Intermittency exponents in *pp* collisions at 400 GeV/*c*

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The pseudorapidity distribution for charged particles produced in *pp* collisions at 400 GeV/*c* has been measured by using the LEBC films offered by the CERN NA27 Collaboration. The scaled factorial moments have been calculated. It is obtained that the intermittency exponents increase with increasing order q of the moments and decrease with increasing average multiplicities. The anomalous fractal dimensions dq increase with increasing order q of the moments. It shows that multiparticle production in hadron-hadron collisions possesses a self-similar cascade property.

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I. INTRODUCTION

A growing interest in studying intermittency and multifractality in multiparticle production has been seen in recent years. The most interesting advancement is the method of scaled factorial moments which was introduced by Białas and Peschanski [1] to search for nonstatistical fluctuation in the rapidity (pseudorapidity) distributions of charged particles produced in high-energy interactions. It was suggested that there is a power-law dependence of the scaled factorial moment on the rapidity bin width when such a nonstatistical fluctuation exists.

The intermittency has been observed in leptonic [2], hadronic [3], and nuclear [4] collisions. What causes intermittency and correlations may lead to the fundamental question about the production mechanism of hadrons, which is highly desirable. Many origins for intermittency have been discussed, such as the self-similar random cascade, jet models with a self-similar branching structure, the second-order phase transition from a quark-gluon plasma to normal hadronic matter, and short-range correlations. But none of them can explain all of the experimental results. On the other hand, the experimental data are not enough to identify the different models. So more experimental data and theoretical analyses are needed. In this paper, we present the analysis of our data of *pp* collisions at 400 GeV/*c* which are obtained by measuring the Lexan Bubble Chamber (LEBC) films, in terms of the factorial moments Fq, to extract out the intermittency behavior in multiparticle production.

II. THE EXPERIMENT

The space-geometry reconstruction was performed for the 3364 events produced in *pp* collisions at 400 GeV/*c* by using the LEBC films offered by the CERN NA27 Collaboration. The space-geometrical acceptance was 4π for the high-resolution LEBC. The diameter of the bubbles was 17 μm. The density of the bubbles was 80/cm. The length of the fiducial volume in the beam direction was 12 cm. Without a magnetic field, the charged-particle tracks were linear and the pictures were clear, which helped to measure the angular distributions of the reactionary products accurately.

The NA27 Collaboration offered the space-coordinate reconstruction program for the interaction vertex of the events. It is used to reconstruct the space direction for each track. The event vertex is considered a point on each track. For the outgoing tracks, the coordinate of the other point should be as far from the vertex as possible. In order to determine the direction of the incident track, we measure several points on the incident track. When the incident track is short, we measure the direction of the parallel incident uninteracted track as the direction of the incident track of the event, thereby increasing the accuracy of the angular measurement.

After the space geometries of the events have been reconstructed, the charged particle pseudorapidity η_L in the laboratory system is calculated according to the formula

$$\eta_L = -\ln\tan\frac{\theta_L}{2}\ , \tag{1}$$

where θ_L is the angle between the outgoing and incident particles in the laboratory system. The c.m.s. pseudorapidity η_c can be calculated with the following formula with good approximation [5]:

$$\eta_c = \eta_L - \ln(\gamma_c + \sqrt{1+\gamma_c^2})\ , \tag{2}$$

where $\gamma_c = 1/\sqrt{1-\beta_c^2}$ and β_c is the velocity of the center of mass.

A total of 3364 events with charge multiplicities $N_{ch} \geq 4$ have been measured. The pseudorapidity distribution of these events in the c.m. frame is shown in Fig. 1. The accuracy of the pseudorapidity in the region of interest $(-2 < \eta_c < 2)$ was about 0.1 units.

III. THE INTERMITTENCY

The horizontally averaged factorial moment Fq of order q is given by

$$\langle Fq \rangle = \frac{1}{\langle \bar{n}_m \rangle^q} \left\langle \frac{1}{M} \sum_{m=1}^{M} n_m(n_m-1)\cdots(n_m-q+1) \right\rangle\ , \tag{3}$$

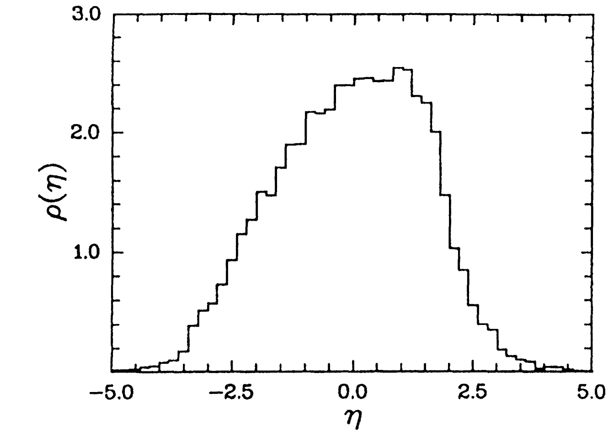

FIG. 1. Pseudorapidity distribution in the center-of-mass frame.

where

$$\langle \bar{n}_m \rangle = \left\langle \frac{1}{M} \sum_{m=1}^{M} n_m \right\rangle . \tag{4}$$

The pseudorapidity window $\Delta\eta$ is divided into M equal bins, each having a width $\delta\eta = \Delta\eta/M$, where n_m is the number of charged particles of a single event in the mth bin and the angular brackets imply an average over all events.

As suggested by Białas and Peschanski [1], if the fluctuations are purely statistical, a saturation of $\langle Fq \rangle$ with decreasing bin width $\delta\eta$ is expected. However, for nonstatistical self-similar fluctuations, the factorial moment of order q is given by

$$\langle Fq \rangle \sim (\Delta\eta/\delta\eta)^{\Phi_q} . \tag{5}$$

The exponent Φ_q is the slope characterizing a linear rise of $\ln\langle Fq \rangle$ with $-\ln(\delta\eta)$. The strength of the intermittency is characterized by the value of Φ_q which increases with the order q.

The factorial moments as described above need to be corrected for the nonuniform shape of the pseudorapidity distribution by dividing by the factor

$$Rq = \frac{1}{M} \sum_{m=1}^{M} M^q \langle n_m \rangle^q / \langle N \rangle^q , \tag{6}$$

and

$$\langle n_m \rangle = \frac{1}{N_{\text{evts}}} \sum_{i=1}^{N_{\text{evts}}} n_{m,i} \tag{7}$$

where N_{evts} is the number of events in the sample, $n_{m,i}$ is the number of particles in the mth bin for the ith event, and $\langle N \rangle$ is the average multiplicity in pseudorapidity window $\Delta\eta$. The qth-order scaled factorial moment is, then, corrected as

$$\langle Fq \rangle^{\text{corr}} = \frac{\langle Fq \rangle}{Rq} . \tag{8}$$

This correction factor was first introduced by Fialkowski, Wosiek, and Wosiek [6] in order to remove the contribution coming from the trivial fluctuations due to the nonuniform shape of the pseudorapidity distribution. In order to avoid introducing the correction factor, we adopted the vertically averaged factorial moment which is defined as

$$\langle Fq \rangle = \frac{1}{M} \sum_{m=1}^{M} \frac{1}{N_{\text{evts}}} \times \sum_{i=1}^{N_{\text{evts}}} \frac{n_{m,i}(n_{m,i}-1)\cdots(n_{m,i}-q+1)}{\langle n_m \rangle^q} . \tag{9}$$

The scaled factorial moments have been calculated according to formula (9) for all events ($N_{\text{ch}} > 4$) and for 2026 events with higher multiplicities ($N_{\text{ch}} > 10$). The pseudorapidity window $\Delta\eta = 4$ ($-2 < \eta < 2$) has been chosen. In Figs. 2(a) and 2(b), $\ln\langle Fq \rangle$ is plotted as a function of $-\ln(\delta\eta)$ in the range $-1.5 < -\ln(\delta\eta) < 2.3$ for $N_{\text{ch}} > 4$ and $N_{\text{ch}} > 10$, respectively. It can be seen from Fig. 2 that $\ln\langle Fq \rangle$ increases with decreasing $\delta\eta$ over the whole range of $\delta\eta$, but there are two distinctive slopes: a steep slope for $\delta\eta > 1$ and a flatter slope for $1 > \delta\eta > 0.1$.

The linear fitting to the data in the region $1 > \delta\eta > 0.1$ is performed according to the relation

$$\ln\langle Fq \rangle = A - \Phi_q \ln(\delta\eta) , \tag{10}$$

where A is a constant, and the slope Φ_q characterizes the strength of the intermittency signal. The obtained intermittency exponents Φ_q are presented in Table I. It is

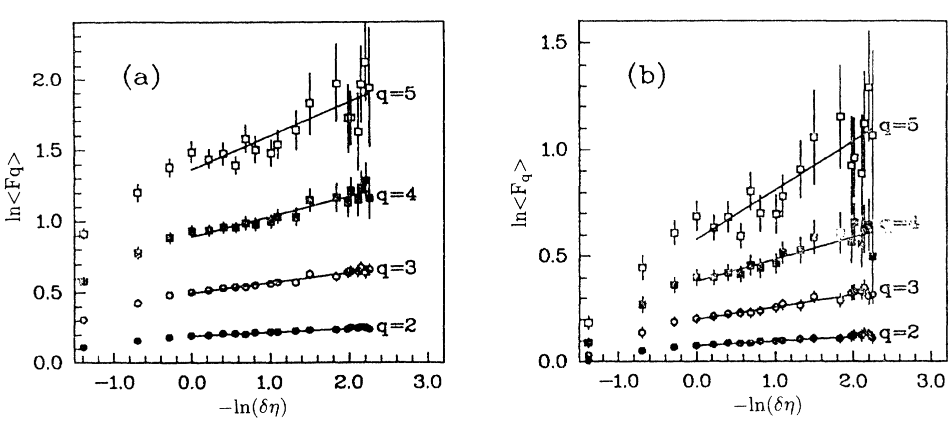

FIG. 2. $\ln\langle Fq \rangle$ as a function of $-\ln(\delta\eta)$ (a) for $N_{\text{ch}} > 4$ events; (b) for $N_{\text{ch}} > 10$ events.

TABLE I. Variation of the intermittency exponents Φ_q with q.

Events	$q=2$	$q=3$	$q=4$	$q=5$
pp (400 GeV/c)				
$N_{ch}>4$	0.0253±0.0014	0.0727±0.0044	0.142±0.013	0.239±0.042
$N_{ch}>10$	0.0220±0.0017	0.0591±0.0047	0.102±0.014	0.230±0.037
pp (360 GeV/c)[a]				
$N_{ch}>0$	0.014±0.001	0.037±0.002	0.139±0.008	0.322±0.02
$N_{ch}>12$	0.0095±0.001	0.026±0.001	0.056±0.002	0.145±0.02
$\pi^{\dagger}P/k^{\dagger}P$[a]				
(250 GeV/c)	0.0127±0.001	0.0499±0.0022	0.148±0.007	0.328±0.02

[a]Reference [3].

found that the intermittency exponents Φ_q increase with the order q of the moments and decrease with increasing average multiplicities.

The dependence of the intermittency exponents on the order q of the moments may reveal a self-similar cascade mechanism for multiparticle production in high-energy collisions. According to the α model, in the log-normal approximation the relationship between Φ_q and Φ_2 can be predicted as

$$\Phi_q=\frac{q(q-1)}{2}\Phi_2 \ . \tag{11}$$

In Table II, a comparison between some experimental results and this prediction is shown. A better approximation which has been obtained by using Levy stable laws to random cascading models is [7]

$$\Phi_q=\frac{\Phi_2}{2^{\mu}-2}[q^{\mu}-q] \ , \tag{12}$$

where μ is the Levy index. If $\mu<1$, the possibility of the formation of a quark-gluon plasma exists. According to our data, the best fitting values of the Levy index are

$$\mu=1.89\pm0.16 \ \text{for} N_{ch}>4 \ ;$$

$$\mu=1.66\pm0.22 \ \text{for} N_{ch}>10 \ .$$

The fitted values of Φ_q/Φ_2 are also shown in Table II.

The intermittency exponents Φ_q are related to the physics of fractal objects through the generalized dimensions Dq, which are commonly used for the description of fractals and multifractals in classical chaotic systems [8]. Dq can be calculated by the relation [9,10]

$$Dq=1-\frac{\Phi_q}{q-1} \ , \tag{13}$$

and

$$\frac{\Phi_q}{q-1}=dq \ ,$$

where dq is the anomalous fractal dimension. Białas and Hwa [11] have argued that for a second-order phase transition from a quark-gluon plasma to normal hadronic matter, dq should be independent of q. If, on the other hand, the final hadron system is created as a result of a cascading process one rather expects dq to be approximately linear in q. Figure 3 shows the values of dq versus the order q of the moments for $N_{ch}>4$ events. In Fig. 3, we also show the results from NA22 data and the simulated result obtained by using the geometrical branching model (ECCO Monte Carlo code) by Hwa and Pan [12]. It is found that the anomalous fractal dimensions have the tendency to grow linearly with increasing order q of the moments. As mentioned above, the observed pattern of dq and the values of Levy index μ both indicate that the self-similar cascade mechanism is the most probable had-

TABLE II. The values of Φ_q/Φ_2.

Events	Φ_3/Φ_2	Φ_4/Φ_2	Φ_5/Φ_2
$\sqrt{S}=27.4$ GeV[a]			
$N_{ch}>4$	2.91±0.24	5.66±0.62	9.58±1.75
$N_{ch}>10$	2.66±0.27	4.59±0.74	10.36±1.83
$\sqrt{S}=22$ GeV[b]			
	3.93±0.35	11.07±1.0	25.6±2.5
$\sqrt{S}=630$ GeV[b]			
$N_{ch}<15$	2.89±0.30	4.64±0.79	10.2±1.8
$q(q-1)/2$	3	6	10
$\mu=1.89$	2.92	5.70	9.35
$\mu=1.66$	2.75	5.15	8.14

[a]This experiment.
[b]Reference [3].

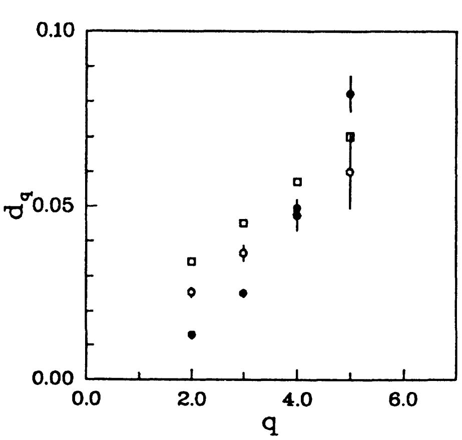

FIG. 3. The anomalous fractal dimension dq as a function of q (order of moments). ○ This experiment; □ the simulated results obtained by Hwa and Pan [12]; ● NA22 data.

ronization process. It seems that we can discard the possibility of the formation of a quark-gluon plasma in *pp* collisions at 400 GeV/*c*.

IV. CONCLUSION

Analysis of the scaled factorial moments has been performed for the experimental data of the pseudorapidity distributions of charged particles produced in *pp* collisions at 400 GeV/*c*. An intermittent pattern is observed. The intermittency exponents increase with increasing order q of the moments and decrease with increasing average multiplicities. The anomalous fractal dimensions dq increase with q. The results as mentioned above indicate that multiparticle production in *pp* collisions at 400 GeV/*c* possesses a self-similar cascade property.

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