Symmetry remnants: Rationale for having two Higgs doublets

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There is a good reason why the standard electroweak SU(2)×U(1) gauge model may be supplemented by two Higgs scalar doublets. They may be remnants of the spontaneous breaking of an SU(2)×SU(2)×U(1) gauge symmetry at a much higher energy scale. In one case, the two-doublet Higgs potential has a custodial SU(2) symmetry and implies an observable scalar triplet. In another, a light neutral scalar becomes possible.

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In the standard SU(2)×U(1) electroweak gauge model, only one Higgs scalar doublet is needed for the spontaneous generation of all particle masses. Yet there are numerous research papers dealing with the possibility of having two (or more) doublets [1]. A good reason is of course supersymmetry, but, in that case, there should be many other particles as well. Nevertheless, a general two-doublet extension of the standard electroweak model without supersymmetry is routinely studied with little theoretical justification other than the obvious fact that it is not known to be wrong [2]. To remedy this situation, we will show in the following that if the standard SU(2)×U(1) electroweak gauge group is the remnant of a larger symmetry, then the appearance of two (or more) doublets at the electroweak energy scale is actually required in some cases [3] and the special form of the corresponding Higgs potential may even be indicative of what the larger theory is.

Consider the following Higgs potential for two doublets:[4]

$$\begin{aligned} V=&\mu_1^2\Phi_1^\dagger\Phi_1+\mu_2^2\Phi_2^\dagger\Phi_2+\mu_{12}^2(\Phi_1^\dagger\Phi_2+\Phi_2^\dagger\Phi_1)\\ &+\tfrac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2+\tfrac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2+\lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)\\ &+\lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)+\tfrac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2+\tfrac{1}{2}\lambda_5^*(\Phi_2^\dagger\Phi_1)^2\ , \end{aligned} \tag{1}$$

where

$$\Phi_{1,2}=\begin{bmatrix}\phi_{1,2}^+\\ \phi_{1,2}^0\end{bmatrix} \tag{2}$$

and μ_{12}^2 has been chosen real by virtue of the arbitrary phase between Φ_1 and Φ_2. This V is invariant under a Z_2 discrete symmetry where $\Phi_1(\Phi_2)$ may be considered even (odd) except for the μ_{12}^2 term, but which breaks it only softly. Consequently, it allows for the natural suppression of flavor-changing neutral currents as long as each fermion gets its mass from only one scalar vacuum expectation value, i.e., either $\langle\phi_1^0\rangle$ or $\langle\phi_2^0\rangle$ but not both.

Such two-doublet extensions of the standard electroweak model have been studied extensively for their phenomenological implications. However, a more fundamental question to be considered is why they should be studied at all. In supersymmetry, two scalar doublets are necessary because each is accompanied by a fermionic partner having a nonzero contribution to the axial-vector triangle anomaly but their sum is zero. The requirement of supersymmetry also constrains the parameters of V as follows:

$$\begin{aligned} &\lambda_1=\lambda_2=\tfrac{1}{4}(g_1^2+g_2^2)\ ,\ \lambda_3=-\tfrac{1}{4}g_1^2+\tfrac{1}{4}g_2^2\ ,\\ &\lambda_4=-\tfrac{1}{2}g_2^2,\ \lambda_5=0\ , \end{aligned} \tag{3}$$

where g_1 and g_2 are the U(1) and SU(2) gauge couplings of the standard model, respectively. The soft terms, i.e., μ_1^2, μ_2^2, and μ_{12}^2, are considered arbitrary because they are allowed to break the supersymmetry. Discovery of scalar particles with a mass spectrum conforming to such a Higgs potential would certainly be a strong indication of supersymmetry.

Consider now a different rationale for the existence of two Higgs doublets. They may be remnants of the spontaneous breaking of a larger gauge symmetry at some higher energy scale. Take for example the gauge group $SU(2)_1\times SU(2)_2\times U(1)$. Let the scalar sector consist of two doublets $\Phi_{1,2}$ and one self-dual bidoublet η transforming as (2,1,1/2), (1,2,1/2), and (2,2,0), respectively:

$$\Phi_{1,2}=\begin{bmatrix}\phi_{1,2}^+\\ \phi_{1,2}^0\end{bmatrix}\ ,\ \eta=\frac{1}{\sqrt{2}}\begin{bmatrix}\overline{\eta}^0 & \eta^+\\ -\eta^- & \eta^0\end{bmatrix}\ . \tag{4}$$

The most general Higgs potential V is then given by

$$\begin{aligned} V=&m_1^2\Phi_1^\dagger\Phi_1+m_2^2\Phi_2^\dagger\Phi_2+m_3^2\mathrm{Tr}(\eta^\dagger\eta)+\tfrac{1}{2}f_1(\Phi_1^\dagger\Phi_1)^2+\tfrac{1}{2}f_2(\Phi_2^\dagger\Phi_2)^2+\tfrac{1}{2}f_3[\mathrm{Tr}(\eta^\dagger\eta)]^2\\ &+f_4(\Phi_1^\dagger\Phi_1)\mathrm{Tr}(\eta^\dagger\eta)+f_5(\Phi_2^\dagger\Phi_2)\mathrm{Tr}(\eta^\dagger\eta)+f_6(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)+t(\Phi_1^\dagger\eta\Phi_2+\Phi_2^\dagger\eta^\dagger\Phi_1)\ , \end{aligned} \tag{5}$$

where t has been chosen real by virtue of the arbitrary relative phase between Φ_1 and Φ_2. Note that because

$$\Phi_1^\dagger\eta\Phi_2+\Phi_2^\dagger\eta^\dagger\Phi_1 = \mathrm{Tr}\begin{pmatrix}\phi_1^0 & -\phi_1^+\\ \phi_1^- & \overline{\phi_1^0}\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}\overline{\eta^0} & \eta^+\\ -\eta^- & \eta^0\end{pmatrix}\begin{pmatrix}\overline{\phi_2^0} & \phi_2^+\\ -\phi_2^- & \phi_2^0\end{pmatrix}, \tag{6}$$

this V has automatically an extra global SU (2) symmetry [5]. As the first step of symmetry breaking, consider only $\langle\eta^0\rangle=\langle\overline{\eta^0}\rangle=u\neq0$, then our $\mathrm{SU(2)_1\times SU(2)_2\times U(1)}$ breaks down to the standard $\mathrm{SU(2)}_L\times\mathrm{U(1)}_Y$, resulting in a massive vector-boson triplet $(g_1W_1^{\pm,0}-g_2W_2^{\pm,0})/\sqrt{g_1^2+g_2^2}$ and preserving the extra global SU(2) symmetry. The reduced V now has the form of Eq. (1) but with the important restriction that $\lambda_4=\lambda_5=0$. [The other parameters are $\mu_1^2=m_1^2+f_4u^2$, $\mu_2^2=m_2^2+f_5u^2$, $\mu_{12}^2=tu/\sqrt{2}$, $\lambda_1=f_1-f_4^2/f_3$, $\lambda_2=f_2-f_5^2/f_3$, and $\lambda_3=f_6-f_4f_5/f_3$.] Both Φ_1 and Φ_2 now transform as doublets under the standard $\mathrm{SU(2)}_L\times\mathrm{U(1)}_Y$ gauge group, as well as the extra global SU(2). As ϕ_1^0 and ϕ_2^0 acquire vacuum expectation values v_1 and v_2, the gauge symmetry $\mathrm{SU(2)}_L\times\mathrm{U(1)}_Y$ breaks down to electromagnetic $\mathrm{U(1)}_Q$, but a custodial SU(2) symmetry remains, in exact analogy to the well-known case of the standard model with only one Higgs doublet. Consequently, of the five physical scalar bosons, three are organized into a triplet:

$$H_3^\pm=-\sin\beta\phi_1^\pm+\cos\beta\phi_2^\pm\ , \tag{7}$$

$$H_3^0=\sqrt{2}(-\sin\beta\,\mathrm{Im}\phi_1^0+\cos\beta\,\mathrm{Im}\phi_2^0)\ , \tag{8}$$

where $\tan\beta\equiv v_2/v_1$, with a common mass given by

$$m_{H_3}^2=\frac{-2\mu_{12}^2}{\sin2\beta}\ . \tag{9}$$

The other two are singlets:

$$H_1=\sqrt{2}(\cos\beta\,\mathrm{Re}\phi_1^0+\sin\beta\,\mathrm{Re}\phi_2^0)\ , \tag{10}$$

$$H_2=\sqrt{2}(-\sin\beta\,\mathrm{Re}\phi_1^0+\cos\beta\,\mathrm{Re}\phi_2^0)\ , \tag{11}$$

with a mass-squared matrix given by

$$\mathcal{M}^2=\begin{pmatrix}2(c^2\lambda_1v_1^2+s^2\lambda_2v_2^2+2sc\lambda_3v_1v_2) & 2sc[(-\lambda_1+\lambda_3)v_1^2+(\lambda_2-\lambda_3)v_2^2]\\ 2sc[(-\lambda_1+\lambda_3)v_1^2+(\lambda_2-\lambda_3)v_2^2] & m_{H_3}^2+2sc(\lambda_1+\lambda_2-2\lambda_3)v_1v_2\end{pmatrix} \tag{12}$$

where $s=\sin\beta$ and $c=\cos\beta$. Since $\lambda_1+\lambda_2>2|\lambda_3|$ is required for V to be bounded from below, the above matrix shows that at least one of the singlet scalars must be heavier than the triplet.

The V of Eq. (1) is in general not invariant under an extra global SU(2) symmetry; hence, the presence of two Higgs doublets is expected to contribute significantly to the radiative correction which makes the electroweak parameter ρ different from one [6]. Experimentally, there is no evidence of any deviation which cannot be accounted for with a t-quark mass of about 150 GeV or so. Hence such a custodial symmetry is desirable for V, but that would require [7] $\lambda_4=\lambda_5$ which cannot be maintained naturally in the context of the standard model because infinite radiative corrections are unavoidable. In our case, the restriction $\lambda_4=\lambda_5=0$ is obtained from the reduction of a larger theory and it can easily be shown that λ_4 and λ_5 have finite radiative corrections which go to zero in the high-energy limit.

The reason for both Φ_1 and Φ_2 to be present in the reduced Higgs potential has to do with the original $\mathrm{SU(2)_1\times SU(2)_2\times U(1)}$ theory. If some of the fermions couple to $\mathrm{SU(2)_1\times U(1)}$ and others to $\mathrm{SU(2)_2\times U(1)}$, then both Φ_1 and Φ_2 are required to allow all fermions to acquire mass [8]. At the 10^2 GeV energy scale, all fermions couple to the standard $\mathrm{SU(2)\times U(1)}$ in the usual way and the only clue to their original difference is the two Higgs doublets with $\lambda_4=\lambda_5=0$ in V. Discovery of the scalar triplet $H_3^{\pm,0}$ would certainly be indicative of such a possibility.

As a second example, consider again the gauge group $\mathrm{SU(2)_1\times SU(2)_2\times U(1)}$ but with an unconventional assignment of fermions [9]. An exotic quark h of electric charge $-1/3$ is added so that $(u,d)_L$ transforms as (2,1,1/6), $(u,h)_R$ as (1,2,1/6), whereas both d_R and h_L are singlets (1,1,−1/3). There are again the two Higgs doublets $\Phi_{1,2}$ but now the bidoublet is not self-dual, i.e.,

$$\eta=\begin{pmatrix}\overline{\eta_1^0} & \eta_2^+\\ -\eta_1^- & \eta_2^0\end{pmatrix}. \tag{13}$$

Assume also a Z_4 discrete symmetry under which $\Phi_1\to\Phi_1$, $\Phi_2\to i\Phi_2$, $\eta\to i\eta$, $(u,d)_L\to(u,d)_L$, $(u,h)_R\to -i(u,h)_R$, $d_R\to d_R$, and $h_L\to -h_L$. This then forces h_L to pair up with h_R via $\langle\phi_2^0\rangle=v_2$, d_L with d_R via $\langle\phi_1^0\rangle=v_1$, and u_L with u_R via $\langle\eta_1^0\rangle=u_1$. It also allows $\langle\eta_2^0\rangle=0$ as shown below. The resulting theory retains an exact Z_2 discrete symmetry under which h, η_2, and $W_R^\pm$ are odd and all the other particles are even. It can be thought of as a residual R parity derived from the original supersymmetric E_6 theory and has many interesting and remarkable phenomenological consequences [10].

The most general Higgs potential V invariant under the assumed Z_4 discrete symmetry is given by

$$
\begin{aligned}
V = {} & m_1^2\Phi_1^\dagger\Phi_1 + m_2^2\Phi_2^\dagger\Phi_2 + m_3^2\mathrm{Tr}(\eta^\dagger\eta) + \tfrac{1}{2}f_1(\Phi_1^\dagger\Phi_1)^2 + \tfrac{1}{2}f_2(\Phi_2^\dagger\Phi_2)^2 + \tfrac{1}{2}f_3[\mathrm{Tr}(\eta^\dagger\eta)]^2 + \tfrac{1}{4}f_4\mathrm{Tr}(\eta^\dagger\tilde{\eta})\mathrm{Tr}(\tilde{\eta}^\dagger\eta) \\
& + \tfrac{1}{8}f_5[\mathrm{Tr}(\eta^\dagger\tilde{\eta})]^2 + \tfrac{1}{8}f_5[\mathrm{Tr}(\tilde{\eta}^\dagger\eta)]^2 + f_6(\Phi_1^\dagger\Phi_1)\mathrm{Tr}(\eta^\dagger\eta) + f_7(\Phi_2^\dagger\Phi_2)\mathrm{Tr}(\eta^\dagger\eta) + f_8(\Phi_1^\dagger\eta\eta^\dagger\Phi_1) + f_9(\Phi_2^\dagger\eta^\dagger\eta\Phi_2) \\
& + f_{10}(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + t(\Phi_1^\dagger\tilde{\eta}\Phi_2 + \Phi_2^\dagger\tilde{\eta}^\dagger\Phi_1) ,
\end{aligned} \tag{14}
$$

where

$$
\tilde{\eta} \equiv \sigma_2\eta^*\sigma_2 = \begin{bmatrix} \overline{\eta_2^0} & \eta_1^+ \\ -\eta_2^- & \eta_1^0 \end{bmatrix} . \tag{15}
$$

The couplings f_5 and t have been chosen real by virtue of the arbitrary relative phases among $\Phi_{1,2}$ and η. As the first step of symmetry breaking, consider now only $\langle\phi_2^0\rangle = v_2 \neq 0$, then $\mathrm{SU}(2)_2\times\mathrm{U}(1)$ breaks down to $\mathrm{U}(1)_Y$, whereas $\mathrm{SU}(2)_1$ remains unbroken and is in fact the standard $\mathrm{SU}(2)_L$. Eliminating the heavy ϕ_2 and η_2 scalar bosons from V, we again obtain Eq. (1) but with $\lambda_5=0$ and Φ_2 replaced by η_1. [The other parameters are $\mu_1^2=m_1^2+f_{10}v_2^2$, $\mu_2^2=m_3^2+f_7v_2^2$, $\mu_{12}^2=tv_2$, $\lambda_1=f_1-f_{10}^2/f_2$, $\lambda_2=f_3-f_7^2/f_2$, $\lambda_3=f_6+f_8-f_7f_{10}/f_2$, and $\lambda_4=-f_8$.] Note that the only term in Eq. (14) involving three different neutral scalar fields is $t(\overline{\phi_1^0}\eta_1^0\phi_2^0+\overline{\phi_2^0}\,\overline{\eta_1^0}\phi_1^0)$ which means that $\langle\eta_2^0\rangle=0$ is allowed. Note also that because η is not self-dual, the V of Eq. (14) does not have an extra global SU (2) symmetry. Hence $\lambda_4\neq\lambda_5$ and $H_3^\pm$ and H_3^0 have different masses. However, because $\lambda_5=0$, the mass of H_3^0 is still given by Eq. (9), whereas

$$
m_{H_3^\pm}^2 = m_{H_3^0}^2 - \lambda_4(v_1^2+u_1^2) . \tag{16}
$$

Since H_3^0 is now the only scalar boson with a mass-squared proportional to μ_{12}^2, it may in fact be light. [If μ_{12}^2 were zero as well as λ_5, then V has an extra global U(1) symmetry, the spontaneous breaking of which would result in a massless H_3^0.] The decay $Z^0\to H_3^0H_3^0$ is absolutely forbidden by angular-momentum conservation and Bose statistics, whereas $Z^0\to H_{1,2}^0H_3^0$ and $W^\pm\to H_3^\pm H_3^0$ may be forbidden kinematically because $H_{1,2}^0$ and $H_3^\pm$ are heavy. However, since $H_{1,2}^0$ couple to $H_3^0H_3^0$ through V, the decay $Z^0\to H_3^0H_3^0H_3^0$ may be possible, although the branching fraction is expected to be very much suppressed [11]. Note that $\lambda_5=0$ also in supersymmetry, but there it may be argued that μ_{12}^2 should not be small. In the Yukawa sector, since d_R only couples to Φ_1 and u_R only to η_1, the usual Z_2 discrete symmetry assumed for the natural suppression of flavor-changing neutral currents is also realized.

It has been shown in the above that the scalar sector accompanying the standard model at the electroweak energy scale may very well consist of two doublets, obeying the Higgs potential of Eq. (1), but with the important restriction that $\lambda_4=\lambda_5=0$ in the first case, and $\lambda_5=0$ in the second. These have interesting phenomenological consequences because of the existence of an unbroken custodial SU(2) symmetry in the former, and a softly broken U(1) symmetry in the latter. A scalar triplet $H_3^{\pm,0}$ with a common mass is then predicted in the first case, and a possibly light H_3^0 in the second. It might be possible to test these two specific scenarios experimentally with future high-energy accelerators such as the Superconducting Super Collider (SSC) and the CERN Large Hadron Collider (LHC), although it would be very difficult to determine all the parameters of the Higgs potential.

In closing, we should point out that with the fermionic content of our second example, it is actually possible to have the same reduced V as in our first example, i.e., with $\lambda_4=\lambda_5=0$, but a rather *ad hoc* assumption is then required. Let us choose the bidoublet η to be self-dual, which means that we cannot impose any additional symmetry to distinguish η from $\tilde{\eta}$ as in our second example. The mass matrix linking $(\bar{d}_L,\bar{h}_L)$ to (d_R,h_R) is no longer restricted to be diagonal. In particular, there is a $\bar{h}_Ld_R$ term. However, if we make the *ad hoc* assumption that this term is small compared to the $\bar{h}_Lh_R$ term which comes from $\langle\phi_2^0\rangle$, then again the heavy particles will decouple and we obtain the V of our first example. Another way to achieve this result is to forbid the $\bar{h}_Ld_R$ term with a discrete symmetry by adding a second Φ_2, the existence of which is of course not very well motivated.

We have also looked at other models, such as the new $\mathrm{SU}(3)\times\mathrm{U}(1)$ extension [12] of the standard electroweak model. We have worked out the details of its scalar sector and we find the reduced Higgs potential to consist of three doublets with two softly broken global U(1) symmetries. There are four singlets and one triplet, but they are all heavy and do not contribute to the breaking of the $\mathrm{SU}(2)\times\mathrm{U}(1)$ gauge symmetry. In general, unless a scalar multiplet participates in the $\mathrm{SU}(2)\times\mathrm{U}(1)$ breaking, it will automatically be heavy. In our second example, η_2 is heavy precisely for this reason. This means that in practice, we should only have doublets at the electroweak mass scale. Singlets are allowed, but they would have to be singlets also under the larger symmetry in which case their mass scale is arbitrary to begin with.

The same V for two Higgs doublets may come from very different models at a much higher energy scale. However, their couplings to the quarks and leptons will generally not be the same. We have not considered these here because they are highly model dependent. If two Higgs doublets are discovered in the future, detailed experimental determination of their properties will likely point to a larger gauge theory at some higher energy scale.

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[2] There are of course models which use an enlarged Higgs sector to obtain *CP* nonconservation or to support some new discrete or global symmetry, etc. The extra scalar multiplets are then there for exactly those purposes and not because the gauge theory, i.e., the standard electroweak $SU(2)\times U(1)$ model, requires them.

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