}}$ is the on-shell quark mass, use of running quark masses decreases the annihilation cross section by a factor of $(m_q^{\text{run}}/m_q^{\text{on shell}})^6$. This results in a dramatic decrease in the cross section for annihilation into quark-antiquark pairs. In practice, unless $M_2$ is extremely large, a neutralino that is primarily Higgsino will contain some gaugino component, and so the actual decrease in the $\bar{q}q$ annihilation cross section will hardly ever be as dramatic as indicated by the pure-Higgsino case.

In Fig. 3 we show the effect of QCD corrections to the cross section for annihilation into $\bar{\tau}\tau$, $\bar{c}c$, and $\bar{b}b$ pairs. The ratio of the cross section using running quark masses to that using on-shell quark masses is plotted against neu-

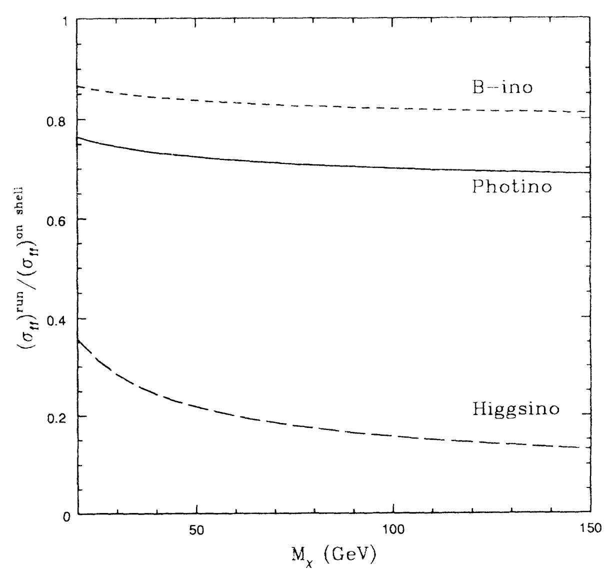


FIG. 3. Effect of QCD corrections to the cross section for annihilation of Higgsinos, photinos, and $B$-inos into light fermions ($\bar{\tau}\tau$, $\bar{c}c$, and $\bar{b}b$ pairs)

tralino masses from 20 to 150 GeV, and we illustrate the results for the cases of a pure photino, $B$-ino, and Higgsino. The graph suggests that the effect of QCD corrections to the tree-level cross sections is at least about 10% and may, in some cases, be as big as 90%. In the next section we will discuss the effect of these results on the energetic-neutrino signals.

In Fig. 4 we illustrate the importance of the $gg$ and $g\bar{q}q$ annihilation channels relative to the light-fermion annihilation channels for a neutralino of more arbitrary composition. In the regions shaded most heavily, the cross section for neutralino annihilation into the $gg$ and $g\bar{q}q$ final states is greater than that for annihilation into light fermions, $\sigma_{gg}+\sigma_{g\bar{q}q}>\sigma_{\bar{f}f}$. In the more lightly shaded regions, the cross sections for the new channels are at least 0.1 times as a big as the cross section for annihilation into light fermions, $\sigma_{gg}+\sigma_{g\bar{q}q}>0.1\sigma_{\bar{f}f}$. In the most lightly shaded regions, the cosmological relic abundance of the neutralino is $\Omega_{\tilde{\chi}}h^2<0.05$, where $\Omega_{\tilde{\chi}}$ is the cosmological mass density of neutralinos in units of the critical density and $h$ is the Hubble parameter in units of 100 km/sec Mpc. In these regions of parameter space, the neutralino is viable, but its abundance is too small to account for the dark matter in galactic halos. In the empty region below and to the left of the solid curve, the mass of the chargino is less than 45 GeV, and so this region is not experimentally viable. In the empty region in the upper right-hand corner of the graph, the neutralino mass is greater than the top-quark mass (which we take to be 150 GeV), and so our results will have little effect. The short-dashed curve indicates the $m_{\tilde{\chi}}=80$ GeV contour, and the diagonal long-dashed curves are of gaugino fractions ($\equiv N_{01}^2+N_{02}^2$) of 0.1, 0.5, and 0.9, as labeled. Therefore, in the bottom right part of the graph, the neutralino is predominantly gaugino, and in the upper left part, the neutralino is predominantly Higgsino. We took $\tan\beta=2$, $m_A=500$ GeV, $m_t=150$ GeV, and $m_{\tilde{q}}=200$ GeV. We also took all squark and slepton masses to be degenerate and assumed no mixing of right and left squarks, and we used running quark masses to implement the leading-logarithmic QCD corrections.

Note that the new channels can be quite important for a large range of masses and neutralino mixings and are especially important if the neutralino is primarily gaugino. Note that in large regions of parameter space, where the neutralino makes a good dark-matter candidate, our results are significant. We should also mention that changes to Fig. 4 should be small if we change the assumed squark mass. This is because in the heavy-squark limit, $\sigma_{gg}/\sigma_{\bar{f}f}$ is squark-mass independent. In addition, in the regions where we show $\Omega_{\tilde{\chi}}h^2\lesssim 0.05$, neutralino annihilation occurs primarily into gauge bosons (where the neutralino is heavier than the $W$ boson and primarily Higgsino) or into light quarks through gauge and/or Higgs bosons (in the regions where the neutralino mass is roughly 50 GeV). The cross sections for these processes do not depend on the squark mass.

In Fig. 5 we illustrate the importance of the three-body final state relative to the $gg$ final state for the same SUSY parameters that were used in Fig. 4. The most heavily shaded regions are those where the cross sections for annihilation into $g\bar{q}q$ are larger than that for annihilation into gluons, $\sigma_{g\bar{q}q}>\sigma_{gg}$. In the more lightly shaded regions, $\sigma_{g\bar{q}q}>0.1\sigma_{gg}$. Again, the most lightly shaded regions are those where the relic abundance of the neutralino is too small to account for the dark matter in galactic halos. Note that the three-body channel seems to be

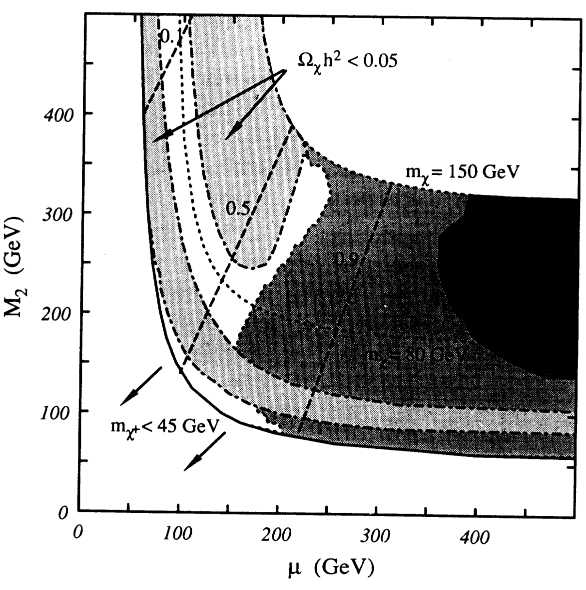


FIG. 4. Strength of $gg$ and $g\bar{q}q$ final states relative to the light-fermion final states. The darkest region corresponds to $\sigma_{gg}+\sigma_{g\bar{q}q}>\sigma_{\bar{f}f}$, the next lightest to $\sigma_{gg}+\sigma_{g\bar{q}q}>0.1\sigma_{\bar{f}f}$, and the lightest region corresponds to $\Omega_{\chi}h^2<0.05$, as indicated. Also shown are contours of $m_{\tilde{\chi}}=80$, 150 GeV and gaugino fraction contours at (0.9,0.5,0.1). See the text for discussion of other parameters.

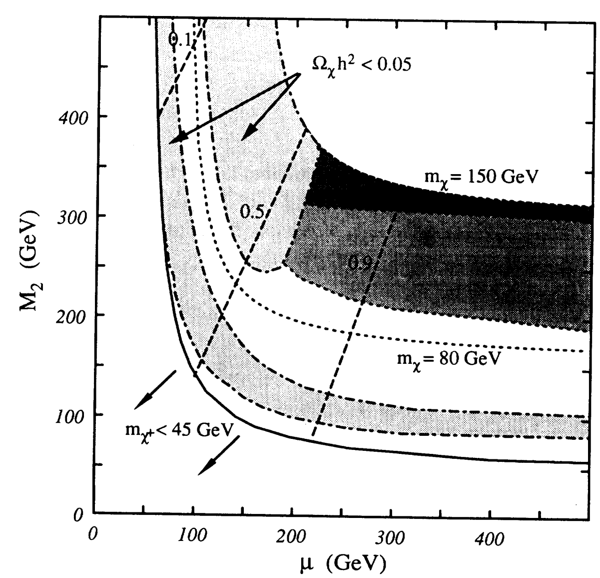


FIG. 5. Strength of the $g\bar{q}q$ final state relative to the $gg$ final state. The darkest region corresponds to $\sigma_{qqg}/\sigma_{gg}>1$, the next lightest to $\sigma_{qqg}/\sigma_{gg}>0.1$, and the lightest to $\Omega_{\chi}h^2<0.05$, as in Fig. 4.

most important only in the regions where the new channels do not have much effect (cf. Fig. 4). If larger squark masses were used, the relative importance of the three-body final state would be even smaller. This suggests that the effect of the three-body final state is generally, although not always, negligible.

We should point out that, in Figs. 4 and 5, we illustrate the importance of the new annihilation channels under specific assumptions about several SUSY parameters. In general, there is a large viable range for several of the parameters, and the relative importance of the new channels for a given set of assumptions may be either greater or smaller than indicated in the limited regions of parameter space that we have explored.

## IV. ENERGETIC NEUTRINOS FROM THE SUN AND EARTH

If neutralinos populate the galactic halo, then they will be captured in the Sun and Earth [12], annihilate therein, and produce high-energy neutrinos that could be detected in underground detectors [10,11]. Neutralinos from the galactic halo are accreted onto the Sun and Earth, and their numbers therein are depleted by annihilation. Typically, the two processes come into equilibrium on a time scale much shorter than the age of the solar system, in which case the rate for neutralino annihilation is equal to the capture rate divided by 2, $C/2$. Then the differential flux of energetic neutrinos of type $i$ (e.g., $i=\nu_\mu,\bar{\nu}_\mu$, etc.) from neutralino annihilation in the Sun or Earth is

$$\left(\frac{d\phi}{dE}\right)_i=\frac{C}{4\pi R^2}\sum_F B_F\left(\frac{dN}{dE}\right)_{Fi} , \qquad (4.1)$$

where $R$ is the distance from the detector to the center of the Sun or Earth, the sum on $F$ is over all annihilation channels, $B_F$ is the branching ratio for annihilation into channel $F$, and $(dN/dE)_{Fi}$ is the differential energy spectrum of neutralino type $i$ at the detector expected from injection of the particles in channel $F$ at the core of the Sun or Earth [19]. Calculation of these spectra can be quite complicated as it involves hadronization of the annihilation products; furthermore, if annihilation takes place in the Sun, interaction of the annihilation products with the solar medium as well as interactions of the neutrinos as they propagate through the Sun must be taken into account.

In the rest of our discussion, we will focus on the neutrino signal from neutralino annihilation in the Earth. If the neutralino is lighter than the top quark, then the neutrino signal from the Earth should be greater than or equal to that from the Sun (unless the neutralino has only axial interactions with nuclei in which case it is captured in the Sun but not in the Earth). Also, calculation of neutrino spectra from annihilation in the Earth is much simpler than the calculation of spectra from the Sun. It should be kept in mind, however, that our conclusions will also apply to neutrino rates from the Sun.

The most promising method of detection of energetic neutrinos is observation of upward moving muons induced by neutrino interactions in the rock below the detector. Given the fluxes $(d\phi/dE)_i$, the rate for neutrino-induced upward-moving muons may be written simply as

$$\Gamma_{\rm detector}=\sum_i D_i\int\left(\frac{d\phi}{dE}\right)_i E^2 dE , \qquad (4.2)$$

where the sum is over $\nu_\mu$, which produce muons, and $\bar{\nu}_\mu$, which produce antimuons. The rate is proportional to some constant $D_i$ times the second moment of the neutrino energy spectrum. This is because the cross section for a neutrino to produce a muon is proportional to the neutrino energy, and the range of the muon is proportional to its energy, giving an overall dependence on the square of the impinging neutrino energy.

If neutralino annihilation takes place in the Earth, then interactions of the annihilation products and neutrinos with the Earth can be neglected; thus, the neutrino-energy spectrum from a given annihilation channel $F$ can be determined in a straightforward way from the results of Monte Carlo calculations [19]. Furthermore, in this case, the energy spectra of neutrinos and antineutrinos are the same, and so the rate for neutrino-induced upward-moving muons from neutralino annihilation in the Earth may be written

$$\Gamma_{\rm detector}=3.9\times10^{-20}\left(\frac{C}{\rm sec^{-1}}\right)\left(\frac{m_{\tilde{\chi}}}{\rm GeV}\right)^2 \times\sum_F B_F\langle Nz^2\rangle_F\ {\rm m^{-2}\,yr^{-1}} . \qquad (4.3)$$

The quantity $\langle Nz^2\rangle_F$ is the second moment of the energy spectrum of neutrinos produced from final state $F$ divided by the neutralino mass squared. Final-state electrons, muons, and $u$, $d$, and $s$ quarks will not produce energetic neutrinos. The weak decays of $\tau$ leptons and $c$ and $b$ quarks produce energetic neutrinos, and expressions for $\langle Nz^2\rangle$ for these final states have been given by Ritz and Seckel [19]. For neutralino masses large compared with the light-fermion masses, $\langle Nz^2\rangle_{\bar{b}b}=0.0195$, $\langle Nz^2\rangle_{\bar{c}c}=0.0084$, and $\langle Nz^2\rangle_{\bar{\tau}\tau}=0.0682$; thus, the $\bar{\tau}\tau$ final state gives the strongest neutrino signal of the fermionic final states. If the neutrino annihilates into gauge-boson pairs, then energetic neutrinos are produced directly by the decays of the gauge bosons, and $\langle Nz^2\rangle_{W^+W^-}=0.035[1-(m_W^2/4m_{\tilde{\chi}}^2)]$ and $\langle Nz^2\rangle_{ZZ}=0.045[1-(m_Z^2/4m_{\tilde{\chi}}^2)]$ [11]. Expressions for $\langle Nz^2\rangle$ from final states with Higgs bosons may also be given [10]. For annihilation in the Earth, the second moment of the neutrino spectrum from a Higgs boson $B$ is roughly $\langle Nz^2\rangle_B\simeq\sum_f\langle Nz^2\rangle_f\Gamma_f^B/2$, where the sum on $f$ is over all the decay channels of the $B$ boson and $\Gamma_f^B$ is the branching ratio for $B$ decay into final state $f$. The branching ratios are given in, e.g., Ref. [20]. Leading-order QCD corrections to these results may be included by using running quark masses instead of the tree-level quark masses. Although we will not need it here, $\langle Nz^2\rangle$ for the top-quark final state can be obtained by noting that the top quark will decay predominantly into a $b$

quark and a $W$ boson and using the results for $\langle Nz^2 \rangle$ for these final states.

Given Eq. (4.3) and the estimates of $\langle Nz^2 \rangle$ above, we can see qualitatively the effect of including the $gg$ and $g\bar{q}q$ annihilation channels, as well as the effect of running quark masses on rates for energetic-neutrino events from neutralino annihilation in the Earth. The gluons will produce essentially no energetic neutrinos. Energetic neutrinos will come from weak decays of heavy quarks, and only a small fraction of the gluon energy goes into heavy quarks. Thus, if the cross section for annihilation into gluons is appreciable, the branching ratios $B_F$ into the annihilation channels that *do* produce energetic neutrinos are decreased, and the event rate, given by Eq. (4.3), is decreased accordingly.

The effect of annihilation into $g\bar{q}q$ ($q=b,c$) is similar, though not as severe, as the effect of annihilation into gluons. Again, the emitted gluon will not produce any energetic neutrinos. The quark and antiquark will produce neutrinos, but their energies will be decreased leading to a smaller event rate. In the heavy-squark limit, $r_{\tilde{q}}=m_{\tilde{q}}^2/m_{\tilde{\chi}}^2\to\infty$, the gluon carries away $\frac{1}{3}$ the available energy, which suggests that the rate for energetic-neutrino events from the three-body final state containing $c$ or $b$ quarks would be roughly $\frac{4}{9}$ what it would be if the neutralino annihilated into $\bar{q}q$. As the ratio $r_{\tilde{q}}$ is decreased toward unity, the gluon carries away a larger fraction of the available energy, thereby weakening the neutrino signal. Moreover, more than 50% of the total $q\bar{q}g$ contribution comes from light ($u,d,s$) quarks. Therefore the energetic-neutrino flux from the three-body final state is generally small: If $r_{\tilde{q}}\sim 1$, then the branching ratio may be significant, but the neutrino signal is small; if $r_{\tilde{q}}$ is large, the branching ratio into the three-body final state is suppressed. We have not done the calculation of the neutrino spectrum from this final state more precisely than indicated here. A more precise calculation would simply involve a convolution of the neutrino energy spectra from quark-antiquark pairs with the differential cross section in Eq. (2.17).

Although the new annihilation channels tend to decrease the event rate, leading-order QCD corrections to the tree-level cross section for neutralino annihilation into $b$ and $c$ quarks have the opposite effect. QCD corrections decrease the cross sections for annihilation into $b$ and $c$ quarks, and therefore increase the branching ratio into $\bar{\tau}\tau$ leptons. $\tau$ leptons provide a stronger signal, and so the neutrino event rate is increased. In Fig. 6 we illustrate this effect by plotting the ratio of the neutrino event rate obtained using running quark masses divided by that obtained using on-shell quark masses. We illustrate the results for the cases of a pure photino, pure $B$-ino, and a pure Higgsino. In all three cases, we have assumed annihilation occurs only into light fermions. The effect is largest for the photino. This is because the cross sections for annihilation of photinos into $\tau$ leptons is comparable to that into quarks. For $B$-inos, the branching ratio to $\tau$ leptons is larger, and for Higgsinos, the branching ratio to the $b$ quark is larger. Recall that the energetic-neutrino flux from a given annihilation channel

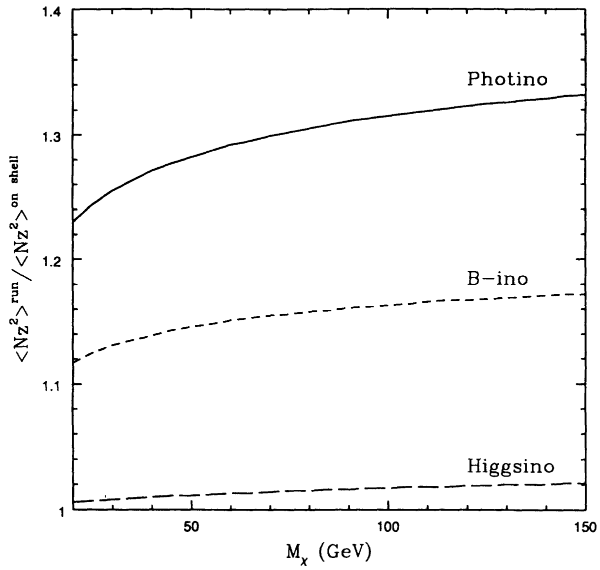


FIG. 6. Effect of QCD corrections to the tree-level cross sections for $\tilde{\chi}\tilde{\chi}\to\bar{q}q$ on energetic-neutrino event rates. We illustrate the results for the cases of a pure photino, pure $B$-ino, and pure Higgsino.

depends on the branching ratio into that channel, and *not* on the total annihilation cross section.

In Fig. 7 we illustrate the combined effect of including the $gg$ and $g\bar{q}q$ final states, as well as the QCD-corrected cross sections for annihilation into $\bar{q}q$ pairs, on the rate for energetic-neutrino events. We consider the ratio of the event rate predicted using our new results to that which would be predicted ignoring the new annihilation channels and the QCD corrections to the light-fermion

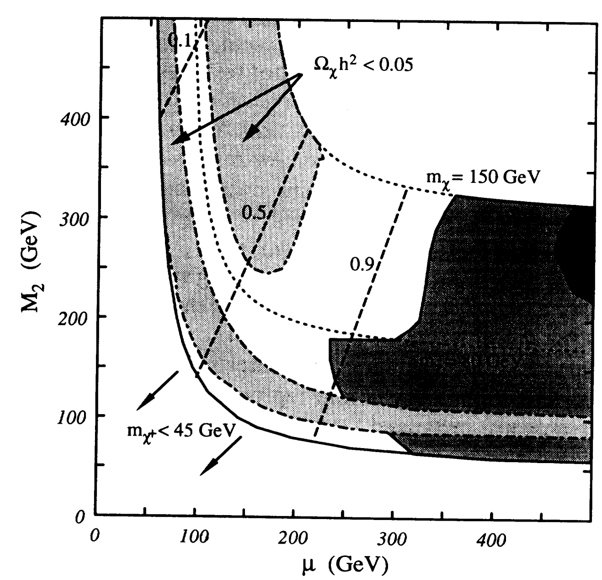


FIG. 7. Combined effect of new results on energetic-neutrino event rates. The darkest region corresponds to $\langle Nz^2\rangle_{QCD}/\langle Nz^2\rangle_{tree}<0.5$, the next lightest to $\langle Nz^2\rangle_{QCD}/\langle Nz^2\rangle_{tree}<0.9$, and the lightest region corresponds to $\Omega_\chi h^2<0.05$, as in the previous plots.

annihilation cross section, $\langle Nz^2\rangle_{\rm QCD}/\langle Nz^2\rangle_{\rm tree}$. In the most heavily shaded regions (where the neutralino is primarily gaugino), the ratio is less than 0.5. In the more lightly shaded regions, the ratio is less than 0.9. In the rest of the graph, the ratio is greater than 0.9, but it is nowhere greater than 1.1. Again, the most lightly shaded regions are those where the cosmological abundance is too small to account for the dark matter in galactic halos. The SUSY parameters used here are the same as those used in Figs. 4 and 5: $m_A=500$ GeV, $m_t=150$ GeV, $\tan\beta=2$, and $m_{\tilde q}=200$.

We have included all annihilation channels in this graph. For our numerical work, we assumed that the $g\bar{q}q$ final state produced no energetic neutrinos. This underestimates the true flux, but we are confident that, if the calculation were performed more precisely, the results in Fig. 7 would not be altered. We believe so because our estimate of the neutrino yield from the three-body final state is small and because Figs. 4 and 5 suggest the new annihilation channels are important only when the three-body final state is subdominant. As was the case for Fig. 4, Fig. 7 will change little qualitatively if the squark mass is changed.

The graph illustrates that our results are most important for gauginos, as discussed in the previous section. Also, note that our results are important in large regions of parameter space where the neutralino makes a good dark-matter candidate. Again, we should point out that we have explored only a restricted region of parameter space. The effect of our results on energetic-neutrino fluxes may be larger or smaller depending on the specific SUSY parameters used.

## V. SUMMARY AND DISCUSSION

We have calculated the cross sections for annihilation of neutralinos into the two-gluon and gluon-quark-antiquark final states in the nonrelativistic limit. We have also calculated QCD corrections to the tree-level matrix elements for annihilation of neutralinos into quark-antiquark pairs. These new results should have little impact on existing neutralino cosmological-abundance calculations, although they will be significant for event rates for indirect-detection schemes.

Since neutralino annihilation into light quarks and leptons is helicity suppressed in the nonrelativistic limit, the rate for annihilation into gluons, although suppressed by $\alpha_s^2$, may be comparable to or greater than that for annihilation into light quarks and leptons. Very few energetic neutrinos are produced by hadronization of gluons in the Sun and Earth, and so annihilation into gluons tends to decrease the rate for energetic-neutrino events. The new annihilation channels should have a significant effect if the neutralino annihilates predominantly into light fermions. If the neutralino is heavier than the top quark or if it is primarily Higgsino and heavier than the $W$ boson, then it will annihilate predominantly into top quarks or gauge bosons, respectively, and the new channels should have little effect.

QCD corrections decrease the rate for annihilation into light quarks. The flux of energetic neutrinos depends on the branching ratios into the various final states, and not on the total annihilation cross section. In addition, the neutrino signal from $\bar{\tau}\tau$ pairs is stronger than that from light quarks. Therefore, since the branching ratio into $\bar{\tau}\tau$ pairs is increased, while the branching ratios into light quarks are decreased, QCD corrections to the process $\tilde{\chi}\tilde{\chi}\rightarrow\bar{q}q$ increase the energetic-neutrino flux.

The combined effect of the new annihilation channels and QCD corrections is greater than 10% in large regions of parameter space. If the neutralino is primarily gaugino, the rate for annihilation into gluons may be greater than the rate for annihilation into light fermions, and the flux of energetic neutrinos will be decreased dramatically. In some regions of parameter space, the two effects yield a slight increase in the neutrino flux, although this increase is no greater than about 10% in any regions of parameter space we explored.

We have not performed a detailed calculation of the flux of energetic neutrinos that come from the $g\bar{q}q$ final state. If the squark is only slightly heavier than the neutralino, then the gluon carries away most of the available energy, and the energetic-neutrino yield should be small. On the other hand, if the squark mass is large, the branching ratio for annihilation into the three-body final state becomes negligible.

In conclusion, the $gg$ and $g\bar{q}q$ final states, as well as QCD corrections to the tree-level amplitudes for annihilation into $\bar{q}q$ pairs, are appreciable for many regions of parameter space. These new results should therefore be included in analyses that constrain SUSY dark-matter candidates from limits on fluxes of energetic neutrinos from the Sun and Earth.

The new results will also have implications for cosmic-ray antiproton searches [9]. If neutralinos populate the galactic halo, they will annihilate and produce low-energy antiprotons. Cosmic-ray antiprotons are produced in standard models of cosmic-ray propagation by spallation of cosmic rays in the interstellar medium, and therefore provide a background to the signal from WIMP's. However, this background decreases dramatically for cosmic-ray antiproton energies less than about 1 GeV. There are many astrophysical uncertainties associated with the predicted fluxes of cosmic-ray antiprotons from WIMP annihilation, and so nonobservation of such cosmic rays cannot be used to eliminate dark-matter candidates; on the other hand, under certain reasonable assumptions about the relevant astrophysics and particle physics, the predicted flux of low-energy cosmic-ray antiprotons will be large enough to be distinguished from background. Observational upper limits to the cosmic-ray antiproton flux are currently about an order of magnitude larger than the background expected [21]. By performing similar cosmic-ray experiments at the South Pole, the sensitivity of these experiments can be improved by several orders of magnitude [22], and so a cosmic-ray–antiproton signature for neutralino dark matter in the halo should be observable, should such a signature exist.

Our results will have an effect on the cosmic-ray antiproton flux from neutralino annihilation in the halo with magnitude similar to that on rates for energetic-neutrino events from annihilation in the Sun and Earth; however, the effect is converse to that discussed above. QCD corrections decrease the cross sections for annihilation into light quarks, and so the number of antiprotons produced from hadronization of light quarks will be decreased. On the other hand, if annihilation into gluons is appreciable, then the antiproton flux will be increased because hadronization of gluons will produce antiprotons (although Monte Carlo results will be needed to determine the precise contribution). The enhancement will be most significant if the neutralino is primarily gaugino. We should also point out that if the new annihilation channels decrease the neutrino rate, then they will increase the cosmic-ray antiproton flux. Therefore, cosmic-ray antiproton searches provide an excellent complement to energetic-neutrino searches.

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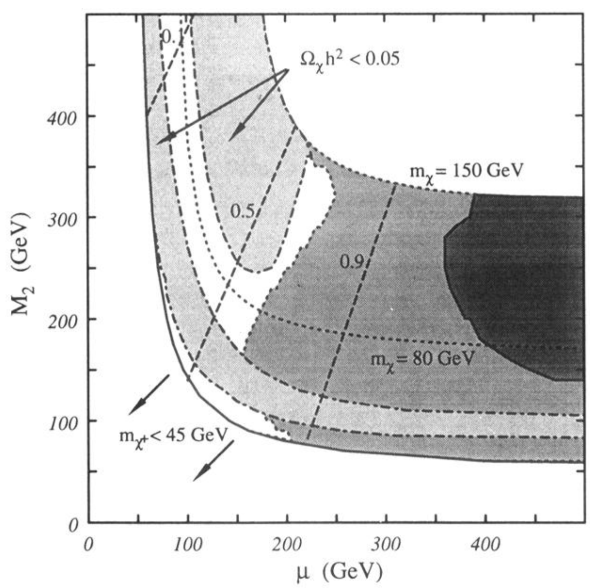


FIG. 4. Strength of $gg$ and $g\bar{q}q$ final states relative to the light-fermion final states. The darkest region corresponds to $\sigma_{gg} + \sigma_{g\bar{q}q} > \sigma_{\bar{f}f}$, the next lightest to $\sigma_{gg} + \sigma_{g\bar{q}q} > 0.1\sigma_{\bar{f}f}$, and the lightest region corresponds to $\Omega_\chi h^2 < 0.05$, as indicated. Also shown are contours of $m_{\tilde{\chi}} = 80$, 150 GeV and gaugino fraction contours at (0.9,0.5,0.1). See the text for discussion of other parameters.

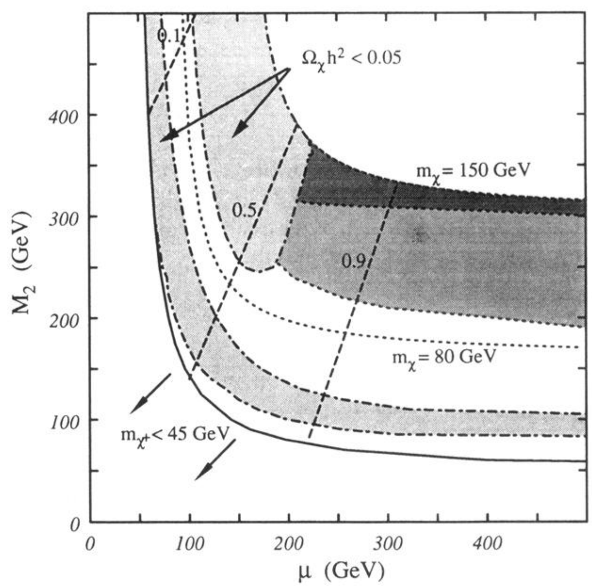


FIG. 5. Strength of the $g\bar{q}q$ final state relative to the $gg$ final state. The darkest region corresponds to $\sigma_{qqg}/\sigma_{gg} > 1$, the next lightest to $\sigma_{qqg}/\sigma_{gg} > 0.1$, and the lightest to $\Omega_\chi h^2 < 0.05$, as in Fig. 4.

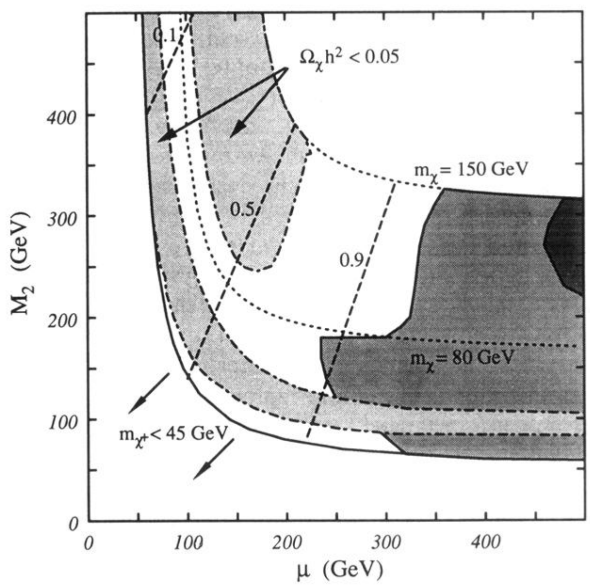


FIG. 7. Combined effect of new results on energetic-neutrino event rates. The darkest region corresponds to $\langle Nz^2\rangle_{QCD}/\langle Nz^2\rangle_{tree} < 0.5$, the next lightest to $\langle Nz^2\rangle_{QCD}/\langle Nz^2\rangle_{tree} < 0.9$, and the lightest region corresponds to $\Omega_\chi h^2 < 0.05$, as in the previous plots.