Phenomenological σ models

N. N. Achasov and G. N. Shestakov
Laboratory of Theoretical Physics, Institute for Mathematics, 630090, Novosibirsk 90, Russia
(Received 2 June 1993)

The question of the phenomenological description of the broad scalar resonances compatible with chiral symmetry and unitarity is discussed within the framework of the linear σ model. It is pointed out that a naive inclusion of the large decay widths in the scalar meson propagators is not an adequate approximation for physical amplitudes. It is shown that the simplest unitarization scheme for the real $\pi\pi \to \pi\pi$ Born amplitude with $l = I = 0$ leads to a broad resonance arising with a mass $m_{\rm res} \approx 420$ MeV and a width $\Gamma_{\rm res}(m^2_{\rm res}) \approx 740$ MeV at the bare σ meson mass $m_\sigma \approx 1$ GeV. The resonance parameters are largely controlled by the nonresonant background amplitude. Some applications to other reactions are also discussed.

PACS number(s): 13.75.Lb, 11.30.Rd, 13.20.Eb, 13.25.Es

I. INTRODUCTION

During more than the past 25 years the various modifications of the classical σ model [1] have been applied to the description of the data. Both before and today the linear SU(2)×SU(2) and U(3)×U(3) σ models are rather popular (see, for example, Refs. [2–11]). The attractiveness of the linear models is caused by two circumstances: (i) scalar resonances are really observed and there is a great temptation to realize the σ model with these states; (ii) the linear σ model is renormalized to allow the approximations to be routinely made which keep many (perhaps all) properties built into the theory such as chiral symmetry, unitarity, and so on.

The theoretical discussions of the ways and means of chiral symmetry realization are also developing [12–14].

The behavior of the amplitudes in a rather wide energy region and also near the resonances cannot be discussed without taking unitarity into consideration [we shall consider the $\pi\pi$ invariant mass ($\sqrt{s}$) region from $2m_\pi$ to 1 GeV]. There are a lot of works in which the unitarized chiral amplitudes are constructed [4,7,10,15–23] (as to the reactions $\pi\pi \to \pi\pi$, $K\pi \to K\pi$, $\pi\pi \to K\bar{K}$, see, for example, Refs. [4,7,10,15,16,18–20]). Today the theoretical applications of such amplitudes lie in the range from $\pi\pi$ scattering and $K \to \pi\pi l\nu$ decays to $\psi' \to J/\psi\pi\pi$ decays and further to $W_L W_L$ scattering at TeV energies (recent surveys of this subject are contained in Refs. [20–23]). There is an interesting physical question about the adequate determination of resonance parameters in the presence of a large background [24] [for example, it is important in connection with an examination of the SU(3) or U(3)×U(3) relations for masses and coupling constants].

In the present work we show that the simplest (obvious) unitarization scheme for the $l = I = 0$ $\pi\pi \to \pi\pi$ tree chiral amplitude leads to a broad resonance arising with the $m_{\rm res} \approx 420$ MeV and the $\Gamma_{\rm res}(m^2_{\rm res}) \approx 740$ MeV, if the bare σ meson mass $m_\sigma \approx 1$ GeV. The large background amplitude plays the crucial role in its formation. From the beginning we point out that a naive inclusion of the large decay widths in the scalar meson propagators is not an adequate approximation for physical amplitudes. Then we construct a unitarized amplitude, which incorporates the above resonance, and discuss its properties. Some applications to other $\pi\pi$ production processes are shortly discussed at the end.

II. CONSIDERATION OF THE SCALAR MESON WIDTH IN THE CHIRAL MODEL

Let us treat the reaction $\pi^+\pi^- \to \pi^0\pi^0$. In the classical linear σ model [1] the tree amplitude [4]

$$T(\pi^+\pi^- \to \pi^0\pi^0) \equiv A(s,t,u) = \frac{m_\pi^2 - m_\sigma^2}{F_\pi^2}\left(1 - \frac{m_\sigma^2 - m_\pi^2}{m_\sigma^2 - s}\right) = \frac{m_\pi^2 - m_\sigma^2}{F_\pi^2}\,\frac{m_\pi^2 - s}{m_\sigma^2 - s} \qquad (1)$$

automatically satisfies the Adler condition [1,25]

$$A(m_\pi^2, m_\pi^2, m_\pi^2) = 0\ , \qquad (2)$$

which guaranties in the σ model the current-algebra prediction for the $\pi\pi$ scattering length. Simple phenomenological applications of the effective Lagrangians of the linear SU(2)×SU(2) and U(3)×U(3) σ models are often reduced (i) to using the tree approximation for the estimates of the scalar meson masses and widths, and (ii) to the representation of the scalar meson propagators in a simplest Breit-Wigner form, i.e., for example, the σ meson propagator is

$$\frac{1}{D_\sigma(s)} = \frac{1}{m_\sigma^2 - s - i\sqrt{s}\Gamma_{\sigma\pi\pi}(s)}\ , \qquad (3)$$

where $\Gamma_{\sigma\pi\pi}(s)$ is the energy-dependent width of the $\sigma \to \pi\pi$ decay:

$$\Gamma_{\sigma\pi\pi}(s) = \frac{3}{2}\frac{g^2_{\sigma\pi^+\pi^-}}{16\pi}\frac{\rho_{\pi\pi}}{\sqrt{s}} , \quad (4)$$

$$g_{\sigma\pi^+\pi^-} = \frac{m_\pi^2 - m_\sigma^2}{F_\pi} , \qquad \rho_{\pi\pi} = \sqrt{1 - 4m_\pi^2/s} . \quad (5)$$

In so doing, the energy region in question is of 1 GeV order. Let us also note that the module of the coupling constant $g_{\sigma\pi^+\pi^-}$ is, as a rule, very large. For example, $g_{\sigma\pi^+\pi^-} \approx -10.5$ GeV and $\Gamma_{\sigma\pi\pi}(s = m_\sigma^2) \approx 3.2$ GeV at $m_\sigma = 1$ GeV ($F_\pi = 93.1$ MeV, $m_\pi = m_{\pi^0} \approx 135$ MeV).

In essence, taking into account the width in the propagator of a scalar particle [see Eq. (3)] implies summing of the infinite chain of the bubble diagrams with the real π mesons in the intermediate states:

$$\frac{1}{D_\sigma(s)} = \frac{1}{D^0_\sigma(s)} + \frac{1}{D^0_\sigma(s)} i\sqrt{s}\Gamma_{\sigma\pi\pi}(s)\frac{1}{D^0_\sigma(s)} + \cdots \quad (6)$$

[where $1/D^0_\sigma(s) = 1/(m_\sigma^2 - s)$].

There arise two questions. (1) Do we sum up well only this chain of the diagrams? (2) Is the Adler condition satisfied in this approximation?

Substituting the propagator (3) instead of $1/(m_\sigma^2 - s)$ in the second term in the parentheses in Eq. (1) we obtain

$$A(s,t,u) = \frac{m_\pi^2 - m_\sigma^2}{F_\pi^2}\left(1 - \frac{m_\sigma^2 - m_\pi^2}{m_\sigma^2 - s - i\sqrt{s}\Gamma_{\sigma\pi\pi}(s)}\right) . \quad (7)$$

The amplitudes (1) and (7) coincide at the $\pi\pi$ threshold. However, if the m_σ is sufficiently large ($m_\sigma \sim 1$ GeV) then the approximation (7) is surprisingly bad at the very threshold. The amplitude does not have a smooth behavior; the Adler condition (2) is destroyed. Indeed, it follows from Eq. (7) at $m_\sigma = 1$ GeV that at the threshold,

$$A(4m_\pi^2, 0, 0) = \frac{m_\sigma^2 - m_\pi^2}{m_\sigma^2 - 4m_\pi^2}\frac{3m_\pi^2}{F_\pi^2} \approx 6.68 , \quad (8)$$

at the Adler point (taking into account analytical continuation) instead of Eq. (2),

$$A(m_\pi^2, m_\pi^2, m_\pi^2) = \frac{m_\pi^2 - m_\sigma^2}{F_\pi^2}\left(1 - \frac{m_\sigma^2 - m_\pi^2}{m_\sigma^2 - m_\pi^2 + 3\sqrt{3}g^2_{\sigma\pi^+\pi^-}/32\pi}\right) \approx -96.7 , \quad (9)$$

at 1 MeV above the threshold,

$$A(s = (2m_\pi + 1 \text{ MeV})^2, t, u) \approx 33.9 e^{i96^\circ} . \quad (10)$$

Owing to the chiral symmetry, for s near the $\pi\pi$ threshold there are cancellations between terms of order m_σ^2/F_π^2 in the real parts of the amplitudes (1) and (7) up to the m_π^2/F_π^2 level. But the imaginary part of the amplitude (7) near threshold is not canceled by anything and turns out as large as $(m_\sigma^2/F_\pi^2)(3m_\sigma^2\rho_{\pi\pi}/32\pi F_\pi^2)$, see Eq. (10).

Let us consider now the $\pi\pi$ scattering partial amplitude with $l = I = 0$, T_0^0. In the tree approximation [4]

$$T_0^{0\,(\text{tree})} = \frac{m_\pi^2 - m_\sigma^2}{F_\pi^2}\left\{5 - 3\frac{m_\sigma^2 - m_\pi^2}{m_\sigma^2 - s} - 2\frac{m_\sigma^2 - m_\pi^2}{s - 4m_\pi^2}\ln\left(1 + \frac{s - 4m_\pi^2}{m_\sigma^2}\right)\right\} . \quad (11)$$

The first, second, and third terms in Eq. (11) correspond to the pointlike diagram, the contribution of the intermediate σ meson in the s channel, and the σ exchanges in the t and u channels, respectively. If the propagator $1/(m_\sigma^2 - s)$ in Eq. (11) is replaced by the expression (3) then there arises the ImT_0^0 for $s > 4m_\pi^2$. However, it is obvious that the amplitude T_0^0 constructed in this way does not satisfy the unitarity condition

$$\text{Im}T_0^0 = \rho_{\pi\pi}|T_0^0|^2/32\pi . \quad (12)$$

For example, the left-hand side of Eq. (12) $\approx$ 101, 165, 111, and the right-hand side of Eq. (12) $\approx$ 9, 372, 1025, respectively, at $\sqrt{s} = 2m_\pi + 1$ MeV, 300 MeV, 500 MeV. It is clear that the naive account of the σ meson width destroys completely good predictions of the σ model at low energies. Therefore, if we want to describe the data with the help of the σ model in a sufficiently wide energy region (for example, from $\sqrt{s} \approx 2m_\pi$ to 1 GeV in the $\pi\pi$ channel), we must try in some definite way to take into account all orders of strong interactions. The improved amplitude must (in particular) satisfy the unitarity condition.

The simplest example for the amplitude containing the most important features of low-energy $\pi\pi$ dynamics, namely, chiral symmetry and unitarity, can be constructed as follows (see, for example, Refs. [4,7,10,16]):

$$T_0^0 = \frac{T_0^{0\,(\text{tree})}}{1 - i\rho_{\pi\pi}T_0^{0\,(\text{tree})}/32\pi} = \frac{(m_\sigma^2 - s)\lambda(s) + 3g^2_{\sigma\pi^+\pi^-}}{(m_\sigma^2 - s)[1 - i\Delta\lambda(s)] - i3\Delta g^2_{\sigma\pi^+\pi^-}} , \quad (13)$$

where $T_0^{0\,(\text{tree})}$ is given by Eq. (11), $\Delta = \rho_{\pi\pi}/32\pi$, and therefore

$$\lambda(s) = \frac{m_\pi^2 - m_\sigma^2}{F_\pi^2}\left\{5 - 2\frac{m_\sigma^2 - m_\pi^2}{s - 4m_\pi^2}\ln\left(1 + \frac{s - 4m_\pi^2}{m_\sigma^2}\right)\right\} . \quad (14)$$

Graphically the amplitude (13) corresponds to the infinite chain of the diagrams in Fig. 1(a) with the real π mesons in the intermediate states. As seen from Fig. 2 this amplitude gives a reasonable description of the data

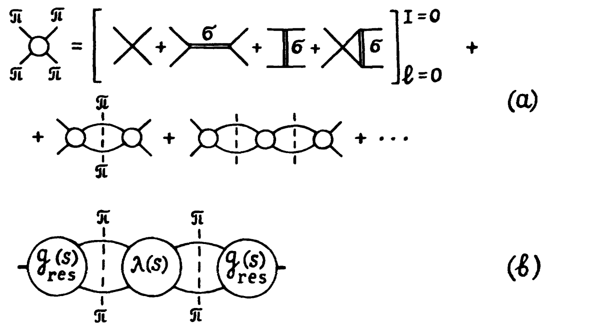

FIG. 1. (a) The graphical representation of the amplitude (13). The vertical dashed lines show that the π mesons in the loops are on mass shell. The circles correspond to the tree $\pi\pi$ amplitude with $l = I = 0$; see Eq. (11). (b) The graphical representation of Eq. (19) for the $\text{Re}\tilde{\Pi}_{\text{res}}(s)$ [see (14) and (21)].

[26–30] on the $l = I = 0$ $\pi\pi$ phase shift δ_0^0 for $\sqrt{s} < 0.9$ GeV at $m_\sigma \approx 1$ GeV.[1]

Let us note that for[2] $\sqrt{s} < 0.4$ GeV our curves do not describe the data worse than the well known curves from the works of Rosselet *et al.* [27] and Gasser [27], which correspond to the $l = I = 0$ $\pi\pi$ scattering length $a_0^0 = 0.28\pm0.05$ and 0.20 ± 0.01, respectively. At the same time our formulas (13) and (11) give $a_0^0 = 7(m_\pi/F_\pi)^2 \times (1+29m_\pi^2/7m_\sigma^2)/32\pi \approx 0.16$ at $m_\sigma \approx 1$ GeV that is close to the current algebra result (0.146). In fact, only future

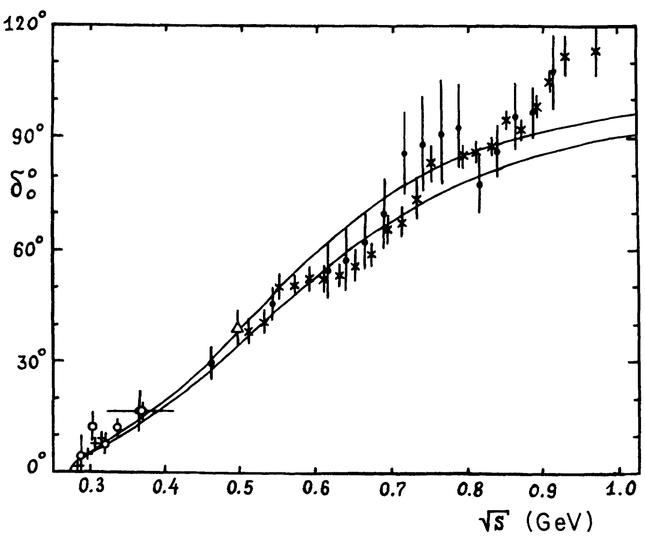

FIG. 2. The $\pi\pi$ phase shift δ_0^0 calculated according to Eqs. (13) and (15). The down (up) line corresponds to $m_\sigma = 1$ (0.862) GeV. The data: $\times$ [26], $\circ$ [27], + [28], $\bullet$ [29], $\triangle$ [30].

[1]Similar calculating gives also the $\pi\pi$ amplitude T_0^2 with $l = 0$ and $I = 2$. The phase difference $\delta = \delta_0^0 - \delta_0^2$ at $\sqrt{s} = m_K$ (which determines the phase of the direct CP violation parameter ϵ' in $K \to \pi\pi$ decays) is 45° and 49° for $m_\sigma = 1$ GeV and 0.862 GeV, respectively, as in Fig. 2. This result agrees with the experiments, according to which $\delta = 47 \pm 6°$; see Ref. [30].

[2]It is domain of validity for the Rosselet and Gasser curves [27].

experiments, for example, at DAΦNE, will allow one to choose one of many interpolations of the data. At present the empirical scattering length, measuring the slope of δ_0^0 at threshold, lies in the interval from 0.17 to 0.3; see, for example, Ref. [30]. [3] The theoretical problems with a_0^0 and interpolation of the data are extensively discussed in Refs. [18,19,21,22,27,30].

Our fit in the wide energy region ($2m_\pi < \sqrt{s} < 0.9$ GeV, see Fig. 2) is not worse than the fits in other unitarization schemes as, for example, the Padé method, to say the least; see Refs. [4,7,18,19,21,22,30]. This suggests that the main features of the above amplitude do not depend on the unitarization schemes.

We emphasize that we do not suggest a new method of the unitarization (a new prescription). We use the field theory which, certainly, is unitary. We show that in the σ model, in general, all orders of strong interaction are essential. But presently it is not realistic to take into account all them. That is why we regard only the special type of diagrams satisfying the unitarity. Of course, such a strategy is possible only in a renormalized theory and has no sense, in our opinion, in the nonlinear realization of chiral symmetry.

Of course, we do not pretend on the description of the region $\sqrt{s} > 0.9$ GeV. As is known there is a narrow structure in δ_0^0 around 1 GeV caused by the puzzling $f_0(975)/S^*$ resonance [31]. We do not take into account the $f_0(975)/S^*$ phenomenon, but our amplitude (13) can be considered as a smooth elastic background for it [31].

In order to understand better the matter of the formula (13) (for example, the specific resonant features and so on), let us rewrite it in the form which is often used for the treatment of the experimental data:

$$T_0^0 = \frac{32\pi}{\rho_{\pi\pi}}\frac{e^{2i\delta_0^0}-1}{2i} = \frac{32\pi}{\rho_{\pi\pi}}\left\{\left(\frac{e^{2i\delta_{\text{bg}}}-1}{2i}\right) + e^{2i\delta_{\text{bg}}}T_{\text{res}}\right\}. \tag{15}$$

According to Eq. (13), $\delta_0^0 = \arctan(\rho_{\pi\pi}T_0^{0(\text{tree})}/32\pi)$, and so δ_0^0 passes through 90° at $\sqrt{s} = m_\sigma$ [see Eq. (11)]. The term in the parentheses in Eq. (15) corresponds to the sum of the all diagrams in Fig. 1(a) without poles [positive degrees of $1/(m_\sigma^2 - s)$]; i.e., it is the background amplitude

$$T_{\text{bg}} = \frac{32\pi}{\rho_{\pi\pi}}\left\{\frac{e^{2i\delta_{\text{bg}}}-1}{2i}\right\} = \frac{\lambda(s)}{1-i\Delta\lambda(s)}\,. \tag{16}$$

The sum of the rest diagrams in Fig. 1(a) gives the term $e^{2i\delta_{\text{bg}}}T_{\text{res}}$, in Eq. (15), i.e., the resonance contribution modified by the background. The additional background phase in front of the T_{res} is a consequence of the unitarity. $\delta_0^0 = \delta_{\text{bg}} + \delta_{\text{res}}$, where δ_{res} is the phase of the amplitude T_{res}, see Eq. (15):

$$T_{\text{res}} = \frac{\sqrt{s}\Gamma_{\text{res}}(s)}{M_{\text{res}}^2 - s + \text{Re}\tilde{\Pi}_{\text{res}}(M_{\text{res}}^2) - \tilde{\Pi}_{\text{res}}(s)}\,. \tag{17}$$

[3]Our approach has some reserve to fit the data. This is connected with virtual intermediate states, which contributions can be investigated in addition when the more precise data on the $\pi\pi$ phase shift δ_0^0 will be obtained.

Here

$$\mathrm{Im}\tilde{\Pi}_{\mathrm{res}}(s)=\sqrt{s}\Gamma_{\mathrm{res}}(s)=\frac{3}{2}\frac{g^2_{\mathrm{res}}(s)}{16\pi}\rho_{\pi\pi}\ , \tag{18}$$

$$\mathrm{Re}\tilde{\Pi}_{\mathrm{res}}(s)=-3\lambda(s)\Delta^2 g^2_{\mathrm{res}}(s)\ . \tag{19}$$

Equation (19) is shown graphically in Fig. 1(b). The nonresonant (background) $\pi\pi$ interaction modifies the tree parameters of the σ meson, its mass m_σ, and its coupling constant $g_{\sigma\pi^+\pi^-}$ [see Eq. (5)], as follows:

$$M^2_{\mathrm{res}}=m^2_\sigma-\mathrm{Re}\tilde{\Pi}_{\mathrm{res}}(M^2_{\mathrm{res}})\ , \tag{20}$$

$$g_{\mathrm{res}}(s)=\frac{g_{\sigma\pi^+\pi^-}}{|1-i\Delta\lambda(s)|}\ , \tag{21}$$

see Eqs. (17)–(19). As we take into account only the π mesons on mass shell in all intermediate states in Fig. 1, then there are not any divergences, and all renormalizations connected with the background interaction are finite. We determine the resonance mass square M^2_{res} as the value of s at which the real part of the inverse σ propagator modified by the background contributions vanishes; see Eqs. (11), (13), (15), and (17)-(19).

Equations (20) and (21) show that if there is a large background then the characteristics of a resonance in the $\pi\pi$ scattering (its mass and width) can bear a rather remote relation to the initial tree parameters m^2_σ and $g_{\sigma\pi^+\pi^-}$. For example, at m_σ = 1 GeV, $M_{\mathrm{res}}\approx$ 417 MeV and $\Gamma_{\mathrm{res}}(M^2_{\mathrm{res}})\approx$ 738 MeV. The latter explains a smooth behavior of the phase shift δ^0_0, which does not have any sharp changes typical for the narrow resonance either near m_σ or near M_{res}, where $\delta_{\mathrm{res}}\approx 90°$; see Fig. 2. At the same time the rather large width[4] $\Gamma_{\mathrm{res}}(s)$ is about an order of magnitude smaller than $\Gamma_{\sigma\pi\pi}(s)$. For example, from Eq. (4) at $\sqrt{s}=M_{\mathrm{res}}\approx$ 417 MeV, the width $\Gamma_{\sigma\pi\pi}(M^2_{\mathrm{res}})\approx$ 6.1 GeV.

As seen from Eqs. (17)-(19), the representation of the amplitude (13) in the form $T^0_0=T_{\mathrm{bg}}+e^{2i\delta_{\mathrm{bg}}}T_{\mathrm{res}}/\Delta$ [see Eqs. (14)-(17)] provides a correct structure of the resonance term: the imaginary part of the polarization operator $\tilde{\Pi}_{\mathrm{res}}(s)$ [see Eqs. (17), (18)] is positively defined and is given by the modulus square of the vertex function $g_{\mathrm{res}}(s)e^{i\delta_{\mathrm{bg}}}$, which is determined by one-particle irreducible diagrams. Note that the imaginary part of the denominator of the whole amplitude T^0_0 [see Eq. (13)] cannot be represented (and interpreted) in this way.

Let us also adduce several equivalent expressions for the matrix element $\langle\pi^+\pi^-|\sigma(0)|0\rangle$ describing the propagation and decay of σ. Like the amplitude T^0_0 [see Eq. (13) and Fig. 1(a)], we obtain

$$\begin{aligned}\langle\pi^+\pi^-|\sigma(0)|0\rangle&=\frac{g_{\sigma\pi^+\pi^-}}{m^2_\sigma-s}\frac{1}{1-i\rho_{\pi\pi}T^{0\,(\mathrm{tree})}_0/32\pi}\\&=\frac{g_{\sigma\pi^+\pi^-}}{(m^2_\sigma-s)[1-i\Delta\lambda(s)]-i3\Delta g^2_{\sigma\pi^+\pi^-}}\\&=\frac{g_{\mathrm{res}}(s)e^{i\delta_{\mathrm{bg}}}}{M^2_{\mathrm{res}}-s+\mathrm{Re}[\tilde{\Pi}_{\mathrm{res}}(M^2_{\mathrm{res}})-\tilde{\Pi}_{\mathrm{res}}(s)]-i\sqrt{s}\Gamma_{\mathrm{res}}(s)}\\&=\frac{g_{\sigma\pi^+\pi^-}}{m^2_\sigma-s}e^{i\delta^0_0}\cos\delta^0_0\ .\end{aligned} \tag{22}$$

It is intriguing that the nonresonant backgrounds in the S-wave $0^-_1 0^-_2$ interactions are large both in the theoretical σ models and in the available experiments. Although the phase shift δ^0_0 keeps a memory about the tree m^2_σ value [in our simplest model, $\delta^0_0(m^2_\sigma)=90°$], neither the amplitude T^0_0 (13) nor the matrix element $\langle\pi^+\pi^-|\sigma(0)|0\rangle$ (22) can be presented (or approximated) by a simple Breit-Wigner formula of the type (3), as is done in Refs. [8,9]; see Eqs. (13)-(22). Therefore the evaluations of the scalar meson widths through the tree coupling constants [see Eqs. (4) and (5)] and their direct comparisons with the effective widths of the observable resonance structures seem to be little (if at all) justified without taking into account the large background contributions.

III. $K^+\to\pi^+\pi^-e^+\nu$, $K_s\to\pi\pi$ DECAYS AND UNITARITY

There are simple applications to the weak, electromagnetic, and Okubo-Zweig-Iizuka violating processes with the $I=0$ S-wave $\pi\pi$ pair production, for example, $K\to\pi\pi e\nu$, $K_s\to\pi\pi$, $\gamma\gamma\to\pi\pi$, $\psi'\to J/\psi\pi\pi,\ldots$. Let us denote the corresponding transition amplitudes as f_{tran}. In the elastic region of the $\pi\pi$ channel, they must satisfy to the unitarity condition

$$\mathrm{Im}f_{\mathrm{tran}}=\rho_{\pi\pi}T^0_0 f^*_{\mathrm{tran}}/32\pi\ . \tag{23}$$

Equation (23) is not satisfied by the tree amplitude $f^{(\mathrm{tree})}_{\mathrm{tran}}$. At the same time, if, as it has been done above, we "dress" the initial tree amplitude by the strong on-shell $\pi\pi$ final-state interaction, the modified amplitude turns out consistent with the unitarity. For example, the σ meson contribution to the form factor f_1

[4]Strictly speaking the width $\Gamma_{\mathrm{res}}(s)$ defined by Eq. (18) is an unrenormalized one. In the present case the renormalization constant of the σ wave function is finite and, at the above M_{res} value, it is not far from one. So, the renormalized width $\Gamma^{(\mathrm{ren})}_{\mathrm{res}}(s)=\Gamma_{\mathrm{res}}(s)/[1+\mathrm{Re}\tilde{\Pi}'_{\mathrm{res}}(M^2_{\mathrm{res}})]\approx 0.8\Gamma_{\mathrm{res}}(s)$.

for $K^+ \to \pi^+\pi^- e\nu$ decay [$f_{\rm tran}(K^+ \to \pi^+\pi^- e^+\nu) \sim \bar{\nu}\gamma_\mu(1+\gamma_5)e(f_1(p_{\pi^+}+p_{\pi^-})_\mu+\cdots)$] is given by Eq. (22): $f_1 = \sqrt{2}\langle\pi^+\pi^-|\sigma(0)|0\rangle = \sqrt{2}g_{\sigma\pi^+\pi^-}e^{i\delta_0^0}\cos\delta_0^0/(m_\sigma^2 - s)$ [32]. In so doing, condition (23) for f_1 is automatically satisfied. Choosing the σ mass [or more exactly, the masses of the σ and σ' mesons in the U(3)×U(3) model [8,9]], as done in Ref. [9], one can obtain the satisfactory description of the $K^+ \to \pi^+\pi^- e^+\nu$ decay.[5]

Let us also consider the $\Delta I = 1/2$ $K_s \to \pi^+\pi^-$ decay. The tree diagrams with the σ, κ^+, and κ^- poles [it means the U(3)×U(3) linear σ model together with the prescription of the correspondence between the quark and meson field operators] give some enhancement of the $\Delta I = 1/2$ transition [9,11]:

$$f_{\rm tran}^{(\rm tree)}(K_s \to \pi^+\pi^-) = C_{pv}\left\{\frac{1}{1-2(R-1)} - 1\right\} . \quad (24)$$

C_{pv} is a known normalization constant [9,11,36]. If $R = F_K/F_\pi = 1.22$ [34], then, according to mass formulas [8,9,11], $m_\sigma \approx 862$ MeV and the factor in the braces in Eq. (24) is equal to 0.786 that is very close to the original estimate [36]. The unitarized amplitude

$$f_{\rm tran}(K_s \to \pi^+\pi^-) = f_{\rm tran}^{(\rm tree)}(K_s \to \pi^+\pi^-) \times \cos\delta_0^0(m_K^2)e^{i\delta_0^0(m_K^2)} . \quad (25)$$

It follows from Eqs. (11), (13), and (15) at $m_\sigma \approx 862$ MeV that $\delta_0^0(m_K^2) \approx 38.5°$ (see Fig. 2), and $\cos\delta_0^0(m_K^2) \approx 0.783$. So, the estimate of the decay rate turns out approximately 1.6 as smaller as compared with the tree case and it is necessary to find additional resources in the model [8,9,11] to obtain the empirical value.

[5]Note that the theoretical value for the $K^+ \to \pi^+\pi^- e^+\nu$ decay rate obtained within the framework of current-algebra and PCAC (partial conservation of axial vector current) [33] is approximately half as much as the experimental one [9,34,35].

IV. CONCLUSION

Using a simplest model, we would mainly like to stress the important role of the background contributions and unitarity effects in considering the chiral amplitudes in the wide energy region from the $\pi\pi$ threshold up to $\sqrt{s} \approx 1$ GeV.

We showed that a naive regard for the decay widths in the scalar meson propagators is not an adequate approximation for the physical amplitudes.

Of course, we would also like to hope that the main features of the above unitarized chiral $\pi\pi$ amplitude containing a low-lying scalar resonance should persist if, for example, two resonances in two coupled channels [7–9] and the real parts of the loop diagrams are accounted for.

[1] M. Gell-Mann and M. Levy, Nuovo Cimento **16**, 705 (1960); S. Weinberg, Phys. Rev. Lett. **17**, 616 (1966); **18**, 188 (1967).
[2] B.W. Lee, Nucl. Phys. **B9**, 649 (1969); J.L. Gervais and B.W. Lee, *ibid.* **B12**, 627 (1969); H.W. Crater, Phys. Rev. D **1**, 3313 (1970).
[3] S. Gasiorowicz and D.A. Geffen, Rev. Mod. Phys. **41**, 531 (1969).
[4] J.L. Basdevant and B.W. Lee, Phys. Lett. **29B**, 437 (1969); Phys. Rev. D **2**, 1680 (1970).
[5] J. Schechter and Y. Ueda, Phys. Rev. D **3**, 168 (1971); **3**, 2874 (1971).
[6] P. Carruthers and R.W. Haymaker, Phys. Rev. D **4**, 1808 (1971); R. Olshansky, *ibid.* **4**, 2440 (1971).
[7] L.-H. Chan and R.W. Haymaker, Phys. Rev. D **7**, 402 (1973); **7**, 415 (1973); **10**, 4143 (1974); **10**, 4170 (1974).
[8] E.P. Shabalin, Yad. Fiz. **42**, 260 (1985) [Sov. J. Nucl. Phys. **42**, 165 (1985)]; **46**, 1312(E) (1987) [**46**, 768(E) (1987)].
[9] E.P. Shabalin, Yad. Fiz. **48**, 272 (1988) [Sov. J. Nucl. Phys. **48**, 172 (1988)]; **49**, 588 (1989) [**49**, 365 (1989)].
[10] W. Lin and B.D. Serot, Phys. Lett. B **233**, 23 (1989).
[11] T. Morozumi, C.S. Lim, and A.I. Sanda, Phys. Rev. Lett. **65**, 404 (1990).
[12] N.N. Achasov, Zh. Eksp. Teor. Fiz. **101**, 1713 (1992) [Sov. Phys. JETP **74**, 913 (1992)]; Phys. Lett. B **287**, 213 (1992).
[13] G.N. Shore and G. Veneziano, Mod. Phys. Lett. A **8**, 373 (1993).
[14] N.N. Achasov, Pis'ma Zh. Eksp. Teor. Fiz. **56**, 345 (1992) [JETP Lett. **56**, 329 (1992)]; Zh. Eksp. Teor. Fiz. **103**, 11 (1993) [JETP **76**, 5 (1993)].
[15] L.S. Brown and R.L. Goble, Phys. Rev. Lett. **20**, 346 (1968); Phys. Rev. D **4**, 723 (1971).
[16] B.W. Lee, *Chiral Dynamics* (Gordon and Breach, New York, 1972).
[17] T.N. Truong, Phys. Lett. **99B**, 228 (1981); Phys. Lett. B **207**, 495 (1988).
[18] T.N. Truong, Acta Phys. Pol. B **15**, 633 (1984); Phys. Rev. Lett. **61**, 2526 (1988); A. Dobado, M.J. Herrero, and T.N. Truong, Phys. Lett. B **235**, 134 (1990).
[19] J. Gasser and U.-G. Meissner, Nucl. Phys. **B357**, 90 (1991); V. Bernard, N. Kaiser, and U.-G. Meissner, *ibid.* **B364**, 283 (1991).
[20] D. Morgan and M.R. Pennington, Phys. Rev. D **12**, 1283 (1975); K.L. Au, D. Morgan, and M.R. Pennington, *ibid.* **35**, 1633 (1987).
[21] T.N. Truong, *Lectures given at Ettore Majorana International School on Low Energy Antiproton Physics*, Erice, Italy (Plenum, New York, 1990); Phys. Rev. Lett. **67**, 2260 (1991); Phys. Lett. B **258**, 402 (1991).
[22] U.-G. Meissner, Comments Nucl. Part. Phys. **20**, 119 (1991); Report No. BUTP-91/16, 1991 (unpublished); Invited talk presented at the 5th International Workshop on Perspectives in Nuclear Physics at Intermediate Energies, Trieste, Italy, 1991 (unpublished).

[23] K. Hikasa, in *Physics and Experiments with Linear Colliders*, Proceedings of the Workshop, Saariselkä, Finland, 1991, edited by R. Orava, P. Eerola, and M. Nordberg (World Scientific, Singapore, 1992), Vol. II, p. 451.

[24] P.W. Coulter, Phys. Rev. Lett. **29**, 450 (1972); Phys. Rev. D **7**, 96 (1973); A.B. Wicklund *et al.*, Phys. Rev. Lett. **45**, 1469 (1980); D. Cohen *et al.*, Phys. Rev. D **22**, 2595 (1980).

[25] S.L. Adler, Phys. Rev. **137B**, 1022 (1965); **139B**, 1638 (1965).

[26] P. Estabrooks and A.D. Martin, Nucl. Phys. **B79**, 301 (1974); **B95**, 322 (1975).

[27] L. Rosselet *et al.*, Phys. Rev. D **15**, 574 (1977); J. Gasser, in *Proceedings of the Workshop on Physics and Detectors for DAΦNE, the Frascati Φ Factory,* Frascati, Italy, edited by G. Pancheri (INFN, Frascati, 1991), p. 291; J.L. Petersen, *The $\pi\pi$ Interaction* (CERN Yellow Report No. CERN 77-04, Geneva, Switzerland, 1977).

[28] A.A. Bel'kov *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. **29**, 652 (1979) [JETP Lett. **29**, 597 (1979)].

[29] K.N. Mukhin *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. **32**, 616 (1980) [JETP Lett. **32**, 601 (1980)].

[30] W. Ochs, πN News Letter **3**, 25 (1991); J. Gasser and U.-G. Meissner, Phys. Lett. B **258**, 219 (1991); G. Ecker, CERN Report No. CERN-TH 6660/92, 1992 (unpublished).

[31] N.N. Achasov, S.A. Devyanin, and G.N. Shestakov, Usp. Fiz. Nauk **142**, 361 (1984) [Sov. Phys. Usp. **27**, 161 (1984)]; Phys. Lett. **102B**, 196 (1981).

[32] N.N. Achasov and G.N. Shestakov, Yad. Fiz. **56**, 206 (1993).

[33] S. Weinberg, Phys. Rev. Lett. **17**, 336 (1966); **18**, 1178(E) (1967).

[34] Particle Data Group, Phys. Lett. B **170**, 1 (1986); Particle Data Group, K. Hikasa *et al.*, Phys. Rev. D **45**, S1 (1992), p. VII.I.

[35] C. Riggenbach, J. Gasser, J.F. Donoghue, and B.R. Holstein, Phys. Rev. D **43**, 127 (1991).

[36] A.I. Vainshtein, V.I. Zakharov, and M.A. Shifman, Zh. Eksp. Teor. Fiz. **72**, 1275 (1977) [Sov. Phys. JETP **45**, 670 (1977)].