

New class of Majoron-emitting double- β decays

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Motivated by the excess events that have recently been found near the end points of the double- β decay spectra of several elements, we reexamine models in which double- β decay can proceed through the neutrinoless emission of massless Nambu-Goldstone bosons (Majorons). Noting that models proposed to date for this process must fine-tune either a scalar mass or a VEV to be less than 10 keV, we introduce a new kind of Majoron which avoids this difficulty by carrying lepton number $L = -2$. We analyze in detail the requirements that models of both the conventional and our new type must satisfy if they are to account for the observed excess events. We find (1) the electron sum-energy spectrum can be used to distinguish the two classes of models from one another, (2) the decay rate for the new models depends on different nuclear matrix elements than for ordinary Majorons, and (3) all models require a (pseudo) Dirac neutrino, having a mass of a several hundred MeV, which mixes with ν_e .

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I. INTRODUCTION AND SUMMARY

Only six years ago the first direct observations of double- β decay were made in the laboratory [1]. The major interest of these experiments is in the search for deviations in the shape of the spectrum from that which is predicted for ordinary neutrino-emitting decays. While most of the attention has gone toward searches for neutrinoless decays that would indicate neutrino masses, a third decay mode has also been discussed, in which a massless Goldstone boson, the Majoron,¹ is emitted in lieu of neutrinos. The main purpose of this paper is to determine whether there is any theoretical hope for the last decay to occur at observable levels. Our conclusion is that there may be, but only if the Majoron has rather different properties than have previously been assumed.

Our more immediate motivation for studying this question is the recent observation of a mysterious excess of high-energy electrons in the electron spectrum for the double- β ($\beta\beta$) decay of several elements. This claim was first made in 1987 for the decay ${}^{76}\text{Ge} \rightarrow {}^{76}\text{Se} + 2e^-$ by Avignone *et al.* [2], although the effect was discounted when they, as well as other groups, subsequently excluded a signal having the original strength [3]. The mysteri-

ous events reappeared, however, when the UC Irvine group found excess numbers of electrons near but below the end points for ${}^{100}\text{Mo}$, ${}^{82}\text{Se}$, and ${}^{150}\text{Nd}$, with a statistical significance of 5σ [4]. Such events also persist in the ${}^{76}\text{Ge}$ data [5,6] at approximately one-tenth of the original rate.

Since these are difficult experiments, it is possible that the anomalous events will turn out to be due to systematic error or to a hitherto unsuspected nuclear physics effect.² But they may also be the fingerprint of the new fundamental interaction [8,9], of Majorons with neutrinos [10,11]. If so, these observations are of vital importance since they provide us with a glimpse of physics beyond the standard electroweak theory.

We assume for the sake of argument that any excess events which might be detected in these experiments are due to Majoron emission, denoted by $\beta\beta_M$. Our goal is to explore the implications of $\beta\beta_M$ taken together with the other known constraints on neutrino physics. In so doing, we have found that the candidate models capable of describing Majoron emission from nuclei fall into two broad classes.

In the first class of models for $\beta\beta_M$, which to our knowledge includes everything that has been proposed until recently [9,12], the Majoron is the Nambu-

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¹The term "Majoron" was originally used for the Nambu-Goldstone boson associated with spontaneous breaking of lepton number, since the same lepton number breaking induced a Majorana mass for the neutrinos. We enlarge the meaning of the name in this paper by applying it even if the scalar is massive or if the model in question does not generate Majorana masses.

²After completing this work, we were informed of evidence that the anomalous events reported by the UC Irvine group may be due to resolution problems for the higher-energy electrons [7]. Even excluding events which can be explained in this way, however, there remains a smaller set of residual events whose magnitude is in agreement with observations of the Moscow-Heidelberg experiment.

Goldstone boson associated with the spontaneous breaking of a U(1) lepton number symmetry. The only way to get an observable rate in this context is to have either a scalar mass or vacuum expectation value (VEV) of the order of 10 keV. We refer to these as “ordinary” Majoron models (OMM’s) and denote their associated β decay by $\beta\beta_{\text{OM}}$. We provide here a first comprehensive analysis of which OMM’s can give a large enough rate of $\beta\beta_{\text{OM}}$.

In addition, we have recently proposed [13] a second, qualitatively different, sort of Majoron that does not require such a small scale. Unlike OM’s, this new Majoron carries a classically unbroken lepton number charge and is the Nambu-Goldstone boson for a symmetry distinct from lepton number. We accordingly call such theories “charged” Majoron models (CMM’s) and denote the associated decay by $\beta\beta_{\text{CM}}$.³

Our main results, briefly summarized in Ref. [13], are (1) the two classes of models predict different electron spectra for Majoron-emitting double- β decay, which may therefore be used to identify the type of process that is being observed⁴ and (2) if the Majorons are electroweak singlets and the couplings are renormalizable, then $\beta\beta_M$ is observable only if there is a neutrino which mixes appreciably with the electron neutrino and whose mass is at least $\sim(50-100)$ MeV. CMM’s are further constrained to have the mass of this neutrino also not much *heavier* than a few hundred MeV.

We start, in the following section, with a brief summary of the experimental situation, parametrizing the size of the effect in terms of the strength of a hypothetical Yukawa coupling between the Majoron and electron neutrino. There follows a formulation of the naturalness problem faced by OMM’s. This motivates the introduction and definition of our alternative: the charged Majoron.

Section III proceeds with an analysis of the $\beta\beta$ decay rate for a theory with generic neutrino masses and Yukawa couplings. We derive the shape of the predicted electron sum energy spectrum for all of the models of interest, as well as a general momentum-space parametrization of the relevant nuclear matrix elements as a sum of six form factors. General formulas for the $\beta\beta$ decay rates are presented in terms of these form factors, which we also translate into nuclear matrix elements in the nonrelativistic impulse approximation for the weak interaction currents.

Sections IV and V then apply the general expressions derived in Sec. III to specific models of the ordinary and charged Majoron type. The properties of the particle spectrum required for a sufficiently large $\beta\beta$ rate are determined, and the necessity of a neutrino with mass $M \gtrsim 100$ MeV is explained. We show that, for CMM’s, M must also not be much heavier than this scale if the

observed anomalous $\beta\beta$ rate is to be accounted for. A similar conclusion follows on less robust grounds from naturalness considerations for OMM’s.

Having established what conditions are necessary for producing the observed decay rate, we turn in Sec. VI to a discussion of the constraints these theories must satisfy to avoid conflict with other experiments. Searches for heavy neutrinos in the decays $K \rightarrow e\nu$ and $\pi \rightarrow e\nu$ currently furnish the most restrictive laboratory limits. Nucleosynthesis is given particular attention in this section, since it would rule out the existence of light scalars that are required in both ordinary and charged theories. We show how these bounds can be evaded by somewhat complicating the various models.

II. GENERAL CONSIDERATIONS

We begin by parametrizing the size of the anomalous effect in the data and expounding the theoretical naturalness issue which provides the biggest challenge in accounting for the excess events. For the purposes of model building, the salient features of the anomalous events are that they are above where standard $\beta\beta_{2\nu}$ decays contribute appreciably, yet below the end point for the decays. These facts preclude their interpretation as either $\beta\beta_{2\nu}$ or the neutrinoless $\beta\beta_{0\nu}$.

Another crucial input comes from e^+e^- annihilation at the CERN e^+e^- collider LEP. The precise measurement of the Z -boson width for decay into invisible particles constrains its couplings to putative light scalars. Any model in which the rate for $Z \rightarrow (\text{light scalars})$ is appreciable, for example, that of Gelmini and Roncadelli [9], is ruled out. We therefore focus on scalars that are electroweak singlets [8]. Although it is possible for Majorons to be an admixture of both singlets and fields carrying electroweak charges, they have no advantages over purely singlet Majorons with respect to the β decay anomalies. In fact, these models suffer even more severely from the naturalness problems outlined later in this section, and so we will not consider them further.

A. Size of the effect

There are currently four experiments measuring double- β decay with sufficient precision to potentially see the excess events observed by the UC Irvine group. Two report no excess, with one of these quoting an upper bound [16] that is marginally in conflict with the Irvine result.

To compute the effect in various nuclei, we follow the experimental practice of quantifying the $\beta\beta_M$ rate using a hypothetical direct Yukawa coupling between the electron neutrino and a massless scalar φ . The rate for Majoron-emitting double- β decay follows from the Feynman graph of Fig. 1, evaluated using the effective interaction

$$\mathcal{L}_{\text{phen}} = \frac{i}{2} g_{\text{eff}} \bar{\nu}_e \gamma_5 \nu_e \varphi. \quad (1)$$

Table I lists the coupling strength g_{eff} needed to produce the observed signals in the various double- β decay

³A variation on this theme in which this broken symmetry is gauged has been discussed in Ref. [14].

⁴A similar spectrum can arise for OMM’s if two Majorons are emitted simultaneously, as in the models of Ref. [15].

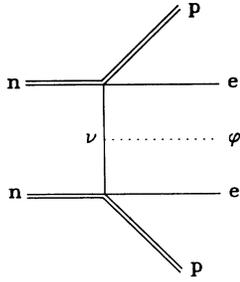


FIG. 1. Feynman graph which gives rise to double- β decay accompanied by the emission of a Majoron. The four-fermion vertices are those of the usual charged-current weak interactions.

experiments. Our analysis used the nuclear matrix elements ($\mathcal{M} = \langle [\sigma_n \sigma_m - (g_V/g_A)^2] h(r) \rangle$) found in Staudt, Muto, and Klapdor-Kleingrothaus [17] to estimate the rates for the two-neutrino and Majoron decay modes. The details of how these matrix elements arise are explained more fully in later sections. Here g_V and g_A are the axial vector and vector couplings of the weak currents to the nucleon and $h(r)$ is a neutrino potential. To quantify the number of excess events, we choose (by eye) a threshold energy E_{th} above which the anomalous events begin and the contribution from ordinary $\beta\beta$ decay is negligible. The data are taken from Ref. [4] for the elements ^{82}Se , ^{100}Mo , and ^{150}Nd and from the published spectrum of ^{76}Ge in Ref. [5]. In all of these cases, the excess events comprise $R = 2-3\%$ of the total number observed. Interestingly, g_{eff} lies in the range $8 \times 10^{-5} - 4 \times 10^{-4}$ for all elements.

Although the coupling apparently needed for ^{100}Mo looks disturbingly large compared to the others, this may be due to uncertainties in the evaluation of the nuclear matrix elements. As described in Ref. [17], the 0ν matrix element for ^{100}Mo in particular suffers from the near collapse of the random phase approximation that was employed.

We also quote here, for comparison, the results of the Heidelberg-Moscow-Gran Sasso group, who claim a 90% C.L. upper bound for ^{76}Ge of $g_{eff} < 1.8 \times 10^{-4}$ in [16]. A similar bound of $g_{eff} < 2.0 \times 10^{-4}$ is reported for Xe decay by the Neuchatel-SIN-Caltech Collaboration [18].

In addition to these laboratory experiments in which

TABLE I. Parameters required for emission of ordinary Majorons in double- β decay. $T_{1/2}^{-1}$ is the inverse half-life of the anomalous events, and R is the ratio of anomalous to the total number of events. E_{th} (MeV) denotes our choice for the threshold value of the sum of the electron energies, above which essentially only excess events appear. g_{eff} is the phenomenological coupling [defined in Eq. (1)] required to explain the excess rate.

Element	$T_{1/2}^{-1}$ (yr $^{-1}$)	R	E_{th}	g_{eff}
^{76}Ge	2×10^{-23}	0.02	1.5	1×10^{-4}
^{82}Se	2×10^{-22}	0.03	2.2	8×10^{-5}
^{100}Mo	3×10^{-21}	0.03	1.9	4×10^{-4}
^{150}Nd	3×10^{-20}	0.02	2.2	2×10^{-4}

the electron energy spectrum is directly measured, there are also several geochemical and radiochemical experiments. In these, the final abundance of daughter products is measured, and so only the total decay rate can be determined. Since the energy spectrum is unknown, it is impossible to directly determine which process is responsible for the decay. For comparison with Table I, we show in Table II the couplings g_{eff} of Eq. (1) that would be allowed assuming the *total* decay rate were due to the Majoron-emitting process. Since these values are comparable with those in Table I, confirmation of the laboratory excess events would likely imply a significant role for Majoron emission in the geophysical observations.

The predictions for Te are of particular interest because of a recent measurement of the ratio of decay rates⁵ $\zeta \equiv \Gamma(^{130}\text{Te})/\Gamma(^{128}\text{Te}) = (2.41 \pm 0.06) \times 10^3$. Taking the ratio of the lifetimes is useful because some of the uncertainties in their experimental determination are expected to cancel. As we will explain in more detail in subsequent sections, the significance of this ratio lies in its strong dependence on the relative phase space for the two decays [20]. It is therefore sensitive to the integrated electron spectrum, which can discriminate between the different possible decay processes.

The allowed coupling for ^{238}U is included here for completeness although we have been informed that the discrepancy between the ^{238}U observations of Turkevich, Economou, and Cowan [21] and the calculations of Staudt, Muto, and Klapdor-Kleingrothaus [17] have now been resolved by improving the theoretical estimates.

B. Naturalness issue

One of the first puzzles that must be addressed by any theory of the anomalous events is how $\beta\beta_M$ could be seen without evidence for the neutrinoless decay $\beta\beta_{0\nu}$. If the emitted scalar is the Nambu-Goldstone boson for spontaneous breaking of lepton number, as in OMM's, then $\beta\beta_{0\nu}$ must exist at some level due to the generation of Majorana neutrino masses. We argue that OMM's answer this question by requiring some dimensional parameter in the scalar potential to be of order 10 keV.

The small scale arises because the same VEV u that breaks lepton number in these models typically also generates a Majorana mass for the electron neutrino whose size is

$$m_{\nu_e \nu_e} \simeq g_{eff} u . \quad (2)$$

The Majorana mass gives rise to $\beta\beta_{0\nu}$ decays which would have been seen if it exceeded the experimental limit

$$m_{\nu_e \nu_e} \lesssim 1 \text{ eV} . \quad (3)$$

Together with the inferred coupling strength $g_{eff} \sim 10^{-4}$,

⁵This result of Bernatowicz *et al.* [19] was used to constrain $\beta\beta_{OM}$ emission by W. Haxton at Neutrino '92, Granada, Spain [20].

this bound implies an upper limit for the lepton-number-breaking VEV of

$$u \lesssim 10 \text{ keV} . \quad (4)$$

One might try to avoid such an artificially small scale simply by having no breaking at all, $u=0$. In this case $\beta\beta_{0\nu}$ is completely forbidden by lepton number conservation. The question then becomes why the emitted scalar in $\beta\beta_M$ should be so light. Since the experiments resolve events within 100 keV of the end point, the scalar must be no heavier than 100 keV. Spontaneous breaking of lepton number naturally satisfies this constraint since the Majoron is an exactly massless Nambu-Goldstone boson, but if lepton number is unbroken, the smallness of the mass would seem to require fine-tuning of parameters in the Lagrangian.

In either case, by kinematics if lepton number is unbroken or from Eq. (2) if it is broken, we are led to a mass scale in the scalar sector of the order of 10–100 keV. Introducing it by hand is at best repugnant. Naturalness demands that the smallness of this new scale, relative to the Higgs VEV, for instance, must be stable under renormalization. Otherwise, we have a new hierarchy problem, which is particularly severe if the light scalars carry electroweak quantum numbers, as in the triplet Majoron model [9]. In that case, loops involving the electroweak gauge bosons generate contributions to the scalar potential that are of order $\sqrt{\alpha/4\pi}M_W \gtrsim 100 \text{ MeV}$.

It has been claimed in Ref. [12] that a Majoron coupling $g_{\text{eff}} \sim 10^{-4}$ is small enough to generally allow such a hierarchy below the weak scale to be stable. But in the OMM that these authors consider, the effective coupling measured in $\beta\beta_M$ decay is $g_{\text{eff}} \sim g\theta^2$, where the mixing angle θ is bounded by neutrino oscillation and decay experiments to be very small. This means that the coupling dominating radiative corrections, g rather than g_{eff} , is *not* small: $g \sim 1$. Therefore the corrections to the small 10 keV scale will tend to be at least four orders of magnitude bigger than the scale itself and fine-tuning must be invoked.

We show in Sec. IV that the scalar hierarchies in these models *can* be made stable under renormalization by taking advantage of the small couplings and masses within the neutrino sector, but only in some corners of parameter space having potentially troublesome phenomenology. For example, the OMM of Sec. IV points to heavy neutrinos in the mass range of several hundred MeV that mix appreciably with ν_e . Even though such models are technically natural, they suffer from the aesthetic problem of requiring mysteriously small dimensionless scalar self-couplings $\xi \lesssim 10^{-14}$.

C. Introducing charged Majoron models

The above comparison suggests a third option in which the light scalar mass and the absence of the neutrinoless decay $\beta\beta_{0\nu}$ can both be naturally understood. To do so, we still assume that the emitted scalar is a Nambu-Goldstone boson in order to ensure its small mass. The absence of $\beta\beta_{0\nu}$ is also guaranteed if the spontaneously broken global symmetry is *not* a lepton number, which

we assume remains conserved. Thus $\beta\beta_{0\nu}$ is completely forbidden because it is a $\Delta L=2$ process. The Majoron-emitting decay is still permitted, however, provided that the massless Nambu-Goldstone boson itself carries lepton charge $L=-2$. We dub such particles “charged Majorons” and show as one of our main results that they lead to qualitatively different features for double- β decay, thus allowing them to be distinguished from ordinary Majorons.

III. GENERAL PROPERTIES OF THE DOUBLE- β DECAY RATE

Next we derive expressions for the rates of the various possible double- β decay processes. Although a number of excellent reviews exist [22,23], detailed formulas are presented here for several reasons. Our first goal is to highlight the differences in predictions between OMM’s and CMM’s, since the CMM’s have not been considered in earlier work. Second, we want to isolate the dependence of our results on the nuclear matrix elements, since these are the most uncertain factors. For generality, we introduce a form-factor parametrization of the decay rate which relies simply on the symmetries of the problem. Expressions for these form factors in the familiar nonrelativistic impulse approximation are subsequently derived.

There are essentially two properties of double- β decay that can be measured or computed: the shape of the spectrum as a function of the energies of the two emitted electrons and the overall normalization of this spectrum, which determines the total decay rate. Only the second of these quantities depends on the size of the nuclear matrix elements.

Consider the differential decay rate for the four processes to which the experiments are potentially sensitive: $\beta\beta_{2\nu}$, $\beta\beta_{0\nu}$, $\beta\beta_{OM}$, and $\beta\beta_{CM}$. The amplitudes for the first two depend on the Feynman graphs of Fig. 2 or 3, respectively. Those for the Majoron-emitting processes require instead the evaluation of Fig. 1 using the appropriate Majoron couplings (more about which later).

It is convenient to write the resulting rates as

$$d\Gamma(\beta\beta) = \frac{(G_F \cos\theta_C)^4}{4\pi^3} |\mathcal{A}(\beta\beta)|^2 d\Omega(\beta\beta) , \quad (5)$$

where G_F is the Fermi constant, θ_C the Cabibbo angle,

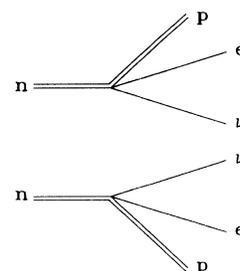


FIG. 2. Feynman graph which gives rise to ordinary two-neutrino double- β decay as occurs in the standard model.

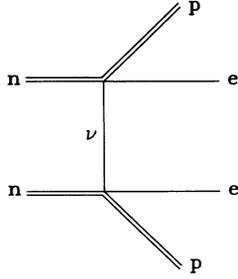


FIG. 3. Feynman graph which gives rise to neutrinoless double- β decay, with no Majoron emission.

$\mathcal{A}(\beta\beta)$ a nuclear matrix element, and $d\Omega(\beta\beta)$ the differential phase space for the particular process. The observables are taken to be the energies of the two outgoing electrons ϵ_k ($k=1,2$). Deriving explicit formulas for $\mathcal{A}(\beta\beta)$ and $d\Omega(\beta\beta)$ is the goal of the remainder of this section.

Equation (5) shows that the decay rate depends on the nuclear matrix elements only as an overall multiplicative constant. The only approximation that must be made to derive this form from the graphs of Figs. 1–3 is the neglect of the dependence of these matrix elements on the final-state lepton energies and momenta. This is a good approximation for the neutrinoless modes in which we are interested because the final leptons (plus Majoron) can carry at most the end point energy $Q \sim 1\text{--}3$ MeV, while the nuclear matrix elements are characterized by the nucleon Fermi momentum $p_F \sim 100$ MeV. Corrections to this approximation thus introduce a relative error of order $Q/p_F \sim$ a few percent.

A. Electron energy spectrum

Consider first the electron energy spectrum $d\Omega(\beta\beta)$ in Eq. (5). This factor is determined solely by the leptonic part of the appropriate Feynman graph. From Figs. 1–3 it is straightforward to find the following results.

The $\beta\beta_{0\nu}$ decay is essentially two body since the nucleus is too heavy to carry away any appreciable kinetic energy. The electron phase space is

$$d\Omega(\beta\beta_{0\nu}) = \frac{1}{64\pi^2} \delta(Q - \epsilon_1 - \epsilon_2) \prod_{k=1}^2 p_k \epsilon_k F(\epsilon_k) d\epsilon_k. \quad (6)$$

Here $p_k = |\mathbf{p}_k|$ is the magnitude of the electron three-momentum and Q is the end point energy for the electron spectrum, determined by the initial and final nuclear energy levels M and M' to be $Q = M - M' - 2m_e$. $F(\epsilon)$ is the Fermi function, normalized to unity in the limit of vanishing nuclear charge.

In contrast, the phase space for the other three processes can be written in a similar form:

$$d\Omega(\beta\beta_i) = \frac{1}{64\pi^2} (Q - \epsilon_1 - \epsilon_2)^{n_i} \prod_{k=1}^2 p_k \epsilon_k F(\epsilon_k) d\epsilon_k. \quad (7)$$

Only the spectral index n_i differs between $\beta\beta_{2\nu}$, $\beta\beta_{0M}$, and $\beta\beta_{CM}$ decays:

$$n_{2\nu} = 5, \quad n_{CM} = 3, \quad n_{0M} = 1. \quad (8)$$

For $\beta\beta_{2\nu}$ and $\beta\beta_{CM}$ these values of n_i simply reflect the phase space for the corresponding process. But for $\beta\beta_{CM}$ there are two extra powers of $(Q - \epsilon_1 - \epsilon_2)$ due to the proportionality of the leptonic matrix element to the Majoron energy, a distinctive and generic feature of CMM's that we elucidate in Sec. III E below. We have assumed that the boson emitted in $\beta\beta_{0M}$ or $\beta\beta_{CM}$ was massless; if it has mass m , one must use $[(Q - \epsilon_1 - \epsilon_2)^2 - m^2]^{1/2}$ in place of $(Q - \epsilon_1 - \epsilon_2)$ in Eq. (7).

The difference between $n_{2\nu} = 5$ and $n_{0M} = 1$ has long been recognized as a way for experimenters to discern a possible admixture of these types of decays; they lead to differently shaped curves for the differential rate $d\Gamma/d\epsilon$ as a function of the sum of the electron energies, $\epsilon = \epsilon_1 + \epsilon_2$. The surprising fact that charged Majorons have an index $n_{CM} = 3$ intermediate between $\beta\beta_{2\nu}$ and $\beta\beta_{0M}$ therefore makes it possible, in principle, to determine whether a distortion in the $\beta\beta_{2\nu}$ spectrum is due to ordinary or charged Majoron emission. Figure 4 shows the shape of the sum-energy spectra for the three possible values of the spectral index.

The spectral shape can also have implications for the total decay rate, which, being an integral over the sum-energy spectrum, depends strongly on n_i . Roughly speaking, each successive power of $(Q - \epsilon)$ in $d\Gamma/d\epsilon$ suppresses the total rate by an additional power of $Q/(100 \text{ MeV})$. Therefore geophysically determined decay rates, such as the ratio $\zeta = \Gamma(^{130}\text{Te})/\Gamma(^{128}\text{Te})$ defined in the previous section, may ultimately prove useful for distinguishing between different models. Once the relative strength of $\beta\beta_{2\nu}$ to $\beta\beta_M$ decays is better determined, a definite prediction for ζ will become possible. If, for example, the decay rate is dominated by the Majoron-emitting process, then $\beta\beta_{0M}$ decay predicts too small a ratio [20]; we find that $\zeta(\beta\beta_{0M}) = 93$. This number includes a factor of $\frac{5}{7}$ due to the ratio of nuclear matrix elements as computed by Ref. 17 and the more significant factor of $(30.4/0.23)$ due to the difference in phase space

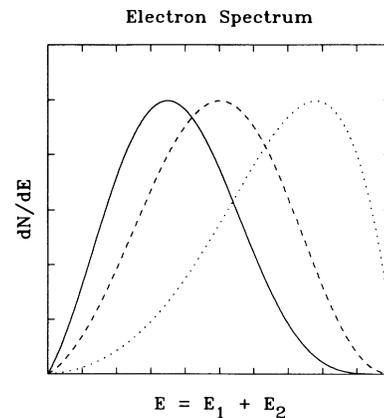


FIG. 4. Number of decay electrons as a function of the sum of the two electrons' energy. The solid curve represents the two-neutrino decay, the dotted curve gives the OMM decay, and the dashed curve gives the CMM decay. All three curves have been arbitrarily assigned the same maximum value for the purposes of comparison.

TABLE II. Parameters consistent with emission of ordinary Majorons in double- β decay. $T_{1/2}^{-1}$ is the total inverse half-life, assumed to consist completely of *anomalous* events. Ω is the total phase space available for each decay measured in units of m_e^7 . As in Table I, g_{eff} is the required coupling, defined in Eq. (1). The changes in Ω relative to Ref. [13] reflect the new measurements of Ref. [19].

Element	$T_{1/2}^{-1}$ (yr $^{-1}$)	Ω	g_{eff}
^{128}Te	1×10^{-25}	0.23	3×10^{-5}
^{130}Te	4×10^{-22}	30	6×10^{-5}
^{238}U	5×10^{-22}	33	2×10^{-4}

for the two decays (see Table II). Because of the small end point energy for ^{128}Te compared to that of ^{130}Te , the same ratio for $\beta\beta_{\text{CM}}$ decay is much larger, $\zeta(\beta\beta_{\text{CM}}) = 770$, and is closer to the experimental value.

B. Nuclear form factors

The other observable constraining models of Majoron-emitting double- β decays is the total rate for any given decay. This requires a knowledge of the matrix element denoted $\mathcal{A}(\beta\beta)$ in Eq. (5), forcing us to deal with the uncertainties in calculating nuclear transition amplitudes. The latter can be written as a sum of six form factors, with which we parametrize the dependence on nuclear physics. The form factors can subsequently be expressed (as we do below) within the context of a given nuclear model. We start by defining the form factors and then use them to specify $\mathcal{A}(\beta\beta)$ for the various decay processes in Secs. III C–III F below.

The nuclear matrix element that appears in the evaluation of Figs. 1–3 is

$$W_{\alpha\beta}(P, P', p) \equiv (2\pi)^3 \left[\frac{EE'}{MM'} \right]^{1/2} \times \int d^4x \langle N' | T^* [J_\alpha(x) J_\beta(0)] | N \rangle e^{ipx}. \quad (9)$$

Here $J_\mu = \bar{u} \gamma_\mu (1 + \gamma_5) d$ is the weak charged current that causes transitions from neutrons to protons and $|N\rangle$ and $|N'\rangle$ represent the initial and final 0^+ nuclei in the decay. E and M are the energy and mass of the initial nucleus N , while E' and M' are the corresponding properties for the final nucleus N' . The prefactor $\sqrt{EE'/MM'}$ is required to ensure that $W_{\alpha\beta}$ transform as a tensor since, as is common in the literature, we use nuclear states which are not covariantly normalized: $\langle \mathbf{p} | \mathbf{p}' \rangle = \delta^3(\mathbf{p} - \mathbf{p}')$. The $(2\pi)^3$ is conventional and is required in order to put our matrix elements into the standard form once the overall center-of-mass motion of the nucleus is separated out.

A priori the tensor $W_{\alpha\beta}$ is a function of the four-momenta P_μ and P'_μ of the initial and final nuclei, as well as four-momentum transfer between the two currents, p_μ . This dependence can be significantly simplified, however. For $\beta\beta_{0\nu}$ and $\beta\beta_M$, p_μ is of the order of the nuclear Fermi momentum $p_F \sim 100$ MeV, whereas the difference $(P - P')_\mu$ is only a few MeV and may therefore be

neglected compared to p_μ . Then the dependence of $W_{\alpha\beta}$ on P_μ and P'_μ may be replaced with the single variable u_μ , the common four-velocity of the initial and final nuclei. For $\beta\beta_{2\nu}$, the momentum transfer p_μ is itself also of order the energy released in the decay, and so in this case $W_{\alpha\beta}$ may be simplified even further by approximating $p_\mu \approx 0$.

It is also straightforward to show that the Bose statistics of the weak currents J_α imply that $W_{\alpha\beta}(u, p) = W_{\beta\alpha}(u, -p)$. Using the aforementioned approximation, the most general possible form for $W_{\alpha\beta}$ is [13]⁶

$$W_{\alpha\beta}(u, p) = w_1 \eta_{\alpha\beta} + w_2 u_\alpha u_\beta + w_3 p_\alpha p_\beta + w_4 (p_\alpha u_\beta + p_\beta u_\alpha) + w_5 (p_\alpha u_\beta - p_\beta u_\alpha) + i w_6 \epsilon_{\alpha\beta\sigma\rho} u^\sigma p^\rho, \quad (10)$$

where the six Lorentz-invariant form factors $w_a = w_a(u \cdot p, p^2)$ are functions of the two independent invariants that can be constructed from p_μ and u_μ . Under the reflection $p \rightarrow -p$, all the w_i are even except for w_4 , which is odd.

By evaluating the leptonic parts of the $\beta\beta$ matrix elements and contracting with $W_{\alpha\beta}$, one can show that, to leading order in lepton energies, only its trace W_α^α enters into the rates for $\beta\beta_{2\nu}$, $\beta\beta_{0\nu}$, and $\beta\beta_{\text{OM}}$. In terms of the Gamow-Teller and Fermi nuclear form factors, which we define in the nuclear rest frame by $w_F = W_{00}$ and $w_{\text{GT}} = \sum_i W_{ii}$, we therefore retrieve the familiar linear combination

$$W_\alpha^\alpha = w_F - w_{\text{GT}}. \quad (11)$$

For $\beta\beta_{2\nu}$ we may to a good approximation neglect p_μ . This permits two important simplifications: (i) We may drop all but the form factors w_1 and w_2 , and (ii) we may approximate these two form factors by constants, $w_i(u \cdot p, p^2) \simeq w_i(0, 0)$. In this limit there is a direct relation between w_1 and w_2 with w_F and w_{GT} , given by $w_1 \simeq \frac{1}{3} w_{\text{GT}}$ and $w_2 \simeq w_F + \frac{1}{3} w_{\text{GT}}$. For $\beta\beta_{0\nu}$ and $\beta\beta_{\text{OM}}$, however, p_μ is large and so w_3 and w_4 may also contribute significantly to w_{GT} and w_F .

The next step is to express $\mathcal{A}(\beta\beta)$, and hence the double- β decay rate $d\Gamma(\beta\beta)$, in terms of the form factors w_a . Before doing so, we pause to present explicit expressions for these form factors, modeling the nuclear decay as the independent decay of its constituent nonrelativistic nucleons. In addition to giving some intuition as to the potential sizes to be expected for these form factors, these expressions allow a connection between our form-factor analysis and the nuclear matrix elements that appear in the literature.

C. Form factors in the nonrelativistic impulse approximation

The common practice in the literature is to provide expressions for the double- β decay rates with the nuclear

⁶The reader should be advised that we define our form factors here differently than in Ref. [13].

matrix elements computed using explicit models of the nucleus. In this section we present expressions for the form factors using such a model. This gives a point of contact between the formalism we present here and the rest of the literature. Besides providing a check on our calculations, the expressions we obtain give some indica-

tion of the size that might be expected for each of the form factors.

Our evaluation starts by inserting a complete set of states, $|X\rangle\langle X|$, into the matrix element of Eq. (9). Working in the nuclear rest frame (where $EE'/MM'=1$) and writing out the time ordering, we have

$$\begin{aligned} W_{\alpha\beta} &= (2\pi)^3 \int d^4x \sum_X e^{ipx} [\langle N'|J_\alpha(x)|X\rangle\langle X|J_\beta(0)|N\rangle\theta(x_0) + \langle N'|J_\beta(0)|X\rangle\langle X|J_\alpha(x)|N\rangle\theta(-x_0)] \\ &= -i(2\pi)^3 \int d^3\mathbf{x} \sum_X e^{-i\mathbf{p}\cdot\mathbf{x}} \left[\frac{\langle N'|J_\alpha(\mathbf{x})|X\rangle\langle X|J_\beta(0)|N\rangle}{p_0 + E_X - M' + i\varepsilon} - \frac{\langle N'|J_\beta(0)|X\rangle\langle X|J_\alpha(\mathbf{x})|N\rangle}{p_0 - E_X + M - i\varepsilon} \right]. \end{aligned} \quad (12)$$

Contact with the literature can be made once we perform the following approximations.

(1) *The closure approximation:* In this approximation a sum over intermediate states of the form $\sum_X F(E_X)|X\rangle\langle X|$ is simplified by replacing the X -dependent prefactor $F(E_X)$ by $F(\bar{E})$ where \bar{E} is the energy averaged over the states that contribute to the matrix element in question. In the present example we may also use the information that $M - M'$ is much less than M and \bar{E} to replace M' with M throughout.

(2) *The nonrelativistic impulse approximation:* The next simplification is to model the nuclear decay in terms of the independent decay of its constituent nucleons, which are taken to be nonrelativistic. We work in the position representation, as is conventional in nuclear physics. In this representation, the weak currents acting on the constituent nucleons takes the form

$$J_0(\mathbf{x}) = \sum_n \delta(\mathbf{x} - \mathbf{r}_n) \tau_n^+ (g_V - g_A C_n) + O(v^2/c^2), \quad (13)$$

$$\mathbf{J}(\mathbf{x}) = \sum_n \delta(\mathbf{x} - \mathbf{r}_n) \tau_n^+ (g_A \vec{\sigma}_n - g_V \mathbf{D}_n) + O(v^2/c^2),$$

where we have included terms up to $O(v/c)$ in the nucleon velocities. The sum here runs over the constituent nucleons, with the position of the n th nucleon denoted by \mathbf{r}_n . The operator $\vec{\sigma}_n$ similarly denotes the Pauli spin matrices acting on the n th nucleon spin, while τ_n^+ is the isospin raising operator for this nucleon. As in Sec. II A, $g_V \simeq 1$ and $g_A \simeq 1.25$ represent the usual vector and axial vector couplings of the nucleon to the weak currents.

The operators C_n and \mathbf{D}_n represent the $O(v/c)$ contributions to the weak currents and are included here since some of the form factors vanish in the limit that $v=0$. They are defined in terms of the initial and final four-momenta of the decaying nucleon, (E_n, \mathbf{P}_n) and (E'_n, \mathbf{P}'_n) , the Pauli spin matrices $\vec{\sigma}_n$, the mass of the pion m_π , and the mass of the proton M_p , by [22]

$$C_n = (\mathbf{P}_n + \mathbf{P}'_n) \cdot \vec{\sigma}_n / 2M_p - (E_n - E'_n)(\mathbf{P}_n - \mathbf{P}'_n) \cdot \vec{\sigma}_n / m_\pi^2, \quad (14)$$

$$\mathbf{D}_n = [(\mathbf{P}_n + \mathbf{P}'_n) + i\mu_\beta(\mathbf{P}_n - \mathbf{P}'_n) \times \vec{\sigma}_n] / 2M_p.$$

Here $\mu_\beta = \frac{1}{2}(g_p - g_n) \simeq 4.7$ is a combination of the proton and neutron spin g factors that originates from the contribution of “weak magnetism.”

The final step is to separate the overall motion of the nucleon center of mass \mathbf{R} out of the nuclear wave function. For a nucleus labeled by its overall momentum \mathbf{P} , as well as its other quantum numbers a , we write

$$\langle \mathbf{r}_1, \dots, \mathbf{r}_A | \mathbf{P}, a \rangle \equiv \frac{e^{i\mathbf{P}\cdot\mathbf{R}}}{(2\pi)^{3/2}} \langle \hat{\mathbf{r}}_1, \dots, \hat{\mathbf{r}}_A | a \rangle, \quad (15)$$

where the “reduced” coordinates $\hat{\mathbf{r}}_n$ are subject to the constraint $\sum_n \hat{\mathbf{r}}_n = 0$.

These approximations give the following results for w_F and w_{GT} :

$$\begin{aligned} w_F &= \frac{2i\mu g_V^2}{p_0^2 - \mu^2 + i\varepsilon} \left\langle N' \left| \sum_{nm} e^{-i\mathbf{p}\cdot\mathbf{r}_{nm}} \tau_n^+ \tau_m^+ \right| N \right\rangle, \\ w_{GT} &= \frac{2i\mu g_A^2}{p_0^2 - \mu^2 + i\varepsilon} \left\langle N' \left| \sum_{nm} e^{-i\mathbf{p}\cdot\mathbf{r}_{nm}} \tau_n^+ \tau_m^+ \vec{\sigma}_n \cdot \vec{\sigma}_m \right| N \right\rangle, \end{aligned} \quad (16)$$

where $\mu \equiv \bar{E} - M$ is the average excitation energy of the intermediate nuclear state and \mathbf{r}_{nm} is the separation in position between the two decaying nucleons. We neglect the $O(v/c)$ corrections to this expression.

The only other combination of form factors which arise for $\beta\beta_{2\nu}$, $\beta\beta_{0\nu}$, $\beta\beta_{OM}$, and $\beta\beta_{CM}$ decays are w_5 and w_6 , and these arise only in $\beta\beta_{CM}$. In the impulse approximation we are using, these expressions vanish at lowest order in v/c , forcing us to go to the next higher order. We find that

$$\begin{aligned} w_5 &= \frac{i\mu\mathbf{p}}{|\mathbf{p}|^2(p_0^2 - \mu^2 + i\varepsilon)} \cdot \langle N' | e^{-i\mathbf{p}\cdot\mathbf{r}_{nm}} [g_A^2(C_n \vec{\sigma}_m - C_m \vec{\sigma}_n) + g_V^2(\mathbf{D}_m - \mathbf{D}_n)] | N \rangle, \\ w_6 &= \frac{\mu g_A g_V \mathbf{p}}{|\mathbf{p}|^2(p_0^2 - \mu^2 + i\varepsilon)} \cdot \langle N' | e^{-i\mathbf{p}\cdot\mathbf{r}_{nm}} [\mathbf{D}_n \times \vec{\sigma}_m + \vec{\sigma}_n \times \mathbf{D}_m] | N \rangle, \end{aligned} \quad (17)$$

in which $\sum_{mn} \tau_m^+ \tau_n^+$ are implicit.

We may now complete the calculation by expressing the various double- β decay amplitudes $\mathcal{A}(\beta\beta_i)$ in terms of the nuclear form factors w_i . For this purpose we must specify the form for the interactions and neutrino masses to be used in evaluating Figs. 1–3. We consider each of the four decay processes separately in the following sections.

D. $\beta\beta_{2\nu}$ rate

For completeness we start with the standard two-neutrino decay $\beta\beta_{2\nu}$. Evaluating the total rate using the leptonic part shown in Fig. 2, and comparing with Eq. (5), we deduce that the nuclear part of the amplitude is approximately

$$\begin{aligned} \mathcal{A}(\beta\beta_{2\nu}) &\approx \frac{2}{\pi\sqrt{15}} [W_\alpha^\alpha]_{p_\mu=0} \\ &\approx \frac{2}{\pi\sqrt{15}} [4w_1(0,0) - w_2(0,0)] \\ &\approx \frac{2}{\pi\sqrt{15}} [w_F(0,0) - w_{GT}(0,0)]. \end{aligned} \quad (18)$$

For simplicity, all final lepton energies and masses have been ignored. Thus only the form factors evaluated at zero argument appear because, for $\beta\beta_{2\nu}$ decay, conservation of momentum and energy determines the nucleon recoil four-momentum p_μ in terms of the energy and momentum of the final-state leptons.

E. $\beta\beta_{0\nu}$ rate

To evaluate Fig. 3 for the $\beta\beta_{0\nu}$ decay rate, one must know the neutrino mass spectrum. We consider a general mass matrix for an arbitrary set of Majorana neutrinos:

$$\mathcal{L}_{\text{mass}} = -\frac{1}{2} \bar{\nu}_i (m_{ij} \gamma_L + m_{ij}^* \gamma_R) \nu_j, \quad (19)$$

where $m_{ij} = m_{ji}$ is the left-handed neutrino mass matrix and γ_L (γ_R) are the usual projectors onto left-handed (right-handed) spinors. The physical masses m_i are given by the square roots of the eigenvalues of the matrix $m^\dagger m$, not necessarily by the eigenvalues of m itself, which may be complex. The electron-flavor row of the associated ‘‘Kobayashi-Maskawa-type’’ matrix for the weak charged-current interactions is denoted by V_{ei} .

With this choice the $\beta\beta_{0\nu}$ decay matrix element becomes

$$\mathcal{A}(\beta\beta_{0M}) = 4\sqrt{2} \sum_{ij} V_{ei} V_{ej} \int \frac{d^4 p}{(2\pi)^4} \left[\frac{W_\alpha^\alpha (a_{ij} m_i m_j + p^2 b_{ij})}{(p^2 - m_i^2 + i\epsilon)(p^2 - m_j^2 + i\epsilon)} \right]. \quad (24)$$

For neutrino masses that are much smaller than p_F , this expression simplifies to the form

$$\mathcal{A}(\beta\beta_{0M}) \simeq -4\sqrt{2} \left[\sum_{ij} V_{ei} V_{ej} b_{ij} \right] \int \frac{d^4 p}{(2\pi)^4} \left[\frac{W_\alpha^\alpha}{p^2 + i\epsilon} \right], \quad (25)$$

$$\mathcal{A}(\beta\beta_{0\nu}) = 8\sqrt{2}\pi \sum_i V_{ei}^2 m_i \int \frac{d^4 p}{(2\pi)^4} \left[\frac{W_\alpha^\alpha}{p^2 - m_i^2 + i\epsilon} \right]. \quad (20)$$

Although the range of integration runs over all possible neutrino four-momenta p_μ , the nuclear form factors w_α act to cut the integrals off at the Fermi momentum and energy p_F and E_F . The contributions from heavy neutrinos thus become suppressed, decoupling as $1/m$, as m starts to exceed this scale.

Using the approximations of Sec. III C for the form factors in W_α^α leads to the familiar Gamow-Teller and Fermi expressions

$$\begin{aligned} &\int \frac{d^4 p}{(2\pi)^4} \left[\frac{W_\alpha^\alpha}{p^2 - m_i^2 + i\epsilon} \right] \\ &= \frac{1}{4\pi} \langle\langle N' | h(\mathbf{r}_{nm}; m_i) (g_V^2 - g_A^2 \vec{\sigma}_n \cdot \vec{\sigma}_m) | \rangle\rangle, \end{aligned} \quad (21)$$

where $h(\mathbf{r}_{nm}; m_i)$ is the neutrino potential function defined by

$$\begin{aligned} h(\mathbf{r}_{nm}; m) &= \frac{1}{2\pi^2} \int d^3 p \frac{\exp(-i\mathbf{p} \cdot \mathbf{r}_{nm})}{\omega(\omega + \mu)}, \\ \omega &= (p^2 + m^2)^{1/2}. \end{aligned} \quad (22)$$

Again, $\sum_{mn} \tau_m^+ \tau_n^+$ is implicit in these expressions.

An important special case is that in which the neutrino masses are negligible compared to the nuclear scale p_F . Then one can use the massless propagator in Eq. (20) and make the replacement $\sum_i V_{ei}^2 m_i = m_{\nu_e \nu_e}$, since the integral is to a good approximation independent of i . Thus the rate vanishes in the absence of a direct Majorana mass for the electron neutrino, as it should.

F. Rate for ordinary Majoron emission

For Majoron-emitting decays, we wish to evaluate Fig. 1, and this requires a knowledge of the neutrino-Majoron coupling. For generality's sake we take the form

$$\mathcal{L}_{\phi\nu\nu} = -\frac{1}{2} \bar{\nu}_i (a_{ij} \gamma_L + b_{ij} \gamma_R) \nu_j \phi^* + \text{c.c.} \quad (23)$$

If the scalar field is real, then (23) still applies, but with the restriction that $b_{ij} = a_{ij}^*$. For example, the phenomenological interaction of Eq. (1) represents the case of a single neutrino with $b_{\nu_e \nu_e}^* = a_{\nu_e \nu_e} = -ig_{\text{eff}}/2$.

Evaluating Fig. 1 using this interaction and neglecting, as before, the final-state lepton energies and momenta leads to the amplitude

which involves the same combination of nuclear matrix elements as appears in Eq. (20) for $\mathcal{A}(\beta\beta_{0\nu})$ with light neutrinos, a result first pointed out in Ref. [11]. The sum over mass eigenstates simply gives the coupling in the flavor basis $b_{\nu_e \nu_e}$, which must vanish in a renormalizable theory if the Majoron comes from an electroweak singlet

field. Thus, in renormalizable singlet Majoron models, it is necessary for at least one neutrino to have a mass $m \gtrsim p_F \sim 50\text{--}100$ MeV. Such a neutrino can generate an effective, nonrenormalizable $b_{\nu_e \nu_e}$ comparing upon being integrated out. Note that this observation rules out the simplest singlet Majoron model [8] (in which the standard model is supplemented by a singlet scalar field without additional intermediate mass neutrino species) as an explanation for the anomalous $\beta\beta$ events.

G. Rate for charged Majoron emission

For charged Majorons the interaction (23) must be further constrained to reflect the fact that φ now carries lepton number. Suppose that the global symmetry for which φ is the Nambu-Goldstone boson acts on the neutrino fields in the following way: $\delta\nu = i(q\gamma_L - q^T\gamma_R)\nu$, with the generator represented by the matrix q . Then, as shown in Appendix A, the Majoron coupling matrix to neutrinos may be written as

$$\begin{aligned} a &= -\frac{i}{f}(q^T m + m q), \\ b &= +\frac{i}{f}(q m^* + m^* q^T), \end{aligned} \quad (26)$$

where f is the decay constant, proportional to the symmetry-breaking scale. Note that Nambu-Goldstone bosons carrying an unbroken charge are associated with non-Hermitian generators (for example, the longitudinal component of the W^\pm bosons), so that $q^T \neq q^*$ in what follows.

Equations (26) are equivalent to the statement that it is possible to redefine the neutrino fields in such a way as to ensure that the neutrino-boson coupling has the derivative form

$$\mathcal{A}(\beta\beta_{\text{CM}}) = 8\sqrt{2} \sum_{ij} V_{ei} V_{ej} b_{ij} \int \frac{d^4 p}{(2\pi)^4} \left[\frac{\mathbf{p}^2(w_5 + w_6)}{(p^2 - m_i^2 + i\epsilon)(p^2 - m_j^2 + i\epsilon)} \right]. \quad (28)$$

Whereas previously it was the trace of $W_{\alpha\beta}$ that arose in the decay rate, here it is the skew-symmetric part, parametrized by the form factors w_5 and w_6 , that appears. In the nuclear rest frame these form factors are given by $w_5 = p_i(W_{0i} - W_{i0})/(2|\mathbf{p}|^2)$ and $w_6 = \epsilon_{ijk} p_i W_{jk}/(2|\mathbf{p}|^2)$.

Using the approximations of Sec. III C for these form factors leads to the formulas of Ref. [13].⁸ We note that

⁷We thank C. Carone for pointing out an error (corrected here) in this equation as it appeared in Ref. [13].

⁸These two matrix elements correspond to what was called A_1 in Ref. [13]. We have corrected the erroneous coefficient of $\frac{7}{9}$ which multiplies A_1^2 there. There is a p -wave contribution to the amplitude which we called A_2 , omitted here because it is expected to be much smaller. The amplitude A_3 is also omitted here because it can be seen to vanish identically.

$$\mathcal{L}_{\varphi\nu\nu} = \frac{i}{2f} \bar{\nu}\gamma^\mu (q\gamma_L - q^T\gamma_R)\nu \partial_\mu \varphi + \text{c.c.} \quad (27)$$

The equivalence of this interaction with the Yukawa formulation is demonstrated explicitly for double- β decay in Appendix B.

The big surprise now comes when Eqs. (26) are substituted into the result (24) for the $\beta\beta_{\text{OM}}$ decay rate. As is shown by brute force using the Yukawa couplings in Appendix C, the result vanishes identically. This is a reflection of the general statement that the amplitude $\mathcal{A}(\beta\beta_{\text{CM}})$ vanishes as the energy of the emitted Majoron goes to zero [recall that we ignored all final-state momenta in deriving (24)], a fact which is most easily seen using the variables for which the neutrino-Majoron coupling takes its derivative form as in Eq. (27).

This result depends crucially on having the emitted Nambu-Goldstone boson carry an unbroken quantum number, in this case lepton number. The same result does not apply to ordinary Majorons, even if they are true Nambu-Goldstone bosons rather than being massive. This statement may be puzzling on reflection, since in this case also one can put the Majoron-neutrino coupling into the derivative form of Eq. (27). The resolution of the paradox is that for OMM's the rest of the amplitude is singular in the limit of vanishing Majoron energy, leaving a nonzero result. For the details of this argument, we refer the reader to Appendix D.

The upshot is that in CMM's one must work to next higher order in the final lepton energies than was done to get Eq. (24). The extra factors of the Majoron momentum can be put into $d\Omega(\beta\beta_{\text{CM}})$, and account for the difference between n_{CM} and n_{OM} in Eq. (8). In the rest frame of the decaying nucleus, the nuclear matrix element turns out to be⁷

the neutrino potential that results from doing the momentum integral in Eq. (28) does not give the usual expression, Eq. (22), because of the different momentum dependence of the form factors (17).

In fact, there are a number of important differences between the charged Majoron amplitude (28) and the corresponding result for ordinary Majorons, Eq. (24).

(1) Equation (28) depends on completely different form factors than the corresponding expression for any other kind of Majoron-emitting double- β decay. In fact, we know of no variety of $\beta\beta$ which depends on w_5 ,⁹ and this matrix element therefore appears not to have been computed by anyone yet. On the other hand, w_6 would ap-

⁹Except for highly subdominant Coulomb or recoil corrections to $\beta\beta_{\text{ov}}$ that would give an $S-P_{1/2}$ final state for the electrons—see Eq. (C.2.12b) of Ref. [22].

pear in $\beta\beta_{0\nu}$, if there were right-handed currents, and it has been calculated in Ref. [24]. An interesting feature of this computation is that the value of w_6 does *not* appear to be suppressed by the nucleon velocity v/c , as would have naively been expected, but is instead rather large.

(2) In the limit where all neutrinos are much lighter than p_F , the flavor dependence of the amplitude becomes proportional to the same combination of couplings as appeared for OMM's: $\sum_{ij} V_{ei} V_{ej} b_{ij} = b_{\nu_e \nu_e}$. This direct coupling to the electron neutrino must vanish in any renormalizable CMM's, because we have assumed the Nambu-Goldstone bosons to all be electroweak singlets. Analogously to OMM's, it follows that in any CMM at least one of the neutrinos must have an appreciable mass: $m_i \gtrsim p_F \sim 50\text{--}100$ MeV.

(3) As may be seen from Eq. (28), if either of the neutrino masses in the graph is large compared to p_F , then the result becomes suppressed by at least *two* powers of the heavy neutrino mass: $1/m^2$. Note that this is a stronger suppression than the $1/m$ behavior that follows from Eq. (24) for OMM's.

IV. ORDINARY MAJORON MODELS

We now construct a viable alternative to the original singlet and triplet Majoron models, since these are not able to yield an observable rate of double- β decay while still satisfying all other experimental constraints. In this section we focus on ordinary Majorons, reminding the reader that here "Majoron" means any light scalar with couplings to neutrinos, regardless of whether it is a Nambu-Goldstone boson. It will be shown that if $\beta\beta_{\text{OM}}$ occurs at the rate suggested by present experiments, one can infer that the masses and mixing angles of the neutrinos are in a range where they are potentially observable by other kinds of experiments.

A. Minimal model

It is easy to invent a minimal ordinary Majoron theory encompassing the low-energy effects of more complicated physics at the electroweak scale. Let φ be a complex electroweak singlet scalar carrying -2 units of lepton number. The lowest-dimensional operator coupling φ to leptons, while respecting the gauge symmetries and global lepton symmetry, is

$$\mathcal{L} = -\frac{\kappa}{2M^2} (\bar{L}H)(H^T L^c)\varphi + \text{c.c.} \quad (29)$$

Here $L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$ and H are the usual left-handed lepton and Higgs doublets. This interaction can be derived from a more fundamental theory, such as the one given in the next section, by integrating out heavy particles of mass M . κ is a dimensionless number that depends on the coupling constants of the underlying theory.

Once the Higgs doublet is replaced by its expectation value, $\langle H \rangle = v = 174$ GeV, Eq. (29) reduces to a coupling of the form of Eq. (23), with strength

$$a_{\nu_e \nu_e} = 0, \quad b_{\nu_e \nu_e} = \frac{\kappa v^2}{M^2}. \quad (30)$$

The imaginary part of φ therefore couples axially as in Eq. (1), with $g_{\text{eff}} = \kappa v^2 / \sqrt{2} M^2$. Because of the requirement $g_{\text{eff}} \simeq 10^{-4}$, it follows that $M/\sqrt{\kappa} \simeq 10$ TeV, consistent with the assumption that the particles of mass M can be integrated out when analyzing double- β decay.

If the light scalar φ should also develop a VEV, then the effective coupling of Eq. (29) also induces a Majorana electron-neutrino mass $m_{\nu_e} = g_{\text{eff}} \langle \varphi \rangle$, which is consistent with the present upper bound only if $\langle \varphi \rangle \lesssim 10$ keV. The simplest assumption is that $\langle \varphi \rangle = 0$. This illustrates the general arguments of Sec. II B in the present example.

The electron spectrum for $\beta\beta_{\text{OM}}$ that would be predicted by this effective coupling can be consistent with the excess events that are seen, provided that at least one scalar mass eigenstate is lighter than 100 keV. This conclusion holds regardless of whether $\langle \varphi \rangle$ is strictly zero or not, since the decay rate found using the scalar couplings of Eq. (29) in the general expression of Eq. (24) is sufficiently large even for massless neutrinos, as would be implied by a vanishing VEV.

The alert reader may wonder how the above model can lead to observable scalar emission even when the neutrinos are massless, since this is in apparent contradiction to the general result for $\beta\beta_{\text{OM}}$ decay that was stated in Sec. III E above. There we claimed that $\beta\beta_{\text{OM}}$ is suppressed if all neutrinos are much lighter than the scale p_F of the nuclear matrix elements. The contradiction is only apparent, however, because the argument of Sec. III E presupposed only dimension-4 (i.e., renormalizable) Yukawa couplings, and so does not include those of Eq. (29). In fact, this effective coupling can be obtained by integrating out the heavy neutrino that is required by the general arguments in a renormalizable theory, as we demonstrate shortly.

An imperative question in this scenario is why the potential for φ should contain such a small scalar mass or vacuum expectation value. But somewhat surprisingly, the hierarchy between this small scale and the weak scale *is* technically natural in the sense of being stable against renormalization, at least within the low-energy effective theory below the heavy scale M . Qualitatively, there are two types of dangerous terms within the scalar potential of the effective theory,

$$\rho^2 \varphi^* \varphi, \quad \xi \varphi^* \varphi H^\dagger H, \quad (31)$$

whose coefficients must be extremely small, $\rho \lesssim 10$ keV and $\xi \lesssim 10^{-14}$, if m_φ is to be kept $\lesssim 10$ keV. If we choose to define the running of these couplings within the decoupling-subtraction renormalization scheme,¹⁰ then both couplings run logarithmically, except for the discontinuous quadratic contributions when a particle is integrated out at its threshold. The initial conditions for the renormalization group (RG) equations in this scheme

¹⁰This scheme consists of the usual modified minimal subtraction ($\overline{\text{MS}}$) scheme, supplemented by the explicit integrating out of any heavy particles as the renormalization point is reduced below the corresponding thresholds [25].

are given by the values of the couplings, e.g., $\rho(M)$ and $\xi(M)$, at the heavy-physics scale $\mu=M$, where the effective theory is matched onto the underlying theory. Provided that these initial values are small, the logarithmic RG evolution through the scales $\mu < M$ in the effective theory keeps them small. The same is true for the nonlogarithmic contributions that arise when the W and Z bosons are integrated out, since these particles couple only very weakly to φ . Furthermore, even though the coupling g_{eff} to light neutrinos is not particularly small, the effects of the operator (29) in loop diagrams are suppressed within the effective theory by the small (or vanishing) ν_e mass.

Although the small scalar mass is stable within the effective theory below the scale $\mu=M$, the difficult issue is whether there exists a model for the physics at $\mu=M$, which can produce the effective coupling, Eq. (29), and still not generate large scalar self-couplings. Such a question can only be addressed within the context of the underlying renormalizable interactions, which are the subject of the next section.

B. Renormalizable model

It is useful to look for a ‘‘fundamental’’ theory whose low-energy limit is the phenomenological model in the previous section. One would like to know whether such a theory exists, whether it has any additional observable consequences, and how much fine-tuning it requires. Naturally, we seek a candidate with the smallest number of new particles. With hindsight, the simplest choice appears to be the addition of a Dirac neutrino, whose mass will turn out to be in the range of $p_F \sim 50\text{--}100$ MeV. Of course, we also must include the singlet scalar that is emitted in $\beta\beta$ decay. The Dirac neutrino can be described as two singlet left-handed neutrinos s_{\pm} , whose lepton number charges are ± 1 , and the singlet scalar field must have lepton charge -2 . The most general renormalizable couplings of the new particles, consistent with the assumed symmetries, are

$$\mathcal{L} = -\lambda \bar{L} H \gamma_R s_- - M \bar{s}_+ \gamma_R s_- - \frac{1}{2} g_+ \bar{s}_+ \gamma_R s_+ + \varphi - \frac{1}{2} g_- \bar{s}_- \gamma_R s_- - \varphi^* + \text{c.c.} \quad (32)$$

$$\frac{\sum V_{ei} V_{ej} b_{ij}}{p^2 + i\varepsilon} \rightarrow g_+ \cos^2 \theta \sin^2 \theta \left[\frac{1}{p^2 + i\varepsilon} - \frac{2}{p^2 - M_h^2 + i\varepsilon} + \frac{p^2}{(p^2 - M_h^2 - i\varepsilon)^2} \right] + g_- \sin^2 \theta \frac{M_h^2}{(p^2 - M_h^2 + i\varepsilon)^2} \quad (35)$$

in Eq. (25), which agrees with $b_{\nu'_e \nu'_e} \cos^2 \theta$ in the limit of large M_h .

C. Naturalness

Having specified the particle content at the intermediate mass scale, we can now return to the question of how natural is the smallness of the scalar masses. We saw in the previous section that if the initial values $\xi(M)$ and $\rho(M)$ are small, then $\xi(\mu)$ and $\rho(\mu)$ remain small as μ

For simplicity, we assume that lepton number is not spontaneously broken: $\langle \varphi \rangle = 0$. The spectrum then contains three massless neutrinos ν'_e , ν_μ , and ν_τ , together with a massive Dirac neutrino ν_h . The relation between the left-handed weak-interaction eigenstates and the left-handed mass eigenstates is

$$\begin{aligned} \nu_e &= \nu'_e \cos \theta + \nu_h \sin \theta, \\ s_- &= \nu_h^c, \\ s_+ &= -\nu'_e \sin \theta + \nu_h \cos \theta, \end{aligned} \quad (33)$$

where $\tan \theta = \lambda v / M$ and ν_h^c is the charge conjugate of ν_h . For ν_h masses M_h in the range of present interest, the universality of leptonic weak interactions requires that $\theta \lesssim 0.1$, very conservatively; thus we have the hierarchy $M / \lambda v \gtrsim 10$ and $M_h = \sqrt{M^2 + \lambda^2 v^2} \simeq M$. If ν_h is very heavy compared to p_F , it may be integrated out, resulting in an effective coupling of the form of Eq. (29), with $\kappa / M^2 = \lambda^2 g_+ / M_h^2$.

There are two light scalars which can be emitted in double- β decay in this model, corresponding to the real and imaginary parts of the complex field φ . The total rate for $\beta\beta_{\text{OM}}$ decay is given by Eq. (25), where the Yukawa couplings of the scalar to the light neutrino are

$$a_{\nu'_e \nu'_e} = 0, \quad b_{\nu'_e \nu'_e} = \sin^2 \theta g_+ . \quad (34)$$

This latter coupling is also equal to g_{eff} , and so the experimentally suggested value of $g_{\text{eff}} = 10^{-4}$ may be obtained by varying the parameters of the renormalizable model in the range $0.01 \lesssim g_+ \lesssim 1$, and $0.01 \lesssim \sin \theta \simeq \lambda v / M \lesssim 0.1$. Setting $\lambda = 1$, we get an upper limit of $M_h \sim 10$ TeV for the heavy neutrino mass. But if $\lambda = 10^{-4}$, for example, then $M_h \sim 100$ MeV, which is the smallest it can be before the amplitude starts to become suppressed by powers of M_h / p_F . In that case we must use the more exact expression for the leptonic part of the matrix element. This is accomplished by making the replacement

runs to lower energies, for which the effective Lagrangian (32) is valid. But this by itself is not enough; in addition, we must establish whether the matching conditions at the heavy neutrino threshold $\mu = M$ are consistent with small values for $\xi(\mu)$ and $\rho(\mu)$ at scales $\mu > M$. We regard the parameters as being naturally small only if no delicate cancellation is needed between their values above $\mu = M$ and the quadratic contribution arising when ν_h is removed from the effective theory.

The contribution to $\rho(M)$ and $\xi(M)$ due to integrating

out the heavy neutrino is easily estimated from the graphs of Fig. 5, in which ν_h is the virtual particle:

$$\begin{aligned}\delta\rho^2(M) &\sim \frac{g_+ + g_-}{16\pi^2} M^2, \\ \delta\xi(M) &\sim \frac{g_-^2 \lambda^2}{16\pi^2}.\end{aligned}\quad (36)$$

Contrary to the prejudice that a $\rho = 10$ keV scalar mass requires extreme fine-tuning, we see that it is possible to keep $\delta\rho \lesssim \rho = 10$ keV and yet have $g_{\text{eff}} = 10^{-4}$ using plausible values of the underlying couplings. For example, $g_+ \simeq 10^{-2}$, $g_- \simeq \lambda \simeq 10^{-4}$, and $\sin\theta \simeq 0.1$ imply $M_h \simeq 100$ MeV, which then implies, from Eqs. (36), $\delta\rho^2(M) \simeq (10 \text{ keV})^2$ and $\delta\xi(M) \simeq 10^{-18}$.

In the above scenario we have ν_e mixing strongly with a neutrino in the mass range of several hundred MeV, a choice with potentially strong phenomenological consequences (see Sec. VI). However, M_h can be pushed to much higher values by letting g_- become smaller, since this has no effect on the effective Majoron coupling, Eq. (34), that is relevant to $\beta\beta$ decay. While it might seem less pleasing aesthetically to have $g_- \ll g_+$, there are no logical grounds for excluding this possibility. In this case the model is safe from any of the constraints to be discussed in Sec. VI that follow from a MeV-scale neutrino.

V. CHARGED MAJORON MODELS

We now repeat the above exercise for charged Majoron models, i.e., to construct the simplest example both as a low-energy effective theory and a renormalizable one. It will be seen that our CMM has some close similarities to the OMM just constructed. In contrast, however, we will find that a heavy neutrino in the 100 MeV mass range is not merely suggested, but required, in order to achieve a high enough $\beta\beta_{\text{CM}}$ rate.

To motivate the specific example, we start with some general considerations. Consider the spontaneous breaking of a global symmetry group G down to a subgroup H . The resulting Nambu-Goldstone bosons can carry quantum numbers with respect to unbroken charges in H only

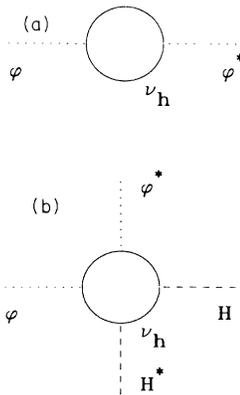


FIG. 5. Two most dangerous Feynman graphs contributing to the light scalar couplings when the heavy neutrino is integrated out.

if the original group G is non-Abelian and the unbroken charges do not all commute with the broken generators of G . In the standard model itself, a global non-Abelian symmetry acting upon the leptons is precluded by their Yukawa couplings to the standard Higgs boson, or equivalently by the charged-lepton masses. We must extend the low-energy particle content in order to devise such a symmetry. In so doing, it is prudent to let the new particles be electroweak singlets lest dangerous couplings arise between the massless Nambu-Goldstone bosons and charged leptons or electroweak bosons.

If the new electroweak singlet neutrinos are integrated out, we obtain a low-energy effective coupling of the charged Majoron to light neutrinos. It is instructive to write down the lowest-dimension such interaction that is possible since this reveals many of the features that are common to all underlying models. For CM's the general result that the $\beta\beta$ amplitude must be proportional to the CM momentum (see Appendix B) suggests using field variables for which the derivative couplings are explicit. Because the CM carries lepton number $L = -2$, the usual interaction of the form $\bar{\nu}_e \gamma_5 \gamma_\mu \nu_e \partial_\mu \varphi$ is not allowed; the current has $L = 0$. Rather, we need an even number of γ matrices:

$$M^{-4} \bar{L} H (a_1 \vec{\partial} \gamma_\mu + a_2 \gamma_\mu \vec{\partial}) H^T L^c \partial^\mu \varphi. \quad (37)$$

These are the lowest-dimension operators that are possible; note that they are suppressed by two more powers of the heavy neutrino mass M than are the OMM effective couplings. It follows that $\mathcal{A}(\beta\beta_{\text{CM}})$ is suppressed by at least the factor $\theta^2 q p / M^2$ relative to the corresponding OMM result, where q and p are, respectively, the average momenta of the Majoron and virtual neutrino.

This estimate gives us constraints on the parameters needed if the underlying heavy-neutrino model is to reproduce the observed anomalous $\beta\beta$ events. Recall the OMM result $g_{\text{eff}}(\text{OMM}) \sim g\theta^2$, where g and θ , respectively, measure the couplings between the heavy neutrino and Majoron and its mixing with ν_e . Roughly, the corresponding CMM result is

$$g_{\text{eff}}(\text{CMM}) \sim g\theta^2 \left[\frac{Q p_F}{M^2} \right]. \quad (38)$$

Using $Q \sim 1$ MeV, $p_F \sim 100$ MeV, and $\theta < 0.1$, we see that $g_{\text{eff}} \sim 10^{-4}$, as required, only if (i) $g \sim 1$ and (ii) $p_F / M \sim 1$.

A. Renormalizable model

The above considerations can be simply illustrated within a renormalizable model. We must first choose the global symmetry group G that will break to give a Majoron carrying a $U(1)$ charge. The standard model itself provides us with an example, since if $SU_L(2) \times U_Y(1)$ were a global rather than a gauged symmetry, the Nambu-Goldstone bosons absorbed by W^+ and W^- would each carry a unit of electric charge. We are therefore led to try an analogous global symmetry $SU_F(2) \times U_L(1)$, which is to be broken down to ordinary lepton (electron) number $U_L(1)$ by scalar fields ϕ_i . These are like the two components of the standard model Higgs

boson in being a doublet under the new $SU_F(2)$ symmetry; however, they are gauge singlets. In further analogy, ϕ_i carries a unit of the $U_{L'}$ charge, just as the standard model Higgs boson carries weak hypercharge. Our field content is completed by adding an $SU_F(2)$ doublet of right-handed gauge-singlet neutrinos N_{\pm} and two sterile $SU_F(2)$ -singlet neutrinos s_{\pm} carrying only the new $U_{L'}$ quantum number, namely, $L' = \pm 1$. The $U_{L'}(1)$ factor is required to permit lepton number to be embedded into the flavor group through the mixing of ν_e and the new singlets. Explicitly, the transformation properties of the new fields under $SU_F(2) \times U_{L'}(1)$ are

$$\begin{aligned} \gamma_R N &\equiv \begin{bmatrix} N_- \\ N_+ \end{bmatrix} \sim (2, 0), \quad \gamma_R s_{\pm} \sim (1, \pm 1), \\ \Phi &\equiv \begin{bmatrix} \phi_{--} \\ \phi_0 \end{bmatrix} \sim (2, -1), \end{aligned} \quad (39)$$

where the subscripts denote the corresponding charges under the ultimately unbroken lepton number $L = -2T_3 + L'$.

We construct the most general renormalizable Lagrangian respecting all the symmetries. The usual standard model particles are taken to be singlets under $SU_F(2)$, and their $U_{L'}(1)$ quantum numbers are chosen to coincide with their lepton (or electron) number. The new mass terms and Yukawa couplings are

$$\begin{aligned} \mathcal{L} = & -\lambda \bar{L} H \gamma_R s_- - M \bar{s}_+ \gamma_R s_- \\ & -g_+ (\bar{N} \gamma_L s_+) \Phi - g_- (\bar{N} \gamma_L s_-) \tilde{\Phi} + \text{c.c.} \end{aligned} \quad (40)$$

[A direct mass term for N is forbidden by the $SU_F(2)$ symmetry.] Here $\tilde{\Phi} = i\tau_2 \Phi^*$ is the conjugate $SU_F(2)$ doublet, with τ_2 the second Pauli matrix acting on flavor indices. The scalar potential is chosen to ensure that Φ gets a VEV, which we assume has been rotated to the form

$$\langle \Phi \rangle = \begin{bmatrix} 0 \\ u \end{bmatrix}. \quad (41)$$

This breaks $SU_F(2) \times U_{L'}(1)$ down to $U_L(1)$, with the unbroken electron-type lepton number symmetry generated by L .

The mass matrix resulting from Eqs. (40) and (41) yields a massless neutrino ν'_e and two heavy Dirac neutrinos ψ_{\pm} , whose masses can be written as

$$M_{\pm} = \frac{1}{2} \{ \tilde{M}^2 \pm \sqrt{\tilde{M}^2 - 4g_+^2 u^2 (\lambda^2 v^2 + g_-^2 u^2)} \}^{1/2}, \quad (42)$$

with

$$\tilde{M}^2 = M^2 + \lambda^2 v^2 + (g_-^2 + g_+^2) u^2.$$

In terms of left-handed neutrino fields, ν'_e , ψ_+ , and ψ_- carry $L = +1$, while ψ'_+ and ψ'_- carry $L = -1$. Only the $L = +1$ fields mix with the electroweak eigenstate ν_e , with a mixing matrix given by

$$\nu_e = \nu'_e c_{\theta} + (\psi_+ s_{\alpha} + \psi_- c_{\alpha}) s_{\theta}, \quad (43)$$

with $s_{\theta} = \sin\theta$, $c_{\theta} = \cos\theta$, etc., denoting mixing angles which are given in terms of model parameters by

$$\begin{aligned} \tan\theta &= \frac{\lambda v}{g_- u}, \\ \tan 2\alpha &= \frac{2M \sqrt{\lambda^2 v^2 + g_-^2 u^2}}{M^2 - \lambda^2 v^2 + (g_+^2 - g_-^2) u^2}. \end{aligned} \quad (44)$$

We are interested in the couplings b_{ij} controlling the $\beta\beta_{\text{CM}}$ decay. Strictly speaking, this is a 5×5 matrix since there are five left-handed neutrinos, but because lepton number conservation only permits the b_{ij} coupling among $L = +1$ neutrinos, the result can be simply expressed in terms of ν'_e and ψ_{\pm} . In the basis (ν'_e, ψ_-, ψ_+) we have

$$b_{ij} = \frac{ig_+}{2} \begin{bmatrix} 0 & s_{\theta} s_{\alpha} & -s_{\theta} c_{\alpha} \\ s_{\theta} s_{\alpha} & -c_{\theta} \sin 2\alpha & c_{\theta} \cos 2\alpha \\ -s_{\theta} c_{\alpha} & c_{\theta} \cos 2\alpha & c_{\theta} \sin 2\alpha \end{bmatrix}. \quad (45)$$

This expression has the property that the $\beta\beta_{\text{CM}}$ amplitude is zero if either of the couplings g_+ or g_- vanishes.

In addition to the charged Majorons, there is also a neutral one φ_3 corresponding to the diagonal generator of $SU_F(2)$. Its Yukawa couplings to neutrinos can be read directly from the Lagrangian once this is expressed in terms of neutrino mass eigenstates. The coupling is

$$\mathcal{L}_{3\nu\nu} = -\frac{c_{ij}}{2} \bar{\nu}_i \gamma_L \nu_j \varphi_3 + \text{c.c.}, \quad (46)$$

where c_{ij} is a 3×2 matrix whose rows are labeled by the $L = +1$ states (ν'_e, ψ_+, ψ_-) and whose columns are labeled by the $L = -1$ states (ψ'_+, ψ'_-) . We find

$$c_{ij} = \frac{1}{\sqrt{2}} \begin{bmatrix} g_- s_{\theta} s_{\beta} & -g_- s_{\theta} c_{\beta} \\ -g_- c_{\theta} c_{\alpha} s_{\beta} - g_+ s_{\alpha} c_{\beta} & g_- c_{\theta} c_{\alpha} c_{\beta} - g_+ s_{\alpha} s_{\beta} \\ -g_- c_{\theta} s_{\alpha} s_{\beta} + g_+ c_{\alpha} c_{\beta} & g_- c_{\theta} s_{\alpha} c_{\beta} + g_+ c_{\alpha} s_{\beta} \end{bmatrix}. \quad (47)$$

β here denotes the mixing angle among the $L = -1$ fields and is given in terms of the model parameters by

$$\tan 2\beta = \frac{2g_+ u M}{M^2 + \lambda^2 v^2 + (g_-^2 - g_+^2) u^2}. \quad (48)$$

We should remark that the mass terms in the ψ_{\pm}, ψ'_{\pm} field variables have the form $\frac{1}{2} M_{\pm} \bar{\psi}_{\pm} \psi'_{\pm} + \text{H.c.}$, leading to matrix propagators

$$\begin{bmatrix} \langle \psi_{\pm} \bar{\psi}_{\pm} \rangle & \langle \psi_{\pm} \bar{\psi}'_{\pm} \rangle \\ \langle \psi'_{\pm} \bar{\psi}_{\pm} \rangle & \langle \psi'_{\pm} \bar{\psi}'_{\pm} \rangle \end{bmatrix} = \frac{i}{p^2 - M_{\pm}^2} \begin{bmatrix} \not{p} & M \\ M & \not{p} \end{bmatrix}. \quad (49)$$

The usual Dirac mass terms $M_{\pm} \bar{\psi}_{\pm} \psi_{\pm}$ and propagators $\langle \psi_{\pm} \bar{\psi}_{\pm} \rangle = i / (\not{p} - M_{\pm})$ can be recovered by making the transformation

$$\begin{bmatrix} \psi_{\pm} \\ \psi'_{\pm} \end{bmatrix} \rightarrow \begin{bmatrix} \gamma_R & \gamma_L \\ \gamma_L & \gamma_R \end{bmatrix} \begin{bmatrix} \psi_{\pm} \\ \psi'_{\pm} \end{bmatrix}, \quad (50)$$

where ψ^c denotes the usual charge conjugate field. In

these more conventional variables, only the chirality projections of ψ_{\pm} have definite lepton numbers.

B. Naturalness

The naturalness issues are much less severe in this model than they are for OMM's. This is because the unbroken lepton number permits the scale u of $SU_F(2)$ breaking to be much higher than ~ 10 keV without including unacceptably large $\beta\beta_{0\nu}$ decay. The first step is to determine how large this scale can be. We saw earlier in this section that the conditions for achieving an acceptable $\beta\beta_{CM}$ rate require heavy neutrino masses $M_{\pm} \sim p_F \sim 100$ MeV, with comparatively large heavy-neutrino-scalar couplings $g_{\pm} \sim 1$. We also found that $gu \sim M$ if the neutrino-scalar coupling is not to be suppressed by additional mixing angles, such as α of our explicit example. Taken together, these conditions imply an $SU_F(2)$ symmetry-breaking scale $u \sim p_F \sim 100$ MeV.

Besides being four orders of magnitude larger than the symmetry-breaking scale that is permitted for OMM's, the CMM scalar sector is also more natural for another reason. In both cases the largest contribution to the scalar potential comes from loops which involve the heavy neutrino, of mass M . This contribution is dangerous for OMM's because this neutrino is itself much heavier than the lepton symmetry-breaking scale. The same is not true of CMM's, however, because for these models both u and M are of the same size. As a result, even though these particles couple with non-negligible strength, $g \sim 1$, the contributions of heavy neutrino loops are

$$\begin{aligned} \delta\rho^2 &\sim \frac{g^2 M^2}{16\pi^2}, \\ \delta\xi &\sim \frac{g^2 \lambda^2}{16\pi^2}. \end{aligned} \quad (51)$$

$\delta\rho^2$ is clearly acceptably small, since all that is required is $\delta\rho^2 \lesssim M^2$. The Majoron-Higgs-boson coupling, on the other hand, must satisfy $\delta\xi \lesssim 10^{-6}$, which is also easily satisfied given the phenomenological constraint that $V_{ei} \sim \lambda v / M \sim 0.1$, which implies $\lambda \sim 10^{-4}$.

VI. OTHER BOUNDS

The couplings of Majorons to matter are seen most directly in the $\beta\beta$ processes which have been the main subject of this work, but there are other constraints which must also be considered. The most serious of these are laboratory searches for the mixing of ν_e with a heavy neutrino, which is one of the generic predictions of the models we have discussed above. In addition, one must take care that Majoron emission from stars or supernovas does not cause them to burn out prematurely or that Majorons make so large a contribution to the energy density of the Universe that they cause too much helium synthesis or cause the Hubble expansion to slow too much. In the following sections we discuss the models proposed above with regard to these issues.

A. Laboratory bounds

It was argued that in order to get an observable rate of $\beta\beta_{OM}$ or $\beta\beta_{CM}$ events, it is necessary to have a neutrino ν_h in the 100 MeV range which mixes with ν_e . Such a neutrino could be inferred from a "spike" it implies for the positron spectra of the decays π^+ , $K^+ \rightarrow e^+ \nu_h$ if it is lighter than the decaying meson. A survey of the Particle Data Group book [26] shows that pion decay experiments limit the mixing angle to values $\theta < 10^{-3}$ for a neutrino with mass $M_h \sim 100$ MeV. On the other hand, the above analysis indicates that the minimum angle needed for observable $\beta\beta_M$ is approximately 0.1 for charged Majorons and 0.01 for ordinary Majorons.

The bound on the mixing angle from the pion decays is easily evaded by taking $M_h > m_{\pi}$ so that the decay is kinematically forbidden. Note that experimental constraints on mixing coming from searches for the decays of ν_h do not apply to our models. These constraints assume the visible decay channel $\nu_h \rightarrow e^+ e^- \nu_e$ due to weak interactions, but in the present situation the weak process is completely subdominant to decays into Majorons, $\nu_h \rightarrow \nu_e \varphi$, which would be undetectable.

One must therefore look to the decays $K \rightarrow e \nu_h$ for limits on the mixing angle when $M_h > 140$ MeV. The Particle Data Group book lists such constraints only up to a mass of 160 MeV, and so one might be misled into thinking that a modest increase in M_h above m_{π} would render large mixing angles safe from being ruled out. Actually, there exist stringent results from KEK [27] that do not appear in Ref. [26]. This experiment also restricts $\theta \lesssim 10^{-3}$ for masses up to $M_h = 350$ MeV. Such a large mass leads to a large suppression of the amplitude for $\beta\beta_{CM}$, although not necessarily for $\beta\beta_{OM}$.

An indirect limit on the coupling of ν_e to heavy neutrinos also comes from tests for universality of the weak interactions of leptons in different families. The most restrictive test comes from the comparison of electron and muon charged current couplings in pion decays [28]. Suppose that ν_e had mixing angle θ to a neutrino with mass $M > m_{\pi}$, so that its weak couplings were suppressed relative to those of ν_{μ} by $\cos\theta$ (assuming for simplicity that ν_{μ} does not mix with anything.) The comparison of theory with experiment shows that the ratio of electron to muon couplings measured in meson decays is

$$\frac{(G_F)_e}{(G_F)_{\mu}} = \cos\theta = 0.9970 \pm 0.0023. \quad (52)$$

Taking the 1σ lower deviation, we get a bound on the mixing angle of

$$\theta < 0.10, \quad (53)$$

which is marginally consistent with having observable $\beta\beta_M$ in our models. Note that a real deviation of (52) from unity would be indirect evidence for the sort of neutrino mixing we need.

A further constraint on the Majoron coupling to neutrinos comes from searches for the decay $\pi \rightarrow e \nu \varphi$ [29]. These yield a comparatively weak limit of $g_{\text{eff}} < 9 \times 10^{-3}$.

B. Cosmology and astrophysics

Because of the weak coupling of the Majoron to matter, one might worry that it could have deleterious cosmological effects, such as contributing too much energy density if it is massive, interfering with the formation of large scale structure due to its decays. Emission of Majorons from stars or supernovas might also shorten the lifetimes of either.

In fact, the effective coupling $g_{\text{eff}} \sim 10^{-4}$ of a massive Majoron to neutrinos is sufficient for avoiding the cosmological problems. The Majoron lifetime due to the decay $\varphi \rightarrow \nu_e \nu_e$ is

$$\tau_\varphi = 16\pi g_{\text{eff}}^{-2} m_\varphi^{-1}, \quad (54)$$

which is 10^{-10} s for $m_\varphi \sim 10$ keV, far less than is required by consideration of the density of the Universe or galaxy formation.

In contrast, the Majoron coupling to ordinary matter such as found in stars is too *weak* to do any harm. Since the thermal background of neutrinos in a star is negligible, Majorons are emitted primarily as bremsstrahlung from electrons. But for the OMM considered above, lepton number conservation prevents a single Majoron from being emitted; rather, they must appear in pairs with zero net lepton number. The effective coupling of two Majorons to electrons is generated by a loop diagram in which a W boson is exchanged (Fig. 6). The resulting effective interaction with electrons can be estimated as

$$\frac{\theta^2 m_e G_F}{16\pi^2} |\varphi|^2 \bar{e} \gamma_5 e, \quad (55)$$

where θ is the mixing angle between ν_e and the heavy neutrino, whose mass does not appear because we have assumed it to be much less than $G_F^{-1/2}$. The amplitude for φ emission proves to be some eight orders of magnitude below the observational limit. We expect similar results for charged Majorons, which must also be emitted in pairs. But in addition we need to check the rate of neutral Majoron (φ_3) emission in the CMM. The coupling of φ_3 to electrons arises at one loop from W - and Z -boson exchange. To make an estimate we have computed only the latter contribution (the two are numerically equal in the singlet Majoron model [8]). It is shown in Fig. 7. Using the couplings of Eq. (47), one can eventually find that the effective interaction has the form

$$\frac{\lambda^2 m_e}{16\sqrt{2}\pi^2 u} f(\theta, \alpha, \beta) [(1+2\epsilon)\ln(1+\epsilon)-2] \varphi \bar{e} \gamma_5 e, \quad (56)$$

$$\epsilon = (M_+^2 - M_-^2) / M_-^2,$$

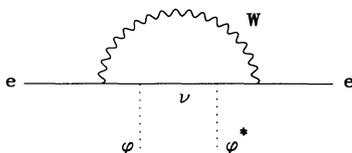


FIG. 6. Feynman graph through which the effective electron-Majoron interaction is induced.

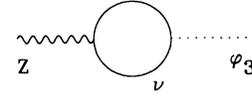


FIG. 7. Feynman graph which mixes the Z boson with the “neutral” Goldstone boson. Once the Z is attached to a fermion line, this induces an effective electron–Goldstone-boson interaction.

where $f(\theta, \alpha, \beta)$ is a function of the three mixing angles of the model (see Sec. V A for their definitions) and which we here conservatively take to be of order 1. Recall that $\lambda\nu \sim \theta_i M_i \sim 10$ MeV from the requirement of getting observable $\beta\beta_{\text{CM}}$. Using the fact that $M_i \sim u$, we get a coefficient of order 10^{-13} . Comparing with the analysis of Ref. [11], one sees that this is somewhat below the limit from red giant lifetimes of 10^{-6} times the electron Yukawa coupling or 3×10^{-12} .

Because of the higher temperatures in supernovas, weak interactions are in equilibrium and there is a thermal population of neutrinos. A coupling of order $g_{\text{eff}} = 10^{-4}$ between neutrinos and Majorons is sufficient for bringing the latter into equilibrium as well [30]. Therefore, in contrast with the situation for stars, in supernovas Majorons are so strongly coupled that they are *trapped* in the core and do not significantly deplete the normal energy flux, in this case due to neutrinos. This will be made more quantitative in the next section, where we examine the equilibration of Majorons when the Universe was at a temperature of 1–100 MeV: conditions similar to those in a supernova.

If Majorons are trapped in supernovas, they can have an adverse effect on the bounce and subsequent explosion [31]. This has only been studied for triplet Majorons, using restrictive assumptions about the energy dependence of the cross section for $\nu\nu \rightarrow \varphi\varphi$, so that no direct conclusions on the models studied here can be drawn.

C. Nucleosynthesis

A difficulty not so easily surmounted is that the Majorons in our models generally change the expansion rate of the Universe enough to have increased the predicted abundance of primordial helium [32]. We will show how this comes about and suggest some possibilities for evading the problem.

Every scalar degree of freedom in equilibrium at MeV temperatures in the early Universe is equivalent to $\frac{4}{7}$ of a neutrino species in its contribution to the energy density and hence the expansion rate. A complex scalar, as in the OMM we have discussed, would thus count as $\frac{8}{7}$, and the CMM would give $\frac{12}{7}$ because it has a total of three Nambu-Goldstone bosons. The current limit on the number of additional neutrino species beyond those of the standard model is 0.4 [33].

In the OMM's, the dominant means for equilibrating massive Majorons is the decay $\varphi \rightarrow \nu\nu$ and its inverse process. The thermally averaged rate is roughly

$$\Gamma \sim 10^{-2} g_{\text{eff}}^2 m_\varphi^2 / T, \quad (57)$$

which comes into equilibrium before a temperature of 1

MeV for all scalar masses greater than a few eV, assuming $g_{\text{eff}} = 10^{-4}$. Since it becomes increasingly unnatural to have scalars lighter than the 10 keV allowed by the $\beta\beta$ experimental anomaly, we expect the decays to be in equilibrium for massive Majorons.

Charged Majorons will suffer fast decays only if they develop a large enough thermal mass. Rather than compute this, we focus on the annihilation process $\nu\bar{\nu} \rightarrow \varphi\varphi^*$. Using the interactions of Eq. (45), we estimate the thermally averaged annihilation rate to be of order

$$\Gamma \sim \frac{1}{\pi^3} \theta^4 T^5 M^{-4} \quad (58)$$

in the limit that $T \ll M$. This is some ten orders of magnitude faster than the expansion rate at temperatures near 1 MeV, assuming masses and mixing angles of $\theta \sim 0.1$ and $M \sim 100$ MeV. In addition, using the neutral Majoron interactions of Eq. (47), we find that the rate for $\nu\nu \rightarrow \varphi\varphi_3$ can be suppressed relative to (58) only by a factor of $(T/M)^2$. Thus all three kinds of Majorons will be in equilibrium at $T \sim 1$ MeV, in contradiction to the nucleosynthesis bound.

We would like to point out two ways in which the nucleosynthesis may proceed as usual, despite the presence of two or three Majoron species. One possibility is that the τ -neutrino mass is close to its experimental upper bound, in the region of 5–30 MeV. If it decays or annihilates into Majorons on time scales faster than 1 s, the time when neutrinos decouple, there will be one less species of neutrinos, making room for two species of Majorons or three with a weak violation of the bound.

A second possibility is that some neutrino decays into ν_e plus φ in such a way as to heat the electron neutrinos relative to the other species. It was shown that this occurs if ν_μ or ν_τ has the desired decay with a lifetime in the range $6 \times 10^{-4} \text{ s} < \tau < 2 \times 10^{-2} \text{ s}$ [34]. The overpopulation of ν_e results in prolonged equilibrium between neutrinos and protons, which compensates for the extra density of the Universe in its effect on helium synthesis. This idea can be generalized to the annihilations of sterile neutrinos in the mass range of a few MeV as well. In fact, it is not necessary that the decaying or annihilating particle go directly into ν_e 's; as long as it produces particles that are in equilibrium with ν_e , after the decoupling of neutrinos from electrons, it will accomplish the same thing.

As an existence proof for these mechanisms, we show how the ordinary Majoron model of Sec. IV B can be generalized to include a heavy τ neutrino. Let there be one additional sterile neutrino s_3 whose lepton number is the same as that of s_- . When we include the other two generations, the straightforward extension of the Lagrangian (32) yields a mass matrix of the form

$$\begin{pmatrix} & m_1 & 0 & m_4 \\ & 0 & m_2 & 0 & m_5 \\ & & m_3 & 0 & m_6 \\ m_1 & m_2 & m_3 & 0 & M_1 & 0 \\ 0 & 0 & 0 & M_1 & 0 & M_2 \\ m_4 & m_5 & m_6 & 0 & M_2 & 0 \end{pmatrix} \quad (59)$$

in the basis $(\nu_e, \nu_\mu, \nu_\tau, s_-, s_+, s_3)$. It is easy to see that the spectrum consists of two massless states which are mostly ν_e and ν_μ , a Dirac neutrino of mass $\sim m_i$ consisting mostly of ν_τ and s_- , and a Dirac neutrino of mass $\sim M_i$ which is mostly s_+ and s_3 . Supposing that the intermediate Dirac mass is of order 10 MeV, for example, we see that the constraints on mixing angles can be satisfied:

$$\theta_{e\tau} = (cm_1 - sm_4)/(cm_3 - sm_6) \lesssim 0.01 \quad (60)$$

from searches for peaks in the $\pi \rightarrow e\nu$ spectrum [28] and

$$\theta_{es} = [(cm_4 + sm_1) - \theta_{e\tau}(cm_6 + sm_3)]/M \sim 0.01 \quad (61)$$

from the requirement that $g_{\text{eff}} \sim 10^{-4}$ in $\beta\beta_{\text{OM}}$. Here $s/c = M_1/M_2$ is the tangent of the mixing angle in the sterile neutrino sector, $M = (M_1^2 + M_2^2)^{1/2}$ is approximately the mass of the heaviest state, and θ_{ec} denotes the mixing angle between ν_e and the mass eigenstates that are mostly ν_τ or the heavy sterile neutrino.

It turns out that the tree level couplings that would cause the decay $\nu_\tau \rightarrow \nu_e \varphi$ vanish; nevertheless, the annihilation process $\nu_\tau \bar{\nu}_\tau \rightarrow \varphi\varphi^*$ goes at a rate comparable to (58), however without the mixing angle suppression. The annihilations are therefore very efficient in depleting the ν_τ population, as long as the heavy neutrino mass scale M is significantly smaller than 100 GeV. Moreover, the resulting Majorons are still in equilibrium with the light neutrinos, and so we have the ν_e -heating mechanism in addition to the elimination of ν_τ . We note that a τ neutrino in this mass range would not necessarily have manifested itself in supernova 1987A through the delayed signal of its decay products. Because it interacts so strongly with Majorons, its neutrinosphere will be farther out in the core where the temperature is lower and the Boltzmann suppression is greater, contrary to the usual case where ν_τ is emitted at a higher temperature. Thus the flux of ν_τ 's, would be greatly reduced relative to the electron neutrinos.

VII. CONCLUSIONS

Motivated by experiments suggestive of Majoron emission in double- β decay, we have proposed two kinds of models that are able to account for this effect without lepton number violation. In the first proposal, the boson is not of the Nambu-Goldstone variety, but rather has a small mass, which can nevertheless be natural in a technical sense discussed above. The second proposal is to let the Majoron be truly massless, but carry lepton number charge. Coincidentally, both of these schemes suggest the existence of heavy isosinglet neutrinos in the mass range of several hundred MeV with significant mixing to the electron neutrino. These heavy neutrinos could manifest themselves in the decays $K \rightarrow \nu e$ or by nonuniversality in the weak interactions of electrons versus other leptons. The models can be consistent with nucleosynthesis constraints if the τ neutrino is in the 10–30 MeV mass range or there exist additional sterile neutrinos with a mass of a few MeV. The anomaly in the double- β decay spectra, if confirmed, would thus be the precursor to several new phenomenon in neutrino physics.

Note added in proof. Since completion of this work, Ref. [35] appeared relating to the effect of a heavy τ neutrino on nucleosynthesis.

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APPENDIX A: YUKAWA FORMULATION OF NAMBU-GOLDSTONE BOSON COUPLINGS

In this appendix we derive the general form for the Yukawa coupling of a Nambu-Goldstone boson: Eq. (26).

To this end consider an arbitrary set of Yukawa couplings between a collection of spin- $\frac{1}{2}$ and spin-0 particles. Such particles may always be cast as Majorana fermions ψ^i and real scalar fields Φ_a . The most general form for their mutual couplings is

$$\mathcal{L}_{\text{Yuk}} = -\frac{1}{2}\bar{\psi}^i \Gamma_{ij}^a \gamma_L \psi^j \Phi_a + \text{c.c.} \quad (\text{A1})$$

Suppose also that this Lagrangian is invariant with respect to the following global symmetry transformations:

$$\delta\psi^i = i\theta^\alpha [(q_\alpha)^i_j \gamma_L - (q_\alpha)^i_j \gamma_R] \psi^j, \quad (\text{A2})$$

$$\delta\Phi_a = i\theta^\alpha (Q_\alpha)_a^b \Phi_b,$$

in which both sets of matrices q_α and Q_α are Hermitian and Q_α must also be imaginary. Invariance of the Yukawa couplings is expressed by the identity

$$(q_\alpha)^T \Gamma^a + \Gamma^a q_\alpha + \Gamma^b (Q_\alpha)_b^a \equiv 0. \quad (\text{A3})$$

Any explicit left-handed fermion mass matrix $(m_0)_{ij}$ must similarly satisfy the relation $q^T m_0 + m_0 q \equiv 0$.

This symmetry is spontaneously broken when the scalar fields acquire their VEV's $v_a = \langle \Phi_a \rangle$, and the resulting Nambu-Goldstone boson directions in scalar-field space, φ_α , are given by the action of the symmetry on v_a :

$$(\delta_{\text{GB}}\Phi)_a \equiv i(Q_\alpha v)_a (F^{-1})^{\alpha\beta} \varphi_\beta. \quad (\text{A4})$$

The real, symmetric normalization matrix F^{-1} is chosen to ensure that the scalar kinetic terms remain properly normalized. That is, $\partial_\mu \Phi_a \partial^\mu \Phi_a = \partial_\mu \varphi_\alpha \partial^\mu \varphi_\alpha + \dots$, provided that

$$(F^{-1})^{\alpha\gamma} [v^T Q_\gamma Q_\lambda v] (F^{-1})^{\lambda\beta} = \delta^{\alpha\beta}. \quad (\text{A5})$$

The Yukawa coupling for φ_α therefore becomes

$$\begin{aligned} \mathcal{L}_{\text{Yuk}} &= -\frac{i}{2} \bar{\psi}^i \Gamma_{ij}^a \gamma_L \psi^j (Q_\alpha v)_a (F^{-1})^{\alpha\beta} \varphi_\beta + \text{c.c.} \\ &= +\frac{i}{2} \bar{\psi}^i (q_\alpha^T \Gamma^a + \Gamma^a q_\alpha) \gamma_L \psi^j v_a (F^{-1})^{\alpha\beta} \varphi_\beta + \text{c.c.}, \end{aligned} \quad (\text{A6})$$

where Eq. (A3) was used in writing the last line. The expression for the right-handed coupling follows simply from taking the complex conjugate of this expression.

Equation (A6) gives the most general form for Nambu-Goldstone boson couplings. It can be recast into the form of Eq. (26) using some additional simplifying features of the models which we consider. Suppose first

that no symmetry-invariant fermion mass terms exist, $m_0 = 0$. Then the Yukawa coupling matrices Γ^a which appear in Eq. (A6) can be traded for a dependence on the fermion mass matrix using

$$m_{ij} = \Gamma_{ij}^a v_a. \quad (\text{A7})$$

Next, we suppose that the Nambu-Goldstone bosons carry an unbroken U(1) charge, as is the case for CMM's. It is then convenient to work with complex combinations of the φ_α 's. If, for example, φ_1 and φ_2 form a multiplet under the unbroken U(1), then the symmetry transformations become diagonal when expressed in terms of $\varphi = (\varphi_1 + i\varphi_2)/\sqrt{2}$. The same steps as before once more lead to Eq. (A6), with the proviso that the corresponding broken charge $q = (q_1 - iq_2)/\sqrt{2}$ need no longer be Hermitian.

The simplest case is if there is only one Nambu-Goldstone boson with a nonzero charge, as in the models we consider. Then the normalization matrix F^{-1} cannot mix φ with any of the uncharged Nambu-Goldstone bosons and must be proportional to the unit matrix in the charged scalar sector. Denoting the proportionality constant by $1/f = (F^{-1})^{11} = (F^{-1})^{22}$, we obtain Eqs. (26), as required.

APPENDIX B: EQUIVALENCE OF DERIVATIVE AND YUKAWA FORMULATIONS

A famous property of Nambu-Goldstone bosons is that they only couple derivatively. Here we make this property explicit for the couplings of the Nambu-Goldstone bosons that are considered in Eq. (26) by showing the equivalence of these two formulations for the double- β decay rate.

Nambu-Goldstone bosons can only couple derivatively, because if these fields are taken to be constants, they completely drop out of the Lagrangian density. This is because the Nambu-Goldstone directions in field space are defined by performing a field-dependent symmetry transformations on the vacuum, as in Eq. (A4). For constant fields these transformations are really symmetries and so produce no effect at all in the Lagrangian. φ_α only appears to the extent that it varies in spacetime, and so it must couple only through its derivatives.

To see this in the present case, consider the following field-dependent redefinition of the fermion fields:

$$\delta\psi \equiv -i(q_\alpha \gamma_L - q_\alpha^T \gamma_R) \psi (F^{-1})^{\alpha\beta} \varphi_\beta. \quad (\text{B1})$$

The fermion mass term changes by

$$\delta\mathcal{L}_{\text{mass}} = -i\bar{\psi} (q_\alpha^T m + m q_\alpha) \gamma_L \psi (F^{-1})^{\alpha\beta} \varphi_\beta + \text{c.c.}, \quad (\text{B2})$$

where $m = \Gamma^a v_a$. Note that this is exactly what is required to cancel the Yukawa coupling of Eq. (A6). A similar cancellation occurs for all of the nonderivative interactions of φ_α . It is important to note in this regard that if the broken symmetry should transform other particles such as the electron, in addition to neutrinos, then these other particles must also participate in the field redefinition, Eq. (B1), in order to remove all nonderivative φ dependence.

The φ_α dependence is not completely eliminated, how-

ever, since the fermion kinetic terms are not invariant under a spacetime-dependent transformation such as that of Eq. (B1). It is a simple exercise to show that, under the assumptions leading to Eq. (26), the variation of the kinetic term is given by Eq. (27). The latter has been expressed in a way that holds even if the generators q are not Hermitian, as is appropriate for charged Nambu-Goldstone bosons. Since these two forms for the Nambu-Goldstone boson interaction are related by a field redefinition, they must give equivalent scattering amplitudes.

To concretely verify the equivalence of these two expressions for the Nambu-Goldstone boson couplings, we compute the double- β decay rate using both Eqs. (26) and (27). Although the graph in which the Nambu-Goldstone boson is emitted by the neutrino line, as in Fig. 1, is *not* equivalent by itself, we will show that the result becomes equivalent once Fig. 1 is added to the remaining graphs of Fig. 8. We denote the leptonic part of the amplitude computed from Figs. 1 and 8 using derivative couplings by $M_5^{\mu\nu}$ and the same amplitude using Yukawa couplings by $M_y^{\mu\nu}$.

First, consider the evaluation of these graphs using derivative couplings. For generality, we work with an arbitrary set of Majorana fermions and real scalar fields, and assume the following form for scalar and charged-current interactions which appear in the Feynman rules for Figs. 1 and 8:

$$\begin{aligned} \mathcal{L}_{\psi\psi\varphi} &= \bar{\psi}^i \gamma^\mu (\mathcal{V}_{ij} + \mathcal{A}_{ij} \gamma_5) \psi^j \partial_\mu \varphi, \\ \mathcal{L}_{\psi\psi W} &= \bar{\psi}^i \gamma^\mu (V_{ij} + A_{ij} \gamma_5) \psi^j W_\mu + \text{c. c.}, \end{aligned} \quad (\text{B3})$$

where, on general grounds, A and \mathcal{A} are both symmetric matrices, while V and \mathcal{V} are antisymmetric. For the charged-current weak interactions, we take $A = V \propto T_-$,

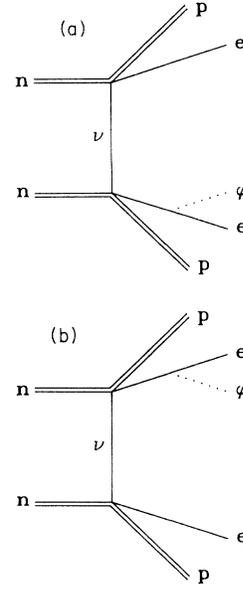


FIG. 8. Remaining Feynman graphs which contribute to double- β decay accompanied by Majoron emission. These graphs only arise if direct electron-Majoron couplings exist, as is the case for OMM's in the variables for which the Majoron is derivatively coupled.

where T_- is the $SU_L(2)$ lowering operator. For the Nambu-Goldstone boson associated with the charge q , we have $\mathcal{A} = \frac{1}{2}(q + q^T)$ and $\mathcal{V} = \frac{1}{2}(q - q^T)$. The corresponding Yukawa couplings will be denoted by the matrix $\Lambda \propto a \gamma_L + b \gamma_R$, with a and b given by Eq. (26).

Up to a common overall normalization, the leptonic part of the integrands for the three graphs become (in an obvious matrix notation)

$$\begin{aligned} M_5^{\mu\nu}(1) &= \bar{u}(p_1) \gamma^\mu (V + iA \gamma_5) S(p_1 - k_1) \not{q} (\mathcal{V} + i\mathcal{A} \gamma_5) S(k_2 - p_2) \gamma^\nu (V + iA \gamma_5) u^c(p_2), \\ M_5^{\mu\nu}(8a) &= \bar{u}(p_1) \not{q} (\mathcal{V} + i\mathcal{A} \gamma_5) S(p_1 + q) \gamma^\mu (V + iA \gamma_5) S(k_2 - p_2) \gamma^\nu (V^* + iA^* \gamma_5) u^c(p_2), \\ M_5^{\mu\nu}(8b) &= \bar{u}(p_1) \gamma^\mu (V + iA \gamma_5) S(p_1 - k_1) \gamma^\nu (V^* + iA^* \gamma_5) S(-p_2 - q) \not{q} (\mathcal{V} + i\mathcal{A} \gamma_5) u^c(p_2), \end{aligned} \quad (\text{B4})$$

where $S(p) = [i\not{p} + m \gamma_L + m^* \gamma_R]^{-1}$ is the fermion propagator, thought of as a matrix in Dirac and flavor space, while $u(u^c)$ is the (conjugate) electron spinor.

These expressions can be related to the Yukawa expressions by applying the easily proven identities

$$\begin{aligned} S(p_1 - k_1) i \not{q} (\mathcal{V} + i\mathcal{A} \gamma_5) S(k_2 - p_2) &= S(p_1 - k_1) \Lambda (\mathcal{V} + i\mathcal{A} \gamma_5) S(k_2 - p_2) \\ &\quad + S(p_1 - k_1) (\mathcal{V} + i\mathcal{A} \gamma_5) - (\mathcal{V} + i\mathcal{A} \gamma_5) S(k_2 - p_2), \\ \bar{u}(p_1) i \not{q} (\mathcal{V} + i\mathcal{A} \gamma_5) S(p_1 + q) &= \bar{u}(p_1) [(\mathcal{V} - i\mathcal{A} \gamma_5) + \Lambda S(p_1 + q)], \\ S(-p_2 - q) i \not{q} (\mathcal{V} + i\mathcal{A} \gamma_5) u^c(p_2) &= [- (\mathcal{V} + i\mathcal{A} \gamma_5) + S(-p_2 - q) \Lambda] u^c(p_2). \end{aligned} \quad (\text{B5})$$

The last two of these identities rely on using the Dirac equation for the initial and final spinors $u(p_1)$ and $u^c(p_2)$.

Using these identities in Eqs. (B4) relates the derivative-coupling and Yukawa-coupling results for each graph:

$$\begin{aligned} M_5^{\mu\nu}(1) &= M_y^{\mu\nu}(1) + \bar{u}(p_1) \gamma^\mu (V + iA \gamma_5) S(p_1 - k_1) (\mathcal{V} + i\mathcal{A} \gamma_5) \gamma^\nu (V^* + iA^* \gamma_5) u^c(p_2) \\ &\quad - \bar{u}(p_1) \gamma^\mu (V + iA \gamma_5) (\mathcal{V} + i\mathcal{A} \gamma_5) S(k_2 - p_2) \gamma^\nu (V^* + iA^* \gamma_5) u^c(p_2), \\ M_5^{\mu\nu}(8a) &= M_y^{\mu\nu}(8a) + \bar{u}(p_1) (\mathcal{V} - i\mathcal{A} \gamma_5) \gamma^\mu (V + iA \gamma_5) S(k_2 - p_2) \gamma^\nu (V^* + iA^* \gamma_5) u^c(p_2), \\ M_5^{\mu\nu}(8b) &= M_y^{\mu\nu}(8b) + \bar{u}(p_1) \gamma^\mu (V + iA \gamma_5) S(p_1 - k_1) \gamma^\nu (V^* + iA^* \gamma_5) (-\mathcal{V} + i\mathcal{A} \gamma_5) u^c(p_2). \end{aligned} \quad (\text{B6})$$

We see that although the result using the two formulations of the Nambu-Goldstone boson couplings do *not* agree graph by graph, their sum is the same provided that the scalar- and charged-current coupling matrices satisfy the conditions

$$[\mathcal{V}, V^*] + [\mathcal{A}, A^*] = [V^*, \mathcal{A}] + [\mathcal{V}, A^*] = [\mathcal{V}, V] + [A, \mathcal{A}] = [\mathcal{V}, A] + [\mathcal{A}, V] = 0. \quad (\text{B7})$$

These are trivially satisfied if the charged-current generators commute with the charge that is associated with the Nambu-Goldstone boson, as is required by the invariance of the charged-current interactions under the spontaneously broken global symmetry. The equivalence of the two formulations for double- β decay is thus established.

APPENDIX C: CHARGED MAJORON ELECTRON SPECTRUM

It is argued in the text that the vanishing of the double- β decay matrix element $\mathcal{A}(\beta\beta_{\text{CM}})$ as the Majoron momentum goes to zero is a key feature of charged Majoron models. Here we demonstrate this property in some detail.

The vanishing of the amplitude is straightforward when the Majoron couplings are expressed in derivative form, as in Eq. (27). In this case the conservation of elec-

tric charge and lepton number precludes any derivative coupling between the electron and charged Majoron, in the absence of exotic, electrically charged fermions. As a result, neither of the graphs of Fig. 8 contribute to the CMM double- β decay rate. The remaining graph, Fig. 1, manifestly vanishes for zero Majoron momentum because of the derivative coupling.

It is more complicated to see this result in the Yukawa-coupling language. Lepton number conservation forbids a direct coupling between the charged Majoron and the electron, and so the only graph to be considered is again that of Fig. 1. As might be expected from Appendix B, however, the result for this graph need not vanish for zero Majoron momenta until the contributions from all of the relevant intermediate neutrinos have been summed.

At zero Majoron energy, the $\beta\beta_M$ decay amplitude is given by Eq. (24), whose integrand is proportional to the $v_e v_e$ element of the following matrix in flavor space:

$$\begin{aligned} \text{integrand} &\propto \left[\frac{1}{p^2 - m^* m} (m^* a m^* + p^2 b) \frac{1}{p^2 - m m^*} \right]_{v_e v_e} \\ &= \frac{b_{v_e v_e}}{p^2} - \sum_{n=0}^{\infty} \frac{1}{(p^2)^{n+2}} \left[(m^* m)^{n+1} b + \sum_{k=0}^n (m^* m)^k (b m m^* - m^* a m^*) (m m^*)^{n-k} \right]_{v_e v_e}. \end{aligned} \quad (\text{C1})$$

As in previous expressions, $m = m^T$ denotes the complex left-handed neutrino mass matrix, while a and b are the Yukawa coupling matrices of Eq. (23).

The last expression simplifies drastically once Eq. (26) is used, which contains the information that the Majoron is a Nambu-Goldstone boson. After a pairwise cancellation of all but one of the terms in the sum over k , we find that

$$\text{integrand} \propto \left[\frac{q m^* + m^* q^T}{p^2} - \sum_{n=0}^{\infty} \frac{1}{(p^2)^{n+2}} [q (m^* m)^{n+1} m^* + (m^* m)^{n+1} m^* q^T] \right]_{v_e v_e}. \quad (\text{C2})$$

The significance of this final result lies in the fact that each term in it is proportional to a v_e matrix element $q_{v_e j}$ of the Nambu-Goldstone boson charge. The final point to be established is that, for CMM's, all such matrix elements are zero. We are therefore forced to work to next order in the Majoron momentum, Eq. (28), in order to get a nonvanishing contribution.

In order to see why $q_{v_e j}$ must vanish in CMM's, consider the symmetry transformations in the basis of weak-interaction eigenstates. Then the invariance of the gauge interactions under the global symmetry implies that q can only transform the entire doublet $(v_e)_L$ into other doublets having the same hypercharge. But since the Nambu-Goldstone boson charge q has embedded in it two units of the unbroken lepton number, such a transformation cannot be made without introducing exotic isodoublet fermions.

APPENDIX D: ORDINARY MAJORON ELECTRON SPECTRUM

Here we wish to show that the OMM double- β decay amplitude, unlike that for CMM's, is nonvanishing even at zero Majoron momentum. This is particularly easy to see using the Yukawa form for the Nambu-Goldstone boson couplings, which can be directly read off from the Lagrangian of a given model. In this form the Majoron typically couples only to neutrinos and not to electrons. Thus only the graph of Fig. 1 contributes. In contrast with CMM's, it is possible to have $q_{v_e j} \neq 0$ in OMM's (see Appendix C), and so the decay rate at zero Majoron momentum need not vanish.

The puzzle is to understand this result when the amplitude is expressed in terms of the derivative couplings, since in this formulation all of the graphs of Figs. 1 and 8 are explicitly proportional to the Majoron momentum k .

The resolution turns out to come from the contributions of Fig. 8. For these graphs, in which the Majoron is emitted from the electron lines, the internal electron goes on shell in the limit as $k \rightarrow 0$, causing a singularity in the propagator. The coefficient of this singularity is proportional to the *vector* part of the electron-Majoron cou-

pling. (The same singularity leads to the familiar infrared divergence of the analogous photon bremsstrahlung graphs in quantum electrodynamics.) Consequently, the electron propagator behaves as $1/k$ for small k , which cancels the explicit k dependence due to the Majoron's derivative coupling.

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