

Model-independent global constraints on new physics

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Using effective-Lagrangian techniques we perform a systematic survey of the lowest-dimension effective interactions through which heavy physics might manifest itself in present experiments. We do not restrict ourselves to special classes of effective interactions (such as “oblique” corrections). We compute the effects of these operators on all currently well-measured electroweak observables, both at low energies and at the Z resonance, and perform a global fit to their coefficients. Despite the fact that a great many operators arise in our survey, we find that most are quite strongly bounded by the current data. We use our survey to systematically identify those effective interactions which are *not* well bounded by the data—these could very well include large new-physics contributions. Our results may also be used to efficiently confront specific models for new physics with the data, as we illustrate with an example.

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I. INTRODUCTION

Where is all the new physics? This, in a nutshell, has become the burning question on most theorists' lips as experimental results from the 100-GeV scale have poured in from the CERN e^+e^- collider LEP, (SLAC) Linear Collider (SLC), and the Fermilab Tevatron. The higher the precision of the experiments being performed, the better seems the agreement with the standard electroweak model. And yet we know that something new, perhaps only the standard model Higgs boson, must almost certainly be found at or below several (tens of?) TeV, since at this scale our description would otherwise fundamentally break down.

If, as now seems quite likely, any new particles are quite massive compared to the electroweak gauge bosons, then their first observable effects can still be sought through the virtual contributions they make to physics at lower, but presently accessible, energies. While we wait for the construction of accelerators powerful enough to directly produce these new particles, theorists can usefully spend their time understanding where the comparatively rare virtual contributions can be expected to take place. It is particularly useful to be able to contrast the detailed predictions of specific models for the physics at high energies with the more model-independent predictions which can be obtained from an effective-Lagrangian viewpoint.

An effective Lagrangian parametrizes in as model independent a way as possible the low-energy implications

of new physics at a much higher scale M . This is done by constructing the most general set of effect interactions that are consistent with the known low-energy particle content and symmetries, and which can arise to a given order in $1/M$. The main goal of an effective-Lagrangian analysis is (i) to determine how large the effective couplings can be without contradicting existing experimental information and (ii) to find where to most fruitfully search for the resulting interactions in future experiments.

This type of search for new physics using effective Lagrangians has been performed in the past, but has tended to be relatively limited in its scope. Traditionally, either the implications of a single type of effective interaction (such as an electric or chromoelectric dipole moment), or a fairly small class of such operators (e.g., anomalous gauge-boson interactions), have been considered. The disadvantage of limiting the investigation to a very few operators is that realistic models of the new physics which underlies the effective Lagrangian typically generate a host of effective operators rather than just a few, and their effects for well-measured observables can be correlated, or even cancel. Recent analyses [1,2] of the implications of new physics for the gauge-boson self-energies, the so-called “oblique” corrections [3], may also be viewed in this way since they can be described [4] in an effective-Lagrangian language in terms of a three-parameter class of effective gauge-boson self-interactions. Although these latter analyses have the virtue of considering the most general effective interactions that might be generated by a given type of TeV-scale physics, they are nevertheless limited in the scope of underlying models that they can encompass by the very restriction to only oblique corrections.

In this paper we wish to extend the confrontation of potential new physics with the present electroweak data

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in a more comprehensive and more systematic way, by analyzing the data in terms of a much broader class of effective interactions than has previously been considered. More specifically, we consider all possible effective interactions which satisfy the following three criteria.

(1) Since we wish to analyze the implications of the present data we restrict ourselves to effective interactions which involve only particles which have already been observed. In particular, we do not assume the existence of a light Higgs boson. For simplicity we do not consider operators involving gluons, although their inclusion into our formalism is conceptually straightforward.

(2) We work up to operator dimension five. That is to say, our effective operators must have dimension $(\text{mass})^d$, with $d \leq 5$. We consider both CP -preserving and CP -violating operators.

(3) We consider only effective interactions which contribute at tree level to presently measured observables.

In practice this means that we include all possible operators of dimension ≤ 5 with the exception of anomalous three- and four-point electroweak boson self-interactions, or interactions involving two fermions and two electroweak bosons. Despite condition (3) above we do not ignore loop-generated bounds completely, however. This is because we do consider constraints on our list of operators which arise from their one-loop contributions to particularly well-measured observables. (We give a more precise justification of which observables are considered in the appropriate sections.)

We present here explicit expressions for a wide class of observables in terms of the couplings of these operators, and systematically constrain their coefficients from the present data. Our results include as special cases some previous analyses, and our formulas reduce to these in the appropriate limits.

Although our results agree with previous workers in the cases of overlap we believe we have streamlined some of the technical details of the calculations in comparison with the procedure of some other authors. Our main improvement lies in our treatment of the new-physics contributions to measured quantities, particularly as regards how the standard model (SM) predictions are altered due to the changes induced in the numerical values that are inferred for the reference input parameters, such as α , M_Z , or G_F . We perform this adjustment once and for all directly in the *Lagrangian*, thereby obviating the need to separately adjust each observable as it is considered. In this way we dispose, at the outset, of many terms which ultimately obscurely cancel in physical predictions in many treatments.

We find that even with the above assumptions we must deal with a large number of new-physics operators, of which many contribute to flavor-changing neutral currents. Our formalism is sufficiently powerful to deal with all of these. Surprisingly, however, we are still able to meaningfully constrain the sizes of most of these operators by performing a global fit to all charged- and neutral-current data. Our aim in doing such an analysis is twofold. First, by considering all interactions, one may discover that certain operators remain poorly constrained

by current data. Their effects might well be large, if only experiments would look for them. We will, in fact, present several examples of such operators.

Our second purpose is to present a comprehensive set of constraints that must be satisfied by all physics beyond the standard model. Any model builder has simply to compute the coefficients of these effective operators in terms of the parameters of the model, and the bounds on these coefficients can be obtained from our analysis. Of course, we have taken a particularly conservative approach—any reasonable model will have far fewer parameters than we have operators, so the true constraints on that model will in general be stronger than those presented here.

We illustrate the simplicity and power of our formalism by using it to constrain a class of models which has been elsewhere directly fit to the data. This example serves two purposes. In addition to providing an illustration of the comparative ease of performing the analysis with our general formalism, we can also see how much weaker our bounds are than those that are found with a direct fit to the parameters of the underlying model. We find that although our approach leads to more conservative constraints on these parameters, as it must, the limits we obtain are not much weaker than those of the direct fit. Thus, for the models we consider, little information is lost by the much simpler procedure of directly using the analysis which we provide in this paper.

We organize our presentation in the following way. In the next section we first illustrate our technique by reproducing the familiar oblique correction analysis. We do so partly in order to demonstrate the simplicity of our approach, but also as a vehicle for explaining the logic of our analysis in this simplest possible case. These same techniques are then applied to the general effective Lagrangian in the following two sections. In Sec. III we describe the most general effective interactions which satisfy our above criteria. We identify in this section how the powers of $1/M$ which can be expected to premultiply each operator in our Lagrangian depend on the assumptions that are made concerning the nature of the underlying physics. This gives an indication of the circumstances under which the interactions we have kept may be expected to dominate. The steps required to make our Lagrangian into an easily used tool are then performed in Sec. IV. Section V contains the main results of our analysis. Here we perform a fit to all charged- and neutral-current experimental data to constrain the new-physics parameters. We find limits on most such parameters, although there are certain directions in parameter space which remain unconstrained. Section VI then applies these results to an illustrative example, namely, the mixing of ordinary and exotic fermions. Our conclusions are summarized in Sec. VII.

II. "OBLIQUE" CORRECTIONS REVISITED

In this section we work through the familiar case of oblique radiative corrections [1,2]. We do so in order to clearly demonstrate the logic of our method in a simple context that is relatively unencumbered by algebra. The

reader interested in diving straight into the full calculation can safely skip directly to Sec. III.

A. The initial Lagrangian

Following Refs. [1,2] we imagine that the hitherto undiscovered new physics that lurks at the high scale M couples more significantly to the electroweak gauge bosons than to the other known light particles. The dominant effects of virtual loops of these heavy particles may therefore be expected to arise among the self-couplings of these gauge bosons. With the intuition, justified, with some qualifications, in more detail in later sections, that the lowest-dimension interactions should be least suppressed by inverse powers of the heavy mass $1/M$ we imagine supplementing the standard model by the following lowest-dimension effective interactions:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}}(\tilde{e}_i) + \hat{\mathcal{L}}_{\text{new}},$$

with

$$\begin{aligned} \hat{\mathcal{L}}_{\text{new}} = & -\frac{A}{4} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} - \frac{B}{2} \hat{W}_{\mu\nu}^\dagger \hat{W}^{\mu\nu} - \frac{C}{4} \hat{Z}_{\mu\nu} \hat{Z}^{\mu\nu} \\ & + \frac{G}{2} \hat{F}_{\mu\nu} \hat{Z}^{\mu\nu} - w \tilde{m}_W^2 \hat{W}_\mu^\dagger \hat{W}^\mu - \frac{z}{2} \tilde{m}_Z^2 \hat{Z}_\mu \hat{Z}^\mu. \end{aligned} \quad (1)$$

Here $\mathcal{L}_{\text{SM}}$ represents the familiar SM Lagrangian, after the top quark and Higgs boson have been integrated out, including loop effects to the extent that experiments are sensitive enough to probe these. $\hat{F}_{\mu\nu}$ and $\hat{Z}_{\mu\nu}$ represent the usual Abelian field strengths, while the $\hat{W}_{\mu\nu}$ is required to be electromagnetically gauge covariant: $\hat{W}_{\mu\nu} = D_\mu \hat{W}_\nu - D_\nu \hat{W}_\mu$ with $D_\mu \hat{W}_\nu = \partial_\mu \hat{W}_\nu + ie \hat{A}_\mu \hat{W}_\nu$.

The new-physics coefficients A through z could be computed within any given underlying theory and should be thought of as (presently unknown) functions of the parameters of this underlying theory. The success of the standard model is equivalent to the statement that all current experiments are consistent with $A=B=C=G=w=z=0$.

Not all six of these parameters are physically significant however, since only three independent combinations of them actually ever appear in expressions for physical observables. Only three independent combinations can have physical content because there is a three-parameter family of changes to the original six parameters in $\hat{\mathcal{L}}_{\text{new}}$ that can be made by redefining the fields, without altering the form of the SM Lagrangian $\mathcal{L}_{\text{SM}}$. The required redefinitions consist of rescalings of the SM electroweak gauge potentials and Higgs doublet: W_μ^a , B_μ , and ϕ . A conventional parametrization of the three physical combinations of the quantities A through z is given by the Peskin-Takeuchi variables S , T , and U . The connection is given explicitly by (we use the notation $s_W = \sin\theta_W$, $c_W = \cos\theta_W$, etc.):

$$\begin{aligned} \alpha S &= 4s_W^2 c_W^2 \left[A - C - \frac{c_W^2 - s_W^2}{c_W s_W} G \right], \\ \alpha T &= w - z, \\ \alpha U &= 4s_W^4 \left[A - \frac{1}{s_W^2} B + \frac{c_W^2}{s_W^2} C - 2 \frac{c_W}{s_W} G \right]. \end{aligned} \quad (2)$$

There are two aspects of our notation that are particularly significant.

(1) The carets that appear on top of the initial Lagrangian and fields in Eq. (1) refer to the fact that these fields are not canonically normalized, since $\hat{\mathcal{L}}_{\text{new}}$ contains kinetic (and mixing) terms for the gauge bosons, in addition to those that are already in $\mathcal{L}_{\text{SM}}$.

(2) The $\tilde{e}_i$ represent all of the parameters appearing in the SM part of the total effective Lagrangian, such as the Higgs Yukawa couplings $\tilde{y}_f$, the electromagnetic fine-structure constant $\tilde{\alpha}$, etc. The tilde is meant to indicate that these parameters do not take their “standard” numerical values, such as $\alpha^{-1} = 137.035989$, when they are inferred from experiment, since the expressions for observables as a function of these parameters are altered by the presence of the new physics.

Our method now consists of diagonalizing and canonically normalizing the gauge-boson kinetic terms, and then eliminating the parameters $\tilde{e}_i$ in favor of parameters e_i which take on the “standard” values. Once we have done so we have used up the freedom to redefine fields, and so we find that the resulting couplings then depend only on the three physical quantities S , T , and U . The resulting Lagrangian, as we shall show, can be readily used to calculate observables in terms of a SM result plus some linear combination of S , T , and U .

B. Diagonalization and canonical normalization

It is a simple matter to canonically normalize and diagonalize the gauge boson kinetic terms, the required field redefinitions being

$$\hat{A}_\mu = \left[1 - \frac{A}{2} \right] A_\mu + G Z_\mu, \quad (3)$$

$$\hat{W}_\mu = \left[1 - \frac{B}{2} \right] W_\mu, \quad (4)$$

$$\hat{Z}_\mu = \left[1 - \frac{C}{2} \right] Z_\mu. \quad (5)$$

Here and elsewhere we work only to linear order in the small coefficients $A, B, \dots, z$. It is straightforward to keep higher-order terms, if desired. After this transformation, the total kinetic and mass terms are of the desired form

$$\begin{aligned} & -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} W_{\mu\nu}^\dagger W^{\mu\nu} - \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} \\ & - (1 + w - B) \tilde{m}_W^2 W_\mu^\dagger W^\mu - \frac{1}{2} (1 + z - C) \tilde{m}_Z^2 Z_\mu Z^\mu. \end{aligned} \quad (6)$$

These field transformations also alter the form of the SM electromagnetic, charged-current, and neutral-current couplings, which now become

$$\mathcal{L}_{\text{em}} = -\tilde{e} \left[1 - \frac{A}{2} \right] \sum_i \bar{f}_i \gamma^\mu Q_i f_i A_\mu, \quad (7)$$

$$\mathcal{L}_{\text{CC}} = -\frac{\tilde{e}}{\tilde{s}_W\sqrt{2}} \left[1 - \frac{B}{2} \right] \sum_{ij} \tilde{V}_{ij} \bar{f}_i \gamma^\mu \gamma_L f_j W_\mu^\dagger + \text{c.c.}, \quad (8)$$

$$\mathcal{L}_{\text{NC}} = -\frac{\tilde{e}}{\tilde{s}_W\tilde{c}_W} \left[1 - \frac{C}{2} \right] \sum_i \bar{f}_i \gamma^\mu [T_{3i} \gamma_L - Q_i \tilde{s}_W^2 + Q_i \tilde{s}_W \tilde{c}_W G] f_i Z_\mu. \quad (9)$$

In these expressions, Q_i is the electric charge of fermion f_i , normalized with $Q_e = -1$. T_{3i} similarly represents the fermion's third component of weak isospin. $\tilde{V}_{ij}$ represents the usual Cabibbo-Kobayashi-Maskawa (CKM) matrix for quarks, and is the unit matrix δ_{ij} for leptons.

C. Reexpressing the Lagrangian in terms of “standard” parameters

The Lagrangian, as we have written it, depends on the three parameters $\tilde{e}$, $\tilde{m}_Z$, and $\tilde{s}_W$ (as well as the fermion and Higgs boson masses $\tilde{m}_i$ and the CKM matrix elements $\tilde{V}_{ij}$). In SM electroweak physics, these three parameters (plus the particle masses and CKM matrix elements) suffice to describe all electroweak observables. We can eliminate $\tilde{e}$, $\tilde{m}_Z$, and $\tilde{s}_W$ in terms of three reference observables, and it is standard to choose the best-measured observables for this purpose: the electromagnetic fine-structure constant α , the physical Z mass M_Z , and the Fermi constant G_F , as measured in muon decay. Using the resulting expressions in the formulas for any other observables then leads to numerical predictions that can be made to any desired accuracy.

Once the standard model is supplemented by $\mathcal{L}_{\text{new}}$, however, the relation between these three parameters and the reference observables changes. As a result the value that is inferred from experiment for a parameter such as $\tilde{e}$, will differ from what would be found for the corresponding parameter, call it simply e , purely within the standard model. Our goal in this section is to compute this difference, for each of the basic three electroweak parameters. It is sufficient for the present purposes to do so at tree level in all interactions, since any loop effects are negligible once multiplied by the already small new-physics parameters.

The program therefore consists of calculating the input observables α , M_Z , and G_F at tree level in the new model as computed using Eqs. (6), (7), (8), and (9). These expressions are then equated to the tree-level SM predictions for the same quantities. The result is a system of three equations that can be inverted to obtain $\tilde{e}_i$ in terms of their “standard” counterparts e_i . These then may be used for predicting any other observable.

Note that, for this choice of new physics (i.e., oblique corrections only), the relations $\tilde{m}_i = m_i$ and $\tilde{V}_{ij} = V_{ij}$ are unchanged. However this is not true in the general case, as we shall see in subsequent sections.

1. Electric charge (e)

The fine-structure constant as determined in electron-electron scattering¹ at very low energies is given at tree level, using the interaction equation (7), by

$$4\pi\alpha = \tilde{e}^2(1 - A). \quad (10)$$

On the other hand, the SM tree-level relation is simply

$$4\pi\alpha = e^2. \quad (11)$$

Comparing Eqs. (10) and (11) gives the relation

$$\tilde{e} = e \left[1 + \frac{A}{2} \right]. \quad (12)$$

2. Z mass (M_Z)

At lowest order, the physical Z -boson mass M_Z is simply the square root of the parameter m_Z^2 that appears as the coefficient of $\frac{1}{2}Z_\mu Z^\mu$ in the SM Lagrangian. At the same order, the Z mass in the new model is similarly given by

$$M_Z^2 = \tilde{m}_Z^2(1 + z - C). \quad (13)$$

Comparing these predictions we deduce

$$\tilde{m}_Z^2 = m_Z^2(1 - z + C). \quad (14)$$

3. Fermi's constant (G_F)

Muon decay is mediated by the low-energy exchange of a W boson. Thus, to calculate the Fermi constant at tree level in the new model, we use the propagator suggested by Eq. (6), and the charged-current interaction expressed in Eq. (8). This results in

$$\begin{aligned} \frac{G_F}{\sqrt{2}} &= \frac{\tilde{e}^2(1 - B)}{8\tilde{s}_W^2\tilde{m}_W^2(1 + w - B)} \\ &= \frac{\tilde{e}^2}{8\tilde{s}_W^2\tilde{c}_W^2\tilde{m}_Z^2}(1 - w). \end{aligned} \quad (15)$$

Note that we are free to use SM relations, such as $\tilde{m}_W = \tilde{m}_Z\tilde{c}_W$, among the “tilded,” or standard-model, parameters. For comparison, the SM tree-level prediction for G_F is simply

$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8s_W^2c_W^2m_Z^2}. \quad (16)$$

We take this last expression as our *definition* of s_W .

Combining Eqs. (12), (14), (15), and (16) we obtain

¹Actually, the fine-structure constant is determined in nonrelativistic condensed-matter systems, such as the quantum Hall effect. However the quantity that is found in this way in the very-low-energy, nonrelativistic effective theory, is ultimately matched onto α as is used at high energies by using electron-electron scattering at energies near the electron mass [5].

$$\tilde{s}_W^2 = s_W^2 \left[1 + \frac{c_W^2}{c_W^2 - s_W^2} (A - C - w + z) \right] \quad (17)$$

as well as the following useful formulas, which we record in passing:

$$\begin{aligned} \tilde{c}_W^2 &= c_W^2 \left[1 - \frac{s_W^2}{c_W^2 - s_W^2} (A - C - w + z) \right], \\ \frac{\tilde{e}}{\tilde{c}_W \tilde{s}_W} &= \frac{e}{c_W s_W} \left[1 + \frac{C + w - z}{2} \right]. \end{aligned} \quad (18)$$

The above expressions achieve our goal of relating $\tilde{e}$, $\tilde{s}_W$, and $\tilde{m}_Z$ to the standard parameters e , s_W , and m_Z . The next step in the process is to reexpress the Lagrangian

itself in terms of these standard parameters. To do so we simply substitute Eqs. (12), (14), (17), and (18) into the various Lagrangian terms.

By construction the Z mass term and electromagnetic interaction take simple forms:

$$\mathcal{L}_Z = -\frac{1}{2} m_Z^2 Z_\mu Z^\mu \quad (19)$$

and

$$\mathcal{L}_{\text{em}} = -e \sum_i \bar{f}_i \gamma^\mu Q_i f_i A_\mu.$$

By contrast, the W mass term gives a more complicated expression

$$\begin{aligned} m_Z^2 c_W^2 \left[1 - B + C + w - z - \frac{s_W^2}{c_W^2 - s_W^2} (A - C - w + z) \right] W_\mu^\dagger W^\mu \\ = m_Z^2 c_W^2 \left[1 - \frac{\alpha S}{2(c_W^2 - s_W^2)} + \frac{c_W^2 \alpha T}{c_W^2 - s_W^2} + \frac{\alpha U}{4s_W^2} \right] W_\mu^\dagger W^\mu, \end{aligned} \quad (20)$$

and the charged-current interaction takes the form

$$\begin{aligned} \mathcal{L}_{\text{CC}} &= -\frac{e}{\sqrt{2}s_W} \left[1 + \frac{1}{2} \left[A - B - \frac{c_W^2}{c_W^2 - s_W^2} (A - C - w + z) \right] \right] \sum_{ij} V_{ij} \bar{f}_i \gamma^\mu \gamma_L f_j W_\mu^\dagger + \text{c.c.} \\ &= -\frac{e}{\sqrt{2}s_W} \left[1 - \frac{\alpha S}{4(c_W^2 - s_W^2)} + \frac{c_W^2 \alpha T}{2(c_W^2 - s_W^2)} + \frac{\alpha U}{8s_W^2} \right] \sum_{ij} V_{ij} \bar{f}_i \gamma^\mu \gamma_L f_j W_\mu^\dagger + \text{c.c.} \end{aligned} \quad (21)$$

Note here that all corrections due to S, T, U are universal. The strength of the charged-current interaction is therefore given by $h_{ij} = h_{ij}^{\text{SM}} + \delta h_{ij}$, with $h_{ij}^{\text{SM}} = V_{ij}$ and

$$\delta h_{ij} = V_{ij} \left[-\frac{\alpha S}{4(c_W^2 - s_W^2)} + \frac{c_W^2 \alpha T}{2(c_W^2 - s_W^2)} + \frac{\alpha U}{8s_W^2} \right]. \quad (22)$$

Finally, the neutral-current interaction becomes

$$\begin{aligned} \mathcal{L}_{\text{NC}} &= -\frac{e}{s_W c_W} \left[1 + \frac{w - z}{2} \right] \sum_i \bar{f}_i \gamma^\mu \left[T_{3i} \gamma_L - Q_i \left[s_W^2 + \frac{s_W^2 c_W^2}{c_W^2 - s_W^2} [A - C - w + z] - s_W c_W G \right] \right] f_i Z_\mu \\ &= -\frac{e}{s_W c_W} \left[1 + \frac{\alpha T}{2} \right] \sum_i \bar{f}_i \gamma^\mu \left[T_{3i} \gamma_L - Q_i \left[s_W^2 + \frac{\alpha S}{4(c_W^2 - s_W^2)} - \frac{c_W^2 s_W^2 \alpha T}{c_W^2 - s_W^2} \right] \right] f_i Z_\mu. \end{aligned} \quad (23)$$

Here there are both universal and nonuniversal corrections due to S, T, U . (We remark that, in the language of Ref. [2], the factor multiplying Q_i in the weak couplings is simply s_W^2 .) The neutral-current couplings g_{iL} and g_{iR} are therefore given by their SM counterparts $g_{iL}^{\text{SM}} = T_{3i} - Q_i s_W^2$ and $g_{iR}^{\text{SM}} = -Q_i s_W^2$, plus the deviations

$$\delta g_{iL(R)} = \frac{\alpha T}{2} g_{iL(R)}^{\text{SM}} - Q_i \left[\frac{\alpha S}{4(c_W^2 - s_W^2)} - \frac{c_W^2 s_W^2 \alpha T}{c_W^2 - s_W^2} \right]. \quad (24)$$

Equations (20)–(24) may now be used to predict the implications for any desired observables.

D. The calculation of observables

The calculation of observables is now straightforward. As has been pointed out before, since the constants which parametrize the new physics are small, we may work to any desired loop order in the SM interactions, and to tree level in the interactions which deviate from the standard model.

Consider, first, the mass of the W boson. In the standard model this mass may be predicted as a function of the three input parameters: $M_W = M_W^{\text{SM}}(M_Z, \alpha, G_F)$. With the new interactions this expression now gets a new con-

tribution which may be read from Eq. (20):

$$(M_W^2)_{\text{phys}} = (M_W^{\text{SM}})^2 \left[1 - \frac{\alpha S}{2(c_W^2 - s_W^2)} + \frac{c_W^2 \alpha T}{c_W^2 - s_W^2} + \frac{\alpha U}{4s_W^2} \right]. \quad (25)$$

Note that because we have eliminated $\bar{e}$, $\bar{s}_W$, and $\bar{m}_Z$ in terms of their untilded counterparts, the SM contribution in this formula takes precisely its usual numerical value. The resulting expression is in agreement with Ref. [2].

The ρ parameter, defined as the ratio of low-energy neutral- and charged-current amplitudes, can be read off from the universal S, T, U corrections to Eqs. (21) and

(23). Taking also into account the corrections to the W mass [Eq. (25)] one finds

$$\rho = 1 + \alpha T, \quad (26)$$

as in Ref. [2].

Finally, consider the left-right (LR) asymmetry at the Z pole. A_{LR} is the sum of the (radiatively corrected) SM expression, plus the direct tree-level contribution from the new interactions. This is

$$A_{LR} = \left[\frac{(g_{eL})^2 - (g_{eR})^2}{(g_{eL})^2 + (g_{eR})^2} \right]. \quad (27)$$

Linearizing this expression about the SM value gives δA_{LR} . Finally, adding this to the SM rate gives

$$\begin{aligned} A_{LR} &= A_{LR}^{\text{SM}} + \frac{4g_{eL}^{\text{SM}}g_{eR}^{\text{SM}}}{((g_{eL}^{\text{SM}})^2 + (g_{eR}^{\text{SM}})^2)^2} (g_{eR}^{\text{SM}}\delta g_{eL} - g_{eL}^{\text{SM}}\delta g_{eR}) \\ &= A_{LR}^{\text{SM}} + \frac{4g_{eL}^{\text{SM}}g_{eR}^{\text{SM}}(g_{eR}^{\text{SM}} - g_{eL}^{\text{SM}})}{((g_{eL}^{\text{SM}})^2 + (g_{eR}^{\text{SM}})^2)^2} \left[\frac{\alpha S}{4(c_W^2 - s_W^2)} - \frac{c_W^2 s_W^2 \alpha T}{c_W^2 - s_W^2} \right], \end{aligned} \quad (28)$$

which again agrees with Ref. [2].

Contrast the ease of application of our Lagrangian with the procedure that is often followed in much of the literature. There, authors instead directly use the Lagrangian expressed with the tilded parameters $\tilde{e}_i$. The direct contribution $\delta\mathcal{O}$ of new physics to a given observable $\mathcal{O}$ is then added to the shift in the SM value for that observable (due to the shift from $\delta e_i = \tilde{e}_i - e_i$) to get the total new-physics effect:

$$\mathcal{O} = \mathcal{O}_{\text{SM}} + \delta\mathcal{O} + \sum_i \left[\frac{\partial\mathcal{O}}{\partial e_i} \right] \delta e_i. \quad (29)$$

The savings in labor in our approach is more striking in the more general Lagrangian we consider in the remainder of the paper.

III. THE GENERAL EFFECTIVE LAGRANGIAN

We now wish to repeat these steps without assuming the particular Lagrangian of Eq. (1). Since our conclusions can only be as general as is the Lagrangian with which we choose to work, the aim of the present section is to justify our Lagrangian's generality. We save its reexpression in terms of the "standard" parameters, and its comparison with observables for subsequent sections.

The only simplifying choice we make is to concentrate on the electroweak sector only. The inclusion of non-standard gluon couplings presents no particular problems, and can be dealt with in our formalism in a straightforward manner.

We start making real physical choices with our remaining two assumptions: (i) the particle content of our low-energy theory—we take only particles which have been

detected to date and (ii) the maximum dimension of the effective interactions which we consider, which we take to be five. Although the first of these assumptions may not provoke much argument, a justification of the second of these turns out to require some thought. We therefore first present the terms that are permitted in our effective Lagrangian by the assumed low-energy particle content, before returning to the question of the validity of the neglect of dimension-six and higher terms in Sec. III B.

A. The effective interactions

We wish to write down the most general effective interactions in $\mathcal{L}_{\text{new}}$ that are consistent with the particle and symmetry content relevant to the energies to which the Lagrangian is to be applied. Our first task is then to decide on precisely what this low-energy particle content is. Since our intended application here is to current experiments whose accessible energy is of order 100 GeV or less, we take our particle content to include only those which already have been detected, namely, most of the SM particles, including precisely three left-handed neutrinos. We do *not* include the Higgs boson and the top quark, which we take to have been integrated out, if they exist. Because of these missing particles, the field content of our effective theory does not fill out linear representations of the electroweak gauge group, and so this symmetry *cannot* be linearly realized, and the resulting Lagrangian must eventually violate unitarity [6] at energies at most of order $4\pi v \sim$ a few TeV. In this case it is simply a matter of convenience whether this gauge symmetry is chosen to be present but nonlinearly realized, or simply

ignored completely [7,8].² Clarity of presentation leads us to choose the second of these options here.

With these comments in mind we may construct the most general Lagrangian that can arise to any order in $1/M$ for the known particles. Our starting point is again the split:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}}(\tilde{e}_i) + \hat{\mathcal{L}}_{\text{new}},$$

with

$$\hat{\mathcal{L}}_{\text{new}} = \sum_{d=2}^{\infty} \hat{\mathcal{L}}_d. \quad (30)$$

Here $\hat{\mathcal{L}}_d$ contains all possible terms that have operator dimension (mass)^d. We wish to list explicitly all terms up to $\hat{\mathcal{L}}_5$. We may freely integrate by parts and use the standard-model equations of motion in order to simplify our Lagrangian, since no operators that can be eliminated in this way can have any physical effects [9].

A word should be said about new-physics operators involving neutrinos. Our low-energy particle content does not include right-handed neutrinos. We can nevertheless continue working with four-component spinors provided that we take the neutrino spinors to be Majorana: $\nu = \nu^C$, where $\nu^C = C\bar{\nu}^T$, with C the charge conjugation matrix. This means that the parameters describing the interactions of neutrinos are subject to more constraints than are those of the other, electrically charged, fermions. We identify these additional conditions, case by case, for the various anomalous interaction terms presented below.

Dimension Two: At dimension two we have the boson mass terms

$$\mathcal{L}_2 = -w\tilde{m}_W^2 \hat{W}_\mu^\dagger \hat{W}^\mu - \frac{z}{2} \tilde{m}_Z^2 \hat{Z}_\mu \hat{Z}^\mu, \quad (31)$$

a particular combination of which also appears in $\mathcal{L}_{\text{SM}}$. Only the combination $w - z$ of these two masses may therefore be detected through the deviation it produces from the standard-model relation between gauge boson masses [cf. Eqs. (20) and (25), for example].

Dimension Three: The fermion mass terms arise at dimension three:

$$\mathcal{L}_3 = -\bar{f}(\delta m_L \gamma_L + \delta m_R \gamma_R) \hat{f}, \quad (32)$$

where f denotes a generic column vector in fermion generation space, and δm_L and δm_R are matrices in this space. Hermiticity of the action requires that $\delta m_L = \delta m_R^\dagger$, and for neutrinos we have the additional condition $\delta m_R^\nu = (\delta m_L^\nu)^*$. We choose our conventions so that $\mathcal{L}_3$ is CP invariant if the matrices δm_L and δm_R are real.

Apart from the neutrino masses, which are zero in the standard model, these fermion mass terms are indistinguishable from the SM ones. They could nevertheless become detectable in the event that a light Higgs particle should be discovered. In this case such interactions could cause deviations from SM relations, such as $y_f = m_f/v$, between the fermion-Higgs Yukawa coupling and the fermion masses.

Dimension Four: Dimension four contains two types of terms, (i) gauge-boson and fermion kinetic terms and (ii) gauge-boson-fermion coupling terms.

We therefore have

$$\hat{\mathcal{L}}_4 = \hat{\mathcal{L}}_{b\text{kin}} + \hat{\mathcal{L}}_{f\text{kin}} + \hat{\mathcal{L}}_{bff} + \hat{\mathcal{L}}_{\text{other}}^{(4)}$$

with

$$\hat{\mathcal{L}}_{b\text{kin}} = -\frac{A}{4} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} - \frac{B}{2} \hat{W}_{\mu\nu}^\dagger \hat{W}^{\mu\nu} - \frac{C}{4} \hat{Z}_{\mu\nu} \hat{Z}^{\mu\nu} + \frac{G}{2} \hat{F}_{\mu\nu} \hat{Z}^{\mu\nu}, \quad (33)$$

$$\hat{\mathcal{L}}_{f\text{kin}} = \bar{f} \gamma^\mu (I_L \gamma_L + I_R \gamma_R) D_\mu \hat{f}, \quad (34)$$

$$\hat{\mathcal{L}}_{bff} = -\frac{\tilde{e}}{\tilde{s}_W \tilde{c}_W} \bar{f} \gamma^\mu (\delta \hat{g}_L \gamma_L + \delta \hat{g}_R \gamma_R) \hat{f} \hat{Z}_\mu - \frac{\tilde{e}}{\sqrt{2} \tilde{s}_W} \bar{f} \gamma^\mu (\delta \hat{h}_L \gamma_L + \delta \hat{h}_R \gamma_R) \tau_+ \hat{f} \hat{W}_\mu^\dagger + \text{c.c.} \quad (35)$$

We use a compact notation in these expressions, in which I_L , I_R , etc., are matrices which act on the indices which label fermion type (or flavor), and where τ_+ is the $SU_L(2)$ raising operator. The matrices I_L , I_R , $\delta \hat{g}_L$, and $\delta \hat{g}_R$ must always be Hermitian, with $I_L = I_R^*$ and $\delta \hat{g}_L = -\delta \hat{g}_R^*$ holding in addition for neutrinos. CP invariance follows if all of these coupling matrices should be real. The derivative

D_μ used in the fermion kinetic terms is covariant with respect to the electromagnetic interactions. [If we also considered nonstandard gluon couplings we would demand covariance with respect to the full unbroken gauge group $SU_c(3) \times U_{\text{em}}(1)$.] Finally, as in Sec. II, the ubiquitous carets indicate that the fields are not yet canonically normalized.

$\hat{\mathcal{L}}_{\text{other}}^{(4)}$ contains all of the other dimension-four operators which we do not consider here. There are two types of such terms, although both involve only the electroweak gauge bosons. The first type consists of a potential electroweak “ Θ -term”—i.e., a term proportional to $\tilde{W}_{\mu\nu} \tilde{W}^{\mu\nu}$. We ignore this here since it produces com-

²This equivalence is an old, in some quarters recently forgotten, result which dates right back to Ref. [6] and beyond.

pletely negligible effects at zero temperature.³ Also lumped into $\hat{\mathcal{L}}_{\text{other}}^{(4)}$ are the dimension-four three- and four-point gauge-boson self-interactions. As explained earlier we have chosen not to include these here since they cannot yet be well bounded at tree level [11]. This

makes them interesting in their own right, since it means that, so far as we know, they could very well contain new physics.

Dimension Five: At dimension five the following combinations can arise:

$$\begin{aligned} \mathcal{L}_5 = & -\bar{e}\hat{f}\sigma^{\mu\nu}(\hat{d}_L\gamma_L + \hat{d}_R\gamma_R)\hat{f}\hat{F}_{\mu\nu} - \frac{\bar{e}}{\bar{s}_W\bar{c}_W}\hat{f}\sigma^{\mu\nu}(\hat{n}_L\gamma_L + \hat{n}_R\gamma_R)\hat{f}\hat{Z}_{\mu\nu} \\ & - \frac{\bar{e}}{\sqrt{2}\bar{s}_W}\hat{f}\sigma^{\mu\nu}(\hat{c}_L\gamma_L + \hat{c}_R\gamma_R)\tau_+\hat{f}\hat{W}_{\mu\nu} + \hat{\mathcal{L}}_{\text{other}}^{(5)} + \text{c.c.} \end{aligned} \quad (36)$$

Again, all coefficients here, $\hat{d}_{L,R}$, $\hat{n}_{L,R}$, and $\hat{c}_{L,R}$, are matrices in flavor space, as is the $\text{SU}_L(2)$ raising operator τ_+ . It is required that $\hat{d}_L = \hat{d}_R^\dagger$ and $\hat{n}_L = \hat{n}_R^\dagger$ for Hermiticity of the action, together with restriction $\hat{d}_L = -\hat{d}_R^*$ and $\hat{n}_L = -\hat{n}_R^*$ for neutrinos. CP conservation requires all of these coupling matrices to be real. $\hat{\mathcal{L}}_{\text{other}}^{(5)}$ here includes all four-point fermion–gauge-boson couplings, such as $\bar{f}fW^\dagger W$, which are also not yet probed in existing experiments.

Dimension Six: Finally, there are a great many operators that can arise at dimension six including a very long list of four-fermion contact interactions. Their inclusion would enormously complicate the present analysis, and so we neglect them throughout what follows. We discuss in the next section the circumstances under which the neglect of these dimension-six interactions can be justified by their suppression by additional powers of $O(1/M^2)$.

B. Power counting

What ultimately makes an effective-Lagrangian analysis useful is the property that only a limited number of effective interactions can arise to any given order in the expansion in the inverse of the heavy mass M of the new physics. Usually, powers of $1/M$ are simply counted by dimensional analysis, with the coefficient c_n of an effective operator of dimension (mass) ^{d_n} being proportional to M^{4-d_n} . (Some exceptions to this common rule of thumb are discussed in Ref. [12].) As has been stated earlier, we choose here to work only up to and including effective interactions of dimension five.

If this were the whole story, then the neglect of dimension-six operators could be simply justified as being due to their suppression by additional powers of $1/M$ relative to those at dimensions four and five. There are two issues which complicate this simple picture, however.

First, it can happen that effective operators are more suppressed than would be indicated by simple dimension

counting. This can occur because of the possibility of suppression by small dimensionless quantities, such as small coupling constants in the underlying theory (such as Yukawa couplings: $y_f = m_f/v$), or by small mass ratios (such as v/M , which is present if M is much larger than the electroweak-breaking scale v).⁴ If this type of additional suppression should arise for the lower-dimension terms which we keep, then their neglect relative to unsuppressed dimension-six terms may no longer be justified.

Second, one might also worry that dimension-six operators may be suppressed by fewer than two powers of M , such as if they were proportional to $1/v^2$ or $1/vM$. As we shall see shortly, such coefficients are indeed possible depending on the nature of the underlying physics that has been integrated out. In such a case the neglect of dimension-six new-physics operators in comparison to those of lower dimension need not be justified.

The bottom line is that the power of v/M which appears in the coefficient of a given effective interaction generically depends on the nature of physics that is associated with the large scale M . As a result, a complete cataloging of effective interactions according to their suppression by $1/M$ cannot be made in an entirely model-independent way. At some point this model dependence may become a good thing: a comparison of the sizes of various effective operators, should they ever be discovered, may ultimately permit the diagnosis of the nature of the underlying new physics.⁵ We therefore neglect dimension-six effective interactions, in the knowledge that an element of model dependence enters in this way into our conclusions. One must simply check, when applying the bounds we obtain below to a particular model, that this neglect is justified in the case of interest.

³See, however, the recent controversy concerning the existence of potential weak-scale baryon-number violation in TeV accelerators [10].

⁴It must be kept in mind here that since our low-energy particle content does not fill out linear representations of $\text{SU}_L(2) \times \text{U}_Y(1)$, v/M cannot be smaller than roughly $1/4\pi$.

⁵For a related, and more detailed, discussion of heavy-mass dependence see Ref. [8].

In order to more concretely illustrate what can be expected for the strength of various effective interactions from differing types of underlying physics at scale M , we next consider explicitly the implications of two types of scenarios—strongly and weakly coupled electroweak symmetry-breaking physics. We do so partly to demonstrate the existence of models for which four-Fermi terms may be neglected, and partly to contrast the sizes of the various terms in the effective Lagrangian for these two cases.

Strongly Coupled New Physics: It is possible that the symmetry-breaking sector of the electroweak theory is strongly coupled, with only the three would-be Goldstone bosons (WBGB's), that is to say, the longitudinal W and Z polarizations, appearing at experimentally accessible energies. In this case the couplings of these WBGB's are completely dictated, at low energies, to be those given by chiral perturbation theory [13]. In the resulting effective Lagrangian successive powers of the WBGB fields are suppressed by inverse powers of the symmetry-breaking scale v . If the Lagrangian were to be applied to energies $E \ll v$, then the powers of v that would be obtained in this way by dimensional analysis would suffice for counting which interactions arise to a given order in E/v .

In practice, however, applications are meant to be for higher energies, $E \simeq v \ll M$. In this case a consistent expansion in powers of E/M is only possible if successive terms in the effective Lagrangian are suppressed by powers of M rather than v . That is to say, an expansion in powers of $1/M$ requires that some couplings in the effective theory must be systematically suppressed by powers of v/M , compared to the powers of v that arise using straight dimensional analysis. This suppression has been formulated in a precise way, based on experience with chiral perturbation theory as applied to low-energy QCD, and is called called “naive dimensional analysis” (NDA) [14]. It states that a term having b WBGB fields, f (weakly interacting) fermion fields, d derivatives, and w gauge fields has a coefficient whose size is

$$c_n(M) \sim v^2 M^2 \left[\frac{1}{v} \right]^b \left[\frac{1}{M^{3/2}} \right]^f \left[\frac{1}{M} \right]^d \left[\frac{g}{M} \right]^w. \quad (37)$$

In this expression the relation $M \lesssim 4\pi v$ must always be kept in mind. (If the fermions were strongly interacting, as would be the case for technifermions or for nucleons in QCD, then the factor is $1/v\sqrt{M}$ for each fermion. This

would lead to a coefficient of order $1/v^2$ for dimension-six four-fermion interactions.) The implications of the above estimate for the various effective interactions are listed in column 2 of Table I.

Weakly Coupled New Physics: A completely opposite point of view is to suppose that the electroweak symmetry-breaking physics is sufficiently weakly coupled to permit a perturbative analysis. In this case one or more physical particles, other than the WBGB's, would be expected to have masses of order λv , where λ is a small dimensionless coupling. Being light, these particles appear in the low-energy theory and, together with the WBGB's, fill out linear realizations of the electroweak gauge group. The standard model itself is an example along these lines, where the physical Higgs scalar plays the role of this new light particle.

In addition to the effects of their direct propagation, these new degrees of freedom can appear within the effective Lagrangian through the powers of v/M that they contribute when their fields are replaced by their vacuum expectation values (VEV's). The precise power which appears in any particular effective interaction therefore depends on the representations in which the Higgs-like fields transform. The most plausible choice is one or more doublets, with the standard hypercharge assignment, since this is what is required to generate masses for the known fermions.

In this scenario the size of any non-Higgs interactions may be found by finding the lowest-dimension interactions which contain the desired term, replacing all Higgs fields by their VEV's, and making up the rest of the dimensions with powers of the heavy mass M . We call the estimate that is obtained in this way “linearly realized dimensional analysis” (LRDA). This estimate is given for the effective operators of interest here in column 3 of Table I.

A Comparison Between NDA and LRDA: As is seen from Table I there are a number of differences between the implications of NDA and LRDA for the lowest-dimension operators we are considering.

Typically the linearly realized gauge symmetry enforces relations among the various coefficients of operators which involve a particular number of fields or derivatives, depending on how these operators can be assembled into linearly realized multiplets. This is best illustrated with a few examples.

Consider the contributions to the W and Z masses:

TABLE I. We tabulate here the estimated sizes that would be expected for the deviations from the standard model among effective operators of the lowest dimension, as is explained in the text. The two columns contrast the implications of two types of assumptions concerning the nature of the underlying physics, either naive dimensional analysis (NDA), or linearly realized dimensional analysis (LRDA). We use the NDA rules for weakly-coupled fermions in obtaining our estimate for four-fermion terms.

Operator	NDA	LRDA
Gauge boson masses	$g^2 v^2$	$g^2 v^4 / M^2$
Neutrino masses	v^2 / M	v^2 / M
Gauge boson kinetic terms	$g^2 v^2 / M^2$	$g^2 v^2 / M^2$
Dim 4 gauge-boson-fermion vertex	gv^2 / M^2	gv^2 / M^2
Dim 5 gauge-boson-fermion vertex	gv^2 / M^3	gv / M^2
Dim 6 four fermion terms	v^2 / M^4	$1 / M^2$

$\mathcal{O}_W = W_\mu^\dagger W^\mu$ and $\mathcal{O}_Z = \frac{1}{2} Z_\mu Z^\mu$. The lowest-dimension operator which contains these terms is simply the dimension-four Higgs kinetic term $(D_\mu \phi)^\dagger (D^\mu \phi)$. Here, because the gauge symmetry is linearly realized, the covariant derivatives are $SU_L(2) \times U_Y(1)$ invariant. Thus, as in the standard model, replacing ϕ by v generates the particular combination $c_W^2 \mathcal{O}_W + \mathcal{O}_Z$ with a coefficient that is of order $g^2 v^2$. There are also dimension-six contributions to the masses, such as $(\phi^\dagger D_\mu \phi) (\phi^\dagger D^\mu \phi) / M^2$. This and similar operators contribute to $\Delta\rho$ (that is, they spoil the mass relation $M_W = M_Z c_W$) by amounts that are of order $g^2 v^4 / M^2$. Therefore $\Delta\rho$ is automatically small in these theories provided only that $v^2 / M^2 \ll 1$. By contrast, if the symmetry-breaking sector is strongly interacting (NDA) generic contributions to both the W and Z boson masses are the same size, $O(gv)$, and so one requires an additional custodial $SU(2)$ symmetry to explain the smallness of $\Delta\rho$.

For the other operators in Table I, however, the NDA estimates are typically *smaller* than or equal to those of LRDA. This need not always be the case, as we have seen for the predictions for $\Delta\rho$ above.

A glance at Table I also shows that, in LRDA, the effective operators we are considering are all suppressed by at most two powers of $1/M$. It is therefore consistent to neglect all operators which are suppressed by more than $1/M^2$. While this rules out any operator of dimension seven or higher, the necessity to include dimension-six operators in general depends on the nature of the underlying theory. As is witnessed by the power counting of NDA, the suppression of four-fermion terms relative to those of lower dimension is possible, even if these lower-dimension terms should be $O(1/M^2)$.

IV. TRANSFORMING TO STANDARD FORM

Having now determined which operators to keep at $O(1/M^2)$, we must recognize that not all of the parameters of our effective Lagrangian need be physically significant. As was the case for the oblique corrections in the previous section, not all of the above interactions can represent a physical deviation from the standard model, since some can be removed without changing the form of $\mathcal{L}_{SM}$ simply by rescaling and rotating the fields. Only those that cannot be removed in this way without violating the symmetries of the standard model can have physical consequences, since these lead to deviations from the predictions that relate SM parameters, such as appears in Eq. (25) in Sec. II.

To determine the physical combinations we follow the logic set out in Sec. II: (i) first rescale all fields to put their kinetic and mass terms into standard form and (ii) eliminate the “tilded” parameters in the Lagrangian in favor of the physical quantities that are extracted from experiment. Only the algebra changes between this more general case and the simpler one studied in Sec. II.

A. Rescaling the fields

The diagonalization of the electroweak boson kinetic and mass terms is identical to that found in Eqs. (3)–(5). The fermion kinetic and mass terms are similarly diagonalized by the transformation:

$$\hat{f} = \left[\left(1 - \frac{I_L}{2} \right) U_L \gamma_L + \left(1 - \frac{I_R}{2} \right) U_R \gamma_R \right] f, \quad (38)$$

where the unitary matrices, U_L and U_R , are chosen to ensure that the mass matrix is diagonal with non-negative entries along the diagonal:

$$\begin{aligned} & \text{diag}(\dots, m_i, \dots) \\ &= U_R^\dagger [\tilde{m}_L + \delta m_L - \frac{1}{2}(I_R^\dagger \tilde{m}_L + \tilde{m}_L I_L)] U_L \\ &= U_L^\dagger [\tilde{m}_R + \delta m_R - \frac{1}{2}(I_L^\dagger \tilde{m}_R + \tilde{m}_R I_R)] U_R. \end{aligned} \quad (39)$$

The matrices $\tilde{m}_{L,R}$ which appear in these expressions denote the left- and right-handed fermion mass matrices in the original fermion basis.

After performing this redefinition, the standard-model and new-physics contributions to the fermion electromagnetic coupling become

$$\begin{aligned} \mathcal{L}_{em} = & -\tilde{e} \left[1 - \frac{A}{2} \right] [\bar{f} \gamma^\mu Q f A_\mu \\ & + \bar{f} \sigma^{\mu\nu} (d_L \gamma_L + d_R \gamma_R) f F_{\mu\nu}], \end{aligned}$$

where

$$d_L \equiv U_R^\dagger \hat{d}_L U_L, \quad d_R \equiv U_L^\dagger \hat{d}_R U_R. \quad (40)$$

Note that for these interactions the unbroken gauge invariance only permits the appearance of dipole-moment couplings, parametrized by the matrices $d_{L,R}$.

The neutral-current interactions similarly become

$$\mathcal{L}_{NC} = -\frac{\tilde{e}}{\tilde{s}_W \tilde{c}_W} \left[1 - \frac{C}{2} \right] [\bar{f} \gamma^\mu (\tilde{g}_L \gamma_L + \tilde{g}_R \gamma_R) f Z_\mu + \bar{f} \sigma^{\mu\nu} (n_L \gamma_L + n_R \gamma_R) f Z_{\mu\nu}], \quad (41)$$

where

$$\begin{aligned} \tilde{g}_L &\equiv g_L^{\text{SM}} + \delta \tilde{g}_L = U_L^\dagger [g_L^{\text{SM}} + \delta \hat{g}_L - \frac{1}{2}(I_L^\dagger g_L^{\text{SM}} + g_L^{\text{SM}} I_L)] U_L + \tilde{s}_W \tilde{c}_W Q G, \\ \tilde{g}_R &\equiv g_R^{\text{SM}} + \delta \tilde{g}_R = U_R^\dagger [g_R^{\text{SM}} + \delta \hat{g}_R - \frac{1}{2}(I_R^\dagger g_R^{\text{SM}} + g_R^{\text{SM}} I_R)] U_R + \tilde{s}_W \tilde{c}_W Q G, \end{aligned}$$

and

$$n_L \equiv U_R^\dagger \hat{n}_L U_L, \quad n_R \equiv U_L^\dagger \hat{n}_R U_R,$$

which in general may involve flavor-changing neutral currents. As discussed in Sec. III A, the left-handed and right-handed neutral-current couplings of neutrinos are not independent, being related by $\bar{g}_R^\nu = -(\bar{g}_L^\nu)^T$.

Finally, the charged-current couplings become

$$\mathcal{L}_{\text{CC}} = -\frac{\bar{e}}{\sqrt{2}\tilde{s}_W} \left[1 - \frac{B}{2} \right] [\bar{f}\gamma^\mu(\tilde{h}_L\gamma_L + \tilde{h}_R\gamma_R)f']W_\mu^\dagger + \bar{f}\sigma^{\mu\nu}(c_L\gamma_L + c_R\gamma_R)f'W_{\mu\nu}^\dagger] + \text{c.c.},$$

where

$$\tilde{h}_L \equiv h_L^{\text{SM}} + \delta\tilde{h}_L = U_L^\dagger [h_L^{\text{SM}} + \delta\hat{h}_L - \frac{1}{2}(I_L^\dagger h_L^{\text{SM}} + h_L^{\text{SM}} I_L')] U_L', \quad \tilde{h}_R \equiv h_R^{\text{SM}} + \delta\tilde{h}_R = U_R^\dagger \delta\hat{h}_R U_R', \quad (42)$$

and

$$c_L \equiv U_R^\dagger \hat{c}_L U_L', \quad c_R \equiv U_L^\dagger \hat{c}_R U_R'.$$

In these expressions, f represents a u -type quark or a neutrino, in which case f' is, respectively, either a d -type quark or a charged lepton. Primes on the matrices $U_{L,R}'$ and I_L' are meant to distinguish the matrices that are associated with f' from those associated with f . There are two qualitatively new features that arise here: (i) the introduction of a right-handed current and (ii) modifications to the left-handed CKM matrix. We elaborate on these in more detail in later sections.

The final remaining step is to determine the shift that is induced by the new physics into the reference parameters in the Lagrangian.

B. Shifting to physical parameters

Because of the present accuracy of the electroweak data, it suffices to work only to linear order in the new-physics parameters of our effective Lagrangian. Keeping higher order terms is conceptually straightforward, though algebraically more complicated.

None of the additional terms in this more general effective Lagrangian alter the connection between $\bar{e}$ and e , or between $\tilde{m}_Z$ and m_Z , so these remain as given in Eqs. (12) and (14):

$$\bar{e} = e \left[1 + \frac{A}{2} \right], \quad \tilde{m}_Z^2 = m_Z^2 (1 - z + C).$$

The really new features arise for the definition of G_F , and so for the expression for $\tilde{s}_W$ in terms of s_W , as well as for the charged-current CKM matrices. This is because each of these quantities is defined with reference to a charged-current fermion decay, and so their determination is affected by the deviations of $h_{L,R}$ from their SM values. We consider these observables here in turn.

Fermi's Constant (G_F): We must compare the tree-level expression for muon decay as computed with the new charged-current interactions, and read off the combination of parameters in the decay rate that is to be identified as the Fermi constant. The result is independent of the induced right-handed currents, since these do not interfere with the left-handed currents to within the accuracy we are interested. The same is true for the

coefficients $c_{L,R}$. The quantity which does arise to linear order in the new physics is

$$\frac{G_F}{\sqrt{2}} = \frac{\bar{e}^2}{8\tilde{s}_W^2 c_W^2 \tilde{m}_Z^2} (1 - w + \Delta_e + \Delta_\mu), \quad (43)$$

where

$$\Delta_f \equiv \left[\sum_i |1 + \delta\tilde{h}_L^{\nu_i f}|^2 \right]^{1/2} - 1 \\ = \sum_i \text{Re}(\delta\tilde{h}_L^{\nu_i f}) + (\text{higher order terms}). \quad (44)$$

Note that only the real part of $\delta\tilde{h}_L^{\nu_i f}$ appears here. This is because we are working to only linear order in the new-physics parameters, and therefore the only operators which can enter into the above expression are those which have SM counterparts with which they can interfere. Since in our conventions the SM leptonic charged-current couplings are purely real, $\text{Im}(\delta\tilde{h}_L^{\nu_i f})$ can never appear at linear order. Note also that, since we do not insist upon lepton-number conservation, the sum is over all light neutrinos. In terms of these variables, the analogue of Eq. (17) for $\tilde{s}_W^2$ is

$$\tilde{s}_W^2 = s_W^2 \left[1 + \frac{c_W^2}{c_W^2 - s_W^2} (A - C - w + z + \Delta_e + \Delta_\mu) \right], \quad (45)$$

where s_W^2 is defined as in Sec. II: $G_F/\sqrt{2} \equiv e^2/(8s_W^2 c_W^2 m_Z^2)$.

The CKM Matrix Elements (V_{ij}): As discussed above, the question of whether or not a new-physics operator contributes to linear order in the expression for an observable depends upon whether or not there is a corresponding SM operator with which it can interfere. For CP -violating new-physics contributions to CKM matrix elements, this appears to be problematic, since, according to this argument, $\text{Im}(\delta\tilde{h}_L^{ij})$ will appear only if the corresponding SM CKM matrix element V_{ij} is complex. However, the phase of a single CKM matrix element is not physically meaningful—any particular matrix element can be made real by phase redefinitions of the quark fields. It is only the phase of the product of four CKM matrix elements $V_{ij} V_{ik}^* V_{lk} V_{lj}^*$ which has a physical mean-

ing. In other words, it is a phase-convention-dependent question whether $\text{Re}(\delta\tilde{h}_L^{ij})$ or $\text{Im}(\delta\tilde{h}_L^{ij})$ (or both) appears in the expression for a particular observable. It is possible to express all observables in terms of the new-physics parameters in a completely general way, with no assump-

tions as to the reality of the CKM matrix elements, but this has the unfortunate effect of rendering the formulas unduly cumbersome. It is therefore useful, for simplicity, to choose a particular form for the SM CKM matrix. We use the approximate parametrization [15]

$$V_{\text{CKM}} \simeq \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\rho\lambda^3 e^{-i\delta} \\ -\lambda(1 + A^2\rho\lambda^4 e^{i\delta}) & 1 - \frac{1}{2}\lambda^2 - A^2\rho\lambda^6 e^{i\delta} & A\lambda^2 \\ A\lambda^3(1 - \rho e^{i\delta}) & -A\lambda^2(1 + \rho\lambda^2 e^{i\delta}) & 1 \end{pmatrix}, \quad (46)$$

in which $\lambda=0.22$ is the sine of the Cabibbo angle, the values of A and ρ are ~ 1 , and δ is constrained to lie between 0 and π (due to the nonzero value of ϵ_K , δ very close to 0 or π is excluded). Note that, in this parametrization, all CKM matrix elements save V_{ub} and V_{td} are essentially real. Therefore we know in advance that the $\text{Im}(\delta\tilde{h}_L^{ij})$, which can contribute to CP -violating processes, will remain virtually unconstrained.

The relation between the $\tilde{V}_{ij}$ and the V_{ij} depends crucially on the manner in which the CKM matrix elements are measured experimentally. For example, V_{ud} is determined from the β -decay rate for superallowed transitions in spinless nuclei. As such, these experiments measure the nuclear matrix element of the vector part of the quark-level transition $d \rightarrow u + e^- + \bar{\nu}$. If we read off the part of the amplitude which appears in this matrix element we find

$$\frac{G_F}{\sqrt{2}} |V_{ud}| = \frac{\tilde{e}^2}{8s_W^2 c_W^2 \tilde{m}_Z^2} |(\tilde{h}_L^{ud} + \tilde{h}_R^{ud})| (1 - w + \Delta_e). \quad (47)$$

Using this result together with expression (43) for G_F as determined in muon decay gives

$$|\tilde{V}_{ud}| \equiv |(h_L^{\text{SM}})_{ud}| = |V_{ud}| [1 + \Delta_\mu] - \text{Re}(\delta\tilde{h}_L^{ud} + \delta\tilde{h}_R^{ud}). \quad (48)$$

Analogous results hold for those elements of the CKM matrix that are determined by measuring the hadronic matrix element of the vector part of the quark-level transition $q_i \rightarrow q_j + e + \nu$. This is true for $|V_{us}|$ as determined in K_{e3} decays, or $|V_{cs}|$ as measured in D_{e3} decays. For these cases we have

$$|\tilde{V}_{ij}| \equiv |(h_L^{\text{SM}})_{ij}| = |V_{ij}| [1 + \Delta_\mu] - \text{Re}(\delta\tilde{h}_L^{ij} + \delta\tilde{h}_R^{ij}). \quad (49)$$

On the other hand, the matrix element V_{cd} is measured in the process $\nu_\mu d \rightarrow cX$, which, to linear order in the new physics, is sensitive only to the left-handed coupling. In this case,

$$|\tilde{V}_{cd}| = |V_{cd}| [1 + \Delta_e] - \text{Re}(\delta\tilde{h}_L^{cd}). \quad (50)$$

In other words, there is no general expression for the relation between $\tilde{V}_{ij}$ and V_{ij} — it must be calculated on a case-by-case basis.

We may now use these parameters in the Lagrangian. The terms of most practical interest are the W mass term, and the gauge-fermion couplings of Eqs. (40)–(42). The coefficient of $W_\mu^\dagger W^\mu$ becomes

$$m_W^2 = m_Z^2 c_W^2 \left[1 - \frac{\alpha S}{2(c_W^2 - s_W^2)} + \frac{c_W^2 \alpha T}{c_W^2 - s_W^2} + \frac{\alpha U}{4s_W^2} - \frac{s_W^2(\Delta_e + \Delta_\mu)}{c_W^2 - s_W^2} \right], \quad (51)$$

where S , T , and U are still defined as in Eq. (2). The electromagnetic interactions are straightforward to write down:

$$\mathcal{L}_{\text{em}} = -e [\bar{f} \gamma^\mu Q f A_\mu + \bar{f} \sigma^{\mu\nu} (d_L \gamma_L + d_R \gamma_R) f F_{\mu\nu}]. \quad (52)$$

The final form for the neutral-current interactions is

$$\begin{aligned} \mathcal{L}_{\text{NC}} = & -\frac{e}{s_W c_W} [\bar{f} \gamma^\mu [(g_L^{\text{SM}} + \delta g_L) \gamma_L + (g_R^{\text{SM}} + \delta g_R) \gamma_R] f Z_\mu + \bar{f} \sigma^{\mu\nu} (n_L \gamma_L + n_R \gamma_R) f Z_{\mu\nu}], \\ \delta g_{L(R)}^{ij} = & \delta_{ij} \frac{g_{i,L(R)}^{\text{SM}}}{2} (\alpha T - \Delta_e - \Delta_\mu) - Q_i \delta_{ij} \left[\frac{\alpha S}{4(c_W^2 - s_W^2)} - \frac{c_W^2 s_W^2 \alpha T}{c_W^2 - s_W^2} + \frac{c_W^2 s_W^2 (\Delta_e + \Delta_\mu)}{c_W^2 - s_W^2} \right] + \delta \tilde{g}_{L(R)}^{ij}. \end{aligned} \quad (53)$$

In the above expression for $\delta g_{L(R)}$, the coefficient of $g_{L(R)}^{\text{SM}}$ represents a universal overall correction to the strength of the interaction. The next term, proportional to the fermion charge Q_i can be considered as a shift in the effective electroweak mixing angle $(s_W^2)_{\text{eff}}$ as measured in neutral-current experiments. The final term consists of any direct new contributions to the current. Of these three types of contributions, this last term, and only this term, can contain

flavor-changing neutral currents (FCNC's).

Finally, the charged-current interaction becomes

$$\mathcal{L}_{\text{CC}} = -\frac{e}{\sqrt{2}s_W} [\bar{f} \gamma^\mu (h_L^{\text{SM}} + \delta h_L) \gamma_L + (h_R^{\text{SM}} + \delta h_R) \gamma_R] f' W_\mu^\dagger + \bar{f} \sigma^{\mu\nu} (c_L \gamma_L + c_R \gamma_R) f' W_{\mu\nu}^\dagger + \text{c.c.}, \quad (54)$$

where, for leptons,

$$\delta h_L^{v_i l_j} = \delta_{ij} \left[-\frac{\alpha S}{4(c_W^2 - s_W^2)} + \frac{c_W^2 \alpha T}{2(c_W^2 - s_W^2)} + \frac{\alpha U}{8s_W^2} - \frac{c_W^2 (\Delta_e + \Delta_\mu)}{2(c_W^2 - s_W^2)} \right] + \delta \tilde{h}_L^{v_i l_j}, \quad \delta h_R^{v_i l_j} = \delta \tilde{h}_R^{v_i l_j}, \quad (55)$$

while, for quarks,

$$\delta h_L^{u_i d_j} = \tilde{V}_{ij} \left[-\frac{\alpha S}{4(c_W^2 - s_W^2)} + \frac{c_W^2 \alpha T}{2(c_W^2 - s_W^2)} + \frac{\alpha U}{8s_W^2} - \frac{c_W^2 (\Delta_e + \Delta_\mu)}{2(c_W^2 - s_W^2)} \right] + \delta \tilde{h}_L^{u_i d_j}, \quad \delta h_R^{u_i d_j} = \delta \tilde{h}_R^{u_i d_j}. \quad (56)$$

As for the neutral currents, in the above equations the coefficients of the δ_{ij} and $\tilde{V}_{ij}$ terms are universal corrections, while all other corrections are nonuniversal. Also, as discussed previously, we have not substituted for $\tilde{V}_{ij}$ in the above equations since there is no general relation between $\tilde{V}_{ij}$ and V_{ij} .

We may now apply these expressions to a number of relevant observables.

V. APPLICATIONS TO OBSERVABLES

The ultimate goal of this analysis is to use current experimental data to constrain the new-physics parameters. In this section we compute expressions for a large number of observables in terms of our various effective couplings. We also report on the results of detailed fits for these couplings where this is appropriate.

Our starting point is the effective Lagrangian we have constructed, which consists only of the standard model supplemented by those effective interactions which have the lowest few dimensions. It is important to keep in mind the existence of a potentially infinite number of terms which we have not written down, and which we imagine are suppressed compared to the ones kept by additional powers of $1/M$. Because of the existence of these other terms, when computing the implications for observables, it would be *inconsistent* to work beyond linear order in our lowest-dimension effective interactions, and to still neglect the higher-dimension operators which we have not included. As a result we limit ourselves to working only to linear order in the couplings of our effective Lagrangian.

We consider only those observables to which our new-physics parameters contribute at tree level for a slightly different reason. In this case any contribution which is obtained by inserting an effective operator into a loop can be canceled by a small correction to the coefficient of the operators which contribute to the same observable at tree level. Alternatively, loop graphs tell us how the effective operators mix as they are renormalized down from the high scale M where the new physics is integrated out, to the lower scales where the observable in question is measured.

Having said this, there is still one situation where

working to higher order in our effective couplings, or going beyond the tree-level calculations, makes sense. This is in the case where measurements of an observable are sufficiently precise to strongly exclude new-physics contributions, even beyond linear order or tree level. Although we cannot ever rule out the possibility that a nonzero contribution from one of our low-dimension operators at quadratic order (say) may cancel with a linear contribution of an operator we have neglected, the likelihood of this becomes more implausible the stronger the cancellation that is required. As a result we can use precision measurements to bound our interactions beyond linear order in their coefficients, and beyond tree level in their contributions, provided that we are aware of this possibility of cancellation.

In practice, sufficiently well-measured observables are usually associated with processes that do not arise, or are highly suppressed, in the standard model due to (approximate) conservation laws or selection rules. For the present purposes we only work beyond linear order for observables which involve flavor-changing neutral currents (FCNC's). These are highly suppressed in the standard model, and so typically first arise to quadratic order in our effective interactions. When computing these bounds we therefore work to this order, but any limits we find that are not very strong must be considered suspect, since they could easily be circumvented through cancellations with higher-dimension operators. For all other processes it suffices to work to linear order in the new-physics parameters. At a practical level this implies that most of the coefficients of operators which are not of the SM form, such as the magnetic terms in Eqs. (53) and (54), or of most *CP*-violating interactions, will not be bounded in this analysis since they do not interfere with the standard model.

Similarly, we only consider the loop-level contributions of our effective operators to neutral-meson mixing, ϵ_K , anomalous magnetic moments, and to particle electric dipole moments (EDM's), all of which are measured (or bounded) with great precision. Again, weak limits should not be taken too seriously, due to possible effects of cancellations among the contributions of various operators.

For FCNC's, and well-measured quantities like $(g-2)_e$ and $(g-2)_\mu$, as well as electric dipole moments (EDM's),

only one (or, sometimes, two) observable is required to bound each effective operator. In this case we simply quote the upper bound that is required for the appropriate effective coupling. Most of the other interactions can contribute to a great many quantities. In this instance we perform a full fit to all of the observables using the entire effective Lagrangian. For comparison purposes we report here on two types of fits. In the first, called the “individual fit,” only one parameter at a time is allowed to be nonzero. This fit will obviously yield the most stringent constraints on the parameter in question, since no possibility exists for cancellations. The second procedure (the “simultaneous fit”) allows all parameters to vary simultaneously. Because of cancellations most parameters are less constrained in this fit, and certain combinations remain unconstrained entirely in this case. As we describe the various observables we also indicate which parameters are not bounded, and hence can be excluded from the simultaneous fit.

Much of the material in this section is adapted from Refs. [16,17]. Where numbers are given we use $\alpha = 1/128$ and $s_W^2 = 0.23$.

A. Flavor-changing neutral currents

As mentioned above, our only excursion past linear order in new physics comes about in this section. In the standard model there are no FCNC's at tree level, and most loop-induced FCNC's are calculated to be extremely small. Thus, FCNC's are a smoking gun for new physics, and it is useful to investigate the prospects for their detection.

The terms in our effective Lagrangian [see Eqs. (52) and (53)] which can lead to FCNC's are

$$\begin{aligned} \mathcal{L}_{\text{FCNC}} = & -\frac{e}{s_W c_W} \left[\bar{f} \gamma^\mu (\delta g_L \gamma_L + \delta g_R \gamma_R) f Z_\mu \right. \\ & \left. + \frac{1}{M} \bar{f} \sigma^{\mu\nu} (n_L \gamma_L + n_R \gamma_R) f Z_{\mu\nu} \right] \\ & - \frac{e}{M} \bar{f} \sigma^{\mu\nu} (d_L \gamma_L + d_R \gamma_R) f F_{\mu\nu}, \end{aligned} \quad (57)$$

where $\delta g^{ij} = \delta \bar{g}^{ij}$ for $i \neq j$, and we introduce a factor of $1/M$ in the “magnetic” terms for dimensional purposes. We discuss the three types of FCNC terms ($\delta g_{L,R}$, $n_{L,R}$, and $d_{L,R}$) in turn.

$\delta g_{L,R}$'s: The strongest constraints on leptonic FCNC's come from the absence of the decays $\mu \rightarrow 3e$ and $\tau \rightarrow 3l$. For this type of decay we find

$$\begin{aligned} \Gamma(L \rightarrow 3l) &= \frac{G_F m_L^5}{48\pi^3} [(g_{l,L}^{\text{SM}})^2 + (g_{l,R}^{\text{SM}})^2] [|\delta \bar{g}_L^{lL}|^2 + |\delta \bar{g}_R^{lL}|^2], \end{aligned} \quad (58)$$

where the masses of the final-state particles have been ignored. Using the experimental bounds on $\mu \rightarrow 3e$ and $\tau \rightarrow 3l$ [18], the limits shown in Table II are obtained.

There are also bounds from $Z \rightarrow lL$. The contribution to this process is

$$\Gamma(Z \rightarrow lL) = \frac{\alpha M_Z}{6s_W^2 c_W^2} [|\delta \bar{g}_L^{lL}|^2 + |\delta \bar{g}_R^{lL}|^2], \quad (59)$$

which, when combined with the experimental limits in [19] leads to the constraints in Table II.

For the ds FCNC, the strongest constraint comes from the decay $K_L \rightarrow \mu^+ \mu^-$. Using the analogue of Eq. (58) for the quarks in the kaon system, and following the analysis of Ref. [21] one finds

$$\frac{B(K_L \rightarrow \mu^+ \mu^-)}{B(K^+ \rightarrow \mu^+ \nu_\mu)} = \frac{\tau(K_L)}{\tau(K^+)} \frac{8[(g_{\mu,L}^{\text{SM}})^2 + (g_{\mu,R}^{\text{SM}})^2] [|\delta \bar{g}_L^{ds}|^2 + |\delta \bar{g}_R^{ds}|^2]}{|V_{us}|^2}, \quad (60)$$

where $\tau(K)$ represents the corresponding K -meson lifetime. Reference [18] gives $B(K_L \rightarrow \mu^+ \mu^-) = (7.0 \pm 0.82) \times 10^{-9}$,

TABLE II. Constraints on the flavor-changing neutral current parameters $\delta g_{L,R}^{ij} = \delta \bar{g}_{L,R}^{ij}$, for $i \neq j$.

Quantity	Upper bound	Source
$ \delta \bar{g}_{L,R}^{e\mu} $	2×10^{-6}	$\mu \rightarrow 3e$ [18]
$ \delta \bar{g}_{L,R}^{e\tau} $	1×10^{-2}	$Z \rightarrow e\mu$ [19]
$ \delta \bar{g}_{L,R}^{\mu\tau} $	6×10^{-3}	$\tau \rightarrow 3l$ [18]
$ \delta \bar{g}_{L,R}^{\mu\tau} $	2×10^{-2}	$Z \rightarrow e\tau$ [19]
$ \delta \bar{g}_{L,R}^{\mu\tau} $	6×10^{-3}	$\tau \rightarrow 3l$ [18]
$ \delta \bar{g}_{L,R}^{ds} $	2×10^{-2}	$Z \rightarrow \mu\tau$ [19]
$ \delta \bar{g}_{L,R}^{ds} $	2×10^{-5}	$K_L \rightarrow \mu^+ \mu^-$ [18]
$ \delta \bar{g}_{L,R}^{ds} $	3×10^{-4}	$\Delta m_{K_L K_S}$ [18]
$[\text{Re}(\delta \bar{g}_L^{ds} \delta \bar{g}_R^{*ds})]^{1/2}$	8×10^{-5}	$\Delta m_{K_L K_S}$ [18]
$ \delta \bar{g}_{L,R}^{uc} $	4×10^{-4}	$D^0 - \bar{D}^0$ mixing [18]
$[\text{Re}(\delta \bar{g}_L^{uc} \delta \bar{g}_R^{*uc})]^{1/2}$	1×10^{-4}	$D^0 - \bar{D}^0$ mixing [18]
$ \delta \bar{g}_{L,R}^{db} , \delta \bar{g}_{L,R}^{sb} $	2×10^{-3}	$B \rightarrow l^+ l^- X$ [20]

and the long-distance contribution from the 2γ intermediate state is found to be [22] $(6.83 \pm 0.46) \times 10^{-9}$. In light of this we assume that the rate for $K_L \rightarrow \mu^+ \mu^-$ is explained by the standard model, and require that the new-physics contribution be smaller than the experimentally measured value plus 1.64σ (which corresponds to 90% C.L.). This gives the bound in Table II.

There is also a constraint on the ds FCNC from the $K_L - K_S$ mass difference. We find

$$\Delta M_K = \frac{G_F}{\sqrt{2}} [|\delta \tilde{g}_L^{ds}|^2 + |\delta \tilde{g}_R^{ds}|^2 + 2(0.77) \text{Re}(\delta \tilde{g}_R^{ds} \delta \tilde{g}_L^{*ds})] \frac{4}{3} f_K^2 m_K B_K, \quad (61)$$

where we have used the results of [23] for the left-right matrix element. We require that this contribution be less than the experimental value ($+1.64\sigma$), leading to the constraints in Table II. Note that these limits are weaker than those from $K_L \rightarrow \mu^+ \mu^-$.

The constraints on uc FCNC's are due to the absence of $D^0 - \bar{D}^0$ mixing. Using Eq. (61) adapted to the D system and taking $B_D = 1$, $f_D = 200$ MeV we find the constraints in Table II.

Finally, the FCNC's involving the b quark are constrained by using the process $B \rightarrow \mu \mu X$. One has [21]

$$\frac{B(B \rightarrow \mu \mu X)}{B(B \rightarrow \mu \nu_\mu X)} = \frac{4[(g_{\mu,L}^{\text{SM}})^2 + (g_{\mu,R}^{\text{SM}})^2][|\delta \tilde{g}_L^{db}|^2 + |\delta \tilde{g}_R^{db}|^2 + |\delta \tilde{g}_L^{sb}|^2 + |\delta \tilde{g}_R^{sb}|^2]}{|V_{ub}|^2 + F_{\text{PS}} |V_{cb}|^2}, \quad (62)$$

where $F_{\text{PS}} \simeq 0.5$ is a phase space factor. The constraints on the FCNC parameters are given in Table II, where we have used $B(B \rightarrow \mu \mu X) < 5 \times 10^{-5}$ [20].

$n_{L,R}$'s: The $n_{L,R}$ FCNC's can be bounded in the same way as the $\delta \tilde{g}_{L,R}$'s. For leptonic FCNC's we use the decays $\mu \rightarrow 3e$ and $\tau \rightarrow 3l$. The contribution of the $n_{L,R}$'s to these decays is found to be

$$\Gamma(L \rightarrow 3l) = \frac{G_F m_L^5}{30\pi^3} [(g_{l,L}^{\text{SM}})^2 + (g_{l,R}^{\text{SM}})^2] \frac{m_L^2}{M^2} [|n_L^{lL}|^2 + |n_R^{lL}|^2]. \quad (63)$$

What is noteworthy here, and indeed in all of the following processes, is the suppression factor m^2/M^2 . From this we can deduce that low-energy limits on FCNC's will

not put terribly strong constraints on the $n_{L,R}$'s. The bounds from $\mu \rightarrow 3e$ and $\tau \rightarrow 3l$ are shown in Table III. We take $M = 1$ TeV.

The contributions of the $n_{L,R}$ terms to $Z \rightarrow lL$ are

$$\Gamma(Z \rightarrow lL) = \frac{\alpha M_Z}{3s_W^2 c_W^2} \frac{M_Z^2}{M^2} [|n_L^{lL}|^2 + |n_R^{lL}|^2]. \quad (64)$$

From these one can deduce the limits shown in Table III. Note that, for FCNC's involving τ 's, in contrast with the $\delta \tilde{g}_{L,R}$'s, the constraints from the absence of leptonic FCNC in Z decays are *stronger* than those from low energy.

Turning to the ds FCNC's, and adapting the results of Ref. [21] we have

$$\frac{B(K_L \rightarrow \mu^+ \mu^-)}{B(K^+ \rightarrow \mu^+ \nu_\mu)} = \frac{192}{15} \frac{\tau(K_L)}{\tau(K^+)} \frac{m_K^2}{M^2} \frac{[(g_{\mu,L}^{\text{SM}})^2 + (g_{\mu,R}^{\text{SM}})^2][|n_L^{ds}|^2 + |n_R^{ds}|^2]}{|V_{us}|^2}, \quad (65)$$

TABLE III. Constraints on the dimension-five coupling parameters $n_{L,R}^{ij}, i \neq j$ using a new-physics scale of $M = 1$ TeV.

Quantity	Upper bound	Source
$ n_{L,R}^{e\mu} $	0.02	$\mu \rightarrow 3e$ [18]
	0.08	$Z \rightarrow e\mu$ [19]
$ n_{L,R}^{e\tau} $	3	$\tau \rightarrow 3l$ [18]
	0.09	$Z \rightarrow e\tau$ [19]
$ n_{L,R}^{\mu\tau} $	3	$\tau \rightarrow 3l$ [18]
	0.1	$Z \rightarrow \mu\tau$ [19]
$ n_{L,R}^{ds} $	0.02	$K_L \rightarrow \mu^+ \mu^-$ [18]
	0.3	$\Delta m_{K_L K_S}$ [18]
$ n_{L,R}^{\mu c} $	0.1	$D^0 - \bar{D}^0$ mixing [18]
$ n_{L,R}^{db} , n_{L,R}^{sb} $	0.2	$B \rightarrow l^+ l^- X$ [20]

giving the bounds in Table III. Extracting constraints from the K_L - K_S mass difference is more problematic. The difficulty is that, using the $n_{L,R}$ operators, new hadronic matrix elements are obtained. Rather than trying to evaluate these we will simply estimate the contribution to ΔM_K as

$$\Delta M_K \sim \frac{4G_F m_K^2}{\sqrt{2}M^2} [|n_L^{ds}|^2 + |n_R^{ds}|^2] \frac{4}{3} f_K^2 m_K B_K, \quad (66)$$

where we have taken the unknown matrix element to be

$$\frac{B(B \rightarrow \mu\mu X)}{B(B \rightarrow \mu\nu_\mu X)} = \frac{192}{30} \frac{m_B^2}{m^2} \frac{[(g_{\mu,L}^{\text{SM}})^2 + (g_{\mu,R}^{\text{SM}})^2][|n_L^{db}|^2 + |n_R^{db}|^2 + |n_L^{sb}|^2 + |n_R^{sb}|^2]}{|V_{ub}|^2 + F_{\text{PS}}|V_{cb}|^2}, \quad (67)$$

which yields the constraints in Table III.

It is noteworthy that the constraints on the couplings $n_{L,R}^{bs}$ from low-energy experiments are very weak. Contrary to the naive expectation that the bounds on $B \rightarrow \mu\mu X$ would preclude any chance of detecting $Z \rightarrow s\bar{b}, \bar{s}b$ at LEP, we see here that the two processes sample completely *different* operators. Should new physics produce terms like $\bar{b}\sigma^{\mu\nu}(n_L\gamma_L + n_R\gamma_R)sZ_{\mu\nu}$, then such FCNC's could be seen at LEP without having been ruled out in B decays—indeed, it would be very foolish to overlook this possibility.

This example beautifully illustrates the power of the effective Lagrangian approach. By systematically listing all operators up to a given order in $1/M$, one can discover terms which can give rise to physically observable effects which might not otherwise have been considered.

$d_{L,R}$'s: The analysis leading to bounds on the $d_{L,R}$ FCNC's is similar to that for the $n_{L,R}$'s, with the following important differences. First, certain $d_{L,R}$'s can be

of the order of the left-left matrix element, and have ignored the left-right mixing term. With this order-of-magnitude estimate one obtains the limits shown in Table III. Note that these are much weaker than those due to $K_L \rightarrow \mu^+\mu^-$.

The same difficulty is encountered in using D^0 - $\bar{D}^0$ mixing to constrain the uc FCNC's. Estimating the contribution to ΔM_D in the same way as was done for the kaon system, we find the constraints shown in Table III.

For $B \rightarrow \mu\mu X$ we have

bounded directly from the process $f \rightarrow f'\gamma$. Second, because of the fact that the photon is massless, decays such as $\mu \rightarrow 3e$ are *not* suppressed by powers of m_μ/m_Z , as they are in the case of the Z FCNC's. In fact, as we shall see, there is a logarithmic enhancement of such decays. Finally, there are no bounds on the $d_{L,R}$'s from FCNC's at the Z peak, since the contribution from photon exchange is very much suppressed in these processes.

The strongest constraints on leptonic $d_{L,R}$ FCNC's come from the experimental limits on the decays $\mu \rightarrow e\gamma$, $\tau \rightarrow e\gamma$, and $\tau \rightarrow \mu\gamma$. The contribution of the $d_{L,R}$'s to these decays is

$$\Gamma(L \rightarrow l\gamma) = \alpha \frac{m_L^3}{M^2} [|d_L^{ll}|^2 + |d_R^{ll}|^2]. \quad (68)$$

Using the experimental limits from Ref. [18] gives the bounds listed in Table IV.

TABLE IV. Constraints on the dimension-five coupling parameters $d_{L,R}^j$, using a new-physics scale of $M=1$ TeV.

Quantity	Upper bound	Source
$ d_{L,R}^{e\mu} $	2×10^{-9}	$\mu \rightarrow e\gamma$ [18]
	3×10^{-9}	$\mu \rightarrow 3e$ [18]
$ d_{L,R}^{e\tau} $	5×10^{-5}	$\tau \rightarrow e\gamma$ [18]
	2×10^{-4}	$\tau \rightarrow 3l$ [18]
$ d_{L,R}^{\mu\tau} $	8×10^{-5}	$\tau \rightarrow \mu\gamma$ [18]
	3×10^{-4}	$\tau \rightarrow 3l$ [18]
$ d_{L,R}^{ds} $	2×10^{-7}	$K_L \rightarrow \mu^+\mu^-$ [18]
	4×10^{-3}	$\Delta m_{K_L K_S}$ [18]
$ d_{L,R}^{\mu c} $	6×10^{-3}	D^0 - $\bar{D}^0$ mixing [18]
$ d_{L,R}^{db} $	3×10^{-5}	$B \rightarrow l^+ l^- X$ [20]
$ d_{L,R}^{sb} $	2×10^{-4}	$b \rightarrow s\gamma$ [24]
	4×10^{-5}	$B \rightarrow l^+ l^- X$ [20]
$ d_L^{ee} + d_R^{ee} $	8×10^{-6}	$a(e)$ [25]
$ d_L^{\mu\mu} + d_R^{\mu\mu} $	1×10^{-4}	$a(\mu)$ [26]
$ d_L^{ee} - d_R^{ee} $	8×10^{-10}	Atomic EDM's [27]
$ d_L^{\mu\mu} - d_R^{\mu\mu} $	0.05	$(g_\mu - 2)/2$ [28]
$ d_L^{\tau\tau} - d_R^{\tau\tau} $	5	$e^+e^- \rightarrow \tau^+\tau^-$ [29]
$ d_L^{v_i v_i} - d_R^{v_i v_i} (i=e,\mu)$	5×10^{-4}	$ve \rightarrow ve$ [29]
$ d_L^{dd} - d_R^{dd} , d_L^{uu} - d_R^{uu} $	6×10^{-8}	Neutron EDM [30]

The leptonic $d_{L,R}$ FCNC's will also lead to the process $L \rightarrow 3l$. As noted earlier, since the photon is massless, there is no suppression of this process relative to $L \rightarrow l\gamma$ due to the photon propagator. The only suppression is due to an additional factor of α , as well as from the three-body phase space, as compared to two-body phase space. And this is partially compensated for by a large logarithm, due to the presence of an infrared mass singularity in the limit $m_l \rightarrow 0$. The contribution of the $d_{L,R}$'s to the process $\mu \rightarrow 3e$ is found to be

$$\Gamma(\mu \rightarrow 3e) = \frac{8\alpha^2}{\pi} \frac{m_\mu^3}{M^2} \left[\frac{1}{24} \ln \left[\frac{m_\mu^2}{m_e^2} \right] - \frac{1}{18} \right] [|d_L^{e\mu}|^2 + |d_R^{e\mu}|^2]. \quad (69)$$

The contribution to the processes $\tau \rightarrow el^+l^-$ and $\tau \rightarrow \mu l^+l^-$ is obtained in the obvious way from the above equation. This leads to the constraints shown in Table IV. As mentioned above, the constraints on the $d_{L,R}^{ll}$ from $L \rightarrow 3l$ are only slightly weaker than those arising from $L \rightarrow l\gamma$.

One constraint on the ds FCNC's comes from the process $K_L \rightarrow \mu^+\mu^-$. Adapting Eq. (69) to the process $s \rightarrow d\mu^+\mu^-$, and using the results of Ref. [21] we have

$$\frac{B(K_L \rightarrow \mu^+\mu^-)}{B(K^+ \rightarrow \mu^+\nu_\mu)} = \frac{3072\alpha^2\pi^2}{m_K^2 M^2 G_F^2} \frac{\tau(K_L)}{\tau(K^+)} \left[\frac{1}{24} \ln \left[\frac{m_s^2}{m_d^2} \right] - \frac{1}{18} \right] \frac{[|d_L^{ds}|^2 + |d_R^{ds}|^2]}{|V_{us}|^2}. \quad (70)$$

We take $m_s = 150$ MeV and $m_d = 5$ MeV, leading to the bounds in Table IV. There are also constraints from the K_L - K_S mass difference. However, as was the case for the $n_{L,R}^{ds}$ FCNC's, we encounter new hadronic matrix elements. Therefore, once again, we simply give a rough estimate of the contribution to ΔM_K :

$$\Delta M_K \sim \frac{e^2}{2M^2} [|d_L^{ds}|^2 + |d_R^{ds}|^2] \frac{4}{3} f_K^2 m_K B_K. \quad (71)$$

This leads to the order-of-magnitude limits in Table IV. As was the case for the $n_{L,R}^{ds}$'s, these limits are much weaker than those due to $K_L \rightarrow \mu^+\mu^-$.

The uc FCNC's are constrained by D^0 - $\bar{D}^0$ mixing. Using the same procedure as was done for the K_L - K_S mass difference we arrive at the bounds in Table IV.

The process $B \rightarrow \mu\mu X$ constrains both the db and sb FCNC's:

$$\frac{B(B \rightarrow \mu\mu X)}{B(B \rightarrow \mu\nu_\mu X)} = \frac{1536\alpha^2\pi^2}{m_B^2 M^2 G_F^2} \left[\frac{1}{24} \ln \left[\frac{m_b^2}{m_q^2} \right] - \frac{1}{18} \right] \frac{[|d_L^{qb}|^2 + |d_R^{qb}|^2]}{|V_{ub}|^2 + F_{PS} |V_{cb}|^2}, \quad (72)$$

in which $q=d,s$. The bounds are shown in Table IV. For the sb FCNC, there is also a limit due to the experimental measurement of $b \rightarrow s\gamma$ [24]. Using Eq. (68) we find the contribution of the $d_{L,R}^{sb}$ to this process to be

$$B(b \rightarrow s\gamma) = \tau_B \alpha \frac{m_b^3}{M^2} [|d_L^{sb}|^2 + |d_R^{sb}|^2]. \quad (73)$$

Taking $\tau_B = 1.49$ psec leads to the constraints in Table IV.

B. Anomalous magnetic moments

Extremely precise measurements exist for the anomalous magnetic moments of the electron and muon: $a_i = (g_i - 2)/2 = (\mu_i/\mu_B) - 1$, for $i=e,\mu$ and $\mu_B \equiv e/2m_i$. The current best experimental values for the electron and positron are [25]

$$\begin{aligned} a(e^-) &= 1\,159\,652\,188.4 (4.3) \times 10^{-12}, \\ a(e^+) &= 1\,159\,652\,187.9 (4.3) \times 10^{-12}, \end{aligned} \quad (74)$$

while that for the muon is [26]

$$\begin{aligned} a(\mu^-) &= 1\,165\,937 (12) \times 10^{-9}, \\ a(\mu^+) &= 1\,165\,911 (11) \times 10^{-9}. \end{aligned} \quad (75)$$

These are in good agreement with the corresponding SM (i.e., QED) predictions [31]:

$$\begin{aligned} a^{\text{th}}(e) &= 1\,159\,652\,140 (5.3) (4.1) (27.1) \times 10^{-12}, \\ a^{\text{th}}(\mu) &= 1\,165\,919\,18 (191) \times 10^{-11}. \end{aligned} \quad (76)$$

The largest error in $a^{\text{th}}(e)$ is due to the determination of α , a fact which presently limits using the comparison with $a^{\text{th}}(e)$ as a precision test of QED.

The quantities $d_{L,R}^{ee}$ and $d_{L,R}^{\mu\mu}$ contribute directly to this observable, by an amount

$$\delta a_i = \frac{2m_i}{M} (d_L^{ii} + d_R^{ii}), \quad (77)$$

where $i=e,\mu$. We obtain our bound by requiring that this contribution be smaller than the corresponding

1.64 σ experimental error. Taking $M=1$ TeV, as before, produces the constraints shown in Table IV.

C. Electric dipole moments

The difference between d_L^{ff} and d_R^{ff} contributes to the corresponding particle's electric dipole moment, defined as the coefficient of the term in the particle's energy shift which is linear in the applied field. For a fundamental fermion such a definition is equivalent to defining d_f as the coefficient of the effective electromagnetic interaction

$$-\frac{d_f}{2}\bar{f}i\sigma^{\mu\nu}\gamma_5 f F_{\mu\nu}. \quad (78)$$

In terms of the interactions in our effective Lagrangian we therefore have

$$\begin{aligned} -id_f &= \frac{e}{M}(d_L^{ff} - d_R^{ff}) \\ &= (2 \times 10^{-17} \text{ e cm})(d_L^{ff} - d_R^{ff}). \end{aligned} \quad (79)$$

In this last line, as in Table IV we take the fiducial value $M=1$ TeV.

Extremely good limits currently exist for the electric dipole moment of various atoms [27,32] and for the neutron [30]. The atomic bounds permit the inference of a very strong bound on the EDM of the electron [27]. Using these limits we arrive at the bounds given in Table IV. The constraints on the EDM's of light quarks are obtained from the experimental limit on the neutron EDM. For both the electron and quark EDM's there is some uncertainty in extracting these bounds since many operators in the underlying theory can generate either atomic or neutron EDM's. For electrons we quote here the bounds as given by the experimental groups themselves. This is not done for the neutron, since here there is the additional uncertainty associated with computing the nucleon matrix element of the quark-level operator. To be conservative we simply use the estimate

$$d_n \sim d_u \sim d_d, \quad (80)$$

and quote a limit on d_q which is ten times weaker than the measured bound on d_n .

The EDM's of other particles may also be constrained.

That for the muon is directly limited by the experiment which measures $(g-2)_\mu$ [28]. One may attempt to obtain a bound for the τ , ν_e , and ν_μ electric moments from the observed absence of the effects that such moments would produce in the reactions $e^+e^- \rightarrow \tau^+\tau^-$ or in νe scattering [29]. Since the EDM enters quadratically into these cross sections, these bounds can only be inferred to the extent that cancellations with other effective interactions can be ignored. As may be seen from Table IV, although this may be plausible for the neutrino moments, it is not justified for the τ lepton.

More indirect limits on neutrino moments also exist in certain circumstances [29]. If neutrinos are Dirac (or pseudo Dirac) particles then right-handed sterile neutrinos likely exist and are light enough to be produced from left-handed neutrinos, via the magnetic moment interactions, in stars, supernovas, and in the early universe. We do not include these bounds here since we have excluded sterile right-handed neutrinos from our low-energy particle content.

D. Charged currents

We next turn to the bulk of the constraints on the effective Lagrangian, charged-current, and neutral-current data. Since many of the effective interactions can contribute to many observables, we evaluate the remaining bounds by performing a global fit.

Some of the low-dimension effective interactions are not bounded to the order we work. This is because many operators do not contribute at all to linear order in their coefficients. This is true, in particular, for terms which do not, on grounds of helicity conservation, interfere appreciably with SM contributions. As a result we will not be bounding the magnetic terms in Eq. (54). The same is true for the right-handed currents in this equation, except insofar as they contribute to linear order to the CKM matrix elements, and in $K \rightarrow 3\pi$ decays. We remind the reader that in what follows we take $\alpha=1/128$ and $s_W^2=0.23$.

The W Mass: In the presence of new physics, the relationship between the W and the Z mass is modified. Inspection of Eq. (51) gives the result

$$\begin{aligned} M_W^2 &= (M_W^2)_{\text{SM}} \left[1 - \frac{\alpha S}{2(c_W^2 - s_W^2)} + \frac{c_W^2 \alpha T}{c_W^2 - s_W^2} + \frac{\alpha U}{4s_W^2} - \frac{s_W^2(\Delta_e + \Delta_\mu)}{c_W^2 - s_W^2} \right] \\ &= (M_W^2)_{\text{SM}} [1 - 0.00723S + 0.0111T + 0.00849U - 0.426(\Delta_e + \Delta_\mu)]. \end{aligned} \quad (81)$$

Recall that the Δ_f are defined in Eq. (44), and since we do not assume the conservation of lepton number, the sum in the definition of Δ_f is over all light neutrinos.

CKM Unitarity: The strongest experimental constraint on new couplings of the W to quarks comes from the unitarity of the CKM matrix. As discussed previously, the relation between the parameters in the Lagrangian $\tilde{V}_{ij}$ and the measured quantities V_{ij} is altered due to new physics. For V_{ud} and V_{us} , the relation is as given in Eq. (49). This is not the case for V_{ub} , which is measured using the end point spectrum of semileptonic B decays. However, in any event, because V_{ub} is so small, we drop terms of order V_{ub}^2 . The three-generation relation $\sum_{i=1}^3 |\tilde{V}_{ui}|^2 = 1$ leads to

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - 2\Delta_\mu + 2|V_{ud}|\text{Re}(\delta\tilde{h}_L^{ud} + \delta\tilde{h}_R^{ud}) + 2|V_{us}|\text{Re}(\delta\tilde{h}_L^{us} + \delta\tilde{h}_R^{us}) + 2[\text{Re}(V_{ub})\text{Re}(\delta\tilde{h}_L^{ub}) + \text{Im}(V_{ub})\text{Im}(\delta\tilde{h}_L^{ub})], \quad (82)$$

where on the right-hand side we have replaced $\tilde{V}_{ij}$ by V_{ij} . Note that the new-physics parameters $\text{Re}(\delta\tilde{h}_L^{us})$, $\text{Re}(\delta\tilde{h}_L^{ub})$, and $\text{Im}(\delta\tilde{h}_L^{ub})$ appear only in the above expression; they contribute to no other charged-current observables (at tree level and to linear order). Therefore, in the simultaneous fit, only the sum of terms $|V_{us}|\text{Re}(\delta\tilde{h}_L^{us}) + [\text{Re}(V_{ub})\text{Re}(\delta\tilde{h}_L^{ub}) + \text{Im}(V_{ub})\text{Im}(\delta\tilde{h}_L^{ub})]$ can ever be constrained, and we present the bound on this combination only.

The second row of the CKM matrix is similar, except that V_{cd} is measured differently, as discussed in Sec. IV B. We find

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1 + 2|V_{cd}|\text{Re}(\delta\tilde{h}_L^{cd}) + 2|V_{cs}|\text{Re}(\delta\tilde{h}_L^{cs} + \delta\tilde{h}_R^{cs}) + 2|V_{cb}|\text{Re}(\delta\tilde{h}_L^{cb}), \quad (83)$$

where we have neglected all $\Delta_{e,\mu}$ terms, as they are much better constrained in other processes. In the simultaneous fit of all parameters, only the sum $|V_{cd}|\text{Re}(\delta\tilde{h}_L^{cd}) + |V_{cs}|\text{Re}(\delta\tilde{h}_L^{cs} + \delta\tilde{h}_R^{cs}) + |V_{cb}|\text{Re}(\delta\tilde{h}_L^{cb})$ arises; the individual new-physics parameters are unconstrained by our fit. As a consequence, as before, we present only the bound on this sum when we perform the simultaneous fit.

Lepton Universality: Lepton universality is tested in pion and τ decays. It is straightforward to calculate

$$\begin{aligned} R_\pi &\equiv \frac{\Gamma(\pi \rightarrow e\nu)}{\Gamma(\pi \rightarrow \mu\nu)} = R_\pi^{\text{SM}}(1 + 2\Delta_e - 2\Delta_\mu), \\ R_\tau &\equiv \frac{\Gamma(\tau \rightarrow e\nu\bar{\nu})}{\Gamma(\mu \rightarrow e\nu\bar{\nu})} = R_\tau^{\text{SM}}(1 + 2\Delta_\tau - 2\Delta_\mu), \\ R_{\mu\tau} &\equiv \frac{\Gamma(\tau \rightarrow \mu\nu\bar{\nu})}{\Gamma(\mu \rightarrow e\nu\bar{\nu})} = R_{\mu\tau}^{\text{SM}}(1 + 2\Delta_\tau - 2\Delta_e). \end{aligned}$$

Universality is also tested in leptonic kaon decays, but the resulting bounds are weaker than those given above.

Right-Handed Currents: Right-handed leptonic charged currents can be constrained through the Michel parameters in muon decay. However, it is necessary to go beyond linear order in the new parameters, so we do not include these measurements in our analysis. For a complete description of muon decay including lepton-number-violating operators, see Refs. [16,33].

Hadronic right-handed currents can be constrained by considering PCAC (partial conservation of axial-vector currents) predictions for $K_{\pi 3}$ decay relative to $K_{\pi 2}$ decay [34]. In terms of our parameters, this gives

$$|\tilde{h}_R^{ud}| < \frac{8 \times 10^{-4}}{|V_{us}|}, \quad |\tilde{h}_R^{us}| < \frac{8 \times 10^{-4}}{|V_{ud}|}. \quad (84)$$

Following Ref. [16], in our fit we consider these upper bounds as 1σ errors.

There are also constraints on right-handed currents in $d \leftrightarrow c$ and $s \leftrightarrow c$ transitions coming from the measurements of the y distributions in $\nu d, \nu s \rightarrow \mu^- c$ and $\bar{\nu} d, \bar{\nu} s \rightarrow \mu^+ \bar{c}$ [35]. Here too, however, the new-physics parameters appear first at quadratic order, so that the (rather weak) bounds extracted in this way are somewhat unreliable, prone as they are to cancellations from dimension-six operators. For this reason we do not include these constraints in our fits.

E. Neutral currents—low energy

As shown in Sec. V A flavor-changing neutral currents involving charged particles are very well constrained, at least for the $\delta\tilde{g}_{L,R}$ couplings. On the other hand, there are no bounds on FCNC's in the neutrino sector, and we will therefore allow for this possibility. In practice, however, since we are working to linear order in the new physics, only the nonstandard flavor-conserving $Z\nu\bar{\nu}$ vertex will be constrained—the flavor-changing couplings always appear quadratically in the expressions for the observables.

The ρ parameter: As was discussed in Sec. II D the ρ parameter, defined as the relative strength of the low-energy neutral- and charged-current interactions, can be read off from the universal corrections to the neutral-current and charged-current couplings [(Eqs. (53) and (54), respectively)], taking also into account the corrections to the W mass [Eq. (51)]. This gives

$$\rho = 1 + \alpha T, \quad (85)$$

as before.

Deep-Inelastic ν Scattering: νq neutral current scattering is measured via the ratios

$$R_\nu = \frac{\sigma(\nu N \rightarrow \nu X)}{\sigma(\nu N \rightarrow \mu^- X)}, \quad R_{\bar{\nu}} = \frac{\sigma(\bar{\nu} N \rightarrow \bar{\nu} X)}{\sigma(\bar{\nu} N \rightarrow \mu^+ X)}. \quad (86)$$

The presence of new physics affects not only the neutral-current process in the numerator, but also the reference charged-current process in the denominator. In principle one must also worry about subsidiary quantities such as the quark distribution functions and the charm threshold. However, it has been argued in Ref. [16] that these are rather insensitive to new-physics effects. The logic of our discussion here follows the lines laid out in this reference.

We wish to compute R/R^{SM} , which we write as

$$\frac{R}{R^{\text{SM}}} = \frac{\sigma(\bar{\nu} N \rightarrow \nu X)/\sigma^{\text{SM}}(\bar{\nu} N \rightarrow \nu X)}{\sigma(\bar{\nu} N \rightarrow \mu X)/\sigma^{\text{SM}}(\bar{\nu} N \rightarrow \mu X)}. \quad (87)$$

We next calculate the numerator and denominator of this expression.

The charged-current process is dominated by $u \leftrightarrow d$ transitions. We therefore compute the corrections only to this process using the effective Lagrangian. A subtlety arises, however, in that our new effective interactions also appear in the reference charged-current SM cross section.

This is because the SM result must be taken as a function of V_{ud} , as it is measured in superallowed β decays, which itself receives corrections from $\text{Re}(\tilde{h}_{L,R}^{ud})$, etc. Whereas one might expect these corrections to cancel with the cor-

responding terms in $\sigma(\bar{\nu}N \rightarrow \mu X)$, this is not the case since V_{ud} from β decay is corrected by the right-handed term $\text{Re}(\tilde{h}_R^{ud})$, while $\sigma(\bar{\nu}N \rightarrow \mu X)$ is not. As a result we find

$$\begin{aligned} \frac{\sigma(\nu N \rightarrow \mu^- X)}{\sigma^{\text{SM}}(\nu N \rightarrow \mu^- X)} &= \frac{(1 - 2\Delta_e - 2\Delta_\mu)(\tilde{V}_{ud} + \delta\tilde{h}_L^{ud})^2 \sum_i (\delta_{i\mu} + \delta\tilde{h}_L^{\nu i\mu})^2}{|V_{ud}|^2} \\ &= 1 + 2\Delta_\mu - 2\Delta_e - 2 \frac{\text{Re}(\delta\tilde{h}_R^{ud})}{|V_{ud}|} . \end{aligned} \quad (88)$$

Note that all of the dependence on the oblique corrections S , T , and U cancels between the corrections to the charged-current couplings, and those to the mass M_W of the virtual W .

We now turn to the neutral-current part of the ratio: $\sigma(\bar{\nu}N \rightarrow \nu X)/\sigma^{\text{SM}}(\bar{\nu}N \rightarrow \nu X)$. The easiest way to make contact with the measurements is through the effective parameters $\epsilon_{L,R}(a)$. These parameters provide the conventional parametrization of the effective neutrino-quark interaction that is probed in deep-inelastic scattering:

$$-\mathcal{L}_{\text{eff}}^{\nu q} = \frac{4G_F}{\sqrt{2}} \bar{\nu}_L \gamma^\mu \nu_L \sum_{a=u,d,\dots} [\epsilon_L(a) \bar{q}_L^a \gamma_\mu q_L^a + \epsilon_R(a) \bar{q}_R^a \gamma_\mu q_R^a] . \quad (89)$$

This is to be compared with the quark-flavor-diagonal piece of the low-energy limit of our general effective Lagrangian:

$$-\mathcal{L}^{\nu q} = \frac{8G_F}{\sqrt{2}} \sum_{ij} \bar{\nu}_i \gamma^\mu (g_L^{\text{SM}} + \delta\tilde{g}_L)^{ij} \gamma_L \nu_j \sum_{a=u,d,\dots} \bar{q}_a \gamma_\mu [(g_L^{\text{SM}} + \delta g_L)^{aa} \gamma_L + (g_R^{\text{SM}} + \delta g_R)^{aa} \gamma_R] q_a . \quad (90)$$

We do not include a right-handed neutrino current in the above equation since this cannot interfere with the SM contribution, and so cannot contribute to linear order. For the same reason, even though FCNC's are allowed, only the flavor-conserving piece $\delta\tilde{g}_L^{\nu\mu\nu\mu}$ contributes to linear order.

Comparing these Lagrangians, and dividing out by the square-root of the charged-current correction factor, $\sqrt{F_{\text{CC}}} = 1 + \Delta_\mu - \Delta_e - \text{Re}(\delta\tilde{h}_R^{ud})/|V_{ud}|$ of Eq. (88), then gives

$$\begin{aligned} \epsilon_{L(R)}(a) &= \frac{2g_L^{\nu\mu\nu\mu} g_L^{aa}}{\sqrt{F_{\text{CC}}}} \\ &= g_{a,L(R)}^{\text{SM}} \left[1 + \alpha T + 2\delta\tilde{g}_L^{\nu\mu\nu\mu} - 2\Delta_\mu + \frac{\text{Re}(\delta\tilde{h}_R^{ud})}{|V_{ud}|} \right] \\ &\quad - Q_a \left[\frac{\alpha S}{4(c_W^2 - s_W^2)} - \frac{c_W^2 s_W^2 \alpha T}{c_W^2 - s_W^2} + \frac{c_W^2 s_W^2 (\Delta_e + \Delta_\mu)}{c_W^2 - s_W^2} \right] + \delta\tilde{g}_{L(R)}^{aa} . \end{aligned} \quad (91)$$

The cross-section ratios R_ν and $R_{\bar{\nu}}$ are finally given by the following expressions [16]: $R_\nu = g_L^2 + r g_R^2$ and $R_{\bar{\nu}} = g_L^2 + g_R^2/\bar{r}$. Here $r=0.383$, $\bar{r}=0.371$ are numbers, and the parameters g_i^2 (not to be confused with the effective neutral-current couplings $g_{L,R}^{ab}$) are related to the $\epsilon_i(a)$ by $g_i^2 \equiv \epsilon_i(u)^2 + \epsilon_i(d)^2$, with $i=L,R$. Combining these results gives the quantities which we use in our fit:

$$\begin{aligned} (g_L^2) &= (g_L^2)_{\text{SM}} - 0.00269S + 0.00663T - 1.452\Delta_\mu - 0.224\Delta_e \\ &\quad + 0.620\text{Re}(\delta\tilde{h}_R^{ud}) - 0.856\delta\tilde{g}_L^{dd} + 0.689\delta\tilde{g}_L^{uu} + 1.208\delta\tilde{g}_L^{\nu\mu\nu\mu} , \\ (g_R^2) &= (g_R^2)_{\text{SM}} + 0.000937S - 0.000192T + 0.085\Delta_e - 0.0359\Delta_\mu \\ &\quad + 0.0620\text{Re}(\delta\tilde{h}_R^{ud}) + 0.156\delta\tilde{g}_R^{dd} - 0.311\delta\tilde{g}_R^{uu} + 0.121\delta\tilde{g}_R^{\nu\mu\nu\mu} . \end{aligned}$$

Neutrino-Electron Scattering: Neutrino-electron scattering data are conventionally expressed in terms of an effective vector- and axial-vector electron coupling, defined by the following effective neutrino-electron interaction

$$-\mathcal{L}^{\nu e} = \frac{2G_F}{\sqrt{2}} \bar{\nu}_L \gamma^\mu \nu_L \bar{e} \gamma_\mu (g_{eV} - g_{eA} \gamma_5) e , \quad (92)$$

which, when compared with our effective Lagrangian [cf. Eq. (90) above] gives

$$\begin{aligned}
g_{eV} &= \frac{2g_L^{\nu\mu}(g_L^{ee} + g_R^{ee})}{\sqrt{F_{CC}}}, \\
g_{eA} &= \frac{2g_L^{\nu\mu}(g_L^{ee} - g_R^{ee})}{\sqrt{F_{CC}}}.
\end{aligned} \tag{93}$$

An additional complication arises here due to the fact that the ν_μ - e scattering cross sections are not all mea-

sured relative to the same charged-current cross section [16]. The high-energy experiments at CERN and Fermilab normalize to $\nu N \rightarrow \mu^- X$ as in deep-inelastic scattering, so that the charged-current correction factor is $1/\sqrt{F_{CC}|_{\text{HE}}} = 1 - \Delta_\mu + \Delta_e + \text{Re}(\delta\tilde{h}_R^{ud})/|V_{ud}|$, as before. The low-energy experiments from BNL, on the other hand, normalize to the quasielastic process $\nu_\mu n \rightarrow \mu^- p$, which gives a slightly different correction factor: $1/\sqrt{F_{CC}|_{\text{LE}}} = 1 - \Delta_\mu + \Delta_e$. Because the global averages of these measurements are dominated by the high-energy experiments, we use $1/\sqrt{F_{CC}|_{\text{HE}}}$ in our fits. We find

$$\begin{aligned}
g_{eV} &= (g_{eV})_{\text{SM}} + 0.00723S - 0.00541T + 0.656\Delta_e + 0.730\Delta_\mu + \delta\tilde{g}_L^{ee} + \delta\tilde{g}_R^{ee} - 0.074\delta\tilde{g}_L^{\nu\mu} - 0.037\text{Re}(\delta\tilde{h}_R^{ud}), \\
g_{eA} &= (g_{eA})_{\text{SM}} - 0.00395T + 1.012\Delta_\mu + \delta\tilde{g}_L^{ee} - \delta\tilde{g}_R^{ee} - 1.012\delta\tilde{g}_L^{\nu\mu} - 0.0506\text{Re}(\delta\tilde{h}_R^{ud}).
\end{aligned} \tag{94}$$

Atomic Parity Violation/Weak-Electromagnetic Interference: The low-energy Lagrangian describing atomic parity violation is conventionally parametrized as

$$-\mathcal{L}^{\text{eq}} = \frac{G_F}{\sqrt{2}} \sum_i [C_{1a} \bar{e} \gamma_\mu \gamma_5 e \bar{q}_a \gamma^\mu q_a + C_{2a} \bar{e} \gamma_\mu e \bar{q}_a \gamma^\mu \gamma_5 q_a], \tag{95}$$

in which

$$C_{1a} = 2(g_L^{ee} - g_R^{ee})(g_L^{aa} + g_R^{aa}), \quad C_{2a}^{\text{SM}} = 2(g_L^{ee} + g_R^{ee})(g_L^{aa} - g_R^{aa}). \tag{96}$$

Inserting our expressions for δg_L 's and δg_R we find

$$\begin{aligned}
C_{1u} &= C_{1u}^{\text{SM}} + 0.00482S - 0.00493T + 0.631(\Delta_e + \Delta_\mu) + 0.387\delta\tilde{g}_L^{ee} - \delta\tilde{g}_L^{uu} - 0.387\delta\tilde{g}_R^{ee} - \delta\tilde{g}_R^{uu}, \\
C_{1d} &= C_{1d}^{\text{SM}} - 0.00241S + 0.00442T - 0.565(\Delta_e + \Delta_\mu) - 0.693\delta\tilde{g}_L^{ee} - \delta\tilde{g}_L^{dd} + 0.693\delta\tilde{g}_R^{ee} - \delta\tilde{g}_R^{dd}, \\
C_{2u} &= C_{2u}^{\text{SM}} + 0.00723S - 0.00544T + 0.696(\Delta_e + \Delta_\mu) + \delta\tilde{g}_L^{ee} - 0.08\delta\tilde{g}_L^{uu} + \delta\tilde{g}_R^{ee} + 0.08\delta\tilde{g}_R^{uu}, \\
C_{2d} &= C_{2d}^{\text{SM}} - 0.00723S + 0.00544T - 0.696(\Delta_e + \Delta_\mu) - \delta\tilde{g}_L^{ee} - 0.08\delta\tilde{g}_L^{dd} - \delta\tilde{g}_R^{ee} + 0.08\delta\tilde{g}_R^{dd}.
\end{aligned} \tag{97}$$

For heavy atoms, the matrix element of this effective interaction within the atomic nucleus, containing N neutrons and Z protons, is proportional to the “weak charge” Q_W defined by

$$Q_W(Z, N) = -2[(2Z + N)C_{1u} + (Z + 2N)C_{1d}]. \tag{98}$$

For cesium we find

$$\begin{aligned}
Q_W(^{133}\text{Cs}) &= [Q_W(^{133}\text{Cs})]_{\text{SM}} - 0.796S - 0.0113T + 1.45(\Delta_e + \Delta_\mu) + 147(\delta\tilde{g}_L^{ee} - \delta\tilde{g}_R^{ee}) \\
&\quad + 422(\delta\tilde{g}_L^{dd} + \delta\tilde{g}_R^{dd}) + 376(\delta\tilde{g}_L^{uu} + \delta\tilde{g}_R^{uu}).
\end{aligned} \tag{99}$$

Note that these expressions are automatically real, even in the presence of CP violation, since the hermiticity of the Lagrangian requires all of the diagonal elements $\delta\tilde{g}_{L,R}^{ii}$ to be real.

F. Neutral currents (Z peak)

Our next class of observables concerns those that are measured in e^+e^- collisions at the Z^0 resonance. Consider first the Z -boson partial widths. Even in the presence of new physics one has (neglecting fermion masses)

$$[\Gamma_{f_i}]_{\text{tree}} = \frac{\alpha M_Z}{6s_W^2 c_W^2} (|g_L^{ii}|^2 + |g_R^{ii}|^2). \tag{100}$$

The contributions from the nonstandard operators can be separated simply by linearizing the above equation about the SM value. This gives

$$\Gamma_f = \Gamma_f^{\text{SM}} \left[1 + \alpha T - \Delta_e - \Delta_\mu + \frac{2g_{f,L}^{\text{SM}} \delta \tilde{g}_L^{ff} + 2g_{f,R}^{\text{SM}} \delta \tilde{g}_R^{ff}}{(g_{f,L}^{\text{SM}})^2 + (g_{f,R}^{\text{SM}})^2} - \frac{2g_{f,L}^{\text{SM}} + 2g_{f,R}^{\text{SM}}}{(g_{f,L}^{\text{SM}})^2 + (g_{f,R}^{\text{SM}})^2} Q_f \left[\frac{\alpha S}{4(c_W^2 - s_W^2)} - \frac{c_W^2 s_W^2 \alpha T}{c_W^2 - s_W^2} + \frac{c_W^2 s_W^2 (\Delta_e + \Delta_\mu)}{c_W^2 - s_W^2} \right] \right]. \quad (101)$$

Note that this expression holds for neutrinos as well as for charged particles since the potentially-present neutrino FCNC's do not contribute to linear order. Using Eq. (101) we find the partial widths

$$\begin{aligned} \Gamma_{l+l-} &= (\Gamma_{l+l-})_{\text{SM}} [1 - 0.00230S + 0.00944T - 1.209(\Delta_e + \Delta_\mu) - 4.29\delta \tilde{g}_L^{ll} + 3.66\delta \tilde{g}_R^{ll}], \\ \Gamma_{u\bar{u}} &= (\Gamma_{u\bar{u}})_{\text{SM}} [1 - 0.00649S + 0.0124T - 1.59(\Delta_e + \Delta_\mu) + 4.82\delta \tilde{g}_L^{uu} - 2.13\delta \tilde{g}_R^{uu}], \\ \Gamma_{d\bar{d}} &= (\Gamma_{d\bar{d}})_{\text{SM}} [1 - 0.00452S + 0.0110T - 1.41(\Delta_e + \Delta_\mu) - 4.57\delta \tilde{g}_L^{dd} + 0.828\delta \tilde{g}_R^{dd}], \\ \Gamma_{b\bar{b}} &= (\Gamma_{b\bar{b}})_{\text{SM}} [1 - 0.00452S + 0.0110T - 1.41(\Delta_e + \Delta_\mu) - 4.57\delta \tilde{g}_L^{bb} + 0.828\delta \tilde{g}_R^{bb}], \\ \Gamma_{\text{had}} &= (\Gamma_{\text{had}})_{\text{SM}} [1 - 0.00518S + 0.0114T - 1.469(\Delta_e + \Delta_\mu) \\ &\quad - 1.01(\delta \tilde{g}_L^{dd} + \delta \tilde{g}_L^{ss} + \delta \tilde{g}_L^{bb}) + 0.183(\delta \tilde{g}_R^{dd} + \delta \tilde{g}_R^{ss} + \delta \tilde{g}_R^{bb}), \\ &\quad + 0.822(\delta \tilde{g}_L^{uu} + \delta \tilde{g}_L^{cc}) - 0.363(\delta \tilde{g}_R^{uu} + \delta \tilde{g}_R^{cc})], \\ \Gamma_{\nu_i \bar{\nu}_i} &= (\Gamma_{\nu_i \bar{\nu}_i})_{\text{SM}} [1 + 0.00781T - (\Delta_e + \Delta_\mu) + 4\delta \tilde{g}_L^{\nu_i \nu_i}]. \end{aligned} \quad (102)$$

The total width is then

$$\begin{aligned} \Gamma_Z &= (\Gamma_Z)_{\text{SM}} [1 - 0.00385S + 0.0105T - 1.35(\Delta_e + \Delta_\mu) + 0.574(\delta \tilde{g}_L^{uu} + \delta \tilde{g}_L^{cc}) \\ &\quad - 0.254(\delta \tilde{g}_R^{uu} + \delta \tilde{g}_R^{cc}) + 0.268(\delta \tilde{g}_L^{e^e \nu_e} + \delta \tilde{g}_L^{\nu_\mu \nu_\mu} + \delta \tilde{g}_L^{\nu_\tau \nu_\tau}) \\ &\quad - 0.144(\delta \tilde{g}_L^{ee} + \delta \tilde{g}_L^{\mu\mu} + \delta \tilde{g}_L^{\tau\tau}) + 0.123(\delta \tilde{g}_R^{ee} + \delta \tilde{g}_R^{\mu\mu} + \delta \tilde{g}_R^{\tau\tau}) \\ &\quad - 0.707(\delta \tilde{g}_L^{dd} + \delta \tilde{g}_L^{ss} + \delta \tilde{g}_L^{bb}) + 0.128(\delta \tilde{g}_R^{dd} + \delta \tilde{g}_R^{ss} + \delta \tilde{g}_R^{bb})]. \end{aligned} \quad (103)$$

Because the $\delta \tilde{g}_L^{e^e \nu_e}$ and $\delta \tilde{g}_L^{\nu_\tau \nu_\tau}$ only contribute to our list of observables through the Z width, only their sum can be bounded in the simultaneous fit.

Various asymmetries are also measured at LEP. In terms of the new-physics parameters, the expression for the left-right asymmetry, Eq. (27), becomes

$$\begin{aligned} A_{LR} &= A_{LR}^{\text{SM}} + \frac{4g_{e,L}^{\text{SM}} g_{e,R}^{\text{SM}}}{[(g_{e,L}^{\text{SM}})^2 + (g_{e,R}^{\text{SM}})^2]^2} (g_{e,R}^{\text{SM}} \delta \tilde{g}_L^{ee} - g_{e,L}^{\text{SM}} \delta \tilde{g}_R^{ee}) \\ &= (A_{LR})_{\text{SM}} - 0.0284S + 0.0201T - 2.574(\Delta_e + \Delta_\mu) - 3.61\delta \tilde{g}_L^{ee} - 4.238\delta \tilde{g}_R^{ee}. \end{aligned} \quad (104)$$

Similarly, we obtain the following expressions for $A_{\text{FB}}(f)$ the forward-backward asymmetries for $e^+e^- \rightarrow f\bar{f}$:

$$\begin{aligned} A_{\text{FB}}^{l+l-} &= \frac{3}{4} A_{LR}^{e^+e-} A_{LR}^{l+l-} \\ &= (A_{\text{FB}})_{\text{SM}} - 0.00677S + 0.00480T - 0.614(\Delta_e + \Delta_\mu) \\ &\quad - 0.430(\delta \tilde{g}_L^{ee} + \delta \tilde{g}_L^{ll}) - 0.505(\delta \tilde{g}_R^{ee} + \delta \tilde{g}_R^{ll}), \\ A_{\text{FB}}(b\bar{b}) &= \frac{3}{4} \left[1 - k_A \frac{\alpha_s}{\pi} \right] A_{LR}^{e^+e-} A_{LR}^{b\bar{b}} \\ &= [A_{\text{FB}}(b\bar{b})]_{\text{SM}} - 0.0188S + 0.0133T - 1.70(\Delta_e + \Delta_\mu) \\ &\quad - 2.36\delta \tilde{g}_L^{ee} - 2.77\delta \tilde{g}_R^{ee} - 0.0322\delta \tilde{g}_L^{bb} - 0.178\delta \tilde{g}_R^{bb}, \\ A_{\text{FB}}(c\bar{c}) &= \frac{3}{4} \left[1 - k_A \frac{\alpha_s}{\pi} \right] A_{LR}^{e^+e-} A_{LR}^{c\bar{c}} \\ &= [A_{\text{FB}}(c\bar{c})]_{\text{SM}} - 0.0147S + 0.0104T - 1.333(\Delta_e + \Delta_\mu) \\ &\quad - 1.69\delta \tilde{g}_L^{ee} - 1.99\delta \tilde{g}_R^{ee} + 0.175\delta \tilde{g}_L^{cc} + 0.396\delta \tilde{g}_R^{cc}. \end{aligned} \quad (105)$$

The factor $(1 - k_A \alpha_s / \pi)$ represents a QCD radiative correction, as in Ref. [2], for which we use the numerical value 0.93.

We can now determine the phenomenological constraints on the new-physics parameters in our electroweak Lagrangian. The observables included in our fit are listed in Table V along with their experimental value and the SM predictions. The standard model values have been calculated with $m_t = 150$ GeV and $M_H = 300$ GeV. The LEP observables in Table V were chosen as they are closest to what is actually measured and their uncertainties are relatively weakly correlated. In our analysis we include the correlations taken from OPAL results [36], but note that all LEP experiments obtain similar results for the correlations.

The expressions for most of the observables in Table V have already been discussed. Of the remaining observables $A_{\text{pol}}(\tau)$, or P_τ , is the polarization asymmetry defined by $A_{\text{pol}}(\tau) = (\sigma_R - \sigma_L) / (\sigma_R + \sigma_L)$, where $\sigma_{L,R}$ is the cross section for the reaction $e^+ e^- \rightarrow \tau \bar{\tau}$ with a correspondingly polarized τ lepton; $A_e(P_\tau)$ is the joint forward-backward/left-right asymmetry as normalized in

Ref. [41]. A_{LR} is the polarization asymmetry which has been measured by the SLD Collaboration at SLC [38]. The expressions for $A_{\text{pol}}(\tau)$ and $A_e(P_\tau)$ are the same as the expression we have already given for A_{LR} . The two remaining observables can be obtained using results already given. In particular the parameter R is defined as $R = \Gamma_{\text{had}} / \Gamma_{\ell\bar{\ell}}$, and $\sigma_p^h = 12\pi \Gamma_{e\bar{e}} \Gamma_{\text{had}} / M_Z^2 \Gamma_Z^2$ is the hadronic cross section at the Z pole.

We first consider the case in which only one of the parameters in our Lagrangian is nonzero. The results of this fit are given in column 2 of Table VI. In this case strong bounds on each of the parameters are obtained since there is no possibility of cancellations. This procedure is commonly used by most practitioners when bounding effective couplings. Although the constraints obtained in this way are the tightest bounds possible, they are clearly artificial in the sense that real underlying physics would change more than one of the parameters. Ideally one could calculate the effects of new physics on the parameters of the global electroweak Lagrangian and then do a global fit on the specific parameters of interest.

Conversely, a simultaneous fit to all of the effective pa-

TABLE V. Experimental values for the electroweak observables included in the global fit. The Z^0 measurements are the 1993 LEP results taken from Ref. [37]. The couplings extracted from neutrino scattering data are the current world averages taken from Ref. [41]. The SM values are for $m_t = 150$ GeV and $M_H = 300$ GeV [44]. We have not shown theoretical errors in the SM values due to uncertainties in the radiative corrections, Δr , and due to uncertainties in M_Z , as they are in general overwhelmed by the experimental errors. The exception is the error due to uncertainty in α_s , shown in square brackets. We include this error in quadrature in our fits. The error in square brackets for Q_W (Cs) reflects the theoretical uncertainty in the atomic wave functions [45] and is also included in quadrature with the experimental error. All other quantities are as defined in the text.

Quantity	Value	Standard model
M_Z (GeV)	91.187 ± 0.007 [37]	Input
Γ_Z (GeV)	2.489 ± 0.007 [37]	$2.490 [\pm 0.006]$
$R_e = \Gamma_{\text{had}} / \Gamma_{e\bar{e}}$	20.743 ± 0.080 [37]	$20.78 [\pm 0.07]$
$R_\mu = \Gamma_{\text{had}} / \Gamma_{\mu\bar{\mu}}$	20.764 ± 0.069 [37]	$20.78 [\pm 0.07]$
$R_\tau = \Gamma_{\text{had}} / \Gamma_{\tau\bar{\tau}}$	20.832 ± 0.088 [37]	$20.78 [\pm 0.07]$
σ_p^h (nb)	41.56 ± 0.14 [37]	$41.42 [\pm 0.06]$
$R_b = \Gamma_{b\bar{b}} / \Gamma_{\text{had}}$	0.2200 ± 0.0027 [37]	$0.2162 [\pm 0.0007]$
$A_{\text{FB}}(e)$	0.0153 ± 0.0038 [37]	0.0141
$A_{\text{FB}}(\mu)$	0.0132 ± 0.0026 [37]	0.0141
$A_{\text{FB}}(\tau)$	0.0204 ± 0.0032 [37]	0.0141
$A_{\text{pol}}(\tau)$	0.142 ± 0.017 [37]	0.137
$A_e(P_\tau)$	0.130 ± 0.025 [37]	0.137
$A_{\text{FB}}(b)$	0.098 ± 0.006 [37]	0.096
$A_{\text{FB}}(c)$	0.075 ± 0.015 [37]	0.068
A_{LR}	0.100 ± 0.044 [38]	0.137
M_W (GeV)	79.91 ± 0.39 [39]	80.18
M_W / M_Z	0.8798 ± 0.0028 [40]	0.8793
$g_L^2(\nu N \rightarrow \nu X)$	0.3003 ± 0.0039 [41]	0.3021
$g_R^2(\nu N \rightarrow \nu X)$	0.0323 ± 0.0033 [41]	0.0302
$g_{eA}(\nu e \rightarrow \nu e)$	-0.508 ± 0.015 [41]	-0.506
$g_{eV}(\nu e \rightarrow \nu e)$	-0.035 ± 0.017 [41]	-0.037
Q_W (Cs)	$-71.04 \pm 1.58 \pm [0.88]$ [42]	-73.20
$ V_{ud} ^2 + V_{us} ^2 + V_{ub} ^2$	0.9992 ± 0.0014 [41]	1
$ V_{cd} ^2 + V_{cs} ^2 + V_{cb} ^2$	1.043 ± 0.40 [18]	1
$R_\pi / R_\pi^{\text{SM}}$	1.003 ± 0.003 [43]	1
$R_\tau / R_\tau^{\text{SM}}$	0.960 ± 0.024 [43]	1
$R_{\mu\tau} / R_{\mu\tau}^{\text{SM}}$	0.968 ± 0.024 [43]	1

rameters gives the most conservative bounds, since cancellations can occur among different parameters. We have performed such a fit. As mentioned in previous sections we have excluded some of the parameters in this simultaneous fit. In particular there are a number of quantities that only appear in particular linear combinations, and so only these combinations can be bounded. Some examples are $|V_{cd}|\text{Re}(\delta\tilde{h}_L^{cd}) + |V_{cs}|\text{Re}(\delta\tilde{h}_L^{cs} + \delta\tilde{h}_R^{cs}) + |V_{cb}|\text{Re}(\delta\tilde{h}_L^{cb})$ in the unitarity of the CKM matrix and $\delta\tilde{g}_L^{v_e v_e} + \delta\tilde{g}_L^{v_\tau v_\tau}$ in the Z width. As more measurements become available the omitted parameters will

be able to be included in the simultaneous fit. The results of this simultaneous fit are given in Table VI.

There are a number of interesting features in Table VI. What is perhaps most surprising is that, despite the large number of parameters, most of them are constrained, and the bounds are fairly tight. This reflects the richness and complementarity of the experimental data. The most significant result of our fit is that every single parameter is consistent with zero, the standard model value. There is no evidence for physics beyond the standard model.

One should be cautioned to not take the central values

TABLE VI. Results of the fits of the new-physics parameters to the data of Table V. Σ_1 and Σ_2 are defined as $\Sigma_1 \equiv \text{Re}(\delta\tilde{h}_L^{us}) + [\text{Re}(V_{ub})\text{Re}(\delta\tilde{h}_L^{ub}) + \text{Im}(V_{ub})\text{Im}(\delta\tilde{h}_L^{ub})]/|V_{us}|$ and $\Sigma_2 \equiv \text{Re}(\delta\tilde{h}_L^{cd}) + |V_{cs}|\text{Re}(\delta\tilde{h}_L^{cs} + \delta\tilde{h}_R^{cs})/|V_{cd}| + |V_{cb}|\text{Re}(\delta\tilde{h}_L^{cb})/|V_{cd}|$.

Parameter	Individual fit	Global fit
S	-0.10 ± 0.16	-0.2 ± 1.0
T	$+0.01 \pm 0.17$	-0.02 ± 0.89
U	-0.14 ± 0.63	$+0.3 \pm 1.2$
Δe	-0.0008 ± 0.0010	-0.0011 ± 0.0041
Δ_μ	$+0.00047 \pm 0.00056$	$+0.0005 \pm 0.0039$
Δ_τ	-0.018 ± 0.008	-0.018 ± 0.009
$\text{Re}(\delta\tilde{h}_L^{ud})$	-0.00041 ± 0.00072	$+0.0001 \pm 0.0060$
$\text{Re}(\delta\tilde{h}_R^{ud})$	-0.00055 ± 0.00066	$+0.0003 \pm 0.0073$
$\text{Im}(\delta\tilde{h}_R^{ud})$	0 ± 0.0036	-0.0036 ± 0.0080
$\text{Re}(\delta\tilde{h}_L^{us})$	-0.0018 ± 0.0032	—
$\text{Re}(\delta\tilde{h}_R^{us})$	-0.00088 ± 0.00079	$+0.0007 \pm 0.0016$
$\text{Im}(\delta\tilde{h}_R^{us})$	0 ± 0.0008	-0.0004 ± 0.0016
$\text{Re}(\delta\tilde{h}_L^{ub}), \text{Im}(\delta\tilde{h}_L^{ub})$	-0.09 ± 0.16	—
Σ_1	—	$+0.005 \pm 0.027$
$\text{Re}(\delta\tilde{h}_R^{ub})$	—	—
$\text{Re}(\delta\tilde{h}_L^{cd})$	$+0.11 \pm 0.98$	—
$\text{Re}(\delta\tilde{h}_R^{cd})$	—	—
$\text{Re}(\delta\tilde{h}_L^{cs})$	$+0.022 \pm 0.20$	—
$\text{Re}(\delta\tilde{h}_R^{cs})$	$+0.022 \pm 0.20$	—
$\text{Re}(\delta\tilde{h}_L^{cb})$	$+0.5 \pm 4.6$	—
Σ_2	—	$+0.11 \pm 0.98$
$\text{Re}(\delta\tilde{h}_R^{cb})$	—	—
$\delta\tilde{g}_{dd}^{cc}$	$+0.0016 \pm 0.0015$	$+0.003 \pm 0.012$
$\delta\tilde{g}_{uu}^{cc}$	$+0.0037 \pm 0.0038$	$+0.007 \pm 0.015$
$\delta\tilde{g}_{uu}^{cc}$	-0.0003 ± 0.0018	-0.002 ± 0.014
$\delta\tilde{g}_{uu}^{cc}$	$+0.0032 \pm 0.0032$	-0.003 ± 0.010
$\delta\tilde{g}_{ss}^{cc}$	-0.0009 ± 0.0017	-0.003 ± 0.015
$\delta\tilde{g}_{ss}^{cc}$	-0.0052 ± 0.00095	$+0.002 \pm 0.085$
$\delta\tilde{g}_{cc}^{cc}$	-0.0011 ± 0.0021	$+0.001 \pm 0.018$
$\delta\tilde{g}_{bb}^{cc}$	$+0.0028 \pm 0.0047$	$+0.009 \pm 0.029$
$\delta\tilde{g}_{bb}^{cc}$	-0.0005 ± 0.0016	-0.0015 ± 0.0094
$\delta\tilde{g}_{bb}^{cc}$	$+0.0019 \pm 0.0083$	0.013 ± 0.054
$\delta\tilde{g}_{L}^{v_e v_e}$	-0.0048 ± 0.0052	—
$\delta\tilde{g}_{L}^{v_\mu v_\mu}$	-0.0021 ± 0.0027	$+0.0023 \pm 0.0097$
$\delta\tilde{g}_{L}^{v_\tau v_\tau}$	-0.0048 ± 0.0052	—
$\delta\tilde{g}_{L}^{v_e v_e} + \delta\tilde{g}_{L}^{v_\tau v_\tau}$	—	-0.004 ± 0.033
$\delta\tilde{g}_{ee}^{cc}$	-0.00029 ± 0.00043	-0.0001 ± 0.0032
$\delta\tilde{g}_{ee}^{cc}$	-0.00014 ± 0.00050	$+0.0001 \pm 0.0030$
$\delta\tilde{g}_{\mu\mu}^{cc}$	$+0.0040 \pm 0.0051$	$+0.005 \pm 0.032$
$\delta\tilde{g}_{\mu\mu}^{cc}$	-0.0003 ± 0.0047	$+0.001 \pm 0.028$
$\delta\tilde{g}_{\tau\tau}^{cc}$	-0.0021 ± 0.0032	0.000 ± 0.022
$\delta\tilde{g}_{\tau\tau}^{cc}$	-0.0034 ± 0.0028	-0.0015 ± 0.019

of this fit too literally. With so many free parameters the central values obtained by the fit are naturally not unique. We find that the errors seem to be stable so that the best values lie within the error bounds irrespective of the search strategy.

In the individual fit, three parameters remain unconstrained: $\text{Re}(\delta\tilde{h}_R^{cd})$, $\text{Re}(\delta\tilde{h}_R^{cb})$, and $\text{Re}(\delta\tilde{h}_R^{ub})$. [As explained in the text, there are in fact (weak) constraints on $\text{Re}(\delta\tilde{h}_R^{cd})$, but they appear only at quadratic order in this parameter, and so could be canceled by higher-dimension operators.] In addition, the constraints on $\text{Re}(\delta\tilde{h}_L^{cd})$, $\text{Re}(\delta\tilde{h}_L^{cb})$, $\text{Re}(\delta\tilde{h}_L^{ub})$, and $\text{Im}(\delta\tilde{h}_L^{ub})$ are quite weak. One physical consequence of this observation is that the chirality of the $b \rightarrow c$ and $b \rightarrow u$ transitions has really not been tested. In other words, this is an ideal area to look for new physics. In fact, models have recently been constructed [46] in which B decays are predominantly right-handed.

In the simultaneous fit, three combinations of the nine parameters $\delta\tilde{g}_L^{v_e v_e}$, $\delta\tilde{g}_L^{v_\tau v_\tau}$, $\text{Re}(\delta\tilde{h}_L^{us})$, $\text{Re}(\delta\tilde{h}_L^{ub})$, $\text{Im}(\delta\tilde{h}_L^{ub})$, $\text{Re}(\delta\tilde{h}_{L,R}^{cs})$, $\text{Re}(\delta\tilde{h}_L^{cd})$, and $\text{Re}(\delta\tilde{h}_L^{cb})$ are also unconstrained, since only three independent combinations enter into well-measured observables. Apart from these exceptional cases, all the other parameters are well bounded. In the individual fit, most of the parameters are constrained at better than the 1% level. In the simultaneous fit the limits are only slightly weakened, to about 2–3% for most new-physics parameters. (Note that, although S , T , and U appear to be poorly constrained, their constraints in fact represent strong bounds on new physics, since a factor of α has been divided out in their definitions [see Eq. (2)].)

The only case in which there is a discrepancy with the standard model is in Δ_τ , which differs from zero by about 2σ . This is a well-known problem, which is due to the apparent breaking of weak universality in τ decays [43]. Many people remain skeptical that this really is a sign of new physics, suggesting instead that the cause of the problem is probably an incorrect measurement of the τ mass. However, recent remeasurements of m_τ have not caused the effect to disappear [47].

Note also that, as expected, the $\text{Im}(\delta\tilde{h}_L^{ij})$ remain virtually unconstrained. Such operators can contribute to CP -violating processes, and could very well be observed in studies of CP violation in the B system. This underlines the significance of CP -violating observables as potent probes for new physics.

One of the interesting conclusions to be drawn from the results of the simultaneous fit is that, although many of the hadronic charged-current experiments are extremely precise, there is still a great deal of room for new physics in this sector—many of the $\delta\tilde{h}$'s are only weakly constrained, if at all. This is due to the fact that, in the standard model, the values of the CKM matrix elements are not predicted. Hence, the only constraints we have are due to the unitarity of the CKM matrix. And, since only the magnitudes of the CKM matrix elements involving the u and c quarks have been measured, the only two constraints which can be used are the normalization of the first two rows [Eqs. (82) and (83)]. This is not very

restrictive. There are, however, a number of ways to constrain new physics in the hadronic charged-current sector more strongly. First, it would be useful to remeasure the known CKM matrix elements, but using methods sensitive to different combinations of the new-physics parameters. Second, measurements of CP violation in the B system allow one to obtain the imaginary parts of the elements of the CKM matrix. Using the unitarity of the CKM matrix, these can be used to extract the magnitudes of the CKM matrix elements, which will help in overconstraining the matrix and putting limits on new physics. Finally, using the fact that the columns of the CKM matrix are orthonormal, accurate measurements of the CKM matrix elements involving the top quark can be used to constrain different combinations of the $\delta\tilde{h}$'s.

Since we have performed this analysis in a model-independent fashion, the constraints presented here must hold for all physics beyond the standard model, provided only that it agree on the low-energy particle content, and that dimension-six operators may be neglected. In any particular model of new physics, one must simply compute the above new-physics parameters in terms of the parameters of the model. The constraints can then be read off from the tables. As an example of how this works we consider in the following section the case of the mixing of ordinary and exotic fermions, first studied in Refs. [16,17]. Before doing so, however, we briefly turn to possible constraints from loop-level processes.

G. Loop constraints

In the previous subsections we found the constraints which current tree-level experimental data put on our new-physics parameters. The bounds on most of these parameters are quite stringent, though there are certain new couplings which are constrained only weakly, if at all. In this subsection we consider the limits which apply to the new-physics parameters due to loop-level processes. For a given observable we have already argued that in general there can be cancellations between the loop-level contributions of certain effective interactions, and the tree-level contributions of other operators. The only possible case where a reasonably reliable bound can be obtained is when the constraint on a new parameter from such loop-induced processes is so strong that cancellations with the higher-dimension operators would require significant fine-tuning. For this reason we need only consider the loop-level contributions to observables which are *extremely* well measured.

Another reason to consider loop-level bounds is that, up to now, almost all CP -violating operators have remained essentially unconstrained. (The only exception are the constraints on the flavor-diagonal $d_{L,R}$'s from EDM's.) Since the only observation of CP violation to date is the parameter ϵ_K in the kaon system, which is a loop-level process, it is interesting to investigate the implications this measurement might have for CP -violating new-physics parameters.

We will therefore consider the contributions of the new-physics parameters to four classes of loop-level ob-

servables: anomalous magnetic moments, EDM's, neutral-meson mixing, and ϵ_K . It must be kept in mind that the only reliable constraints from this analysis are those which are extremely stringent—weak bounds are suspect due to the possibility of cancellation with effects from other operators. (This last point is frequently glossed over when only one effective interaction is considered at a time.) For the purposes of argument we will arbitrarily consider here any bound which is greater than 10^{-3} to be too weak to preclude its cancellation by other operators.

1. Anomalous magnetic moments

Although the measurements of a_e and a_μ are extremely precise, they turn out to be sensitive only to comparatively few of our effective interactions [48]. The reason for this is fairly easy to see. Consider first a dimension-four fermion-gauge boson interaction, such as $\delta g_{L,R}^{ff}$ of Eq. (35). These can contribute to a fermion anomalous magnetic moment through Feynman graphs such as that of Fig. 1. An order-of-magnitude estimate for the contribution to a_i , $i=e,\mu$ due to this graph is

$$\delta a_i \sim \delta g_{L,R}^{ii} \left[\frac{\alpha}{4\pi s_W^2 c_W^2} \right] \left[\frac{m_i}{m_W} \right] F \left[\frac{m_i}{m_W} \right]. \quad (106)$$

Here the second term on the right-hand side is the usual loop factor, and the third term arises because a_i is defined relative to the corresponding Bohr magneton $\mu_{B_i} = e_i/2m_i$. Largely due to the suppression by the small electron or muon mass, the product of these two terms is already very small: $\sim 2 \times 10^{-8}$ for the electron and $\sim 3 \times 10^{-6}$ for the muon. As a consequence, no useful bound on the couplings $\delta g_{L,R}^{ee}$ or $\delta g_{L,R}^{\mu\mu}$ is possible unless the remaining function $F(x_i)$ of the small mass ratio $x_i = m_i/m_W$ is not itself suppressed by a power of x_i for small x_i .

For the dimension-four interactions, helicity conservation along the fermion line shows that $F(x_i)$ is always suppressed by at least one power of x_i , and so no useful bound for these operators is obtained in this way. For the same reason current anomalous-magnetic-moment experiments are not yet sensitive to ordinary SM weak-interaction effects.

The same need not be true for the dimension-five interactions. The only effective couplings whose contribu-

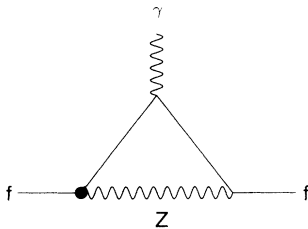


FIG. 1. The Feynman diagram through which an anomalous fermion-Z-boson coupling (blob) can contribute at one loop to the anomalous magnetic moment of the electron or muon.

tion to a_e and a_μ is not further suppressed by light fermion masses, together with the order-of-magnitude of their corresponding bounds, are

$$n_{L,R}^{ee}, c_L^{vee} \lesssim 5 \times 10^{-3} \quad n_{L,R}^{\mu\mu}, c_L^{v\mu\mu} \lesssim 0.08. \quad (107)$$

Given the ever-present possibility of cancellations that is inherent in these loop-generated bounds, we do not consider these limits to be particularly severe.

2. Electric dipole moments

Some light-fermion EDM's are also extremely well bounded, so one might expect these to also give significant bounds for operators which contribute at the loop level. This turns out to be true, but only for those comparatively few operators which can contribute to the electron or u - and d -quark EDM's unsuppressed by small fermion masses.⁶ We consider here each type of effective coupling separately.

The analysis for dimension-four interactions follows closely that for the anomalous magnetic moments of the previous section. Helicity conservation always implies a suppression by at least one factor of a light fermion mass. The only bounds which we can infer in this way are

$$\begin{aligned} \text{Im}[\delta g_{L,R}^{ee}], \quad \text{Im}[\delta h_{L,R}^{vee}] &\lesssim 4 \times 10^{-3}, \\ \text{Im}[\delta g_{L,R}^{\mu\mu}], \quad \text{Im}[\delta g_{L,R}^{dd}], \quad \text{Im}[\delta h_{L,R}^{\mu d}] &\lesssim 0.08. \end{aligned} \quad (108)$$

The suppression of flavor changes in the SM by factors of $\lambda = \sin\theta_c \simeq 0.2$ precludes obtaining significant bounds for other quark operators, e.g., we find $\text{Im}[\delta h_{L,R}^{us}]$, $\text{Im}[\delta h_{L,R}^{cd}] \lesssim 0.4$.

At dimension five there are three kinds of effective couplings: $d_{L,R}^{ij}$, $c_{L,R}^{ij}$, and $n_{L,R}^{ij}$. It turns out that no new bounds arise for $d_{L,R}^{ij}$ from loop-level EDM's, however. These operators might have potentially contributed through the Feynman diagram of Fig. 2, but the following argument shows that this graph leads to no new limits. There are two cases to consider, depending on whether or not the exchanged gauge boson is a photon, a W , or a Z . For the two neutral bosons, the absence of

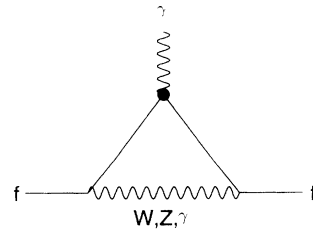


FIG. 2. The Feynman diagram through which an effective fermion-photon coupling (blob) can contribute at one loop to a light-quark or electron electric dipole moment.

⁶Because of our neglect of gluon operators, we are unable to consider some loop contributions to the neutron EDM, such as those of Ref. [49].

SM flavor-changing vertices only permits contributions from the same operators which are already directly bounded at tree level, such as $d_{L,R}^{ee}$, and so no new bounds are obtained. For the graph with a W boson, the result must always be suppressed by one factor of the mass of *both* the external and internal fermions, and so gives too small a result to furnish a useful bound.

It is the remaining couplings, $c_{L,R}^{ij}$ and $n_{L,R}^{ij}$, that can receive nontrivial constraints from loop-generated EDM's. We find that the only contributions which are unsuppressed by too many powers of light masses and mixing angles are

$$\begin{aligned} \text{Im}[c_L^{vee}] , \text{Im}[n_{L,R}^{ee}] &\lesssim 3 \times 10^{-7} , \\ \text{Im}[c_{L,R}^{ud}] , \text{Im}[n_{L,R}^{dd}] , \text{Im}[n_{L,R}^{uu}] &\lesssim 2 \times 10^{-5} , \\ \text{Im}[c_{L,R}^{us}] , \text{Im}[c_{L,R}^{cd}] &\lesssim 1 \times 10^{-4} , \\ \text{Im}[c_{L,R}^{ub}] , \text{Im}[c_{L,R}^{td}] &\lesssim 3 \times 10^{-3} . \end{aligned} \quad (109)$$

Again, keeping in mind the potential for cancellations, we regard only the first three of these as being of real significance.

3. Neutral meson mass differences

In the standard model the short-distance contributions to neutral meson (M^0) mass differences (ΔM_M) are due to the box diagrams which mix M^0 and $\bar{M}^0$. These SM box diagrams predict values for the mass differences in the K -, B -, and D -meson systems which are in agreement with the experimental values, within significant hadronic uncertainties. Because of these uncertainties we can regard this agreement as only to within an order of magnitude, and so in order to obtain estimates of the loop-level bounds on the new-physics parameters, we therefore require that their contributions to ΔM_M be less than those of the SM.

a. ΔM_K . The SM contribution to ΔM_K is

$$\Delta M_K^{\text{SM}} = \left[\frac{G_F}{\sqrt{2}} \frac{\alpha}{6\pi s_W^2} f_K^2 B_K M_K \right] \frac{m_c^2}{m_W^2} \text{Re}(V_{cd}^* V_{cs})^2 . \quad (110)$$

Consider now the case in which $\delta \tilde{h}_L^{ud}$ replaces one of the SM ud couplings. One finds a partial failure of the GIM mechanism in the calculation of the box diagram, leading to the appearance of a logarithmic enhancement:

$$\Delta M_K \sim \left[\frac{G_F}{\sqrt{2}} \frac{\alpha}{6\pi s_W^2} f_K^2 B_K M_K \right] \frac{m_c^2}{m_W^2} \ln \left[\frac{m_W^2}{m_c^2} \right] \text{Re}(\delta \tilde{h}_L^{*ud} V_{cs} V_{cd}^* V_{cs}) . \quad (111)$$

A comparison of this contribution with that of the SM leads to the bound

$$|\text{Re}(\delta \tilde{h}_L^{*ud})| \lesssim |\text{Re}(V_{ud})| / \ln \left[\frac{m_W^2}{m_c^2} \right] . \quad (112)$$

Similar constraints exist for the new-physics parameters $\delta \tilde{h}_L^{us}$, $\delta \tilde{h}_L^{cd}$, and $\delta \tilde{h}_L^{cs}$, yielding

$$\begin{aligned} |\text{Re}(\delta \tilde{h}_L^{ud})| &\lesssim 0.1 , \\ |\text{Re}(\delta \tilde{h}_L^{us})| &\lesssim 0.03 , \\ |\text{Re}(\delta \tilde{h}_L^{cd})| &\lesssim 0.03 , \\ |\text{Re}(\delta \tilde{h}_L^{cs})| &\lesssim 0.1 . \end{aligned} \quad (113)$$

As these constraints are quite weak, they cannot be considered at all reliable, due to the possibility of cancellations with the contributions from other operators.

Consider now the case of right-handed currents, in which $\delta \tilde{h}_R^{ij}$ ($i=u,c$, $j=d,s$) is the new-physics parameter in the box diagram. We find

$$\Delta M_K \sim 7.7 \left[\frac{G_F}{\sqrt{2}} \frac{\alpha}{6\pi s_W^2} f_K^2 B_K M_K \right] \frac{m_{\text{int}}^i m_{\text{ext}}}{m_W^2} \ln \left[\frac{m_W^2}{m_c^2} \right] \text{Re}(\delta \tilde{h}_R^{*ij} V_{ij} V_{cd}^* V_{cs}) , \quad (114)$$

where the factor 7.7 arises from the enhancement of the LR matrix element relative to the LL matrix element [23], and $m_{\text{int}}(m_{\text{ext}})$ is the mass of an internal (external) quark. Comparing this contribution with that of the SM

[Eq. (110)] we see that there are no significant bounds on $\text{Re}(\delta \tilde{h}_R^{ud})$ and $\text{Re}(\delta \tilde{h}_R^{us})$, due to the smallness of m_u . Taking $m_{\text{ext}} \sim M_K/2$, the bounds on $\text{Re}(\delta \tilde{h}_R^{cd})$ and $\text{Re}(\delta \tilde{h}_R^{cs})$ are of the same order of magnitude as their left-handed

counterparts [Eq. (113)].

Finally, the contributions of the parameters c_L and c_R to ΔM_K should be of the same order as those of $\delta\tilde{h}_{L,R}^{ij}$, with an additional suppression of a factor of m/M , where m is a light quark mass. Since the constraints on the $\delta\tilde{h}_{L,R}^{ij}$ are relatively weak, there are thus no limits on the $\text{Re}(c_{L,R})$.

b. ΔM_B . In the SM, B^0 - $\bar{B}^0$ mixing is dominated by the t -quark contribution in the box diagram:

$$\Delta M_B^{\text{SM}} = \left[\frac{G_F}{\sqrt{2}} \frac{\alpha}{6\pi s_W^2} f_B^2 B_B M_B \right] x_t f(x_t) \text{Re}(V_{td}^* V_{tb})^2, \quad (115)$$

in which $x_t \equiv m_t^2/m_W^2$ and $f(x_t)$ takes values ~ 1 for $100 \text{ GeV} < m_t < 200 \text{ GeV}$.

We now estimate the new-physics contributions to ΔM_B . We begin by considering the case in which one of the internal t -quark lines is replaced by a u quark, and $\delta\tilde{h}_L^{ud}$ replaces the SM ud coupling. A calculation of the box diagram yields

$$\Delta M_B = \left[\frac{G_F}{\sqrt{2}} \frac{\alpha}{6\pi s_W^2} f_B^2 B_B M_B \right] x_t f'(x_t) \text{Re}(\delta\tilde{h}_L^{*ud} V_{ub} V_{td}^* V_{tb}), \quad (116)$$

in which $f'(x_t)$ is a different function from that in Eq. (115). It also takes values ~ 1 for $100 \text{ GeV} < m_t < 200 \text{ GeV}$. A comparison of this contribution with that of the SM produces the constraint

$$|\text{Re}(\delta\tilde{h}_L^{*ud} V_{ub})| \lesssim |\text{Re}(V_{td}^* V_{tb})|, \quad (117)$$

with similar expressions for $\delta\tilde{h}_L^{ub}$, $\delta\tilde{h}_L^{cd}$, and $\delta\tilde{h}_L^{cb}$. Using the estimates of the sizes of the CKM matrix elements given in Eq. (46), this gives

$$\begin{aligned} |\text{Re}(\delta\tilde{h}_L^{ud})| &\lesssim 1, \\ |\text{Re}(\delta\tilde{h}_L^{ub})| &\lesssim O(\lambda^3) \sim 0.01, \\ |\text{Re}(\delta\tilde{h}_L^{cd})| &\lesssim O(\lambda) \sim 0.2, \\ |\text{Re}(\delta\tilde{h}_L^{cb})| &\lesssim O(\lambda^2) \sim 0.05. \end{aligned} \quad (118)$$

As was the case for ΔM_K , these constraints are weak and are therefore not reliable.

For the $\delta\tilde{h}_R^{ij}$ ($i=u,c, j=d,b$), the contributions to ΔM_B are suppressed relative to those of the $\delta\tilde{h}_L^{ij}$ by a factor $m_{\text{int}} M_B/m_t^2$. This leads to virtually no bounds on the $\delta\tilde{h}_R^{ij}$.

Finally, the contributions from the $c_{L,R}$ to ΔM_B are suppressed, as in the kaon system, by a factor of m/M relative to those of the $\delta\tilde{h}_{L,R}^{ij}$, leading to no constraints.

c. ΔM_D . The analysis of D^0 - $\bar{D}^0$ mixing proceeds completely analogously to that in the kaon or B system. As in these two systems, no significant bounds are obtained on any of the new-physics parameters.

ϵ_K : In the SM, ϵ_K is calculated from the imaginary part of the K^0 - $\bar{K}^0$ mixing box diagram. There are contributions from diagrams with two internal c quarks, and with one c and one t quark, but the largest effect comes from the diagram with two internal t quarks:

$$\epsilon_K \simeq \left[\frac{G_F}{\sqrt{2}} \frac{\alpha}{6\pi s_W^2} f_K^2 B_K M_K \right] x_t f(x_t) \text{Im}(V_{td}^* V_{ts})^2. \quad (119)$$

Note that, according to Eq. (46), V_{td} has a large imaginary piece, and $\text{Im}(V_{td}^* V_{ts})^2 \sim \lambda^{10}$.

Consider now the diagram in which there is one internal c quark and one t quark, and where the SM cd coupling is replaced by $\delta\tilde{h}_L^{cd}$. A calculation of the contribution of this diagram to ϵ_K yields

$$\sim \left[\frac{G_F}{\sqrt{2}} \frac{\alpha}{6\pi s_W^2} f_K^2 B_K M_K \right] x_t g(x_t) \text{Im}(\delta\tilde{h}_L^{*cd} V_{cs} V_{td}^* V_{ts}), \quad (120)$$

where $g(x_t)$ is another function which takes values ~ 1 for the allowed range of m_t . Comparing this contribution with that of the SM yields $\text{Im}(\delta\tilde{h}_L^{*cd} V_{cs} V_{td}^* V_{ts}) \lesssim \lambda^{10}$. There are similar expressions for the parameters $\delta\tilde{h}_L^{us}$, $\delta\tilde{h}_L^{cs}$, and $\delta\tilde{h}_L^{cb}$. These lead to the constraints

$$\begin{aligned} |\text{Re}(\delta\tilde{h}_L^{ud})|, |\text{Im}(\delta\tilde{h}_L^{ud})|, |\text{Re}(\delta\tilde{h}_L^{cs})|, |\text{Im}(\delta\tilde{h}_L^{cs})| &\lesssim O(\lambda^4) \sim 2 \times 10^{-3}, \\ |\text{Re}(\delta\tilde{h}_L^{us})|, |\text{Im}(\delta\tilde{h}_L^{us})|, |\text{Re}(\delta\tilde{h}_L^{cd})|, |\text{Im}(\delta\tilde{h}_L^{cd})| &\lesssim O(\lambda^5) \sim 5 \times 10^{-4}. \end{aligned} \quad (121)$$

The second of these two constraints is perhaps sufficiently stringent to be taken seriously. However, one must always be aware of the possibility of evading such bounds via (fine-tuned) cancellations with the contributions of other operators to ϵ_K .

On the other hand, the constraints on the $\delta\tilde{h}_R^{ij}$ ($i=u,c$,

$j=d,s$) are much weaker. As was the case in the calculation of ΔM_K , there is a suppression of the contribution of the $\delta\tilde{h}_R^{ij}$ to ϵ_K by a factor $\sim m_{u,c} M_K/m_t^2$ relative to that of the corresponding $\delta\tilde{h}_L^{ij}$. Even taking into account the enhancement of the LR matrix element [23], $\text{Re}(\delta\tilde{h}_R^{ij})$ and $\text{Im}(\delta\tilde{h}_R^{ij})$ are essentially unconstrained by ϵ_K .

Similarly, there are no constraints on the $c_{L,R}$, whose contributions to ϵ_K are suppressed by a factor m/M .

To summarize, the only loop-level observables which yield significant constraints on the new-physics parameters are the CP -violating electron and neutron EDM's [Eq. (109)]. The bounds on these parameters are $\sim 10^{-7}$ – 10^{-5} . There are also limits of $\sim 10^{-4}$ – 10^{-3} on other new-physics parameters from the CP -violating quantity ϵ_K [Eq. (121)]. However, one cannot discount the possibility of evading these latter (weaker) constraints through cancellations with contributions of other operators.

VI. APPLICATIONS TO EXOTIC-FERMION MIXING

In this section we illustrate how the above constraints, which have been obtained in a model-independent way, might be applied to a specific model of new physics. The class of models we consider here are those containing exotic fermions. “Ordinary” fermions are defined as transforming in the standard way under $SU_L(2)$ [left-handed (LH) doublets, right-handed (RH) singlets]. “Exotic” fermions have noncanonical $SU_L(2)$ assignments. Here we restrict ourselves to LH singlets and/or RH doublets. These exotic fermions can mix with the ordinary fermions and, in so doing, change the couplings of the ordinary fermions to the $W^\pm$ and Z^0 . (In the effective-Lagrangian language, these mixings induce new operators.) The precision measurements described in the previous sections have been used to put constraints on these mixings [16,17].

Our aim here is to simply show how the formalism introduced above could be used to bound ordinary-exotic fermion mixing. We do not wish to perform a complete update of the limits on such mixings. As a result we keep the description of the mixing formalism to a minimum. Those wishing more details should refer to Ref. [16]. In addition, we do not present a complete analysis of all the constraints, preferring instead to focus on a few illustrative examples.

We begin by considering mixing between charged particles—neutrinos are treated separately below. For each type of charged particle ($Q_{em} = -1, -\frac{1}{3}, \frac{2}{3}$), we put the LH and RH eigenstates of both ordinary (O) and exotic (E) fermions into a single vector

$$\psi_{L(R)}^0 = \begin{pmatrix} \psi_O^0 \\ \psi_E^0 \end{pmatrix}_{L(R)}, \quad (122)$$

in which the superscript 0 indicates the weak-interaction basis. Similarly, the light (l) and heavy (h) mass eigenstates can be written

$$\psi_{L(R)} = \begin{pmatrix} \psi_l \\ \psi_h \end{pmatrix}_{L(R)}. \quad (123)$$

The weak and mass eigenstates are related by a unitary transformation

$$\psi_a^0 = U_a \psi_a, \quad (124)$$

in which $a = L, R$. The matrix U can be written in block

form as

$$U_a = \begin{pmatrix} A_a & E_a \\ F_a & G_a \end{pmatrix}. \quad (125)$$

Although U_a is unitary, A_a and F_a are not by themselves unitary. These matrices describe the overlap of the light eigenstates with the ordinary and exotic fermions, respectively. We henceforth restrict ourselves to the light eigenstates only.

The effects of mixing on the couplings of the light fermions can now be seen. In the weak basis, the charged-fermion neutral current can be written

$$\begin{aligned} \frac{1}{2} J_Z^\mu = & \bar{\psi}_{iL} \gamma^\mu T_{3L} A_L^\dagger A_L \psi_{iL} + \bar{\psi}_{iR} \gamma^\mu T_{3L} F_R^\dagger F_R \psi_{iR} \\ & - \bar{\psi}_l \gamma^\mu Q_{em} \sin^2 \theta_W \psi_l. \end{aligned} \quad (126)$$

The important implication of the above equation is that, since neither A_L nor F_R is unitary, $A_L^\dagger A_L$ and $F_R^\dagger F_R$ are not necessarily diagonal, and thus mixing in general induces FCNC's among the light particles. In order to avoid these problems, the assumption which is usually made is that each ordinary left- and right-handed fermion mixes with its own exotic partner. In this case, $A_L^\dagger A_L$ and $F_R^\dagger F_R$ are diagonal, thus eliminating FCNC's.

With this assumption one can write

$$(A_a^\dagger A_a)_{ij} = (c_a^i)^2 \delta_{ij}, \quad (F_a^\dagger F_a)_{ij} = (s_a^i)^2 \delta_{ij}, \quad a = L, R, \quad (127)$$

in which $(s_a^i)^2 \equiv 1 - (c_a^i)^2 \equiv \sin^2 \theta_{L(R)}^i$, where $\theta_{L(R)}^i$ is the mixing angle of the i th LH (RH) ordinary fermion and its exotic partner. Therefore, in the presence of mixing the neutral current takes the form

$$\begin{aligned} \frac{1}{2} J_Z^\mu = & \sum_i [\bar{\psi}_{iL} \gamma^\mu (T_{3L} (c_L^i)^2 - Q_{em}^i \sin^2 \theta_W) \psi_{iL} \\ & + \bar{\psi}_{iR} \gamma^\mu (T_{3L} (s_R^i)^2 - Q_{em}^i \sin^2 \theta_W) \psi_{iR}], \end{aligned} \quad (128)$$

where the sum is over the known particles. Similarly, for quarks the charged current is

$$\frac{1}{2} J_W^\mu = \bar{\psi}_{uL} \gamma^\mu V_L \psi_{dL} + \bar{\psi}_{uR} \gamma^\mu V_R \psi_{dR}, \quad (129)$$

in which ψ_{uL} and ψ_{dL} are column vectors of the light LH u -type and d -type quarks, respectively. The CKM matrix V_L is nonunitary in the presence of mixing. It can, however, be decomposed as

$$V_{Lij} = c_L^{u_i} c_L^{d_j} \tilde{V}_{Lij}, \quad (130)$$

where, as before, $\tilde{V}_L$ is the usual (unitary) CKM matrix. The second term in Eq. (129) is a RH charged current. Like V_L , the apparent RH CKM matrix V_R is nonunitary, but can be written

$$V_{Rij} = s_R^{u_i} s_R^{d_j} \tilde{V}_{Rij}, \quad (131)$$

where $\tilde{V}_R$ is unitary.

It is now straightforward to make contact with our general formalism. To do so we first imagine integrating out all of the heavy particles in the model which have not

yet been discovered. This produces the low-energy effective theory with which the earlier sections of this paper have been concerned. At tree level the removal of heavy fermions is very easy: one simply transforms to a basis of mass eigenstates, and sets all heavy fields equal to zero. We are led in this way to interpret Eqs. (128) and (129) as the resulting low-energy effective weak interactions. Other terms, such as contributions to the oblique corrections, are generated once loop effects are included. Although these contributions can be phenomenologically interesting, for ease of presentation we do not pursue them here. We focus instead on the tree-level case, and accordingly set $A=B=C=G=w=z=0$, which leads to $S=T=U=0$.

The key observation to now make is that Eqs. (128) and (129) are the expressions for the effective charged and neutral currents after diagonalization of the fermion fields, but *before* shifting to the physical parameters. They should therefore be compared to Eqs. (41) and (42) (remembering that B and C in these equations are zero). This gives

$$\delta\tilde{g}_L^{ii} = -T_{3L}^i (s_L^i)^2, \quad \delta\tilde{g}_R^{ii} = +T_{3R}^i (s_R^i)^2, \quad (132)$$

and

$$\begin{aligned} \delta\tilde{h}_L^{u_i d_j} &= -\frac{1}{2} V_{ij} [(s_L^{u_i})^2 + (s_L^{d_j})^2], \\ \delta\tilde{h}_R^{u_i d_j} &= s_R^{u_i} s_R^{d_j} \tilde{V}_{Rij}. \end{aligned} \quad (133)$$

The formalism in the case of neutrinos is somewhat different. As before we denote all LH neutrino states as n_L and all RH states as n_R . In the weak basis there are three types of LH neutrinos: those with $T_{3L} = +\frac{1}{2}(n_{OL}^0)$, those with $T_{3L} = -\frac{1}{2}(n_{EL}^0)$, and those which are $SU_L(2)$ singlets (n_{SL}^0). These can be put into a single vector:

$$n_L^0 = \begin{pmatrix} n_{OL}^0 \\ n_{EL}^0 \\ n_{SL}^0 \end{pmatrix}. \quad (134)$$

The mass eigenstates can be classified according to whether the neutrinos are “light” (i.e., essentially massless) or “heavy”:

$$n_L = \begin{pmatrix} n_{lL} \\ n_{hL} \end{pmatrix}. \quad (135)$$

The unitary transformation which relates the weak and mass bases can be written $n_L^0 = U_L n_L$, in which

$$U_L = \begin{pmatrix} A & E \\ F & G \\ H & J \end{pmatrix}_L. \quad (136)$$

This matrix, then, describes the mixing of ordinary and exotic neutrinos. Note that we do not require that each ordinary neutrino mix with only one exotic neutrino. This is because there is no experimental evidence against FCNC's in the neutrino sector.

In the presence of fermion mixing, the leptonic charged current takes the form

$$\frac{1}{2} J_W^\dagger = \bar{n}_L \gamma^\mu A_L^\dagger c_L^e e_L + \bar{n}_R^c \gamma^\mu F_R^\dagger s_R^e e_R \quad (137)$$

in which $e_{L(R)}$ represents a column vector of charged LH (RH) leptons. Following the previous analysis for the charged fermions, it is straightforward to compare Eqs. (137) and (42) to obtain the relations

$$\begin{aligned} \delta\tilde{h}_L^{v_i e_a} &= (A_L^{v_i})_{ia} c_L^{e_a} - \delta_{ia}, \\ \tilde{h}_R^{v_i e_a} &= (F_R^{v_i})_{ia} s_R^{e_a}. \end{aligned} \quad (138)$$

It is useful to write $A_L^v = 1 + \delta A_L^v$, where the new-physics contribution δA_L^v is assumed to be small. As a result the quantity $\Delta_a = \text{Re} \sum_i \delta\tilde{h}_L^{v_i e_a}$ which appears in all physical observables becomes

$$\Delta_a = -\frac{1}{2} (s_L^{e_a})^2 + \text{Re} \sum_i (\delta A_L^{v_i})_{ia}, \quad (139)$$

to linear order in the new physics. These represent the correspondence between our parameters and those of the mixing formalism before the shift to the physical parameters.

Note, however, that it is conventional to parametrize the mixing in the neutrino sector in terms of the mixing angles $(c_L^{v_a})^2 = (A_L^v A_L^{v\dagger})_{aa}$, since these are the only quantities which arise in the rates for realistic reactions in which the final-state neutrinos are unobserved. (There is also a piece coming from the right-handed current in Eq. (137), but this is of higher order in the mixing.) Recall that this is precisely the same reason that only the combination Δ_a appears in our expressions in earlier sections. Linearizing $(c_L^{v_a})^2$ in the new physics we have $\text{Re} \sum_i (\delta A_L^{v_i})_{ia} = -\frac{1}{2} (s_L^{v_{e_a}})^2$, yielding the correspondence

$$\Delta_a = -\frac{1}{2} [(s_L^{e_a})^2 + (s_L^{v_{e_a}})^2] \quad (140)$$

to leading order in the square of the mixing angles.

(In the original exotic-fermion mixing paper [16], mixing in the neutrino sector is not assumed to be small. However, this does not significantly change the above analysis. If the new-physics parameters $(\delta A_L^{v_i})_{ia}$ (and hence the $\delta\tilde{h}_L^{v_i e_a}$) are allowed to be big, then one uses Eq. (138) and the exact definition of Δ_a given in Eq. (44) to again arrive at Eq. (140).)

For the neutrino neutral current, the relations between the mixing angles and our parameters are somewhat more complicated to derive, so for the sake of brevity we do not include them here.

There is one other point we would like to reemphasize. In Refs. [16,17], the analysis of ordinary-exotic fermion mixing was done observable by observable. This led to a certain amount of confusion since mixing affects not only each observable, but also such parameters as G_F and s_W^2 which appear in the theoretical expressions for each process. While it is true that the analyses in these papers ultimately dealt correctly with these problems, our formalism avoids such headaches altogether by incorporating all new-physics effects at the level of the Lagrangian.

The translation from ordinary-exotic fermion mixing angles to our parameters has been summarized in Eqs.

(140), (133), and (132). It is now a simple matter to bound the mixing angles using these relations and the constraints in Table VI. As mentioned already, the bounds obtained in this way are in fact weaker than those which would be obtained in a direct fit to the mixing angles themselves. This is simply because there are more independent parameters in our fit. In this sense our results can be considered the most conservative bounds possible. Nevertheless, the constraints on the mixing angles are really quite restrictive.

One minor complication is that, while our parameters are allowed to be *a priori* either positive or negative, the mixing angles $(s_{L,R}^i)^2$ are necessarily ≥ 0 . This should be taken into account in a proper fit (see Refs. [16,17]). Ignoring this detail we find the following limits at 90% C.L. (defined as 1.64σ):

$$\begin{aligned}
 \Delta_{e,\mu}: \quad & (s_L^e)^2, (s_L^{\nu e})^2 < 0.016, \\
 & (s_L^\mu)^2, (s_L^{\nu \mu})^2 < 0.012, \\
 \delta \tilde{h}_L^{ud}: \quad & (s_L^u)^2, (s_L^d)^2 < 0.02, \\
 \delta g_{L,R}^{ii}: \quad & (s_R^e)^2 < 0.01, \\
 & (s_R^\mu)^2 < 0.09, \\
 & (s_R^\nu)^2 < 0.03, \\
 & (s_R^d)^2 < 0.05, \\
 & (s_L^s)^2 < 0.05, \\
 & (s_L^b)^2 < 0.03,
 \end{aligned} \tag{141}$$

where the numbers have been obtained using the constraints from the simultaneous fit (Table VI), and we have indicated which of our parameters has been used to obtain the limit on the mixing angle. We have not presented all the limits since our purpose was simply to show how our results could be used to bound a specific model of new physics. A comparison of the above numbers with those found in Eq. [17] reveals that the bounds obtained in this way are very similar to those found in a fit to the mixing angles themselves. Of course, our analysis applies to *all* models of new physics, not just the particular case of the mixing of ordinary and exotic fermions.

VII. CONCLUSIONS

New physics can manifest itself in one of two ways: either new particles will be discovered or their presence will be detected via the virtual effects they induce in low-energy processes. Until the next generation of accelerators comes on line, we will probably have to content ourselves with the second possibility. Given this, it is fruitful to study, in as model-independent a manner as possible, the various virtual effects which might be detectable using today's colliders.

A useful framework in which to perform such an analysis is using an effective Lagrangian. It has the principal merit of being completely systematic, so that one is sure that no potential low-energy effects of new physics are accidentally missed. Here the new-physics operators

can be classified according to their dimension, i.e., the number of powers of $1/M$ which are required by dimensional analysis. One subset of operators which has already been studied consists of the new-physics contributions to gauge-boson propagators—the “oblique” corrections. In this paper we have extended the analysis to include all operators of the same dimension, including corrections to the $Z\bar{f}f$ and $W\bar{f}f$ vertices.

We have developed a formalism which can deal with all these new operators in a relatively straightforward way. One of the main effects of new physics is to shift the relationships between the input parameters to the standard model, α , G_F , and M_Z , and the measured values of these quantities. We take these shifts into account in the Lagrangian itself. Having done this, it is no longer necessary to separately adjust each observable as it is considered. This facilitates the calculation, and removes a considerable amount of confusion from the analysis.

We find a great many operators which satisfy the following three assumptions: (i) we concentrate on the electroweak sector alone; (ii) we only keep interactions with dimension ≤ 5 , both CP preserving and CP violating; (iii) we consider only those operators which contribute at tree level in well-measured processes. Despite the large number of operators, most of these are well constrained by the current experimental data. There are a few interesting exceptions.

(1) Of the FCNC operators, dimension five terms of the form $\bar{f}\sigma^{\mu\nu}f'Z_{\mu\nu}$ are quite poorly bounded, their effects could easily be visible at LEP.

(2) With a few exceptions [see Eq. (109)], the constraints on the other dimension-five operators, the flavor-conserving neutral current couplings $\bar{f}\sigma^{\mu\nu}f'Z_{\mu\nu}$ and the charged current $\bar{f}\sigma^{\mu\nu}f'W_{\mu\nu}$, are also quite weak.

(3) There is still a great deal of room for new physics in the hadronic charged-current sector. For example, the chirality of b decays has not yet been tested. There are a number of ways to constrain new physics in this area—remeasurements of the known CKM matrix elements using different methods, CP violation in the B system, and measurements of the CKM matrix elements involving the t quark are a few examples.

(4) Universality violation in τ decays remains a puzzle.

(5) Most CP -violating operators are virtually unconstrained. Their effects might well be seen when CP violation in the B system is studied.

All other operators are well constrained, particularly the neutral current couplings, most to at least the 2–3 % level. The utility of such a global, model-independent analysis is that it presents limits which must be satisfied by *all* models of new physics. For any particular choice of physics beyond the standard model, it is only necessary to compute the coefficients of the above operators in terms of the parameters of that particular model. The constraints presented in this paper then serve to constrain that model. As an example of how this works, we considered mixing of ordinary and exotic fermions. For this case we have shown that, indeed, our constraints reproduce the results of previous analyses, but frequently in a simpler way. It is our hope that this work will serve as a guide to future model builders.

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