Heavy quark solitons: Strangeness and symmetry breaking

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We discuss the generalization of the Callan-Klebanov model to the case of heavy quark baryons. The light flavor group is considered to be SU(3) and the limit of heavy spin symmetry is taken. The presence of the Wess-Zumino-Witten term permits the neat development of a picture, at the collective level, of a light diquark bound to a "heavy" quark with decoupled spin degree of freedom. The consequences of SU(3) symmetry breaking are discussed in detail. We point out that the SU(3) mass splittings of the heavy baryons essentially measure the "low energy" physics once more and that the comparison with experiment is satisfactory.

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I. INTRODUCTION

A natural method for describing a baryon containing a single heavy quark in the "soliton picture" is to consider the heavy baryon to be a bound state of a heavy-meson and a light "baryon as soliton." This was extensively applied by Callan and Klebanov [1] and others [2] to the case where the K meson is considered heavy. A fairly literal transcription of this approach was given for the charm and bottom baryons too [3]. More recently, it has been recognized that it is necessary to take into account the Isgur-Wise heavy spin symmetry [4] when dealing with the chiral interactions of the heavy mesons. This feature was then incorporated, with somewhat different results, by two groups [5,6]. In the present paper, we will study further a possibly simpler method [7], based on an explicit presentation of the ansatz for the "classical" bound state. It was also noticed in [7] that the effect of including light vector mesons in the underlying chiral Lagrangian was important for estimating the semiclassical binding energy.

The new points here are mainly concerned with generalizing the treatment of Ref. [7] to light SU(3) so as to be able to treat heavy baryons with strangeness. A surprising feature is that this generalization actually simplifies the procedure. The reason is the existence of the Wess-Zumino-Witten (WZW) term [8] in the light SU(3) case but not in the light SU(2) case. We will see that the interplay of the WZW term and the heavy meson kinetic term gives an important constraint on the allowed states of the collective Hamiltonian. What emerges is that the collective Hamiltonian describes a bosonic (light diquark) rotator in addition to a decoupled (in the heavy spin symmetry limit) spinor representing the heavy quark. The heavy quark symmetry is then essentially manifest and the entire treatment is rather simple. It is amusing that, although the underlying Lagrangian is a theory of mesons, the collective picture looks quarklike. Of course this is implicit in [1], but here it will be seen to follow in a particularly neat way.

We will also use this formalism to discuss the SU(3) mass splittings of the heavy baryons. It seems natural to do so in the limit of heavy spin symmetry. This is because the heavy spin splittings vanish as the heavy quark mass, $M \to \infty$, in contrast with the SU(3) splittings which remain finite in this limit. The physics which is being probed by these mass splittings is very similar to that determining the mass splittings of the light baryons. In the latter case, a treatment [9] based on lowest order perturbation theory is inadequate, as may be seen by a comparison with the exact diagonalization of the collective Hamiltonian [10]. It was pointed out [11] that second-order perturbation theory does provide an adequate approximation to the exact solution and that is what will be used here. While it would be most desirable to compare our predictions with data on the bottom baryons, there is, at present, sufficient reliable information only for the charmed baryons. Comparing our results with the likely experimental $J^P = \frac{1}{2}^+$ states Λ_c, Σ_c, Ξ_c, and Ω_c, gives values for the basic coefficients of the collective Hamiltonian which are reasonably close to those obtained from studies of the light baryon spectrum. We also predict the mass of another expected Ξ_c state and note that, in the limit of heavy spin symmetry, it should not mix with the already observed one.

For the reader's convenience, the underlying fields and chiral Lagrangians are briefly reviewed in Sec. II. Both the Lagrangians with and without vector fields are given since the form of the SU(3) invariant collective Lagrangian is the same in each case. In Sec. III, the classical soliton solution for the light meson fields as well as the heavy meson bound state ansatz are listed. The discussion of the collective mode quantization and its interpretation is given in Sec. IV. Furthermore, the low lying SU(3) multiplets of heavy baryons are identified and discussed. In Sec. V we introduce the SU(3) symmetry breaking and find its effects on the heavy baryon mass splittings. We note the fact that SU(3) symmetry breaking among the heavy mesons has a rather small effect at the collective Hamiltonian level; it is the light pseudoscalar meson breaking which actually dominates the heavy baryon splittings.

II. THE MESON FIELDS AND THE CHIRAL LAGRANGIANS

In this section we briefly summarize the SU(3) chiral Lagrangians under consideration. The total action consists of a "light" part describing the first three flavors (namely u, d, s) and a "heavy" part which describes the "heavy" multiplet H and its interaction with the light sector:

$$\Gamma_{\text{eff}} = \Gamma_{\text{light}} + \int d^4x \, \mathcal{L}_{\text{heavy}}. \tag{2.1}$$

The relevant light fields belong to the 3×3 matrix of pseudoscalars, ϕ, and to the 3×3 matrix of vectors, ρ_μ. It is convenient to define objects which transform simply under the action of the chiral group:

$$\xi = \exp\left(\frac{i\phi}{F_\pi}\right), \qquad U = \xi^2,$$

$$A_\mu^L = \xi \rho_\mu \xi^\dagger + \frac{i}{\tilde{g}} \xi \partial_\mu \xi^\dagger,$$

$$A_\mu^R = \xi^\dagger \rho_\mu \xi + \frac{i}{\tilde{g}} \xi^\dagger \partial_\mu \xi,$$

$$F_{\mu\nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu - i\tilde{g}[\rho_\mu, \rho_\nu], \tag{2.2}$$

where $F_\pi \approx .132$ GeV and $\tilde{g} \approx 3.93$ for a typical fit.

The interactions of the heavy meson fields can be encoded in a compact way by using the so-called "heavy superfield" [12] which combines the heavy pseudoscalar P' and the heavy vector Q'_μ, both moving with a fixed four-velocity V_μ :

$$H = \frac{1 - i\gamma_\mu V_\mu}{2}(i\gamma_5 P' + i\gamma_\nu Q'_\nu), \quad \bar{H} \equiv \gamma_4 H^\dagger \gamma_4. \tag{2.3}$$

In our conventions the superfield H has the canonical dimension one. It is a 4×4 matrix in the Dirac space and it also carries an unwritten flavor index for the light quark bound to the heavy quark. The chiral interactions of H with the light pseudoscalars were discussed in [13]. The inclusion of the light vector mesons were given in [14,15]. Here we follow the notation of Ref. [14].

Using (2.3), $\mathcal{L}_{\text{heavy}}$ can be simply written as

$$\frac{\mathcal{L}_{\text{heavy}}}{M} = iV_\mu \text{Tr}\left[H(\partial_\mu - i\alpha\tilde{g}\rho_\mu - i(1-\alpha)v_\mu)\bar{H}\right] + id\,\text{Tr}\left[H\gamma_\mu\gamma_5 p_\mu \bar{H}\right] + \frac{ic}{m_v}\text{Tr}\left[H\gamma_\mu\gamma_\nu F_{\mu\nu}(\rho)\bar{H}\right], \tag{2.4}$$

where $m_v \approx 0.77$ GeV is the light vector mass and

$$v_\mu, p_\mu \equiv \frac{i}{2}(\xi\partial_\mu\xi^\dagger \pm \xi^\dagger\partial_\mu\xi). \tag{2.5}$$

Furthermore M is the heavy meson mass and α, c, d are dimensionless coupling constants for the heavy-light interactions. It seems appropriate not to include terms in (2.4) which are higher order in $1/M$ or contain more derivatives of the light meson fields.

The action involving the light pseudoscalar and vector mesons, Γ_{light}, can be written as the sum of a usual piece,

$$\int d^4x \left[-\frac{F_\pi^2}{8}\text{Tr}(\partial_\mu U \partial_\mu U^\dagger) - \frac{1}{4}\text{Tr}\left[F_{\mu\nu}(\rho)F_{\mu\nu}(\rho)\right] - \frac{m_v^2}{2\tilde{g}^2}\text{Tr}\left[(\tilde{g}\rho_\mu - v_\mu)^2\right]\right], \tag{2.6}$$

and a piece proportional to the Levi-Civita symbol. The latter is most conveniently written, using the differential form notation, in terms of the one-forms $\alpha_\mu \equiv \partial_\mu U U^\dagger \to \alpha$ and $A_\mu^L \to A^L$:

$$\Gamma_{\text{WZW}} + \int \text{Tr}\left[ic_1(A^L\alpha^3) + c_2(dA^L\alpha A^L - A^L\alpha dA^L + A^L\alpha A^L\alpha) + c_3\left\{-2i(A^L)^3\alpha + \frac{1}{\tilde{g}}A^L\alpha A^L\alpha\right\}\right], \tag{2.7}$$

where the Wess-Zumino-Witten term [8] is given by

$$\Gamma_{\text{WZW}} = -\frac{i}{80\pi^2}\int_{\mathcal{M}^5}\text{Tr}\,(\alpha^5). \tag{2.8}$$

Note that the c_1, c_2, c_3 terms in (2.7) perform the function of stabilizing the Skyrme soliton in this model. More details are given in [16] (wherein $\tilde{g}$ is denoted by g).

We shall also consider here a simpler light Lagrangian in which the vectors are absent. [In this case the constants α and c in (2.4) should also be set to zero.] We then have the standard SU(3) Skyrme model,

$$\Gamma_{\text{light}} = \int d^4x \text{Tr}\left(\frac{F_\pi^2}{8}\alpha_\mu\alpha_\mu + \frac{1}{32e^2}\left[\alpha_\mu, \alpha_\nu\right]^2\right) + \Gamma_{\text{WZW}}, \tag{2.9}$$

wherein e is the Skyrme constant.

The discussion of SU(3) symmetry-breaking terms is deferred to Sec. V.

III. BARYON STATES AT THE CLASSICAL LEVEL

Following the Callan-Klebanov strategy [1], we first find the classical solution of Γ_{light} and then obtain the classical approximation to the wave function in which this "baryon as soliton" is bound to a heavy meson (yielding a *heavy* hyperon). To avoid confusion we remark that whereas the original approach [1] dealt with a two-flavor light action and considered the strange quark as "heavy," in the present work we are dealing with a three-flavor light action, considering the strange quark as light. When one improves the treatment of flavor symmetry breaking for the SU(3) soliton one obtains results for the strange hyperons of comparable accuracy to those of the Callan-Klebanov approach.

The "hedgehog" ansatz for the classical light baryon in the SU(3) case simply corresponds to embedding the two-flavor ansatz as follows:

$$\xi_c = \begin{pmatrix} \exp \frac{i}{2}[\hat{\boldsymbol{x}} \cdot \boldsymbol{\tau} F(r)] & 0 \\ 0 & 1 \end{pmatrix}. \tag{3.1}$$

In a model with vectors present, we have similarly the classical solutions

$$\rho_{c\,\mu} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\omega_{c\,\mu} + \tau^a \rho^a_{c\,\mu}) & 0 \\ 0 & 1 \end{pmatrix}, \tag{3.2}$$

with (see [16] for example)

$$\rho^a_{i\,c} = \frac{1}{\sqrt{2}\tilde{g}r}\epsilon_{ika}\hat{x}_k G(r), \qquad \rho^a_{0\,c} = 0,$$
$$\omega_{i\,c} = 0, \quad \omega_{0\,c} = \omega(r). \tag{3.3}$$

The boundary conditions for a finite energy light baryon are

$$F(0) = -\pi, \quad G(0) = 2, \quad \omega'(0) = 0,$$
$$F(\infty) = G(\infty) = \omega(\infty) = 0. \tag{3.4}$$

For describing the soliton-heavy meson bound state it is convenient to define the grand spin $\mathbf{G}$ as the sum of isospin and the angular momentum:

$$\mathbf{G} = \mathbf{I} + \mathbf{J} \tag{3.5}$$

In [1] it was found that the attractive channel for the ordinary hyperon, treated as a bound state of the nucleon as soliton and the kaon, was the one with orbital angular momentum $l = 1$ and this was combined with the kaon isospin to give $G = \frac{1}{2}$. In our previous work [7], it was shown that the same situation persisted in the heavy meson-nucleon bound state. In this case it is more intuitive to think of the heavy meson as being at rest and the soliton bound to it. When the heavy meson is at rest ($V_i = 0$) the 4×4 matrix heavy superfield $\bar{H}$ given in (2.3) has nonvanishing elements only in the lower-left 2×2 subblock:

$$\bar{H} = \begin{pmatrix} 0 & 0 \\ \bar{H}^b_{l\,h} & 0 \end{pmatrix}. \tag{3.6}$$

The first lower index l of the submatrix $\bar{H}^b_{l\,h}$ represents the spin of the light degrees of freedom within the heavy meson, while the second lower index h represents the spin of the heavy quark. The wave function for the heavy field is then written as

$$\bar{H}^b_{l,h} = \begin{cases} \frac{1}{\sqrt{8\pi M}}(\hat{\boldsymbol{x}} \cdot \boldsymbol{\tau})_{bm}\epsilon_{lm}u(r)\chi_h & \text{if} \quad b = 1, 2, \\ 0 & \text{if} \quad b = 3. \end{cases} \tag{3.7}$$

This represents an embedding of (3.1) of [7] into the three-dimensional representation of SU(3). The radial wave function $u(r)$ is taken in the classical approximation to be localized at the origin, $r^2|u(r)|^2 \approx \delta(r)$. Clearly, this is reasonable at the limit $M \to \infty$. Note that the quantity $\hat{\mathbf{x}}$ represents the angular part of the spatial wave function and the first factor couples it to the isospin index b to give $G = \frac{1}{2}$. In turn, this is coupled to the light spin index l with the Clebsch-Gordan coefficient $\frac{1}{\sqrt{2}}\epsilon_{lm}$ to give $G = 0$. Finally, h is left uncoupled (as appropriate to the heavy spin symmetry) to give the desired net result $G = \frac{1}{2}$. The two-component heavy quark spinor χ_h [which was implicit in (3.1) of [7]] basically carries the heavy spin.

Substituting the ansatz (3.7) into $\mathcal{L}_{\text{heavy}}$ in (2.4) yields a classical binding potential ($V_0 = -\int d^3x \mathcal{L}_{\text{heavy}}$) given in (4.3) of [7]:

$$V_0 = -\frac{3}{2}dF'(0) + \frac{3c}{m_v \tilde{g}}G''(0) - \frac{\alpha\tilde{g}}{\sqrt{2}}\omega(0). \tag{3.8}$$

The experimental determination of the light-heavy coupling constants d, c, and α is at a very primitive stage. Some information about the light-heavy coupling constants d and c can be obtained from the semileptonic $D \to K$ and $D \to K^*$ transitions, respectively. This suggests that the first two terms of (3.8) are negative, suggesting that V_0 is also negative. Taking into account a simple model for the effect of the quantum fluctuations on (3.8) (see Sec. V of [7]) as well as estimates based on semileptonic data gives an approximate fit [see Eq. (2.15) of [17]]:

$$d = 0.53, \qquad c = 1.6, \qquad \alpha = -2. \tag{3.9}$$

V_0 remains negative (attractive) in the model with pseudoscalars only, in which just the first term of (3.8) is kept. We remark that, as previously mentioned [14] a natural notion of light vector dominance for the light heavy interaction would suggest that $\alpha = 1$. It is hoped that more experimental data from both meson and baryon sector of the theory will clarify the situation.

IV. COLLECTIVE MODE QUANTIZATION

In the soliton approach, the particle states with definite rotational and flavor quantum numbers do not appear until the so-called "rotational collective modes" are introduced and the theory is quantized. This is conveniently done [18] by first finding the time independent parameters which leave the theory invariant. Then those "collective" parameters are allowed to depend on time. Specifically, we set

$$\begin{aligned} \xi(\mathbf{x}, t) &= A(t)\xi_c(\mathbf{x})A^\dagger(t), \\ \rho_\mu(\mathbf{x}, t) &= A(t)\rho_{\mu\,c}(\mathbf{x})A^\dagger(t), \\ \bar{H}(\mathbf{x}, t) &= A(t)\bar{H}_c(\mathbf{x}), \end{aligned} \tag{4.1}$$

where the "classical" bound state wave function $\bar{H}_c$ is to be taken from (3.7). Here $A(t)$ is an SU(3) matrix which acts on the isospin index of $\bar{H}$. A is conventionally considered as a matrix of angle-type variables; generalized angular velocities Ω_k are defined by

$$A^\dagger \dot{A} = \frac{i}{2}\sum_{k=1}^{8}\lambda_k\Omega_k, \tag{4.2}$$

where the λ_k are the usual Gell-Mann matrices. Now substituting (4.1) into the total effective Lagrangian (2.1) and performing a spatial integration eventually yields the following collective Lagrangian:

$$L_{\rm coll} = -M_c - (M+V_0)P + \frac{\alpha^2}{2}\sum_{i=1}^{3}\Omega_i\Omega_i + \frac{\beta^2}{2}\sum_{j=4}^{7}\Omega_j\Omega_j - \frac{\sqrt{3}}{2}\Omega_8 + \frac{\sqrt{3}}{6}\Omega_8\chi^\dagger\chi P. \tag{4.3}$$

Here, M_c is the classical soliton mass, α^2 is the ordinary moment of inertia, and β^2 is the "strange" moment of inertia. Expressions for these quantities are given in [19] for the full model including the vectors and in [20] for the minimal SU(3) Skyrme model containing the pseudoscalars only. The factor P is a projection operator onto the heavy baryon subspace of the theory and appears in those terms in the collective Lagrangian which originate from terms involving the heavy fields. V_0 is the classical binding energy given in (3.8). (Actually the quantum corrections to V_0 discussed in Sec. V of [7] are also important.) The last term in (4.3) comes from the heavy meson kinetic term $iV_\mu\,{\rm Tr}(H\partial_\mu\bar{H})$; we have included in it a factor $\chi^\dagger\chi = \delta_{hh'}$ pertaining to the heavy spin indices which makes manifest the fact that the heavy spin index in (3.6) has *not* been summed over in arriving at this term. Thus the heavy spin represents a dynamical degree of freedom of the collective Lagrangian. The fact that the particular value of the heavy spin is not communicated to the soliton variables is a reflection of the underlying heavy quark symmetry. This means that a term of the form

$$\boldsymbol{\Omega}\cdot\chi^\dagger\boldsymbol{\sigma}\chi \tag{4.4}$$

cannot appear in the present model, which represents a substantial difference from the Callan-Klebanov paper [1]. Notice that the form of $L_{\rm coll}$ is the same regardless of whether or not the light vector mesons are included. However, the specific values of the numerical parameters then differ.

The next step is to quantize (4.3). The canonical momenta (for an implicit parametrization of the matrices A) may be taken as

$$-R_k = \frac{\partial L_{\rm coll}}{\partial\Omega_k}. \tag{4.5}$$

For $k = 1, 2, \ldots, 7$, (4.5) yields true dynamical momenta. However, for $k = 8$ one gets

$$R_8 = \begin{cases} \frac{\sqrt{3}}{2} & \text{for light baryons,} \\ \frac{1}{\sqrt{3}} & \text{for heavy baryons.} \end{cases} \tag{4.6}$$

The above numerical difference together with the presence of the heavy quark spin degree of freedom are the main differences between the present heavy baryon case and the usual light baryon case. Hence we can make use of the usual quantization [21] of the SU(3) Skyrme model. An operator R_8 which obeys the canonical commutation relations can be introduced but we must demand, in the manner of *Dirac*, the following constraint on the allowed states $|\quad\rangle$ of the model :

$$\frac{2}{\sqrt{3}}R_8|\quad\rangle = \begin{cases} |\quad\rangle & \text{for light baryons} \\ \frac{2}{3}|\quad\rangle & \text{for heavy baryons.} \end{cases} \tag{4.7}$$

Then the R's can be seen to obey an SU(3) algebra $[R_i, R_j] = -if_{ijk}R_k$. The spatial components $\mathbf{R}$ (i.e., for $i = 1, 2, 3$) can be identified as rotation generators for the soliton rotator while the quantity $\frac{2}{\sqrt{3}}R_8$ in (4.7) is a conventionally normalized hypercharge generator. We also define "left" generators $L_j = D_{jk}(A)R_k$, where the adjoint representation matrix $D_{jk}(A)$ can be written as

$$D_{jk}(A) = \frac{1}{2}{\rm Tr}\left(\lambda_j A\lambda_k A^\dagger\right). \tag{4.8}$$

They obey a separate SU(3) algebra $[L_i, L_j] = if_{ijk}L_k$ and can be identified as SU(3) flavor generators. Note that $\sum_{m=1}^{8}L_mL_m = \sum_{n=1}^{8}R_nR_n$. The collective Hamiltonian is

$$H_{\rm coll} = M_c + (M+V_0)P + \frac{1}{2}\left(\frac{1}{\alpha^2} - \frac{1}{\beta^2}\right)\mathbf{R}^2 + \frac{1}{2\beta^2}\sum_{m=1}^{8}R_m^2 - \frac{1}{2\beta^2}R_8^2, \tag{4.9}$$

which can be written as

$$H_{\rm coll} = M_c + (M+V_0)P + \frac{1}{2}\left(\frac{1}{\alpha^2} - \frac{1}{\beta^2}\right)J_s(J_s+1) + \frac{1}{2\beta^2}C_2[{\rm SU}(3)_L] - \frac{1}{2\beta^2}R_8^2, \tag{4.10}$$

where J_s is the soliton angular momentum and $C_2[{\rm SU}(3)_L]$ is the SU(3) quadratic Casimir operator. Irreducible representations of SU(3) may be specified by a traceless tensor with p symmetric quark type indices (say, upper) and q symmetric antiquark type (say, lower) indices. Then,

$$C_2[{\rm SU}(3)] = \frac{1}{3}(p^2 + pq + q^2) + (p+q). \tag{4.11}$$

It is evident that $H_{\rm coll}$ is SU(3) flavor invariant. For the *light* baryon subspace, the space of the "angular" wave functions is spanned by SU(3) representation matrices $D^{(\mu)}(A)$, where μ denotes the irreducible representation under consideration. With conventional [22] normalization the light baryon wave functions are

$$\Psi_{\text{light}}(\mu, YII_3, JJ_3; A) = (-1)^{J-J_3}\sqrt{\dim \mu}\ D^{(\mu)*}_{Y,I,I_3;Y_R\,J,\,-J_3}(A). \quad (4.12)$$

The composite indices were labeled in accordance with the fact that the flavor generators L_k act on the left composite index while the generators R_k, which include the space rotation ones, act on the right composite index. It is crucial to note that the constraint (4.7) implies that the index Y_R in (4.12) must be set equal to 1. In turn, this implies that only those irreducible representations $\{\mu\}$ are allowed which contain a state with $Y = +1$. Such states, as we will see below, have half odd-integral spin and so must represent fermions. The rotational spin of the wave function (4.12) equals the isospin of the desired state belonging to $\{\mu\}$ which has $Y = 1$.

The *heavy* baryon wave functions may be constructed analogously. Because of the constraint (4.7), now the angular wave function must have $Y_R = \frac{2}{3}$. In this case, the allowed $D^{(\mu)}(A)$'s must have integral spin, since SU(3) states with $Y = \frac{2}{3}$ necessarily have integral isospin. To see this, let us consider a (p,q) tensor component described above which has n_i=the number of ith-type quark indices minus the number of ith-type antiquark indices, where $i = u, d, s$. Such a state is labeled by

$$I_3 = \frac{1}{2}(n_u - n_d),$$
$$Y = \frac{1}{3}(n_u + n_d - 2n_s). \quad (4.13)$$

Substituting $Y = \frac{2}{3}$ into the last equation above gives $n_u + n_d = 2(1 + n_s)$. Subtracting $2n_d$ from both sides finally yields

$$I_3 = 1 + n_s - n_d = \text{ integer}. \quad (4.14)$$

[If we had substituted $Y = 1$ into (4.13) we would have obtained $I_3 = \frac{3}{2} + (n_s - n_d) = \frac{1}{2}$ odd integer.] The fact that $D^{(\mu)}(A)$ has integral spin meshes perfectly with the need to include in the overall heavy baryon wave function the Pauli spinor χ_h, which remains in the collective Lagrangian (4.3) (even though it decouples in accordance with the heavy quark symmetry). Now, we can write the heavy baryon wave function as

$$\Psi_{\text{heavy}}(\{\mu\}, Y\,I\,I_3; J\,J_3, J_s; A) = \sum_{h=1}^{2} C^{J_s\ \frac{1}{2}\ J}_{M_s\ h\ J_3}(-1)^{(J_s - M_s)}\sqrt{\dim \mu}\,\chi_h D^{(\mu)*}_{Y,I,I_3;\frac{2}{3},J_s,-M_s}(A), \quad (4.15)$$

wherein J_s and M_s are the soliton spin and its z component while the first factor on the right-hand side is an ordinary SU(2) Clebsch-Gordan coefficient. Notice that for a given J_s there are two possible values of the total spin $J = J_s \pm \frac{1}{2}$ which yield degenerate states of H_{coll}. These two states comprise a heavy quark spin symmetry multiplet. This symmetry is essentially manifest in the present treatment.

It is interesting to observe that, even though the starting model describes the interactions of heavy and light mesons, at the collective level a picture more like the quark model emerges; namely, a heavy quark spinor is compounded with a light diquark wave function $D^{(\mu)}(A)$. This picture is not a matter of choice but is forced upon us by the constraint in (4.7). This constraint results from the presence of both the three flavor Wess-Zumino-Witten term [which gives the next to last term in (4.3)] and the heavy field kinetic term with the "classical" ansatz (3.7) [which gives the last term in (4.3)].

Now let us apply this approach to the low lying baryons. The simplest SU(3) multiplet with a $Y = \frac{2}{3}$ member is the $\{\bar{\mathbf{3}}\}$ [with $(p,q) = (0,1)$]. Its (Y,I) content is $\{(\frac{2}{3},0),(-\frac{1}{3},\frac{1}{2})\}$. Since the $Y = \frac{2}{3}$ state has $I = 0$ we conclude that the soliton spin $J_s = 0$. Combining this with the heavy quark spinor in (4.15) yields net spin $\frac{1}{2}$ baryons, which are denoted $\{\Lambda_Q, \Xi_Q(\bar{\mathbf{3}})\}$. The subscript Q indicates that one s quark in the ordinary hyperon has been replaced by the heavy quark Q.

The next simplest SU(3) multiplet with a $Y = \frac{2}{3}$ member is the $\{\mathbf{6}\}$ [with $(p,q) = (2,0)$]. Its (Y,I) content is $\{(\frac{2}{3},1),(-\frac{1}{3},\frac{1}{2}),(-\frac{4}{3},0)\}$. Since the $Y = \frac{2}{3}$ state has $I = 1$, we conclude that the soliton spin $J_s = 1$. Combining this with the heavy quark spinor in (4.15) yields degenerate multiplets with net spins $J = \frac{1}{2}$ and $\frac{3}{2}$. These are denoted as $\{\Sigma_Q, \Xi_Q(\mathbf{6}), \Omega_Q\}$ and $\{\Sigma^*_Q, \Xi^*_Q(\mathbf{6}), \Omega^*_Q\}$, respectively.

The 15 states mentioned are all the low-lying (s wave) baryon states in the quark model containing a single heavy quark. In the limit of exact flavor SU(3) symmetry it is easy to evaluate the mass splittings by acting with H_{coll} given in (4.10) on the wave function (4.15). For example, one can readily see that

$$m(\mathbf{6}, \tfrac{1}{2}) = m(\mathbf{6}, \tfrac{3}{2}), \quad (4.16)$$

since

$$\left(C^{1\ \frac{1}{2}\ J}_{M-\frac{1}{2}\ \frac{1}{2}\ M}\right)^2 + \left(C^{1\ \frac{1}{2}\ J}_{M+\frac{1}{2}\ -\frac{1}{2}\ M}\right)^2 = 1.$$

Equation (4.16) is just the expression of heavy quark symmetry. In addition, using $C_2(\bar{\mathbf{3}}) = \frac{4}{3}$ and $C_2(\mathbf{6}) = \frac{10}{3}$ from (4.11), we find

$$m(\mathbf{6}) - m(\bar{\mathbf{3}}) = \frac{1}{\alpha^2}. \quad (4.17)$$

Similarly treating the light baryons using (4.10) we find

$$m(\Delta) - m(N) = \frac{3}{2\alpha^2} \quad (4.18)$$

where Δ and N stand for the usual decuplet and octet baryons. From the last two formulas we get the structural relation

$$m(\mathbf{6}) - m(\bar{\mathbf{3}}) = \frac{2}{3}[m(\Delta) - m(N)]. \tag{4.19}$$

Not surprisingly, (4.19) agrees with the SU(2) relation given, e.g., in [5,7], where it was noted to be reasonably well satisfied experimentally for $m(\Sigma_c) - m(\Lambda_c)$.

V. SU(3) SYMMETRY BREAKING

In this section we will use perturbation theory to discuss the mass splittings within the heavy baryon SU(3) multiplets mentioned in the last section. We restrict ourselves to the limit of degenerate heavy quark spin multiplets. Of course, the spin splittings (between states of the same flavor) should vanish as $M \to \infty$. In contrast, the flavor splittings within a multiplet of given spin should not vanish as $M \to \infty$ and thus are a characteristic feature of the present model.

At the level of the fundamental QCD Lagrangian the flavor splittings are induced by the light quark mass terms,

$$\mathcal{L}_{\rm mass} = -\hat{m}\bar{q}\mathcal{M}q, \tag{5.1}$$

where q is the column vector of u, d, s quark fields, $\hat{m} = \frac{m_u + m_d}{2}$ and $\mathcal{M}$ is a dimensionless, diagonal matrix which can be expanded as follows:

$$\mathcal{M} = y\lambda_3 + T + xS, \tag{5.2}$$

with λ_3 = diag $(1, -1, 0)$, T = diag $(1, 1, 0)$, and S = diag $(0, 0, 1)$. x and y are the quark mass ratios:

$$x = \frac{m_s}{\hat{m}}, \qquad y = \frac{m_u - m_d}{2\hat{m}}. \tag{5.3}$$

It will be assumed as usual that all the *effective* symmetry breaking terms are proportional to $\mathcal{M}$. In Ref. [23] it was shown that a rather detailed fit to both the light pseudoscalar and light vector systems required six different terms—three quark-line rule conserving terms for the pseudoscalars and three analogous terms for the vectors. For simplicity, we shall keep here just the dominant term involving only the pseudoscalars:

$$\mathcal{L}_{\rm SB} = \delta' \ {\rm Tr}[\mathcal{M}(U + U^\dagger - 2)] + \cdots, \tag{5.4}$$

where $\delta' \approx 4.04 \times 10^{-5}$ GeV^4, $x \approx 31.5$ and $y \approx -0.42$ [23]. We shall also neglect the small isospin violation by assuming $y = 0$. Equation (5.4) may alternatively be regarded as an appropriate symmetry breaking term for the minimal SU(3) Skyrme model with pseudoscalars only, as given in Eq. (2.9). The contribution of (5.4) to the collective Lagrangian is obtained by substituting Eq. (4.1) and the result is

$$L^{\rm SB}_{\rm coll} = \frac{16\pi\delta'}{3}[3 + (x-1)(1 - D_{88}(A))] \times \int r^2 dr(\cos F(r) - 1), \tag{5.5}$$

where $D_{88}(A)$ is defined in (4.8). Conventionally [10], the term with the overall factor of 3 is included in M_c.

It is reasonable to expect that the effective symmetry breaking terms for the heavy meson multiplet H [in (2.3)] should also influence the heavy baryon mass splittings. The leading term of this type has the form [24]

$$\mathcal{L}_{\rm SB,\mathit{h}} = \epsilon M \left[{\rm tr}\left(H\xi\mathcal{M}\xi\bar{H}\right) + {\rm H.c.}\right]. \tag{5.6}$$

The parameter ϵ may be obtained in terms of the mass difference between the strange heavy meson and the nonstrange heavy meson:

$$\epsilon = \frac{M_s - M}{2(x-1)}. \tag{5.7}$$

The charmed meson case yields [25]

$$m(D_s^{+*}) - m(D^{+*}) = m(D_s^+) - m(D^+) = 100 \text{ MeV} \tag{5.8}$$

which is accurate up to 2%. Evidently, the heavy quark spin prediction, implicit in (5.6), works quite well. The contribution of (5.6) to the collective Lagrangian is obtained with the substitutions (3.1) and (3.7) followed by a spatial integration:

$$L^{h\rm SB}_{\rm coll} = \frac{2\epsilon}{3}[(2+x) + (1-x)D_{88}(A)]P + \cdots, \tag{5.9}$$

where the projector P maps into the heavy baryon subspace as usual.

We shall take the sum of terms proportional to $D_{88}(A)$, from (5.5) and (5.9), as our perturbation operator, H' (which transforms like λ_8 in the flavor space):

$$L' = -H' = -\tau D_{88}(A), \quad \tau = \tau_{\rm light} + \tau_{\rm heavy}, \tag{5.10}$$

where $\tau_{\rm heavy} = \frac{1}{3}(M_s - M) \approx 0.034$ GeV. $\tau_{\rm light}$ depends on the soliton profile $F(r)$, as can be seen from (5.5), and is sensitive in its details to the parameters of the light effective Lagrangian [e.g., the value of the Skyrme parameter e in (2.9)]. Typical values for $\tau_{\rm light}$ are in the -0.6 ± 0.2 GeV range [10,19,20], which shows that the effect of (5.6) involving the heavy fields on the heavy baryon mass splittings is in fact rather small.

Before going further, we remark that the earliest treatments [9] of the SU(3) Skyrme model for the light baryons gave predictions which did not compare well with experiment. There were several reasons for this. The first is that perturbation theory was carried out only to the first order. Later, Yabu and Ando [10] showed that the collective Hamiltonian with the symmetry breaker (5.10) can be diagonalized exactly, by numerical means, and the results were considerably improved. It was then noted [11] that an adequate approximation to the exact solution can be obtained by using perturbation theory to the second order. The results could be further improved by taking a kind of "strangeness cranking" [20,19] into account which had the effect of increasing the strange moment of inertia β^2. With these improvements, quite reasonable predictions for the many baryon octet and decuplet mass splittings were obtained. Still, it was necessary to accept a rather large overall baryon mass (assuming that the well-known quantities such as F_π take on their experimental values). More recently, it has been noted [26] that $O(N_c^0)$ corrections are likely to lower the overall baryon mass drastically, without modifying the mass splittings. Keeping these lessons in mind, we will carry out perturbation theory to second order and focus our attention on mass splittings rather than overall masses. In fact we will see that the structural relations for the mass splittings require going beyond first-order perturbation theory.

Denoting the matrix elements of (5.10) between the heavy baryon states in (4.15) by H'_{ab}, we compute the mass corrections as

$$\Delta m_a = H'_{aa} - \sum_n \frac{|H'_{na}|^2}{m_n - m_a} + \cdots . \tag{5.11}$$

The possible intermediate states n which can contribute to the sum in (5.11) are determined from the SU(3) decompositions,

$$\begin{aligned} \bar{\mathbf{3}} \otimes \mathbf{8} &= \bar{\mathbf{3}} \oplus \mathbf{6} \oplus \overline{\mathbf{15}}, \\ \mathbf{6} \otimes \mathbf{8} &= \bar{\mathbf{3}} \oplus \mathbf{6} \oplus \overline{\mathbf{15}} \oplus \mathbf{24}. \end{aligned} \tag{5.12}$$

Now, as we see from (4.15), because of the conservation of heavy spin in the effective theory, the heavy baryon states are also labeled by the soliton spin, J_s (which can be called the spin of the light degree of freedom). The $\{\overline{\mathbf{15}}\}$ has $Y = \frac{2}{3}$ states with both $I = 0$ and $I = 1$, so there are two different eigenstates of (4.10), namely $(\overline{\mathbf{15}}, 0)$ and $(\overline{\mathbf{15}}, 1)$. Because H' does not alter the soliton spin, $H'_{\bar{3},6} = 0$ and also H' will have a vanishing matrix element between $\bar{\mathbf{3}}$ and $(\overline{\mathbf{15}}, 1)$. However, H' has a nonvanishing matrix element between $\bar{\mathbf{3}}$ and $(\overline{\mathbf{15}}, 0)$. The required matrix elements can be expressed in terms of the SU(3) isoscalar factors by the formula [27]

$$\langle \{\mu'\}, YII_3; JMJ_s | D_{88}(A) | \{\mu\}, YII_3; JMJ_s \rangle$$

$$= \sqrt{\frac{\dim \mu}{\dim \mu'}} \begin{pmatrix} \mathbf{8} & \mu & \mu' \\ 00 & YI & YI \end{pmatrix} \begin{pmatrix} \mathbf{8} & \mu & \mu' \\ 00 & \frac{2}{3}J_s & \frac{2}{3}J_s \end{pmatrix} . \tag{5.13}$$

The states here correspond to the heavy baryons in (4.15). Note that, due to the heavy spin conservation, the SU(2) Clebsch-Gordan coefficients in (4.15) do not show up in the final result. The needed SU(3) isoscalar factors are given in Ref. [28]. We then find the matrix elements in Table I. Last, the "energy denominators" in (5.11) can be read off from (4.10) to be

$$\begin{aligned} m(\overline{\mathbf{15}}, 0) - m(\bar{\mathbf{3}}) &= \frac{2}{\beta^2}, \\ m(\overline{\mathbf{15}}, 1) - m(\mathbf{6}) &= \frac{1}{\beta^2}, \\ m(\mathbf{24}, 1) - m(\mathbf{6}) &= \frac{15}{6\beta^2}. \end{aligned} \tag{5.14}$$

Putting things together gives the mass corrections for the low lying baryons containing a single heavy quark:

TABLE I. Matrix elements in (5.11).

	Λ_Q	$\Xi_Q(\bar{\mathbf{3}})$	
$H'_{\bar{3},\bar{3}}$	$\frac{\tau}{4}$	$-\frac{\tau}{8}$	
$H'_{\overline{15},\bar{3}}$	$\frac{3\tau}{4\sqrt{5}}$	$\sqrt{\frac{27}{320}}\tau$	
	Σ_Q	$\Xi_Q(\mathbf{6})$	Ω_Q
$H'_{6,6}$	$\frac{\tau}{10}$	$-\frac{\tau}{20}$	$-\frac{\tau}{5}$
$H'_{\overline{15},6}$	$\frac{\tau}{\sqrt{10}}$	$\sqrt{\frac{3}{80}}\tau$	0
$H'_{24,6}$	$\frac{\tau}{5}$	$\frac{\sqrt{6}\tau}{10}$	$\frac{\sqrt{6}\tau}{10}$

$$\begin{aligned} \Delta m(\Lambda_Q) &= \frac{\tau}{4} - \frac{9}{160}\tau^2\beta^2, \\ \Delta m(\Xi_Q(\bar{\mathbf{3}})) &= -\frac{\tau}{8} - \frac{27}{640}\tau^2\beta^2, \\ \Delta m(\Sigma_Q) &= \frac{\tau}{10} - \frac{29}{250}\tau^2\beta^2 + \frac{1}{\alpha^2}, \\ \Delta m(\Xi_Q(\mathbf{6})) &= -\frac{\tau}{20} - \frac{123}{2000}\tau^2\beta^2 + \frac{1}{\alpha^2}, \\ \Delta m(\Omega_Q) &= -\frac{\tau}{5} - \frac{3}{125}\tau^2\beta^2 + \frac{1}{\alpha^2}, \end{aligned} \tag{5.15}$$

where the $\{\mathbf{6}\}$-$\{\bar{\mathbf{3}}\}$ splitting of $1/\alpha^2$ discussed in Sec. IV is also included. There is one additional prediction of the model. Generally, one would expect the states $\Xi_Q(\bar{\mathbf{3}})$ and $\Xi_Q(\mathbf{6})$ to mix under a λ_8 type perturbation. However, because our states also conserve the soliton spin J_s, this mixing cannot occur to any order in H'. To obtain the $\Xi(\mathbf{6})$-$\Xi(\bar{\mathbf{3}})$ mixing one must include additional terms which take account of the "hyperfine interactions."

How well do the predictions (5.15) agree with experiments? Considering that we have worked throughout to the leading order in M, it would be best to test them for the b baryons. However, at present, sufficient data exists only for the c baryons. The $J^P = \frac{1}{2}^+$ states will be taken to have the masses (all in GeV)

$$\begin{aligned} m(\Lambda_c) &= 2.285, \qquad m(\Xi_c(\bar{\mathbf{3}})) = 2.470, \\ m(\Sigma_c) &= 2.453, \qquad m(\Omega_c) = 2.706. \end{aligned} \tag{5.16}$$

The first three masses (averaging over members of the isomultiplets where necessary) were taken from the particle data tables [25], while the Ω_c mass was taken from [29]. Only one Ξ_c state has apparently been confirmed; we have assigned it to the $\{\bar{\mathbf{3}}\}$ rather than $\{\mathbf{6}\}$. The reason is that the observed state lies very far from the average of the Σ_c and the Ω_c masses. Since the $\{\mathbf{6}\}$ is a "triangular" representation of SU(3), the Gell-Mann–Okubo mass formula (which approximately holds in the light SU(3) Skyrme model even though the second-order terms are important [11]) does predict equal spacing of the levels, which would be badly contradicted by the $\{\mathbf{6}\}$ assignment of $\Xi_c(2470)$.

For orientation, let us first examine the predictions of (5.15) when second-order $\tau^2\beta^2$ terms are excluded. We would then have the relation between the $\{\mathbf{6}\}$ and $\{\bar{\mathbf{3}}\}$ splittings:

$$m(\Omega_c) - m(\Sigma_c) = \frac{4}{5}\left[m(\Xi_c(\bar{\mathbf{3}})) - m(\Lambda_c)\right],$$

which reads numerically as $0.253 = \frac{4}{5}(0.185)$. Clearly, first-order perturbation theory is inadequate. At the second order, we have three known mass differences given in terms of three parameters whose range of values are known from the study of the light SU(3) Skyrme model:

$$m(\Xi_c(\bar{\mathbf{3}})) - m(\Lambda_c) = -\frac{3}{8}\tau + \frac{9}{640}\tau^2\beta^2,$$
$$m(\Omega_c) - m(\Sigma_c) = -\frac{3}{10}\tau + \frac{23}{250}\tau^2\beta^2,$$
$$m(\Sigma_c) - m(\Lambda_c) = -\frac{3}{20}\tau - \frac{239}{4000}\tau^2\beta^2 + \frac{1}{\alpha^2}. \quad (5.17)$$

From these three equations we extract the parameters of the collective Hamiltonian:

$$\tau = -0.542 \text{ GeV}, \qquad \alpha^2 = 6.08 \text{ GeV}^{-1},$$
$$\beta^2 = 4.43 \text{ GeV}^{-1}. \quad (5.18)$$

We also predict the mass of the other Ξ_c state, belonging to $\{\mathbf{6}\}$:

$$m(\Xi_c(\mathbf{6})) = m(\Sigma_c) - \frac{3}{20}\tau + \frac{109}{2000}\tau^2\beta^2 = 2.603 \text{ GeV}, \quad (5.19)$$

which is not too far from the equal spacing prediction of 2.580 GeV. It is in fact of great interest to compare the parameters given in (5.18) to those obtained (see Table I of [20]) by using a nearly minimal SU(3) Skyrme model for the light baryons including both Yabu-Ando [10] and "K-cranking" improvements:

$$\tau = -\frac{\gamma}{2} + 0.034 = -0.635 \text{ GeV},$$
$$\alpha^2 = 6.74 \text{ GeV}^{-1}, \qquad \beta^2 = 5.23 \text{ GeV}^{-1}. \quad (5.20)$$

(We have indicated the connection between our parameter τ and the parameter γ given in [20].) Considering the fact that we are working in the leading M limit and the general accuracy of the Skyrme approach, the agreement between (5.18) and (5.20) is quite encouraging. Note especially that the value of β^2 in (5.18) is larger than that which can be gotten without K cranking. We are persuaded to believe that further improvement of the present model will be able to provide another useful window on nonperturbative QCD.

The relatively simple form of the present model should facilitate the investigation of other issues including the relaxation of the pointlike treatment of the heavy mesons, fine-tuning of the SU(3) symmetry breaking and the inclusion of the "hyperfine " interactions. It would be interesting to study the electromagnetic and weak properties of the heavy baryons and to examine the other channels like $J^P = \frac{1}{2}^-$ in more detail.

Note added. After this paper was submitted we learned of recent interesting papers [30] which discuss other approaches to the quantization rules for heavy quark baryons.

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