COMMENTS

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New wrinkle in the potential for heavy quarkonium

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Two new quark-antiquark potentials have recently been applied to heavy quarkonium. The new potentials have the feature that they are concave downward as a function of the interquark separation R except in a small interval at intermediate values of R. I comment on this new "wrinkle" in the potentials.

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According to QCD perturbation theory, the static potential between a heavy quark and antiquark is Coulomb-like at short distances between the particles. At large distances, QCD perturbation theory fails, but nonperturbative treatments with lattice and string models indicate that the potential is approximately linear. An early potential incorporating these two features is the Cornell potential [1]. Since then, many authors have considered potentials having (exact or approximate) Coulomb behavior at short distances and linear behavior at large distances, sometimes with a phenomenological interpolated behavior at intermediate distances. Rather than list a large number of papers on the subject, I refer to two reviews [2,3], which contain additional references.

A basic feature of many of the quark-antiquark potentials considered in the literature, including the Cornell potential [1], is that they are monotonically increasing and are everywhere concave downward as a function of the separation R between the quark and antiquark. This means that the potential $V(R)$ has the property

$$\frac{dV}{dR}>0, \quad \frac{d^2V}{dR^2}<0, \quad 0<R<\infty \ . \tag{1}$$

This behavior appears to be a general feature of gauge-field theory [4–6], including QCD. Phenomenological consequences of a concave-downward potential have been discussed [7].

Recently, Bambah *et al.* [8] (BDKS) introduced two new potentials for heavy quarkonia, which violate the concavity condition. These authors, and also Kaur and Bambah [9] (KB) used the new potentials to calculate masses and leptonic decay widths of heavy quarkonia. The potentials of BDKS are approximately Coulomb-like at small distances and approximately linear at large distances, with stringlike corrections [10–12]. The authors start with a zeroth-order potential of the form

$$\begin{aligned} V_0(R) &= -\frac{4\alpha_s(R)}{3R}, \quad R<R_c \ , \\ V_0(R) &= K\sqrt{R^2-R_c^2}, \quad R>R_c \ , \end{aligned} \tag{2}$$

where $\alpha_s(R)$ is the strong-interaction coupling strength, K is the string tension, and R_c is a critical distance, which is related to K. This zeroth-order potential is discontinuous at $R=R_c$.

In the next step, BDKS obtain two continuous potentials, which they call V_1 and V_2, by smoothing the potential phenomenologically in two ways in a small interval near $R=R_c$. Each of the potentials V_1 and V_2 has a wrinkle (actually, more like a rounded step) near $R=R_c$, and thus violates the concavity condition (1), as can be seen from Figs. 2 and 3 of BDKS. The second of these potentials is also used by KB (see Fig. 2 of KB), although there it is called V_r.

Of course, the potentials of BDKS are not the only ones that violate the concavity condition. A number of phenomenological potentials are not concave, including all power-law potentials of the form

$$V=aR^b, \quad a>0, \quad b>1 \ . \tag{3}$$

In particular, the simple harmonic oscillator potential, which has been frequently used, is everywhere convex as a function of R. However, the BDKS potentials are the only ones I have seen, which have a wrinkle, i.e., are concave downward everywhere except in a small interval at intermediate values of R.

Although the BDKS potentials V_1 and V_2 may be inconsistent with QCD [4–6], it is interesting to ask whether there is any phenomenological evidence for a wrinkle in the potential for heavy quarkonium. It is to this question that I now turn.

So far, no one has been able to obtain agreement with the observed energy levels in charmonium and bottomonium by using a concave-downward potential in the Schrödinger equation. One reason is that the energy levels of concave-downward potentials appear to satisfy an inequality, which does not hold among the experimental levels, as I now discuss.

When I use a concave-downward potential to calculate numerically the energy eigenvalues E_n (with zero orbital

TABLE I. Observed differences in charmonium and bottomonium $^3S_1$ wave energy levels in MeV, adapted from the tables of the Particle Data Group [14].

States	Energy differences charmonium	Energy difference bottomonium
$2\,^3S_1-1\,^3S_1$	589	563
$3\,^3S_1-2\,^3S_1$	354	332
$4\,^3S_1-3\,^3S_1$	375	225
$5\,^3S_1-4\,^3S_1$	?	285
$6\,^3S_1-5\,^3S_1$	?	154

angular momentum), I find that the separation in energy between two adjacent energy levels decreases as the level number n increases. The condition is

$$E_{n+2}-E_{n+1}<E_{n+1}-E_n \ . \tag{4}$$

I have not been able to prove that the inequality (4) holds for all concave-downward potentials or even for all n in the potentials I have considered, but I believe the inequality is true.

For a power-law potential, Quigg and Rosner [13] have proved in the WKB approximation that the inequality (4) holds for any orbital angular momentum, provided $-2\leq b<2$. If $b>2$, the inequality sign in (4) is reversed.

However, the data [14] are not in accord either with the inequality (4) or with inequality obtained when the inequality sign in (4) is reversed. Although the inequality (4) holds for most levels in heavy quarkonia, it is violated for $n=2$ in charmonium and for $n=3$ in bottomonium. This can be seen from Table I. In charmonium,

$$E_4-E_3>E_3-E_2 \ , \tag{5}$$

while in bottomonium,

$$E_5-E_4>E_4-E_3 \ . \tag{6}$$

I conclude that if heavy quarkonium energy levels are to be explained by a static potential within the framework of the nonrelativistic Schrödinger equation, the potential can neither be everywhere concave downward, as seemingly required by QCD, nor behave like a power. (A power-law potential with $b<1$ is a special case of a concave-downward potential.)

If a concave-downward or power-law potential is inadequate to explain the charmonium and bottomonium data, then what is the explanation? The wrinkles in the potentials V_1 and V_2 of BDKS, while perhaps marginally lessening the disagreement with experiment, do not reproduce the features (5) and (6) of the experimental charmonium and bottomonium levels. Nevertheless, in principle, as far as I can tell, one should be able to reproduce both the observed charmonium and bottomonium spectra with a potential that violates the concavity condition in some interval or intervals of R. Such a phenomenological potential, if sufficiently simple in form, might be a useful correlator of data.

Despite the possibility that one might be able to fit the data with a potential with one or more wrinkles, one should not conclude that the quark-antiquark interaction is in fact a static potential with wrinkles. There are, of course, other mechanisms, which can affect the relative positions of the energy levels. For example, the existence of decay processes, which are not taken into account in the static potential model discussed by BDKS, can shift energy levels from their potential-model values so as to violate the inequality (4) even if the static potential is concave downward. In particular, the observed violation of the inequality (4) occurs in charmonium and bottomonium near the thresholds for the opening of new decay channels. Some calculations [15–17] have already demonstrated the qualitative effect of the opening of decay channels on the spectra of heavy quarkonia. But if one insists on a phenomenological description of heavy quarkonium energy levels with a static potential and no other interactions, one may need a wrinkle or wrinkles in the potential.

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