COMMENTS

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Comment on SU(16) grand unification

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(Received 3 May 1993)

In a recent paper on SU(16) grand unification, because of the presence of intermediate-energy gauge groups containing products of U(1) factors which are not orthogonal among themselves, the renormalization-group treatment has a few small errors. I correct it. I emphasize that one should not switch gauge couplings at the various thresholds. It is easier, and it avoids errors, to use throughout the gauge couplings of the standard model, and compute at each threshold, in the usual way, the extra contributions to the β functions from the extra nondecoupled fields. I also point out that the SU(16) grand unification theory, due to the large number of scalars present in it, is not asymptotically free. It becomes a strong-coupling theory at energies only slightly larger than the unification scale.

PACS number(s): 12.10.Dm, 11.10.Hi

This is a Comment on the paper of Ref. [1], which is on the grand unified theory (GUT) SU(16). The points made in this Comment are however also relevant to the GUT SU(15), notably its analysis in the paper of Ref. [2], and also other GUT's. Moreover, the first two paragraphs of this Comment are of a general nature, and one does not need to have read any of the above papers in order to understand them.

In the context of GUT's, when intermediate-energy gauge groups containing products of U(1) factors arise, people usually use the set of U(1) factors which is more intuitive (which has some simple physical interpretation). Those U(1) factors are usually not, however, orthogonal relative to the active (nondecoupled) set of fields at each energy scale. Orthogonality is defined as follows: two U(1) charges are orthogonal if the sum over all the active fields of the product of their values, weighted by the factors $11/3$ for gauge-boson fields, $-2/3$ for fermion fields, and $-1/3$ for complex scalar fields, vanishes. This nonorthogonality leads [3] to errors in the renormalization-group (RG) treatment of the theory. More or less erroneous conclusions may then be extracted from the RG analysis; I have given [3] as examples two GUT's, the first one based on the group SO(10), where the error has led only to small quantitative imprecisions in the results of the RG analysis, the second one based on the group SU(8), where the error has led to qualitatively false conclusions being extracted from the RG analysis. I find the same error in Ref. [1], this time in the context of the GUT SU(16). It is a much more complex case. The breaking chain of SU(16) in Ref. [1] has several intermediate-energy gauge groups, three of which contain products of two U(1) factors. Moreover, the authors of Ref. [1] have, at each threshold energy, traded the gauge couplings of one intermediate-energy gauge group by the ones of the next intermediate-energy gauge group, the procedure of which involves a matching of the gauge couplings at each threshold, described in detail in Eqs. (2.11)–(2.29) of Ref. [1]. This procedure, although correct, is complicated and unnecessary. I emphasize here an argument at the end of Ref. [3]. The β functions of the gauge couplings of the standard model can only change in response to more fields becoming active, i.e., nondecoupled, at some energy in the middle of the RG evolution. That change is always computed from the Dynkin indices of the representations of newly active fields. For the purpose of the RG evolution, it is irrelevant whether the effective gauge group changes or not at the threshold (i.e., whether at the threshold some gauge bosons become active, or only fermions and/or scalars become active). One just needs to compute the color representations and the weak-isospin representations and the weak hypercharges of all the fields which have their masses at each threshold energy, and take them into account in the normal fashion in the computation of the β functions. This procedure is transparent, and it avoids both the pitfall of the intermediate-energy gauge groups with products of U(1) factors, and the complication of having to trade some gauge couplings for others with the use of matching functions.

What was said above should be physically intuitive. Its mathematical justification is in the fact that all the gauge groups at each energy range, including the U(1) factors, must be orthogonal to each other, relative to the set of fields active at the particular energy. The sum over the active fields of the product of the eigenvalues of any two generators of the gauge group must always vanish. Otherwise, the gauge bosons coupled to those generators mix by means of the one-loop vacuum-polarization diagrams. Then, it is meaningless to talk about the one-loop RG evolution of the gauge couplings of each of those gauge bosons, because at the one-loop level those gauge bosons are mixing during their propagation. Now, if all the generators of the intermediate-energy group are duly orthogonal, then the argument at the end of Ref. [3] shows that the following two procedures lead to the same result: performing the RG evolution in terms of the intermediate-energy groups, or performing all the RG evolution in terms of the gauge couplings of the standard model. The second procedure is much simpler.

In the breaking chain of SU(16) studied in Ref. [1] there are three intermediate-energy gauge groups containing factors U(1)⊗U(1). If we consider Eqs. (2.31) and (2.32) of Ref. [1], we observe that the β functions of the U(1) factors are not needed in the computation of the evolution of α_{3c} and of α_{2L}. Therefore, that computation is correct (I have checked it). The problem is with the computation of the evolution of the hypercharge gauge coupling. Table I gives the set of scalar fields active at each particular energy. It is clear that, because of the representation 560_1, the charges $U(1)_B^q$ and $U(1)_\Lambda^q$ in the intermediate-energy gauge group $4^l 3_L^q 2_L^q 3_R^u 3_R^d 1_B^q 1_\Lambda^q$ are not orthogonal. As a consequence, the contribution in Ref. [1] of the 560_1 to the RG coefficient for the hypercharge-coupling RG running between energies M_B and M_{6R} (9/10) is wrong: the correct value of that coefficient is 0, as is evident from the fact that all the scalars of the 560_1 active at that energy scale have a vanishing hypercharge. Similarly, the contribution of the 560_2 to the running of the hypercharge used in Ref. [1] is wrong in the energy range $M_Y < M < M_{4l}$. In the first part of that range, $M_{3l} < M < M_{4l}$, the correct coefficient is 9/10 instead of 9/2, and in the second part of that range, $M_Y < M < M_{3l}$, the correct coefficient is 0 instead of 18/5 (once again, because all the scalars of the 560_2 active at that energy scale have zero hypercharge).

As a consequence of these small errors, Eqs. (2.35) and (2.36) of Ref. [1] should be replaced by

$$n_G = \frac{2\pi}{\ln 10}\left[-\frac{457}{12\,096}\alpha_{3c}^{-1}(M_Z) + \frac{179}{1792}\alpha_{2L}^{-1}(M_Z) - \frac{3005}{48\,384}\alpha_{1Y}^{-1}(M_Z)\right] - \frac{4943}{16\,128}n_Z$$
$$+\frac{397}{3024}n_Y - \frac{3439}{16\,128}n_{3l} + \frac{1171}{1728}n_{4l} + \frac{2285}{6048}n_B + \frac{47\,245}{16\,128}n_{6R} - \frac{41\,875}{16\,128}n_{6L}\,, \tag{1}$$

$$n_{12} = \frac{2\pi}{\ln 10}\left[-\frac{1}{24}\alpha_{3c}^{-1}(M_Z) + \frac{3}{32}\alpha_{2L}^{-1}(M_Z) - \frac{5}{96}\alpha_{1Y}^{-1}(M_Z)\right] - \frac{7}{32}n_Z$$
$$+\frac{1}{6}n_Y - \frac{7}{32}n_{3l} + \frac{13}{24}n_{4l} + \frac{5}{12}n_B + \frac{85}{32}n_{6R} - \frac{75}{32}n_{6L}\,. \tag{2}$$

From these equations the correct versions of Eqs. (2.38) and (2.39), and of Eqs. (2.41) and (2.42), can easily be derived. It may also be useful to give the value of $\alpha_G(M_G)$:

$$\alpha_G^{-1}(M_G) = -\frac{37\,343}{72\,576}\alpha_{3c}^{-1}(M_Z) + \frac{5029}{10\,752}\alpha_{2L}^{-1}(M_Z) + \frac{86\,165}{290\,304}\alpha_{1Y}^{-1}(M_Z)$$
$$+\frac{\ln 10}{2\pi}\left(\frac{419\,735}{96\,768}n_Z + \frac{54\,035}{18\,144}n_Y - \frac{218\,825}{96\,768}n_{3l} - \frac{54\,523}{10\,368}n_{4l}\right.$$
$$\left.+\frac{186\,715}{36\,288}n_B + \frac{208\,283}{96\,768}n_{6R} - \frac{686\,405}{96\,768}n_{6L}\right)\,. \tag{3}$$

It is easy to check that these corrections to the results in Ref. [1] may sometimes be highly relevant. For instance, for $n_Y = 2.5$, $n_{3l} = 2.6$, $n_{4l} = 12.0$, $n_B = 13.1$, $n_{6R} = 13.5$, and $n_{6L} = 13.6$, Eqs. (1) and (2) give $n_{12} = 13.71$ and $n_G = 13.81$, i.e., M_{12} and M_G about 2 orders of magnitude smaller than what would have been found from Ref. [1].

A second comment that I want to make is that the SU(16) theory has such an enormous number of scalars that it is not asymptotically free and it is strongly coupled. Let us calculate the precise extension of this effect. The Dynkin indices l_R of the relevant representations R of SU(16) are

$$l_{16} = \frac{1}{2}\,,\ l_{136} = 9\,,\ l_{255} = 16\,,\ l_{560} = \frac{91}{2}\,,\ l_{1820} = 182\,,$$
$$l_{2160} = \frac{423}{2}\,,\ l_{14144} = 1664\,. \tag{4}$$

The 2160 is contained in the product $\overline{16} \times 136$, and I will refer to its use later. The β function for SU(16) is computed as in Eq. (2.4):

$$\beta_{16} = \frac{11}{3} \times 16 - \frac{1}{3} \times 6$$
$$- \frac{1}{6}\Bigg(2 \times 182 + 16 + 1664 + 16 + 2 \times \frac{91}{2}$$
$$+16 + 2 \times \frac{1}{2} + 2 \times \frac{91}{2} + 2 \times 9\Bigg)$$
$$= -\frac{1937}{6} \,. \tag{5}$$

The β function is negative, which has as a consequence the existence of a Landau pole [divergence of the gauge coupling of SU(16)]. The energy M_L at which that pole occurs is given as a function of the unification energy M_G by

$$\ln \frac{M_L}{M_G} = \frac{12\pi}{1937}\alpha_G^{-1}(M_G) \,. \tag{6}$$

For $\alpha_G^{-1}(M_G) \sim 10$, one obtains $M_L/M_G \sim 1.2$. This means that the SU(16) theory becomes strongly coupled almost immediately after M_G. It is not clear to me what the cosmological consequences of this fact might be. But it is clear that the threshold effects at M_G are enormous and that they are not calculable in perturbation theory.

Notice that in the calculation of β_{16} in Eq. (5) I have taken into account the existence of mirror fermions (by writing, in the second term of the right-hand side of that equation, a factor 6 corresponding to six fermion families), but I have not taken into account the extra Higgs fields which are needed in order to give mass to those mirror fermions. Those extra Higgs representations will render β_{16} even more negative, and therefore make M_L/M_G even smaller.

In order to lessen the problem (but not eliminate it), one might break $SU(12)_q$ to $SU(3)_c \otimes SU(2)_{qL} \otimes U(1)_{qY}$ directly by means of a complex 2160 of SU(16), which contains a (1,924) of $SU(4)_l \otimes SU(12)_q$.[1] This would avoid the 14 144 of scalars, which is the largest single responsible for the negativeness of β_{16}. This is however not enough to render β_{16} positive. It appears that, if we want to have GUT's based on such large groups as SU(15) or SU(16), we must accept that the GUT is nonperturbative for practically the whole range of its validity, from M_G up to the Planck energy.

After completion of this work I became aware of the paper in Ref. [4], which anticipated some of the results in Ref. [3].

I acknowledge useful discussions with Professors L.-F. Li and L. Wolfenstein. I also thank L. Wolfenstein for reading the manuscript. This work was supported by the United States Department of Energy, under Contract No. DE-FG02-91ER-40682.

[1]I do not claim that this way of breaking of $SU(12)_q$ will preserve desirable features of the theory, like the possibility of a low unification scale. I have not done a RG analysis of SU(16) with breaking via the 2160 because that breaking scheme by itself alone is not enough to solve the problem of the negativeness of β_{16}.

TABLE I. Scalar representations relevant for the symmetry breaking.

	16	4^l12^q	$4^l6^q_L6^q_R1^q_B$	$4^l3^q_L2^q_L6^q_R1^q_B$	$4^l3^q_L2^q_L3^u_R3^d_R1^q_B1^q_V$	$4^l3^q_L2^q_L3^q_R1^q_Y$	$3^l3^q_L2^q_L3^q_R1^l_X1^q_Y$	$2^l_L3^q_L2^q_L3^q_R1^l_Y1^q_Y$	$3_c2_L1_Y$	3_c1_Q
Complex	1820	(1,1)								
Real	255_1	(1,143)	(1,1,1)[0]							
Real	14144	(1,4212)	(1,189,1)[0]	(1,1,1,1)[0]						
Real	255_2	(1,143)	(1,1,35)[0]	(1,1,1,35)[0]	(1,1,1,1,1)[0,0]					
Complex	560_1	(1,220)	$(1,1,20)[-\frac{3}{2\sqrt{6}}]$	$(1,1,1,20)[-\frac{3}{2\sqrt{6}}]$	$(1,1,1,\bar{3},3)[-\frac{3}{2\sqrt{6}},-\frac{1}{2\sqrt{3}}]$	(1,1,1,1)[0]				
Real	255_3	(15,1)	(15,1,1)[0]	(15,1,1,1)[0]	(15,1,1,1,1)[0,0]	(15,1,1,1)[0]	(1,1,1,1)[0,0]			
Complex	16	(4,1)	(4,1,1)[0]	(4,1,1,1)[0]	(4,1,1,1,1)[0,0]	(4,1,1,1)[0]	$(3,1,1,1)[\frac{1}{2\sqrt{6}},0]$	(1,1,1,1)[0,0]		
Complex	560_2	(4,66)	(4,6,6)[0]	(4,3,2,6)[0]	$(4,3,2,1,\bar{3})[0,-\frac{1}{2\sqrt{3}}]$	$(4,3,2,\bar{3})[\frac{3}{2\sqrt{33}}]$	$(3,3,2,\bar{3})[\frac{1}{2\sqrt{6}},\frac{3}{2\sqrt{33}}]$	$(2,3,2,\bar{3})[-\frac{1}{2\sqrt{3}},\frac{3}{2\sqrt{33}}]$	(1,1)[0]	
Complex	136	(10,1)	(10,1,1)[0]	(10,1,1,1)[0]	(10,1,1,1,1)[0,0]	(10,1,1,1)[0]	$(3,1,1,1)[-\frac{2}{2\sqrt{6}},0]$	$(2,1,1,1)[\frac{1}{2\sqrt{3}},0]$	$(1,2)[\frac{1}{2}\sqrt{\frac{3}{5}}]$	(1)[0]

[1] N. G. Deshpande, E. Keith, and P. B. Pal, Phys. Rev. D **47**, 2897 (1993).

[2] B. Brahmachari, U. Sarkar, R. B. Mann, and T. G. Steele, Phys. Rev. D **45**, 2467 (1992).

[3] L. Lavoura and L. Wolfenstein, Phys. Rev. D **48**, 264 (1993).

[4] F. del Aguila, G. D. Coughlan, and M. Quirós, Nucl. Phys. **B307**, 633 (1988).