Remarks on $D_s^* \to D_s \pi^0$ decay

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We calculate the rate for $D_s^* \to D_s \pi^0$ decay using chiral perturbation theory. This isospin-violating process results from π^0-η mixing, and its amplitude is proportional to $(m_d - m_u)/[m_s - (m_u + m_d)/2]$. Experimental information on the branching ratio for $D_s^* \to D_s \pi^0$ can provide insight into the pattern of SU(3) violation in radiative D^* decays.

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The strong and radiative decays of D^* mesons have been studied in Refs. [1–6] using a synthesis of chiral perturbation theory and the heavy quark effective theory. In this Brief Report, we extend the analysis of such transitions to include the isospin-violating mode $D_s^* \to D_s \pi^0$. We describe how experimental information on this process can yield insight into the pattern of SU(3) breaking in radiative D^* decays.

Isospin violation enters into the low energy strong interactions of the π, K, and η pseudo Goldstone bosons through their mass term

$$\mathcal{L}_{\text{mass}} = \frac{\mu f^2}{4} \text{Tr}(\xi m_q \xi + \xi^\dagger m_q \xi^\dagger) \tag{1}$$

in the chiral Lagrange density. Here $\xi = \exp(iM/f)$ represents a 3×3 special unitary matrix that incorporates the meson octet,

$$M = \begin{bmatrix} \pi^0/\sqrt{2} + \eta/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2} + \eta/\sqrt{6} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\tfrac{2}{3}}\eta \end{bmatrix} , \tag{2}$$

and m_q denotes the light quark mass matrix:

$$m_q = \begin{bmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{bmatrix} . \tag{3}$$

The exponentiated Goldstone field and Lagrange density in (1) transform as $\xi \to L \xi U^\dagger = U \xi R^\dagger$ and $(3_L, \bar{3}_R) + (\bar{3}_L, 3_R)$ under the chiral symmetry group $SU(3)_L \times SU(3)_R$. The Goldstone boson mass term contains the off-diagonal interaction

$$\mathcal{L}_{\text{mixing}} = \mu \frac{m_d - m_u}{\sqrt{3}} \pi^0 \eta , \tag{4}$$

which mixes the $I=1$ neutral π with the $I=0$ η. This isospin-violating mixing vanishes in the limit of equal up and down quark masses.

The low energy interactions of pseudo Goldstone bosons with mesons containing a single heavy quark are described by the leading order chiral Lagrange density

$$\mathcal{L} = -i\,\text{Tr}\bar{H}^a v_\mu \partial^\mu H_a + \frac{i}{2}\text{Tr}\bar{H}^a H_b v_\mu (\xi^\dagger \partial^\mu \xi + \xi \partial^\mu \xi^\dagger)^b_a + \frac{ig}{2}\text{Tr}\bar{H}^a H_b \gamma_\mu \gamma_5 (\xi^\dagger \partial^\mu \xi - \xi \partial^\mu \xi^\dagger)^b_a . \tag{5}$$

The 4×4 matrix

$$H_a = \frac{1+\not{v}}{2}(P_a^{*\mu}\gamma_\mu - P_a \gamma_5) \tag{6}$$

combines together velocity-dependent pseudoscalar and vector meson fields that carry a light quark flavor index a. When the heavy quark inside the meson is charm, the individual components of the fields in H are $(D_1^{(*)}, D_2^{(*)}, D_3^{(*)}) = (D^{(*)0}, D^{(*)+}, D_s^{(*)})$. H transforms under heavy quark spin symmetry $SU(2)_v$ and chiral $SU(3)_L \times SU(3)_R$ as $H_a \to S(HU^\dagger)_a$ where $S \in SU(2)_v$.

The interaction term proportional to g in the Lagrange density (5) mediates the strong decays $D^{*+} \to D^0 \pi^+$, $D^{*+} \to D^+ \pi^0$, and $D^{*0} \to D^0 \pi^0$. The tree level rate for the charged π mode is

$$\Gamma(D^{*+} \to D^0 \pi^+) = \frac{g^2}{6\pi f_\pi^2} |\mathbf{p}_{\pi^+}|^3 , \tag{7}$$

while the corresponding widths in the neutral π channels are a factor of 2 smaller due to isospin. Unfortunately, the value for g has not been extracted from these single-π partial widths since the total widths of charmed vector mesons are too narrow to be experimentally resolved. The coupling constant can therefore only be estimated in various models. In the nonrelativistic constituent quark model, one finds $g=1$,[1] whereas $g=0.75$ in the chiral quark model.

Charmed vector mesons can also decay to their pseudoscalar counterparts via single-photon emission. The matrix elements for such electromagnetic transitions have the general form

$$\mathcal{A}(D_a^* \to D_a \gamma) = e \mu_a \epsilon^{\mu\nu\sigma\lambda} \varepsilon_\mu^*(\gamma) v_\nu k_\sigma \varepsilon_\lambda(D^*) , \tag{8}$$

[1]This estimate is similar to the result $g_A = \frac{5}{3}$ for the π-nucleon coupling.

where $e\mu_a/2$ is the transition magnetic moment, k is the photon momentum, $\varepsilon(\gamma)$ is the photon polarization, and $\varepsilon(D^*)$ is the D^* polarization. After squaring the radiative amplitude, averaging over the initial state polarization, and summing over the final state polarization, one finds the electromagnetic partial width

$$\Gamma(D_a^* \to D_a\gamma) = \frac{\alpha_{\rm EM}}{3}|\mu_a|^2|\mathbf{k}|^3 \ . \tag{9}$$

The transition magnetic moments which enter into the radiative matrix element (8) receive contributions from photon couplings to both the heavy charm and light quark electromagnetic currents $\frac{2}{3}\bar{c}\gamma_\mu c$ and $\frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s$. They consequently decompose as $\mu_a = \mu^{(h)} + \mu_a^{(l)}$. The charm magnetic moment is fixed by heavy quark spin symmetry to be $\mu^{(h)} = 2/(3m_c)$. The $\mu_a^{(l)}$ moments on the other hand are *a priori* undetermined. In the SU(3) symmetry limit, they are proportional to the electric charges Q_a of the light quarks:

$$\mu_a^{(l)} = \beta Q_a \ . \tag{10}$$

The proportionality constant β has dimensions of inverse mass and represents an unknown reduced matrix element.

The strong and electromagnetic interactions compete in D^* meson decays. The inherently weaker strength of the radiative modes is offset by the limited phase space available in the strong interaction channels. This competition is unusual in the strange charmed sector where the small $D_s^* \to D_s\gamma$ process dominates over the even smaller isospin-violating transition $D_s^* \to D_s\pi^0$. The neutral π mode proceeds at tree level via virtual η emission, as illustrated in Fig. 1. The intermediate η converts into a π^0 through the mixing term in Eq. (4). The η propagator effectively renders the $D_s^* \to D_s\pi^0$ amplitude inversely proportional to the strange quark mass. Consequently, the diagram contains the isospin violation factor [8]

$$(m_d - m_u)/[m_s - (m_u + m_d)/2] \simeq 1/43.7 \ , \tag{11}$$

which is larger than one might have naively guessed. There is also an electromagnetic contribution to the $D_s^* \to D_s\pi^0$ amplitude. But since it is reduced by $\alpha_{\rm EM}/\pi \simeq 1/430$, the electromagnetic term is expected to be less important than the strong interaction contribution

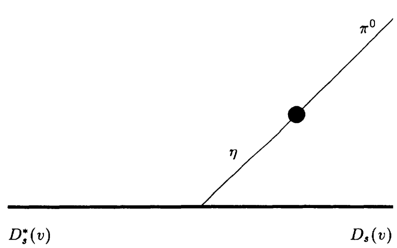

FIG. 1. Tree graph that mediates $D_s^* \to D_s\pi^0$. The solid circle represents the isospin-violating π^0-η mixing vertex proportional to $(m_d - m_u)$.

which we focus upon here.

A straightforward computation of the rate for the isospin-violating decay yields

$$\Gamma(D_s^* \to D_s\pi^0) = \frac{g^2}{48\pi f_\eta^2}\left[\frac{m_d - m_u}{m_s - (m_u + m_d)/2}\right]^2 |\mathbf{p}_{\pi^0}|^3 \ . \tag{12}$$

This partial width sensitively depends upon the D_s^*-D_s mass splitting which determines the magnitude of the neutral π's three-momentum. Using the improved value for this splitting recently reported by the CLEO Collaboration

$$M_{D_s^*} - M_{D_s} = 144.22 \pm 0.60 \text{ MeV}$$

[9], we find $|\mathbf{p}_{\pi^0}| = 49.0$ MeV. Note that we have set the parameter f in Eq. (12) equal to $f_\eta = 171$ MeV rather than $f_\pi = 132$ MeV as in Eq. (7). The difference between these two decay constants represents an SU(3)-breaking effect that is higher order in chiral perturbation theory. Our use of f_η diminishes the magnitude of $\Gamma(D_s^* \to D_s\pi^0)$ and provides a conservative estimate for the impact of higher order terms in the chiral Lagrangian on the decay rate.

It is instructive to determine the order of magnitude of the $D_s^* \to D_s\pi^0$ branching fraction in the SU(3) symmetry limit. Taking the ratio of Eq. (12) to (7) and using SU(3) to set $\Gamma(D_s^* \to D_s\gamma) = \Gamma(D^{*+} \to D^+\gamma)$ and $f_\eta = f_\pi$, we find

$$\begin{aligned} B(D_s^* \to D_s\pi^0) &\simeq \frac{\Gamma(D_s^* \to D_s\pi^0)}{\Gamma(D_s^* \to D_s\gamma)} \\ &= \frac{1}{8}\left[\frac{m_d - m_u}{m_s - (m_u + m_d)/2}\right]^2 \\ &\quad \times \frac{|\mathbf{p}_{\pi^0}|^3}{|\mathbf{p}_{\pi^+}|^3}\,\frac{B(D^{*+} \to D^0\pi^+)}{B(D^{*+} \to D^+\gamma)} \ . \end{aligned} \tag{13}$$

We then insert the experimentally measured branching fraction $B(D^{*+} \to D^0\pi^+) = 68.1\%$ [7] and the quark mass ratio value from Eq. (11) to deduce

$$B(D_s^* \to D_s\pi^0) \simeq 8 \times 10^{-5}/B(D^{*+} \to D^+\gamma) \ . \tag{14}$$

SU(3) corrections to this result are likely to be significant. In particular, the rate for $D_s^* \to D_s\gamma$ is very sensitive to SU(3) breaking in the transition magnetic moment μ_3. We can investigate the general impact of SU(3) violation upon radiative D^* decays in the nonrelativistic constituent quark model. In this model, the constant β which enters into $\mu_a^{(l)}$ becomes dependent upon the subscript a and is replaced by the inverse of the constituent quark mass. The down and strange quark magnetic moments $\mu_2^{(l)}$ and $\mu_3^{(l)}$ are then negative while the charm quark moment $\mu^{(h)}$ is smaller but positive. A partial cancellation between the heavy and light magnetic moments thus occurs in the radiative decay $D^{*+} \to D^+\gamma$, which is reflected in the small experimental upper bound on its branching fraction $B(D^{*+} \to D^+\gamma) < 4.2\%$ (90% C.L.)

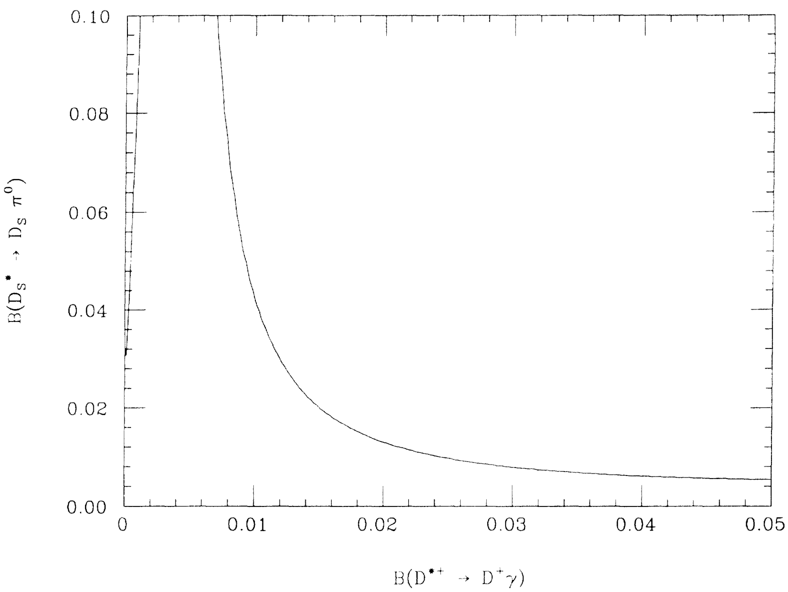

FIG. 2. Isospin-violating branching fraction $B(D_s^* \to D_s \pi^0)$ plotted against $B(D^{*+} \to D^+\gamma)$. SU(3) corrections of order $m_q^{1/2}$ to the light quark magnetic moments are included in this graph.

[7]. The cancellation is even stronger for $D_s^* \to D_s\gamma$. Setting the constituent down, strange, and charm masses equal to 330, 550, and 1600 MeV, respectively, we obtain the constituent quark model prediction $\mu_3/\mu_2 = 0.32$. The $D_s^* \to D_s\gamma$ rate therefore appears to be quite suppressed.

In chiral perturbation theory, the leading SU(3) corrections to the transition magnetic moments are calculable and are of order $m_q^{1/2}$. They arise from one-loop Feynman diagrams that modify the light transition moments [5]:

$$\mu_1^{(l)} = \frac{2}{3}\beta - \frac{g^2 m_K}{4\pi f_K^2} - \frac{g^2 m_\pi}{4\pi f_\pi^2} , \tag{15a}$$

$$\mu_2^{(l)} = -\frac{1}{3}\beta + \frac{g^2 m_\pi}{4\pi f_\pi^2} , \tag{15b}$$

$$\mu_3^{(l)} = \frac{1}{3}\beta + \frac{g^2 m_K}{4\pi f_K^2} . \tag{15c}$$

The loop contributions to $\mu_a^{(l)}$ do not occur in the ratio $2:-1:-1$ since $m_K \neq m_\pi$, and therefore violate SU(3).[2] The modified magnetic moments depend upon the two variables β and g which can be related to the two independent radiative branching fractions $B(D^{*0} \to D^0\gamma) = 36.4\%$ and $B(D^{*+} \to D^+\gamma) < 4.2\%$ [7]. Once β and g are known as functions of $B(D^{*+} \to D^+\gamma)$, we can determine the dependence of the isospin-violating branching ratio $B(D_s^* \to D_s\pi^0)$ upon $B(D^{*+} \to D^+\gamma)$ as well. The result is plotted in Fig. 2. As can be seen in the figure, $B(D_s^* \to D_s\pi^0)$ generally lies in the 1–2 % range. However, for $B(D^{*+} \to D^+\gamma) \lesssim 1\%$, there is a very strong cancellation between $\mu^{(h)}$ and $\mu_3^{(l)}$ which dramatically enhances $B(D_s^* \to D_s\pi^0)$. So these two branching fractions are strongly correlated.

SU(3) corrections to the transition magnetic moments of order m_q may also be important. Unfortunately, such corrections are not completely calculable. They have the general structure

$$A_a m_q \ln(m_q/\Lambda) + B_a(\Lambda) m_q , \tag{16}$$

where Λ denotes the subtraction point. While the coefficients A_a of the chiral logarithms may readily be extracted from one-loop Feynman diagrams [10], the coefficients B_a come from higher dimension operators in the chiral Lagrangian and are unknown. The subtraction point dependence of the B_a coefficients cancels that of the logarithms in Eq. (16). In Ref. [10], it was noted that if the B_a terms are neglected and the subtraction point is chosen to be approximately 1 GeV, then the $O(m_q)$ terms considerably enhance the magnitude of $\mu_3^{(l)}$. In this case, the branching ratio for $D_s^* \to D_s\pi^0$ is very small. However, the strange quark mass is not small enough to argue that the logarithm in (16) dominates over the analytic term. We therefore believe it is more reasonable to regard the $O(m_q)$ terms as unknown.

In conclusion, the branching ratio for $D_s^* \to D_s\pi^0$ that we have found lies in an experimentally interesting range. If this isospin-violating decay is observed, its branching ratio will provide information on the pattern of SU(3) violation in radiative $D^* \to D\gamma$ transitions.

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[2]The difference between $f_\pi = 132$ MeV and $f_K = 160$ MeV represents a higher order SU(3) breaking effect in Eq. (15). We have chosen to set the parameter f equal to f_π and f_K in graphs with π and kaon loops, respectively.

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