Direct tests of *CP*-violating triple gauge boson couplings in photonic linear colliders

G. Bélanger*
Laboratoire d'Annency-le-Vieux de Physique des Particules, Boîte Postale 110, Chemin de Bellevue, F74941 Annecy-le-vieux, France

G. Couture
Département de Physique, Université du Québec à Montréal, Case Postale 8888 Montréal, PQ, Canada H3C 3P8

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Possible tests of the *CP*-violating triple gauge bosons couplings in $\gamma\gamma \to WW$ and $e\gamma \to W\nu$ are discussed. Using either circular or linear polarization of the initial photon laser beam, direct measurements of *CP*-odd observables in $\gamma\gamma \to W^+W^-$ are possible. We obtain limits on the coefficient of the *CP*-violating coupling of $\mathcal{R}(f_6) < 0.9\times10^{-2}$ and $\mathcal{I}(f_6) < 5.4\times10^{-4}$ for $\sqrt{s} = 1$ TeV. These are then compared with the limits obtained from the total cross section with or without polarized beams in $\gamma\gamma$ and with different asymmetries in $e\gamma$.

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I. INTRODUCTION

The next generation of e^+e^- colliders will offer interesting possibilities for *W*-boson physics either with *W* pair production in e^+e^- or $\gamma\gamma$ or with single *W* production in $e\gamma$ collisions. That the *W*-pair production in e^+e^- is a good process for measuring the properties of the *W* bosons such as its mass and couplings has been emphasized for some time [1]. More recently, it became clear that the $\gamma\gamma$ and $e\gamma$ version of a linear collider offer an interesting complement to this process [2]. Both the *W* pair cross section in $\gamma\gamma$ and single *W* production cross section in $e\gamma$ are large. At energies between 0.5 and 1 TeV, the former is approximately 85 pb while the latter is near 50 pb [2]. With design luminosities between 10 and 100 fb^{-1}, a large number of *W*'s can be produced in these processes making it possible to test the electromagnetic properties of the *W* bosons. Most of the previous studies dealing with the couplings of the *W* boson were concerned with the *CP*-even part of these and how one could measure possible deviations from the SM predictions. The results of the different analyses indicate that these couplings will be tested at best at the per mil level [2,3] the level where some new physics could set in. The *CP*-violating couplings have received less attention mainly because of existing constraints and theoretical biases. The present limits on *CP*-violating couplings coming from their one-loop contribution to electric dipole moments (EDM's) are typically $\approx 10^{-2}$ from the electron EDM or 10^{-4}–10^{-1} from the more model-dependent neutron EDM [4]. With approximately the same level of precision achievable for the *CP*-violating couplings at a linear collider as for the *CP*-conserving ones, some improvement on the present limits are expected especially if one ignores the more stringent bounds from neutron EDM. In any case, we emphasize the importance of obtaining direct unambiguous limits on the *CP*-violating couplings. The second point concerns theoretical bias which is reflected in an estimation of the most likely anomalous couplings, those of lowest dimension and those which respect the approximate symmetry of the standard model such as *CP* or custodial SU(2). If indeed this is the case *CP*-conserving dimension-4 anomalous couplings such as κ_γ or κ_Z, δg_z (for the *ZWW* vertex) should be discovered first while *CP*-violating ones should be suppressed as well as the ones coming from higher-dimension operators [5]. Here we ignore these biases and rather adopt the point of view that *CP* violation is small because it comes from a nonstandard source. Like all anomalous couplings, the one violating *CP* are small compared to standard couplings; in general, no additional suppression should be expected.

Considering the importance of an eventual discovery of *CP* violation outside the *K* system and the breakthrough it would provide in the explanation of *CP* violation we want to stress the necessity of searching for *CP* violation in every possible way, including through anomalous couplings. After all any indication of a *CP* violating coupling in the γWW vertex would be a clear sign of physics beyond the standard model since the standard contribution enters at most at the three-loop level and is expected to be extremely small and unmeasurable in the near future. This has been established by Khriplovich and Pospelov when proving that the *W* EDM vanishes exactly at the two-loop level implying also that the EDM of leptons vanishes at the three-loop level [6]. In extensions of the standard model however anomalous couplings can be generated at the one-loop level and could possibly fall in the interesting range for linear colliders [7,8]. In a particular model, the new particles that generate the anomalous couplings could possibly be detected in other ways, such effects will not be considered here as they cannot be

*On leave of absence from Laboratoire de Physique Nucléaire, Université de Montréal, C.P. 6128, Succursal A, Montréal, Québec, Canada H3C 3J7.

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discussed in the model-independent parametrization used here.

In this paper we discuss ways to determine the size of the *CP* violating part of the γWW coupling in both the $e\gamma$ and $\gamma\gamma$ colliders while never observing the polarization of the massive final particles. Several observables may be used to constrain anomalous couplings: total cross sections, energy and angular distributions for example. These *CP* even observables receive contributions from both *CP*- conserving and -violating anomalous couplings and furthermore depend only weakly (like the square) on the coefficient of the *CP* violating coupling. This last point can be compensated by the fact that anomalous cross-sections grow quickly with energy so that a precise measurement of the coupling is still possible especially with a TeV collider. In order to reach maximum sensitivity to *CP* violating couplings, and more importantly to establish *CP* violation one must rely on observables that are explicitly *CP* odd and depend linearly on the anomalous coupling. It is easy to check that a sum over helicities of all particles will automatically lead to a *CP*-even result, information on the helicity of either the photon or the W must therefore be kept. Here we choose to use that of the photon [9]. To have a *CP*-odd observable we need to generate a phase so that an interference with the standard amplitude can occur. This can be done either with the imaginary part of the anomalous coupling or with the azimuthal angle of the W.

In linear colliders, the high-energy photon beam is obtained by backscattering a laser off a high-energy electron beam [10]. It was shown that with a circularly (transversely) polarized laser one could obtain a high energy polarized photon beam, this property will be used to suggest direct measurements of anomalous *CP*-violating couplings in $\gamma\gamma$. While similar measurements can be performed in e^+e^- colliders [11,12], the $\gamma\gamma$ mode has the advantage that it is possible to isolate the contribution of the γWW vertex. However the disadvantage is that polarization of the photon beams is not directly measured so will make it harder to control systematic errors. We will also show that polarization can be useful in enhancing the signal of anomalous couplings in either $\gamma\gamma\to W^+W^-$ or $e\gamma\to W\nu$ even when not looking directly at a *CP*-odd observable. The paper is organized as follows. After discussing our observables (Sec. II) and the spectrum of the photon beams (Sec. III), we present results for circularly polarized and transversely polarized photon beams in $\gamma\gamma$ collisions in Sec. IV. In Sec. V we discuss the sensitivity of various observables to the anomalous couplings in $e\gamma$ collisions. Section VI will be for discussion and comments of the results.

II. OBSERVABLES IN $\gamma\gamma$ AND $e\gamma$

The most general γWW vertex for on-shell W bosons contains seven independent form factors [12] three of which are C, P, and *CP* conserving (these are the usual g, κ, and λ), while three more violate *CP* (g_4, f_6, f_7) and the last (g_5) conserves *CP* but violates both C and P. The coupling g_4 must vanish for real photons in order to respect gauge invariance. The triple gauge boson vertex, including only the standard and *CP*-violating couplings for one real photon is then written as

$$\Gamma^{\alpha\beta\mu}(q_1,q_2,k)=-ie\left[(q_1-q_2)^\mu g^{\alpha\beta}+(q_2-k)^\alpha g^{\mu\beta}+(k-q_1)^\beta g^{\nu\alpha}+f_6\epsilon^{\mu\alpha\beta\rho}k_\rho+\frac{f_7}{M_W^2}(q_1-q_2)^\mu\epsilon^{\alpha\beta\rho\sigma}k_\rho(q_1-q_2)_\sigma\right], \quad (1)$$

where all momenta are ingoing, q_1,q_2 are the momenta of W^-, W^+, and k corresponds to the photon. Here we will consider only the effect of f_6. This simplification is justified by our *exploratory* approach: what observables are sensitive to this *CP*-violating coupling? Furthermore, the coupling f_7 which arises from a higher dimensional term in the Lagrangian is expected to be suppressed, relative to f_6, by a factor M_W^2/Λ^2 where Λ is the scale of new physics responsible for the interaction.[1]

In a weakly coupled theory, f_6 should be almost purely real. Nevertheless it is worth taking into account the possibility of an imaginary part that could arise from final state interactions. Such efforts are expected to be significant in a strongly interacting theory. This possibility cannot be excluded altogether since models with strongly interacting W's are among the ones that generate anomalous couplings. One has to remember however that if the underlying new physics comes from such a source, not only the vertex would be modified; in a complete calculation all new effects should be taken into account. Within our phenomenological approach, which is model independent, we cannot incorporate all of these. Even though the observables we discuss will undoubtedly be a sign of nonstandard physics, this new physics might not be due exclusively to f_6. We will show that both the real and imaginary part of anomalous couplings can be measured using either circularly or linearly polarized photons.

In $\gamma\gamma$ collisions, *CP* invariance is specified by the condition

$$\mathcal{M}^{\lambda_1\lambda_2}_{\lambda_3\lambda_4}=\mathcal{M}^{-\lambda_2-\lambda_1}_{-\lambda_4-\lambda_3}, \quad (2)$$

[1]In the language of chiral Lagrangians describing a strongly interacting Higgs sector, f_6 is related to two dimension-four operators,

$$f_6^\gamma=\frac{e^2}{16\pi^2\sin^2\theta}(L_{13}+L_{14}),$$

where the operators are written as [13]

$$\mathcal{L}_{13}=\frac{L_{13}}{16\pi^2}gg'\epsilon^{\mu\nu\rho\sigma}B_{\mu\nu}\,\mathrm{Tr}(TW_{\mu\nu}),$$

$$\mathcal{L}_{14}=\frac{L_{14}}{16\pi^2}g^2\epsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}(TW_{\mu\nu})\mathrm{Tr}(TW_{\rho\sigma}).$$

The coupling f_6 is also equivalent to the often-used $\tilde{\kappa}$.

where $\lambda_1,\lambda_2(\lambda_3,\lambda_4)$ are the helicities of incoming (outgoing) particles. *CPT* invariance, in the absence of absorptive parts in the amplitude is given by

$$\mathcal{M}^{\lambda_1\lambda_2}_{\lambda_3\lambda_4}=\mathcal{M}^{*-\lambda_2-\lambda_1}_{-\lambda_4-\lambda_3} . \tag{3}$$

We will also use the notion of $CP\hat{T}$ which characterizes a *CPT* invariant theory that contains some absorptive part so that Eq. (3) above is not satisfied [11]. The difference in the total cross sections when both photons have the same helicities is a typical $CP\hat{T}$ odd observable; it is defined as[2]

$$\hat{A}^{\gamma\gamma}_{LR}=\frac{\hat{\sigma}_{LL}-\hat{\sigma}_{RR}}{\hat{\sigma}_{LL}+\hat{\sigma}_{RR}} . \tag{4}$$

A nonvanishing asymmetry will automatically be proportional to the imaginary part of f_6. To observe directly *CP* violation in the absence of absorptive parts one must generate a phase so that interference with the standard model is possible. For this, linearly polarized laser beams can be used. The *CP*-odd observable will depend on the azimuthal angle of the outgoing $W(\phi)$ relative to the direction of the laser polarization. In a photon-photon collider, the center-of-mass energy of the two photons is not fixed, it is clearly advantageous then to choose a variable such as the azimuthal angle that is a boost-invariant quantity. The signal will correspond to a shift in the distribution $d\hat{\sigma}/d\phi$. Rather than using directly the distribution, it is more convenient to define the asymmetries

$$\hat{A}^{ij}_{\gamma\gamma}=\frac{D^{ij}}{S^{ij}} , \tag{5}$$

where

$$D^{ij}=\left[\int_0^{\pi/2}d\phi\frac{d\hat{\sigma}^{ij}}{d\phi}+\int_{\pi}^{3\pi/2}d\phi\frac{d\hat{\sigma}^{ij}}{d\phi}\right] - \left[\int_{\pi/2}^{\pi}d\phi\frac{d\hat{\sigma}^{ij}}{d\phi}+\int_{3\pi/2}^{2\pi}d\phi\frac{d\hat{\sigma}^{ij}}{d\phi}\right] \tag{6}$$

and

$$S^{ij}=2\int_0^{\pi}d\phi\frac{d\hat{\sigma}^{ij}}{d\phi} \tag{7}$$

the indices i,j denote the direction of the linear polarization of the photons. This asymmetry will measure the $\sin 2\phi$ term in $d\hat{\sigma}/d\phi$. As will be seen explicitly in the next section such a term is directly proportional to $\mathcal{R}(f_6)$. With this definition of D^{ij} and S^{ij} it is clear that charge identification of the W's is not necessary since the W^- and W^+ will always scatter at angle ϕ and $\phi+\pi$, respectively. This way the statistics can be increased by using the full angular distribution.

In addition to these direct *CP*-violating signals, many more observables are sensitive to anomalous couplings. While observables that are not explicitly *CP* odd cannot unambiguously establish *CP* violation, they are usually more straightforward to measure and can serve as a complement to the direct measurements. Total cross sections with or without polarized beams will depend on the coefficient $|f_6|^2$. Since the cross section for anomalous couplings diverges at high energies, under such conditions, this observable become very sensitive to the anomalous couplings. The process $e\gamma\to W\nu$ also offer numerous possibilities to probe the γWW coupling. In this case we use the fact that f_6 corresponds to a parity-violating operator and define *P*-odd observables. We consider, for instance, the difference in the total cross section with the photon helicities $\xi=\pm1$:

$$\hat{A}^{e\gamma}_{LR}=\frac{\hat{\sigma}_L-\hat{\sigma}_R}{\hat{\sigma}_L+\hat{\sigma}_R} . \tag{8}$$

Since the standard model violates P maximally we expect nonzero result even in the standard model, in that sense we are not directly probing this particular anomalous coupling. Still it will be seen that a precise measurement can be performed. Other observables are possible if linear polarization is available, in analogy with the $\gamma\gamma$ process we define an asymmetry computed from the distribution over the azimuthal angle of the W relative to the electron beam,

$$\hat{A}^{e\gamma}_{+-}=\frac{\hat{\sigma}_+-\hat{\sigma}_-}{\hat{\sigma}_++\hat{\sigma}_-} \tag{9}$$

with

$$\begin{aligned}\hat{\sigma}_+&=\left[\int_{7\pi/4}^{\pi/4}d\phi\frac{d\hat{\sigma}}{d\phi}+\int_{3\pi/4}^{5\pi/4}d\phi\frac{d\hat{\phi}}{d\phi}\right] ,\\ \hat{\sigma}_-&=\left[\int_{\pi/4}^{3\pi/4}d\phi\frac{d\hat{\sigma}}{d\phi}+\int_{5\pi/4}^{7\pi/4}d\phi\frac{d\hat{\sigma}}{d\phi}\right] .\end{aligned} \tag{10}$$

This asymmetry will measure the coefficient of $\cos 2\phi$. We will see explicitly in Sec. V that this coefficient will contain a standard part together with a term proportional to $\mathcal{I}(f_6)$ and $|f_6|^2$.

III. PHOTON POLARIZATION

The ability to polarize the high-energy photons is crucial to our discussion of *CP*-odd observables.[3] The general setup for realizing such a collider was first discussed a decade ago by Ginzburg *et al.* [14]. The formulas necessary for our discussion will be summarized here. We first define the quantities

$$x=\frac{4E\omega_0}{m_e^2}, \quad y=\frac{\omega}{E}, \quad r=\frac{y}{x(1-y)}\le 1 , \tag{11}$$

[2]The carets denote quantities before the integration over the energy spectrum of the photons is performed, i.e., for the subprocesses $\gamma\gamma\to W^+W^-$ or $e\gamma\to W\nu$.

[3]Other *CP*-odd observables which do not require polarization of the photon beams exist, they involve the measurement of the W polarization. A complete analysis will be presented elsewhere.

where E is the energy of the initial electron (or positron), ω_0 is the energy of the initial laser photon, m_e is the mass of the electron, and y is the ratio of the back-scattered photon's energy to the initial electron energy. For the type of physics that is of concern to us here, one would like to have y as large as possible since the anomalous coupling contribution to the total cross section grows rapidly with energy while the standard one is practically constant for energies above 300 GeV. For observables that are linear in the anomalous coupling, this rapid growth does not occur. Nevertheless a large y is still useful since a large fraction of the photon spectrum falls in the kinematically allowed range, increasing the effective luminosity of the photonic subprocess. The maximum value of y that can be obtained is dictated by the variable x characteristic of the laser chosen. In order to avoid pair production, one must have $x \leq 4.85$. In our numerical results, we used $x = 4.82$, so that the back-scattered photon beam can have up to 83% of the initial electron energy. The energy spectrum of the photons is given by

$$C_{00}(y)=\frac{1}{1-y}+1-y-4r(1-r) -2\lambda_e P_c rx(2r-1)(2-y) , \tag{12}$$

where λ_e is the average helicity of the initial electron and P_c is the degree of circular polarization of the initial laser beam. This spectrum has to be properly normalized such that $\int_0^{y_m} C_{00} dy=1$, where $y_m = x/(x+1)$ is the maximum value for y. With different choices for the polarizations, the energy spectrum of the photons can be modified; for example when polarizing both the electron (λ) and the laser (P_c), the photon spectrum will be either flat ($2\lambda P_c=1$) or peaked towards high energies ($2\lambda P_c=-1$). The back-scattered photons will retain a certain amount of the polarization of the laser photon beam. This polarization, which is energy dependent is determined by the three Stokes parameters ξ_i:

$$\langle \xi_i \rangle = \frac{C_{i0}}{C_{00}} \quad \text{with } \langle \xi_0 \rangle = 1 \tag{13}$$

and where the functions C_{i0} are written as

$$C_{10}=0, \quad C_{30}=2r^2 P_t , \tag{14}$$

and

$$C_{20}=2\lambda_e rx[1+(1-y)(2r-1)^2] -P_c(2r-1)\left[\frac{1}{1-y}+1-y\right] , \tag{15}$$

where now P_t is the degree of transverse polarization of the laser photon beam. The mean helicity of the high energy photon is given by $\langle \xi_2 \rangle$ while its degree of transverse polarization by $\langle \xi_1 \rangle$ and $\langle \xi_3 \rangle$. We will choose the transverse polarization of the laser to lie in the direction of the third axis (called X) in order to avoid unnecessary complications in the formulas.

It is then clear how to calculate the total cross section for the two processes of interest:

$$d\sigma(e\gamma \to W\nu_e) = \int_{y_{\min}}^{y_{\max}} \sum_{i=0,3} \langle \xi_i \rangle C_{00} d\hat{\sigma}_i(e\gamma \to W\nu_e)|_{\hat{s}=ys} dy , \tag{16}$$

where $d\hat{\sigma}_i$ are the cross sections corresponding to different polarization states of the photon. Note that instead of working with differential cross-sections, one could also work with integrated rates.

In the case of $\gamma\gamma$ collisions, things are slightly more complicated because the photon spectrum enters twice:

$$d\sigma(s)=\int_{4M_W^2/s}^{y_m^2} d\tau \int_{\tau/y_m}^{y_m} \frac{dy}{y} \sum_{i,j=0,3} \langle \xi_i \tilde{\xi}_j \rangle C_{00}(y) C_{00}(\tau/y) d\hat{\sigma}_{ij}(\gamma\gamma \to W^+W^-) , \tag{17}$$

where s is the total energy of the electron collider, $\tau=\hat{s}/s$ is the fraction of the initial energy for the subprocess $\gamma\gamma \to W^+W^-$, $\hat{\sigma}_{ij}$ is the cross section for a given polarization of the photons and is evaluated at $\hat{s}=ys$. $\xi_i, \tilde{\xi}_j$ are the Stokes parameters for photons 1 and 2. In the following we will always assume the factorization $\langle \xi_i \tilde{\xi}_j \rangle \approx \langle \xi_i \rangle \langle \tilde{\xi}_j \rangle$, $\langle \xi_i \rangle = C_{i0}/C_{00}$, and $\xi_0=1$. The formulas for unpolarized beams are easily recovered by putting $\langle \xi_i \tilde{\xi}_j \rangle = 0$ for $i \neq 0$ and $j \neq 0$. The choice of different polarizations for initial beams and the consequences for probing anomalous couplings will be discussed in the next sections in connection with the two different processes.

IV. TESTING ANOMALOUS COUPLINGS IN $\gamma\gamma \to W^+W^-$

For the process $\gamma\gamma \to WW$, we want to determine how polarization could be used to measure the CP-odd asymmetries defined in Sec. II. The two cases of circular and transverse polarization of the photons will be presented.

A. Transverse polarization

With a linearly polarized laser ($P_t=1$, $P_c=0$) and unpolarized electrons, one obtains a transversely polarized photon beam. Here we always define the direction of the laser polarization such that only the Stokes parameter $\langle \xi_3 \rangle$ differs from zero. The results for the helicity amplitudes with both photons polarized in the same direction (XX) or at right angle to each other (XY) are given in the appendix. All the amplitudes are written in terms of 2ϕ where ϕ is the azimuthal angle of the outgoing W^- relative to the direction of the polarization of one of the lasers (say laser 1). This direction will always be defined as the X axis. The differential cross section for the two sets of polarization chosen are then written, keeping only the terms in f_6, as

$$\frac{d\sigma^{XX}}{d\cos\theta\, d\phi}=\frac{\alpha^2\beta}{4s}\frac{1}{1-\beta^2\cos^2\theta}[64(1-2z+32z^2)-4\beta^2\sin^2\theta(8+24z\cos2\phi-3\beta^2\sin^2\theta\cos^2 2\phi)] \tag{18}$$

and

$$\frac{d\sigma^{XY}}{d\cos\theta\, d\phi}=\frac{\alpha^2\beta}{4s}\frac{1}{1-\beta^2\cos^2\theta}\Bigg[32(1+\cos^2\theta)+12\sin^2 2\phi\sin^4\theta-32z(4\cos^2\theta+3\sin^4\theta\sin^2 2\phi)$$
$$+192z^2\sin^2 2\phi\sin^4\theta-4\mathcal{R}(f_6)\sin^2\theta\sin2\phi$$
$$\times\left[\frac{\sin^2\theta}{z}-4(1-2\cos^2\theta+4z\cos^2\theta)\right]\Bigg], \tag{19}$$

where $\beta^2=1-4z$ and $z=M_W^2/s$. As one would expect, when both back-scattered photons are polarized in the same direction and after summation over the polarization of the W, the interference term between the anomalous contribution and the standard one vanishes, to measure a CP-odd observable the photons must be linearly polarized at some angle. Notice that σ^{XY} only depends on the real part of the CP-violating coupling. The coefficient of $\mathcal{J}(f_6)$ disappears after summation over the polarizations of the W (as it did for σ^{XX}). It is straightforward to verify that the asymmetry defined in the previous section picks up the $\sin2\phi$ term and that such a term appear only in $d\sigma^{XY}$. In fact the interference term in $d\sigma^{XY}$ is directly proportional to $\sin2\phi$ so the asymmetry we have chosen would give the maximum sensitivity to this parameter. After integration over $|\cos\theta|<c$ and over the azimuthal angle we obtain

$$D^{XX}=0\ , \tag{20}$$

$$D^{XY}=\mathcal{R}(f_6)\frac{8\alpha^2\beta}{M_W^2}\left[-c+\frac{2z}{\beta}\ln\left[\frac{1+\beta c}{1-\beta c}\right]\right], \tag{21}$$

while the polarized cross sections are given by

$$S^{XX}=\frac{2\pi\alpha^2\beta}{s}\left[\frac{c}{1-\beta^2c^2}(19-3c^2+12c^2z+72z^2)+\frac{12z^2}{\beta}(-1+3z)\ln\left[\frac{1+\beta c}{1-\beta c}\right]\right], \tag{22}$$

$$S^{XY}=\frac{2\pi\alpha^2\beta}{s}\left[\frac{c}{1-\beta^2c^2}(19-3c^2+12c^2z+24z^2)+\frac{12z^2}{\beta}(-1+z)\ln\left[\frac{1+\beta c}{1-\beta c}\right]\right], \tag{23}$$

where again only terms up to order f_6 only are written explicitly. Note that although σ^{XX} depends on the anomalous couplings through a term in $|f_6|^2$, such a term is independent of $\sin2\phi$ so that the asymmetry D^{XX} vanishes even to that order. This can be obtained easily from the complete expressions for the helicity amplitudes given in the Appendix.

For energies $s\gg M_W^2$, i.e., $z\ll1$ the cross sections for both sets of polarizations are practically equal and tend to a constant as the energy increases. In this region the numerator of the asymmetry (D^{XY}) is also mostly independent on the energy. So we expect both A^{XX} and A^{XY} to vary little with energy. This behavior would be altered when imposing a cut on $\cos\theta$ ($c<1$). Since most of the standard events near the beam, the polarized cross section will be reduced significantly by the cuts. On the other hand, the cuts will not reduce much D^{XY} since the interference terms between the standard and the anomalous contribution are proportional to $\sin^2\theta$, so that they vanish in the forward and backward direction. Imposing a cut on the angle θ would therefore enhance the signal from the anomalous coupling by reducing the "background" from standard events.

Since the polarization of the backscattered photon is governed by Eq. (13) and can make an arbitrary angle relative to the direction of the laser polarization, the measurement of $\mathcal{R}(f_6)$ can be done with the laser linearly polarized at any angle. We present results for two cases: parallel or perpendicular. When integrating over the energy spectrum of the back scattered photons, the asymmetry $\hat{A}^{ij}_{\gamma\gamma}$ defined in Eq. (5) becomes, for both laser polarized along the X axis,

$$A^{XX}=\frac{\int d\tau\frac{dy}{y}(C_{00}\tilde{C}_{00}+C_{00}\tilde{C}_{30}+C_{30}\tilde{C}_{00})D^{XY}}{\int d\tau\frac{dy}{y}(C_{00}\tilde{C}_{00}+C_{00}\tilde{C}_{30}+C_{30}\tilde{C}_{00})(S^{XX}+S^{XY})+2C_{30}\tilde{C}_{30}S^{XX}}\ . \tag{24}$$

The same asymmetry with the incoming laser having linear polarizations perpendicular to each other is written as

$$A^{XY}=\frac{\int d\tau\frac{dy}{y}(C_{00}\tilde{C}_{00}+C_{00}\tilde{C}_{30}+C_{30}\tilde{C}_{00}+2C_{30}\tilde{C}_{30})D^{XY}}{\int d\tau\frac{dy}{y}(C_{00}\tilde{C}_{00}+C_{00}\tilde{C}_{30}+C_{30}\tilde{C}_{00})(S^{XX}+S^{XY})+2C_{30}\tilde{C}_{30}S^{XY}}\ , \tag{25}$$

TABLE I. Asymmetry with transversely polarized photons beams and 3σ upper bounds on $\mathcal{R}(f_6)$.

Process	$\sqrt{s}$ (TeV)	A^{XX}	A^{YY}	Luminosity (fb^{-1})	Upper bound $\mathcal{R}(f_6)$
$\gamma\gamma \to WW$	0.5	$-0.080\mathcal{R}(f_6)$	$-0.083\mathcal{R}(f_6)$	20	2.3×10^{-2}
	0.5	$-0.080\mathcal{R}(f_6)$	$-0.083\mathcal{R}(f_6)$	100	1.0×10^{-2}
	1	$-0.116\mathcal{R}(f_6)$	$-0.118\mathcal{R}(f_6)$	100	0.6×10^{-2}
	2	$-0.137\mathcal{R}(f_6)$	$-0.139\mathcal{R}(f_6)$	100	0.5×10^{-2}

with the shorthand notation, $C_{i0}=C_{i0}(y)$ and $\tilde{C}_{i0}=C_{i0}(\tau/y)$. Both these quantities depend linearly on $\mathcal{R}(f_6)$ and are approximately the same since the contribution from the term in $C_{30}\tilde{C}_{30}$ is not very significant and $S^{XX}\approx A^{XY}$ at high energies.

In Table I we give the values for the two asymmetries and the limit on $\mathcal{R}(f_6)$ that can be derived from them. The energy dependence is also presented. Although slightly obscured by the fact that an integration over the energy spectrum of the photons was performed, the energy behavior of the two asymmetries correspond to the one of D^{XY}, S^{XX}, and S^{XY} just discussed. As expected, the asymmetries are almost constant when the energy is increased and integration over the full angular distribution is performed. The limits on f_6 are all given at 3σ taking into account only the statistical error.[4] Since the precision that can be achieved depends mainly on the ability to measure the standard contribution which does not increase significantly with energy, there is only a slight improvement when going at higher energies, on the other hand a higher luminosity would improve the limits. Concerning the effect of an angular cut, we found that the coefficient of $\mathcal{R}(f_6)$ in the asymmetry A^{XY} increased, at $\sqrt{s}=0.5$, 1, and 2 TeV, from -0.083, -0.118, -0.139 to -0.159, -0.400, -0.962, respectively, with a cut $|\cos\theta|<0.8$. The enhancement is clearly all the more effective for higher energy accelerators. In practice this cut is not a straightforward one to impose since the two photon beams are not monochromatic and the angle θ is not easily defined in the laboratory frame. A cut on p_T, which is related to $\cos\theta$ would have the same effect but with a lower efficiency.

B. Circular polarization

Circularly polarized photons offer the opportunity to measure $\mathcal{I}(f_6)$. As was shown by Ginzburg *et al.*, circular polarization can be obtained by polarizing either the laser, the electron beam or both [14]. The energy spectrum of the backscattered photons is governed by Eq. (12) and the circular polarization is specified by the Stokes parameter ξ_2 in Eq. (13). The shape of the different polarization spectra are given in [10]. The first thing to notice is that the Stokes parameter $\xi_2,\tilde{\xi}_2$ will each flip sign if the helicities of the initial beams and lasers are reversed $\lambda\to-\lambda, P_c\to-P_c$ while λP_c is kept constant and similarly for the lepton and laser beam on the other arm of the collider. This will allow for the measurement of the CP-odd asymmetry $\hat{A}_{LR}^{\gamma\gamma}$ defined in Eq. (4). To incorporate the energy spectrum of the photons we define a new asymmetry:

$$A_{LR}^{\gamma\gamma}=\frac{\sigma(s,\lambda,P_c,\lambda',P_c')-\sigma(s,-\lambda,-P_c,-\lambda',-P_c')}{\sigma(s,\lambda,P_c,\lambda',P_c')+\sigma(s,-\lambda,-P_c,-\lambda',-P_c')}\,. \tag{26}$$

Writing explicitly the integral over the energy spectrum of the photons we find

$$A_{LR}^{\gamma\gamma}=\frac{\int d\tau\int\frac{dy}{y}(C_{00}\tilde{C}_{20}+C_{20}\tilde{C}_{00})(d\hat{\sigma}_{LL}-d\hat{\sigma}_{RR})}{\int d\tau\int\frac{dy}{y}[C_{+}(d\hat{\sigma}_{LL}+d\hat{\sigma}_{RR})+C_{-}(d\hat{\sigma}_{LR}+d\hat{\sigma}_{RL})]}\,, \tag{27}$$

where $C_{+}=C_{00}\tilde{C}_{00}+C_{20}\tilde{C}_{20}$ and $C_{-}=C_{00}\tilde{C}_{00}-C_{20}\tilde{C}_{20}$. Clearly the numerator will vanish with CP invariance, and it must be directly proportional to $\mathcal{I}(f_6)$. This can be seen easily by making use of the helicity amplitudes for circularly polarized photons given in the appendix to obtain the total cross section for specific photon helicities:

$$\hat{\sigma}_{LL}+\hat{\sigma}_{RR}=\frac{16\pi\alpha^2\beta}{M_W^2}(1-z)(1-3z)\left[1+\frac{2z}{\beta}\ln\left[\frac{1+\beta}{1-\beta}\right]\right]\,, \tag{28}$$

[4]For a generic asymmetry $A=(a_+-a_-)/(a_++a_-)$ which vanish in the standard model, the precision to which it can be evaluated is given by $\Delta A=\Delta a_+/a_+$. If $A=a_6f_6$, the upper limit on the parameter f_6 at $n\sigma$ is given by $f_6\le(n/a_6)(\Delta a_+/a_+)=n/(a_6\sqrt{2N_+^{\text{st}}})$ where N_+^{st} is the number of standard events.

$$\hat{\sigma}_{LR}+\hat{\sigma}_{RL}=\frac{16\pi\alpha^2\beta}{M_W^2}\left[1+\frac{11}{2}z+3z^2-\frac{2z}{\beta}(1-z-3z^2)\ln\left[\frac{1+\beta}{1-\beta}\right]\right], \tag{29}$$

$$\hat{\sigma}_{LL}-\hat{\sigma}_{RR}=-\frac{32\pi\alpha^2\beta}{M_W^2}\mathcal{J}(f_6)\left[1-2z+\frac{z(3-8z)}{2\beta}\ln\left[\frac{1+\beta}{1-\beta}\right]\right], \tag{30}$$

where terms in $|f_6|^2$ are not written explicitly but were included in the calculations. *A priori* one would expect the anomalous cross section to grow with energy; this is indeed the case for the terms in $|f_6|^2$ as will be discussed in the next section. For the specific coupling we are considering however the interference term proportional to $\mathcal{J}(f_6)$ has the same high-energy behavior as the standard cross section; it tends to a constant. This can be verified easily with the helicity amplitudes. The term proportional to $1/z$ that should come from the amplitude for longitudinal W's is absent because the standard amplitude and the anomalous one contribute to different photon helicity amplitudes. From this we deduce that although it is important to have high-energy photons so that a large fraction of the back-scattered photon spectrum is in the kinematically allowed region, there is no obvious advantage in using polarization to maximize the number of hard photons. Rather, it is the convolution of the polarization and energy spectra that should be optimized. Although many choices of polarizations are possible to be able to have a sizable asymmetry, it is crucial to always set-up the polarization of the beams on each arm of the collider such that $\lambda P_c=\lambda' P_c'$. We found that the largest asymmetry was obtained with $\lambda P_c=\frac{1}{2}$ while $\lambda P_c=-\frac{1}{2}$ gave a slightly smaller symmetry. Another interesting possibility is to polarize only the lepton beams. Although the photon polarization is not as good, ξ_2 is always positive so no cancellation can occur. For the two most favorable choices of polarizations, the values for the asymmetry and the limits on $\mathcal{J}(f_6)$ that can be derived from them are given in Table II. The same method as for the transversely polarized beams is used to derive the limits. Again, only the statistical errors are included. The limits are roughly an order of magnitude better than the ones on $\mathcal{R}(f_6)$ for the same energy. Note that in this case there would be no interest in introducing cuts in $\cos\theta$ or p_T since the anomalous and standard cross sections have similar angular distribution.

C. Total cross section

We now compare the limits just obtained from *CP*-odd observables with limits from total cross-section measurements. One important caveat in this type of analysis is that even if something is observed it will be hard to tell if the nonstandard effect comes from f_6 or from some other non-standard physics such as other *CP* conserving anomalous couplings for example. Certainly there could not be any claim of observation of *CP* violation. Furthermore if nothing is observed the limit on the parameter f_6 could always be ignored by appealing to some cancellation between various anomalous contributions. With these warnings in mind, we proceed with the analysis of this type of observables since it is useful to determine the maximum sensitivity to the anomalous coupling on its own.

With the total cross section, only the term in $|f_6|^2$ can be measured since the helicity amplitudes with two longitudinal W's grow with energy, it will clearly be advantageous to use polarization to increase the average effective energy of the $\gamma\gamma$ subprocess. We write

$$\sigma_{\text{tot}}=\sigma_{\text{st}}(1+R_{\text{ano}}|f_6|^2), \tag{31}$$

where $R_{\text{ano}}=\sigma_{\text{ano}}/\sigma_{\text{st}}$ and σ_{ano} is the coefficient of $|f_6|^2$ in the total cross section. In Table III we give the limits on $|f_6|^2$ obtained from a measurement of the total cross section using either unpolarized photon beams or circularly polarized ones, with the polarizations that give the most favorable energy spectrum, $\lambda P_c=\lambda' P_c'=-\frac{1}{2}$. Again only statistical errors are taken into account. We find that R_{ano} increases with energy and the use of polarized beams. We see that at 2 TeV the results are within a factor of 2 of the ones obtained on $\mathcal{R}(f_6)$ directly but are never competitive with the better limits on $\mathcal{J}(f_6)$. Comparisons between the different methods should however be done with care until a complete analysis of errors, including systematics, is performed. Note that the limit

TABLE II. Asymmetry with circularly polarized photons and 3σ upper bounds on $\mathcal{J}(f_6)$.

Process	$\sqrt{s}$ (TeV)	A_{LR}	Luminosity (fb^{-1})	Upper bound $\mathcal{J}(f_6)$
$\gamma\gamma\to WW$	0.5	$-2.11\mathcal{J}(f_6)$	20	1.2×10^{-3}
$\lambda=\lambda'=1/2, P_c=P_c'=1$	0.5	$-2.11\mathcal{J}(f_6)$	100	5.2×10^{-4}
	1	$-1.99\mathcal{J}(f_6)$	100	4.0×10^{-4}
	2	$-1.94\mathcal{J}(f_6)$	100	3.7×10^{-4}
$\gamma\gamma\to WW$	0.5	$1.53\mathcal{J}(f_6)$	20	1.4×10^{-3}
$\lambda=\lambda'=-1/2, P_c=P_c'=0$	0.5	$1.53\mathcal{J}(f_6)$	100	6.2×10^{-4}
	1	$1.36\mathcal{J}(f_6)$	100	5.6×10^{-4}
	2	$1.25\mathcal{J}(f_6)$	100	5.6×10^{-4}

TABLE III. Total cross section and 3σ upper bounds on $|f_6|^2$.

Process	$\sqrt{s}$ (TeV)	$R_{\rm ano}$	Luminosity ($\rm fb^{-1}$)	Upper bound $|f_6|^2$
$\gamma\gamma\to WW$	0.5	1.8	20	1.7×10^{-3}
$\lambda=\lambda'=0, P_c=P_c'=0$	1	3.8	100	0.3×10^{-3}
	2	10.7	100	0.1×10^{-3}
$\gamma\gamma\to WW$	0.5	2.1	20	1.4×10^{-3}
$\lambda=\lambda'=-1/2, P_c=P_c'=1$	1	6.3	100	0.2×10^{-3}
	2	16.5	100	0.09×10^{-3}

shown in Table III would improve with a cut on $\cos\theta$ since such a cut reduces the standard cross section which is very forward peaked while affecting much less the anomalous one. At 500 GeV the effect of a cut is marginal while at TeV a cut $|\cos\theta|<0.8$ improves the limit on $|f_6|^2$ by 25%.

V. TESTING ANOMALOUS COUPLINGS IN $e\gamma\to W\nu$

We now turn to the $e\gamma\to W\nu$ process. Again, we will consider two polarizations of the photon: transverse and circular.

A. Transverse polarization

We assume that the experimental setup is such that only C_{00} and C_{30} are nonzero. In this transverse basis, the X axis corresponds to the polarization axis of the initial laser photons and

$$d\hat{\sigma}_0=\tfrac{1}{2}(\hat{\sigma}_X+\hat{\sigma}_Y)\quad\text{and}\quad d\hat{\sigma}_3=\tfrac{1}{2}(\hat{\sigma}_X-\hat{\sigma}_Y) \tag{32}$$

where

$$\hat{\sigma}_X=\hat{\sigma}(e\gamma\to W\nu)\quad\text{with } \epsilon_\gamma=(0,1,0,0) \tag{33}$$

and

$$\hat{\sigma}_Y=\hat{\sigma}(e\gamma\to W\nu)\quad\text{with } \epsilon_Y=(0,0,1,0)\ . \tag{34}$$

Upon summation over the polarizations of the W boson and integration over $\cos\theta$, we obtain the following expression for the angular distribution in ϕ, the azimuthal angle of the W relative to the direction of the electron beam:

$$\begin{aligned}\frac{d\hat{\sigma}_X}{d\phi}=\frac{\alpha^2}{32\sin^2\theta_W\delta(1-\delta)}\Bigg\{&14\delta^4-38\delta^3+32\delta^2+(8\delta^3-24\delta^2+32\delta)(1-\delta)\ln(1-\delta)\\&+\cos2\phi*4\delta(1-\delta)^2[4\delta+(4-2\delta)\ln(1-\delta)]+\frac{|f_6|^2}{4}\frac{\delta}{1-\delta}\{3\delta^2-5\delta^3-4\delta(1-\delta)\ln(1-\delta)\\&+4\cos2\phi*(1-\delta)[2\delta+(2-\delta)\ln(1-\delta)]\}+\mathcal{J}(f_6)[12\delta^3-24\delta^2+16\delta+(8\delta^2-16\delta+16)(1-\delta)\ln(1-\delta)\\&+\cos2\phi*(16\delta^3-28\delta^2+16\delta+(8\delta^2-20\delta+16)(1-\delta)\ln(1-\delta))]\Bigg\}\ ,\end{aligned} \tag{35}$$

where $\delta=1-M_W^2/\hat{s}$. The substitution $\cos2\phi\to-\cos2\phi$ and $\mathcal{J}(f_6)\to-\mathcal{J}(f_6)$ allows one to go from $d\hat{\sigma}_X/d\phi\to d\hat{\sigma}_Y/d\phi$. It is a straightforward exercise to show that this unpolarized cross section [i.e., $\frac{1}{2}(\sigma_X+\sigma_Y)$] agrees with the known result

$$\sigma(e\gamma\to W\nu_e)=\frac{\pi\alpha^2}{8z^3M_W^2\sin^2\theta_W}[8z^3+2z^2+4z-14-(16z^2+8z-8)\ln(z)]\ , \tag{36}$$

where, as before, $z=M_W^2/\hat{s}=1-\delta$ and the other constants are standard.

The main features of the process under consideration are that upon summation over the polarizations of the outgoing W-boson, the terms that depend on $\mathcal{R}(f_6)$ cancel exactly and that the radiation amplitude zero (the vanishing of the standard amplitude in the forward direction) is spoiled by the $|f_6|^2$ term, as can be seen from Eqs. (B7)–(B9). If one were to measure the polarization of the outgoing W boson, the dependence on $\mathcal{R}(f_6)$ would be retained. The radiation amplitude zero (RAZ) however is more difficult to exploit because it is the distribution of the W boson in the center of mass frame that contains this information and not its decay products. It has been shown previously, albeit in a slightly different context, that such a RAZ is washed out when one considers the complete decay chain and appropriate backgrounds [15].

Using these expressions, we can calculate the asymmetry defined earlier that is sensitive to $\cos2\phi$. To have a more compact notation, Eq. (35) is first rewritten as

$$\begin{aligned}\frac{d\hat{\sigma}_X}{d\phi}=&[a+b\cos2\phi]+|f_6|^2[d+e\cos2\phi]\\&-\mathcal{J}(f_6)[f+g\cos2\phi]\end{aligned} \tag{37}$$

and similarly for $d\hat{\sigma}_Y$. Both Eqs. (16) and (32) can be used to obtain the angular distribution after integrating over the photon spectrum. Using Eqs. (9) and (10) the

TABLE IV. Asymmetry with transversely polarized photons beams and 3σ upper bounds on $\mathcal{I}(f_6)$.

Process	$\sqrt{s}$ (TeV)	A_{+-}	$\int \mathcal{L}\, dt$ (fb^{-1})	Upper bound $\mathcal{I}(f_6)$
$e\gamma \to W\nu$	0.5	$0.322\mathcal{I}(f_6)$	20	2.4×10^{-2}
	0.5	$0.322\mathcal{I}(f_6)$	100	1.1×10^{-2}
	1.0	$0.304\mathcal{I}(f_6)$	100	1.0×10^{-2}
	2.0	$0.295\mathcal{I}(f_6)$	100	9.6×10^{-3}

asymmetry A_{+-} is easily computed:

$$A_{+-}=\frac{\sigma_+-\sigma_-}{\sigma_++\sigma_-}=-\frac{1}{\pi}\frac{\mathcal{I}(f_6)g_{00}+b_{30}+|f_6|^2e_{30}}{a_{00}+|f_6|^2d_{00}-\mathcal{I}(f_6)f_{30}}\,, \tag{38}$$

where we have included the 00 and 30 indices to explicitly indicate that integration over the photon spectrum is included. The main features of this asymmetry are that the coefficients a_{00}, d_{00}, and g_{00} are much larger than the coefficients b_{30}, e_{30}, and f_{30}. In the standard model, this asymmetry is unmeasurably small, -1.1×10^{-4} at $\sqrt{s}=500$ GeV with a statistical error of $\pm5.8\times10^{-4}$ at 100 fb^{-1}. These figures become -9.7×10^{-6} and $\pm5\times10^{-4}$ at 1 TeV and 100 fb^{-1}. Even the addition of the $|f_6|^2=1$ term increases the asymmetry by 50%; still unmeasurable. In setting a bound on $\mathcal{I}(f_6)$, we then feel justified to drop the $|f_6|^2$ term in the denominator and write, approximately,

$$A_{+-}=-\frac{1}{\pi}\frac{\mathcal{I}(f_6)g_{00}}{a_{00}}\,. \tag{39}$$

The bounds obtained on $\mathcal{I}(f_6)$ are listed in Table IV. We see clearly that an intense beam is much more advantageous than a beam at high energy at least with this kind of analysis which includes only statistical errors.

We also considered the total cross-section in the limit where $\mathcal{I}(f_6)=0$. This allows us to put constraints on $|f_6|^2$. As before, we define

$$\sigma_{\text{tot}}=\sigma_{\text{st}}(1+R_{\text{ano}}|f_6|^2)=2\pi a_{00}\left[1+\frac{d_{00}}{a_{00}}|f_6|^2\right]\,. \tag{40}$$

As we can see in Table V, a high energy beam is preferable for this observable although the level of precision is only $\approx10^{-1}$ for $\mathcal{R}(f_6)$. One could use the total cross-section to set a bound on $|f_6|^2$ and then the asymmetry (with the $|f_6|^2$ term set to 0 in the denominator) to set a bound on $\mathcal{I}(f_6)$ *independently*. Admittedly, a complete two-parameter analysis with χ^2 methods would be more appropriate but we would expect that the bounds obtained from such an analysis would be similar to the bounds obtained in our one-parameter analysis; our results represent the best limits one could reach.

Considering Eqs. (B7)–(B9) one sees that the SM contribution to this process peaks in the backward direction (i.e., in the direction of the incoming photon). The *left* and *right* anomalous contributions to the cross section have similar angular distributions while the *longitudinal* anomalous contribution has a *constant* term. It is clear then, that imposing a cut on $\cos\theta$ would increase the sensitivity to the anomalous term. However, this angle is not well defined in the laboratory frame since we are dealing with a photon beam of broad spectrum.

B. Circular polarization

Here we assume that the experimental set-up is such that only C_{00} and C_{20} are nonzero. In this longitudinal basis,

$$d\hat\sigma_0=\tfrac{1}{2}(\hat\sigma_R+\hat\sigma_L)\quad\text{and}\quad d\hat\sigma_2=\tfrac{1}{2}(\hat\sigma_R-\hat\sigma_L)\,, \tag{41}$$

where $\hat\sigma_{R(L)}$ are the cross sections with right-(left-)handed photons. As before, we sum over the polarization of the W boson. After integration over $\cos\theta$ we obtain the cross sections

TABLE V. Total cross sections with transversely polarized beams and 3σ upper bounds on $|f_6|^2$.

Process	$\sqrt{s}$ (TeV)	R_{ano}	$\int \mathcal{L}\, dt$ (fb^{-1})	Upper bound $|f_6|^2$
$e\gamma \to W\nu$	0.5	3.8	20	2.1×10^{-1}
	0.5	3.8	100	1.4×10^{-1}
	1.0	18	100	6.3×10^{-2}
	2.0	73	100	6.6×10^{-2}

$$\hat{\sigma}_L=\frac{\pi\alpha^2}{4\hat{s}\sin^2\theta_W}\frac{1}{(1-\delta)}\{10\delta^3-38\delta^2+32\delta-(32-24\delta+4\delta^2)(1-\delta)\ln(1-\delta)$$
$$-\frac{|f_6|^2}{4}[\delta(2-\delta)+\ln(1-\delta)]+\mathcal{I}(f_6)4[(3\delta-2\delta^2)+(3-\delta)(1-\delta)\ln(1-\delta)]\} \tag{42}$$

and

$$\hat{\sigma}_R=\frac{\pi\alpha^2}{4\hat{s}\sin^2\theta_W}\frac{1}{1-\delta}\left[4\delta[\delta+(1-\delta)\ln(1-\delta)][\delta-\mathcal{I}(f_6)]+\frac{|f_6|^2}{4}[2\delta+(2-4\delta)\ln(1-\delta)]\right]. \tag{43}$$

The well-known result for unpolarized cross section $[\frac{1}{2}(\sigma_L+\sigma_R)]$ is easily recovered from these and was also checked numerically. These results also agree with the ones of Baur and Zeppenfeld [16] at the helicity amplitude level.

Experimentally, one has control over the average helicity of the initial electron and the helicity of the initial laser photon. To measure, for the full process, the asymmetry defined in Eq. (8), two different polarizations setups will be needed. For example, we consider $\lambda_e=-\frac{1}{2}$ and $P_c=0$ which gives $\langle\xi_2\rangle\le 0$ over the whole energy range of the photon and $\lambda_e=\frac{1}{2}$ and $P_c=0$ which gives $\langle\xi_2\rangle\ge 0$ also over the whole energy range of the photon. We call these states $\bar{\sigma}_L$ and $\bar{\sigma}_R$, respectively. Using Eq. (16) one obtains

$$\bar{\sigma}_{L,R}=\frac{\hat{\sigma}_R+\hat{\sigma}_L}{2}C_{00}\mp\frac{\hat{\sigma}_R-\hat{\sigma}_L}{2}C_{20},$$

where C_{20} is defined with $\lambda_e=\frac{1}{2}$, $P_c=0$. The left-right asymmetry is then defined as

$$A_{LR}^{e\gamma}=\frac{\bar{\sigma}_L-\bar{\sigma}_R}{\bar{\sigma}_L+\bar{\sigma}_R}=\frac{\int dy(\hat{\sigma}_L-\hat{\sigma}_R)C_{20}}{\int dy(\hat{\sigma}_L+\hat{\sigma}_R)C_{00}}. \tag{44}$$

It is clear that this asymmetry will not vanish in the SM, since the cross section for left- and right-handed photons differ.

In order to evaluate the bounds that can be derived for both $|f_6|^2$ and $\mathcal{I}(f_6)$, we write a typical cross section as

$$\sigma_L=\sigma_L^S+\mathcal{I}(f_6)\sigma_L^i+|f_6|^2\sigma_L^6. \tag{45}$$

We assume that any deviation from the expectation values of the SM is due to either $|f_6|^2$ or $\mathcal{I}(f_6)$. It is then straightforward to reach the following expression for the upper limit of $|f_6|^2$:

$$|f_6|^2=-\frac{(A_{LR}\pm\Delta_{LR})C_{00}(\sigma_L^S+\sigma_R^S)-(\sigma_L^S-\sigma_R^S)C_{20}}{(A_{LR}\pm\Delta_{LR})C_{00}(\sigma_L^6+\sigma_R^6)-(\sigma_L^6-\sigma_R^6)C_{20}}, \tag{46}$$

where Δ_{LR} is the error on A_{LR}. A similar result holds for $\mathcal{I}(f_6)$ with $\sigma_{L,R}^6\to\sigma_{L,R}^i$. When considering the cross sections with a given laser photon polarization independently, the limit on $|f_6|^2$ is given by

$$|f_6|^2\le\frac{2\Delta_L}{(\sigma_R^6+\sigma_L^6)C_{00}-(\sigma_R^6-\sigma_L^6)C_{20}} \tag{47}$$

or

$$|f_6|^2\le\frac{2\Delta_R}{(\sigma_R^6+\sigma_L^6)C_{00}+(\sigma_R^6-\sigma_L^6)C_{20}}, \tag{48}$$

where $\Delta_{L(R)}$ is the error on $\bar{\sigma}_L(\bar{\sigma}_R)$. Similar expressions are obtained for $\mathcal{I}(f_6)$. The limits on f_6 and $\mathcal{I}(f_6)$ derived with this method are given in Tables VI and VII. Slightly better limits are obtained from the polarized cross section than from the asymmetry. In an experimental situation, the asymmetry might still be better because of lower systematics.

Again, considering Eqs. (B5) and (B6) one sees that a cut on $\cos\theta$ could improve the sensitivity to anomalous couplings. Since our c.m. is not well defined, the sensitivity in a concrete experiment would be degraded.

VI. DISCUSSION

The results of the previous sections show that when taking into account only statistical errors, the sensitivity of $\gamma\gamma\to W^+W^-$ and $e\gamma\to W\nu$ to the CP-violating coupling are typically 10^{-3} for $\mathcal{I}(f_6)$ and 10^{-2} for $\mathcal{R}(f_6)$. The calculations clearly show that a precise knowledge of

TABLE VI. 3σ limits on $|f_6|$ from cross sections and asymmetry with circularly polarized photons in $e\gamma\to W\nu_e$.

$\sqrt{s}$ (TeV)	$\int\mathcal{L}\,dt$ (fb^{-1})	$|f_6|$ from $\bar{\sigma}_R$	$|f_6|$ from $\bar{\sigma}_L$	$|f_6|$ from A_{LR}
0.5	20	4.6×10^{-2}	5.2×10^{-2}	3.6×10^{-1}
0.5	100	3.0×10^{-2}	3.4×10^{-2}	2.4×10^{-1}
1.0	100	2.8×10^{-2}	3.0×10^{-2}	2.4×10^{-1}
2.0	100	2.4×10^{-2}	2.6×10^{-2}	2.3×10^{-1}

TABLE VII. 3σ limits on $I(f_6)$ from cross sections and asymmetry with circularly polarized photons in $e\gamma \rightarrow W\nu_e$.

$\sqrt{s}$ (TeV)	$\int \mathcal{L}\, dt$ (fb^{-1})	$I(f_6)$ from $\bar{\sigma}_R$	$I(f_6)$ from $\bar{\sigma}_L$	$I(f_6)$ from A_{LR}
0.5	20	3.4×10^{-3}	2.2×10^{-3}	2.3×10^{-2}
0.5	100	1.5×10^{-3}	9.4×10^{-4}	1.0×10^{-2}
1.0	100	1.4×10^{-3}	8.8×10^{-4}	9.6×10^{-3}
2.0	100	1.3×10^{-3}	8.4×10^{-4}	9.8×10^{-3}

the photon spectrum is very important in trying to assess the sensitivity of a particular process to an anomalous coupling. Both the cross section and the polarization state of the back-scattered photon will depend on that spectrum. A complete study with detector simulations should therefore include all these variables. The results given here represent the best limits that could be obtained with both a precise knowledge of the spectrum and the full identification of the final state. Other effects such as beamstrahlung have not been taken into account because they are very much machine dependent and accelerator physicists are seriously discussing the possibility of getting rid *completely* of beamstrahlung.

For the two processes considered, high-energy beams are preferable when measuring observables dependent on $|f_6|^2$, due to the rapid growth of the anomalous cross section with energy. This applies in particular to *CP*-even observables such as the total cross section in $\gamma\gamma$ collisions and to the determination of the real part of f_6 in $e\gamma$ using transversely polarized beams (possible only by measuring $|f_6|^2$). On the other hand, it is advantageous to achieve a high luminosity to determine $\mathcal{I}(f_6)$ in $e\gamma$ colliders or to measure a *CP*-odd observable in $\gamma\gamma$. For the latter, the precision on both $\mathcal{R}(f_6)$ and $\mathcal{I}(f_6)$ increases somewhat with the energy of the collider until 1 TeV, but the gain in precision is marginal compared to a gain due to an increase in luminosity. However, one should keep in mind that an increase in luminosity is very useful inasmuch as only the statistical error is concerned. Even with 20 fb^{-1}, the processes we have studied will be statistics limited since the limits we have quoted correspond to an error better than 1%. With a 2% systematic error at 1 TeV, the best limit on $\mathcal{I}(f_6)$ and $\mathcal{R}(f_6)$ would go up to 10^{-2} and 10^{-1}, respectively, with no angular cuts. Only the limit on $\mathcal{R}(f_6)$ could improve with judicious cuts on $\cos\theta$ or p_T of the W.

Throughout this discussion we have concentrated on a gauge boson final state. Clearly, these gauge bosons will decay well within the detector and one should consider a full decay chain in order to have a complete picture. In the case of $e\gamma \rightarrow W\nu_e$, previous work where the nonresonant backgrounds were taken into account has shown that one could loose a factor of 2–5 in sensitivity to an anomalous coupling [15]. Clearly, when dealing with the hadronic decay mode of the W boson, an invariant mass close to M_W greatly reduces the background. Considering the full decay chain also makes the cuts on $\cos\theta$ less effective: the cuts are clearly effective when imposed in the c.m. frame on the W boson. This would require very good reconstruction of the jets in order to get to the W boson but the missing neutrino means that we cannot get to the c.m. frame; thereby degrading the effectiveness of the cut.

In the case of $\gamma\gamma \rightarrow WW$, the hadronic decay channels are a bit confusing because one has four jets to work with. One could think of requiring two jets relatively close to each other in one hemisphere (these would come from a *fast W*) and no cut on the other two jets (these could come from a *fast* or a *slow W*). Presumably, such techniques could allow almost complete reconstruction of the hadronic decay channels. One leptonic and one hadronic decay channel is also a nice signal. A potential background to these signals is pair production of heavy quarks that then decay into hadrons and leptons. Likely, requiring a large invariant mass on the jets would reduce this background almost to zero. Two leptonic decay channels can lead to some confusion when one considers the τ-pair production followed by its decay into lighter leptons. There is a relatively simple way out of this background though. It has been shown [17] that requiring a large invariant mass for the pair of charged leptons will not reduce much the signal while it will completely destroy this type of background. We then expect that these techniques will lead to a clear signal. In both modes, the very clean environment of an e^+e^- collider is a definite advantage.

The bounds we have obtained using direct measurements of the triple vector bosons vertex are comparable to indirect ones from electron EDM. They are also comparable to those that can be reached in some particular extensions [8] of the SM. The physics potential of a high energy and high luminosity e^+e^- collider used in the $e\gamma$ or $\gamma\gamma$ mode is then very relevant to probe models beyond the SM that try to accommodate the phenomenon of *CP* violation.

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APPENDIX A

The formulas for the helicity amplitudes for the process $\gamma(k_1^\mu,\lambda_1)\gamma(k_2^\nu,\lambda_2)\rightarrow W^-(q_1^\alpha,\lambda^3)W^+(q_2^\beta,\lambda_4)$ are listed here. The spinor technique used is explained in a number of publications [18]. The incoming photons are taken to be along the z axis while θ and ϕ define the solid angle for the W^-:

$$k^{\mu}_{1,2}=\frac{\sqrt{s}}{2}(1;0,0,\pm1) \tag{A1}$$

and

$$q^{\mu}_{1,2}=\frac{\sqrt{s}}{2}(1;\pm\beta\sin\theta\cos\phi\,,\ \pm\beta\sin\theta\sin\phi,\pm\beta\cos\theta)\,, \tag{A2}$$

where $\beta=\sqrt{1-4z}$ and $z=m_W^2/s$. The polarization vectors of the photons are chosen as follows in a Cartesian basis:

$$\epsilon^{x}_{1,2}=(0,\pm1,0,0),\quad \epsilon^{y}_{1,2}=(0,0,1,0) \tag{A3}$$

for photons 1 and 2, respectively, and where the X axis is defined by the polarization vector of photon 1. The helicity basis vectors are related to the above by

$$\epsilon_{\lambda_i}=\frac{-\lambda_i\epsilon^x_i-i\epsilon^y_i}{\sqrt{2}}\,, \tag{A4}$$

where $\lambda_i=\pm1$ refers to the two helicity states of photon i. For the gauge bosons the transverse polarization vectors are defined as

$$\epsilon^{\lambda_3 *}_{W^-}=\frac{1}{\sqrt{2}}(0,-\lambda_3\cos\theta\cos\phi-i\sin\phi,-\lambda_3\cos\theta\sin\phi+i\cos\phi,\lambda_3\sin\theta)\,, \tag{A5}$$

$$\epsilon^{\lambda_4 *}_{W^+}=\frac{1}{\sqrt{2}}(0,\lambda_4\cos\theta\cos\phi-i\sin\phi,\lambda_4\cos\theta\sin\phi+i\cos\phi,-\lambda_4\sin\theta)\,, \tag{A6}$$

where $\lambda_3,\lambda_4=\pm1$ denote the helicities of the transverse W^- and W^+, respectively. The longitudinal polarization vectors are

$$\epsilon^{0}_{W^-,W^+}=\frac{1}{2\sqrt{z}}(\beta,\pm\sin\theta\cos\phi,\pm\sin\theta\sin\phi,\pm\cos\theta)\,. \tag{A7}$$

The helicity amplitudes are written as

$$M^{i,j}_{\lambda_3,\lambda_4}=\frac{e^2\overline{M}^{i,j}_{\lambda_3,\lambda_4}}{1-\beta^2\cos^2\theta}\,, \tag{A8}$$

where $i,j=x,y$ refers to the photon polarization vectors in the Cartesian basis. With these conventions, the helicity amplitudes for both photons polarized in the same direction are given by

$$\overline{M}^{XX}_{00}=8z+2(1+4z)\sin^2\theta\cos2\phi-\frac{f_6^2}{4z}\{4z+\sin^2\theta[2-6z-(1+2z)\cos2\phi]\}\,, \tag{A9}$$

$$\begin{aligned}\overline{M}^{XX}_{\lambda_3,0}=&-4\sqrt{2z}\,\sin\theta(\lambda_3\cos\theta\cos2\phi+i\sin2\phi)+f_6\frac{\beta}{\sqrt{2z}}\sin\theta[-\lambda_3\sin2\phi+i\cos\theta(-1+\cos2\phi)]\\&+\frac{f_6^2}{4\sqrt{2z}}\sin\theta\{2\lambda_3\cos\theta[1-2z-(1+2z)\cos2\phi]-i(3-\beta^2\cos^2\theta)\sin2\phi\}\,,\end{aligned} \tag{A10}$$

$$\overline{M}^{XX}_{0\lambda_4}=M^{XX}_{\lambda_3 0}(\lambda_3\to\lambda_4,\cos\theta\to-\cos\theta,\sin\theta\to-\sin\theta)\,, \tag{A11}$$

$$\begin{aligned}\overline{M}^{XX}_{\lambda_3\lambda_4}=&\,2(1+\lambda_3\lambda_4)[1-2z(1+\sin^2\theta\cos2\phi)]-(1-\lambda_3\lambda_4)(1+\cos^2\theta)\cos2\phi\\&-2i(\lambda_3-\lambda_4)\cos\theta\sin2\phi+if_6\beta(\lambda_3+\lambda_4)(1+\cos^2\theta+\sin^2\theta\cos2\phi)+\frac{f_6^2}{16z}\{-(1+\lambda_3\lambda_4)g(z,\theta)+4\lambda_3\lambda_4z\sin^2\theta\\&-[(1-\lambda_3\lambda_4)g(z,\theta)+12\lambda_3\lambda_4z\sin^2\theta]\cos2\phi-i(\lambda_3-\lambda_4)g(z,\theta)\cos\theta\sin2\phi\}\,.\end{aligned} \tag{A12}$$

When both photon polarizations are orthogonal, the helicity amplitudes are simply given by

$$\overline{M}^{XY}_{00}=-2\sin^2\theta\left[(1+4z)\sin2\phi+\frac{f_6^2}{8z}(1+2z)\sin2\phi\right]+f_6\left[\frac{1-4z}{z}\sin^2\theta+4\right]\,, \tag{A13}$$

$$\begin{aligned}\overline{M}^{XY}_{\lambda_3 0}=&-4\sqrt{2z}\,\sin\theta[-\lambda_3\cos\theta\sin2\phi+i\cos2\phi]-\frac{f_6\sin\theta}{\sqrt{2z}}[\lambda_3\cos\theta(1-4z)+\lambda_3\beta\cos2\phi+i\beta\cos\theta\sin2\phi]\\&-\frac{f_6^2\sin\theta}{4\sqrt{2z}}\{i[2\beta\cos\theta+(3-\beta^2\cos^2\theta)\cos2\phi]-2\lambda_3(1+2z)\cos\theta\sin2\phi\}\,,\end{aligned} \tag{A14}$$

$$\overline{M}^{XY}_{0\lambda_4}=M^{XY}_{\lambda_3 0}(\lambda_3\to\lambda_4,\cos\theta\to-\cos\theta,\sin\theta\to-\sin\theta)\,, \tag{A15}$$

$$\overline{M}^{XY}_{\lambda_3\lambda_4}=-2i[(\lambda_3+\lambda_4)\beta+(\lambda_3-\lambda_4)\cos\theta\cos2\phi]+[(1-\lambda_3\lambda_4)(1+\cos^2\theta)+4z(1+\lambda_3\lambda_4)\sin^2\theta]\sin2\phi$$
$$+f_6[-i(\lambda_3+\lambda_4)\beta\sin^2\theta\sin2\phi+2(1+\lambda_3\lambda_4)]-\frac{f_6^2}{16z}(-[(1-\lambda_3\lambda_4)g(z,\theta)+12z\lambda_3\lambda_4\sin^2\theta]\sin2\phi$$
$$+i\{\beta(\lambda_3+\lambda_4)[1-(1+4z)\cos^2\theta]-(\lambda_3-\lambda_4)g(z,\theta)\cos\theta\cos2\phi\})\ , \tag{A16}$$

where $g(z,\theta)=\sin^2\theta(1-4z)+12z$. The helicity amplitudes for other photon polarizations are simply related to these:

$$M^{YX}=M^{XY}(\sin2\phi\to-\sin2\phi,\cos2\phi\to-\cos2\phi)\ , \tag{A17}$$

$$M^{YY}=-M^{XX}(\sin2\phi\to-\sin2\phi,\cos2\phi\to-\cos2\phi)\ . \tag{A18}$$

The amplitudes in the helicity basis can easily be obtained from the above using Eq. (A4) which relate the two sets of polarization vectors in the limit $\phi\to0$:

$$\overline{M}^{\lambda_1,\lambda_2}=\tfrac{1}{2}(\lambda_1\lambda_2M^{XX}+i\lambda_1M^{XY}+i\lambda_2M^{YX}-M^{YY})\ , \tag{A19}$$

which leads to the amplitudes (keeping terms in f_6 only)

$$\overline{M}^{\lambda_1\lambda_2}_{00}=(1+\lambda_1\lambda_2)4z-(1-\lambda_1\lambda_2)(1+4z)\sin^2\theta+i\frac{f_6}{2z}(\sin^2\theta+4z\cos^2\theta)(\lambda_1+\lambda_2)\ , \tag{A20}$$

$$\overline{M}^{\lambda_1\lambda_2}_{\lambda_30}=2\sqrt{2z}\,\sin\theta[(\lambda_1-\lambda_2)+\lambda_3(1-\lambda_1\lambda_2)\cos\theta]-i\frac{f_6\sin\theta}{2\sqrt{2z}}[2\beta\cos\theta-4\lambda_3\cos\theta z(\lambda_1+\lambda_2)+(\lambda_1-\lambda_2)\lambda_3(\beta+\cos\theta)]\ , \tag{A21}$$

$$\overline{M}^{\lambda_1\lambda_2}_{0\lambda_4}=\overline{M}^{\lambda_1\lambda_2}_{\lambda_30}(\lambda_3\to\lambda_4,\cos\theta\to-\cos\theta,\sin\theta\to-\sin\theta)\ , \tag{A22}$$

$$\overline{M}^{\lambda_1\lambda_2}_{\lambda_3\lambda_4}=(\lambda_3+\lambda_4)\beta(\lambda_1+\lambda_2)+(\lambda_3-\lambda_4)\cos\theta(\lambda_1-\lambda_2)+(1+\lambda_1\lambda_2)(1+\lambda_3\lambda_4)(1-2z)$$
$$+\tfrac{1}{2}(1-\lambda_1\lambda_2)[(1-\lambda_3\lambda_4)(1+\cos^2\theta)+4z\sin^2\theta(1+\lambda_3\lambda_4)]+if_6[(\lambda_3+\lambda_4)\beta(\cos^2\theta+\lambda_1\lambda_2)+(\lambda_1+\lambda_2)(1+\lambda_3\lambda_4)]\ . \tag{A23}$$

Using these formulas, and after summation over the polarizations of the W's and integration over the angle θ we get the different asymmetries and cross sections mentioned in the text.

APPENDIX B

We now give some details on the equations in the $e\gamma\to W\nu_e$ process. These were also obtained using a spinor technique [18]. The four-momentum and polarization vectors of the W boson are defined as

$$Q_W=(E_W,P_W\sin\theta\cos\phi,P_W\sin\theta\sin\phi,P_W\cos\theta) \tag{B1}$$

and

$$\epsilon_W^{\text{long}}=(P_W,E_W\sin\theta\cos\phi,E_W\sin\theta\sin\phi,E_W\cos\theta)/M_W \tag{B2}$$

and

$$\epsilon_W^{\pm}=(0,\cos\theta\cos\phi-i\lambda_W\sin\phi,\cos\theta\sin\phi+i\lambda_W\cos\phi,-\sin\theta)/\sqrt{2}\ , \tag{B3}$$

where λ_W is the helicity of the W boson.

In the process $e\gamma\to W\nu_e$, the angle θ is defined as the angle of the W boson with respect to the initial electron. In this two-body process, the energy and momentum of the W boson are given by

$$E_W=\frac{\hat{s}+M_W^2}{2\sqrt{\hat{s}}}\quad\text{and}\quad P_W=\frac{\hat{s}-M_W^2}{2\sqrt{\hat{s}}}\ . \tag{B4}$$

In doing an experiment one might want to impose some cuts on the θ distribution of the W boson; this might even improve the bounds we obtained on the anomalous couplings. The equations we listed in the text do not allow this since we integrated over θ in order to have a boost-invariant expression for the cross section; this simplified the integration over the photon spectrum. We will now give our results before the integration over θ is performed.

In the process $e\gamma\to W\nu_e$ we obtain

$$\frac{d\sigma_L}{d\Upsilon}=\frac{\pi\alpha^2\delta^2}{4\hat{s}\sin^2\theta_W}\frac{1}{(2-\delta\Upsilon)^2}\left\{\Upsilon[\delta\Upsilon+2(1-\delta)]^2+\frac{|f_6|^2\Upsilon}{4(1-\delta)}[-\tfrac{1}{2}\delta^2\Upsilon^2+\delta^2\Upsilon+4(1-\delta)]\right.$$
$$\left.+2\mathcal{I}(f_6)\Upsilon[\delta\Upsilon+2(1-\delta)]\right\} \tag{B5}$$

and

$$\frac{d\sigma_R}{d\Upsilon}=\frac{\pi\alpha^2\delta^2}{4\hat{s}\sin^2\theta_W}\frac{1}{(2-\delta\Upsilon)^2}\left\{4\delta^2\Upsilon-\mathcal{I}(f_6)4\delta\Upsilon+\frac{|f_6|^2}{2(1-\delta)}[2(1-\delta)\Upsilon-\Upsilon+2]\right\}, \tag{B6}$$

where $\Upsilon=1-\cos\theta$. For transversely polarized photons we will give the three different polarization states of the W boson in order to show how the $\mathcal{R}(f_6)$ term enters. The index attached to the matrix element refers to the polarization of the W boson:

$$|M|^2_-=\frac{e^2g^2\delta}{2}\left\{1+\frac{|f_6|^2}{4}-\mathcal{I}(f_6)\right\}I_3\ , \tag{B7}$$

$$|M|^2_+=\frac{e^2g^2\delta}{2}\left[I_1[4-8\delta+8\delta^2]-I_2[4-8\delta+4\delta^2]+I_3(1-\delta)^2\right.$$
$$+\cos2\phi[I_1(4\delta^2-8\delta)-I_2(4\delta^2-4\delta)]+\frac{|f_6|^2}{4}[I_3-4I_2+8I_1+\cos2\phi(8I_1-4I_2)]$$
$$-\mathcal{I}(f_6)\{I_3(1-\delta)+I_2(4\delta-8)+I_1 8(1-\delta)+\cos2\phi[I_2(4\delta-6)+I_1 8(1-\delta)]\}$$
$$\left.-2\mathcal{R}(f_6)\sin2\phi(2I_1-I_2)\right]\ , \tag{B8}$$

$$|M|^2_{\rm long}=\frac{e^2g^2\delta}{2}\left[(4I_2-2I_3)(1-\delta)+\frac{|f_6|^2}{4(1-\delta)}[4I_0-2I_1+(1/2)(2-\delta)^2(2I_2-I_3)\right.$$
$$\left.-\cos2\phi(2-\delta)(4I_1-2I_2)]-\mathcal{I}(f_6)[(2-\delta)(2I_2-I_3)-\cos2\phi(4I_1-2I_2)]+2\mathcal{R}(f_6)\sin2\phi(2I_1-I_2)\right]\ , \tag{B9}$$

where $I_n=[\Upsilon^n/(2-\delta\Upsilon)^2]$. Recall that $\cos\phi\leftrightarrow\sin\phi$ and a sign change in front of the real and imaginary parts of f_6 allows one to go from the X polarization to the Y polarization state of the photon. These equations now allow one to use the angular distribution of the W.

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