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Curvature energy effects on strange quark matter nucleation at finite density

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We consider the effects of the curvature energy term on thermal strange quark matter nucleation in dense neutron matter. Lower bounds on the temperature at which this process can take place are given and compared to those without the curvature term.

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Strange quark matter (SQM) [1,2] remains an interesting and intriguing idea which could be important for the proper understanding of several astrophysical and cosmological processes [3]. Among them, its appearance in dense astrophysical objects has been considered [4,5] as a precondition for the "burning" of the host [6,7], a phenomenon that is still being studied and may reveal rich novel physics [8–11]. So far, the critical issue of the formation of SQM has been only preliminary addressed to understand when and how it may happen in actual situations [12]. We shall consider in this work the effects of the so-called curvature term on the nucleation rate and clarify our earlier work [5] on the subject.

One of the many possibilities of SQM appearance in dense objects [13] we have previously addressed is the (simple) hypothesis of a thermal nucleation [5]. Generally speaking, the knowledge of the free energy as a function of the thermodynamic quantities is the starting point of the thermodynamic approach and it is usual for most situations in which a nucleation takes place just to keep the first two terms (volume and surface) of the expansion around the bulk limit. However, for this specific problem, Mardor and Svetitsky [14] have recently shown that the third (linear in the radius r) term should also be kept in order to achieve a proper description of the process. The appropriate form of this contribution, referred as the "curvature" energy, may be written as [14,15]

$$E_c=\frac{gr}{3\pi}\int_0^\infty dk\, k\{1+\exp[(k-\mu)/T]\}^{-1} = \frac{g\mu^2 r}{6\pi}[1+O((T/\mu)^2)] , \quad (1)$$

where g is the statistical weight, r is the radius (we assume a spherical region), and μ is the chemical potential. Note that the first line expression is valid for massless quarks in a MIT bag [16] and the second line is the integrated form valid for the case $T \ll \mu$, which is always satisfied in our strongly degenerate environment.

Because of this term the problem of SQM nucleation in a protoneutron star [4,5] must be reconsidered to address how the previous results are affected. Consider the work done to form a bubble of SQM (labeled as "i") in a nuclear matter medium (labeled "e"), the system $(i+e)$ assumed to be in thermal equilibrium (common temperature T) but not in chemical or mechanical equilibrium

$$W=-\tfrac{4}{3}\pi r^3(P_i-P_e)+4\pi\sigma r^2-8\pi\gamma r+\tfrac{4}{3}\pi r^3 n_i(\mu_i-\mu_e) , \quad (2)$$

where P is the pressure, σ the surface tension coefficient, γ the curvature coefficient, and n_i the particle number density [8]. Extremization of W as a function of the radius and the knowledge of the coefficients permits the evaluation of the critical size of the bubbles beyond which they are able to grow at the expense of the metastable nuclear phase; but before discussing the existence of the critical points it is important to note that, as the coefficients σ and γ are linear combinations of the contributions from each side, they should be properly defined. Clearly $\sigma=\sigma_i+\sigma_e>0$ is required to avoid an unstable QCD vacuum [17], but $\gamma=\gamma_e-\gamma_i$ is found to be negative for the problem since the expected γ_e is small [18] compared to the dominant γ_i. This is consistent with earlier work [19] on the "inverse" problem of hadron bubble nucleation in cosmological SQM nuggets. The value of γ can be found directly by comparing Eqs. (1) and (2) and may be written as

$$\gamma=\frac{3}{8\pi^2}\mu^2 \sim 18 \text{ MeV fm}^{-1}\left[\frac{\mu}{300 \text{ MeV}}\right]^2 . \quad (3)$$

We shall assume hereafter that σ and γ are due to the SQM contribution since the respective nuclear matter values may be included in the uncertainties of the former due to their smallness.

Defining $\Delta P=P_i-P_e$ and $\Delta\mu=\mu_i-\mu_e$ and putting

$F=-n_i\Delta\mu+\Delta P>0$ the work of Eq. (2) can be minimized with respect to the radius; i.e., $\partial W/\partial r=0$ to find the critical points

$$r_c=\frac{\sigma}{F}(1\pm\sqrt{1+\beta}) , \tag{4}$$

where $\beta\equiv 2F|\gamma|/\sigma^2$ is a positive definite quantity that includes all the effects of the curvature term. We see that unless the nuclear matter curvature term exceeds γ_i (which is certainly not expected) both roots are real and correspond to a maximum r_+ and a minimum $r_-<0$ of W. For the former

$$W_c=W(r_+)=\frac{4\pi\sigma^3}{3F^2}[2+2(1+\beta)^{3/2}+3\beta] . \tag{5}$$

It is the magnitude of β which determines the increase of the critical radius as compared to the $\gamma=0$ case and suppresses the nucleation rate. Typically

$$\beta\simeq 6\left[\frac{F}{20\,\mathrm{MeV\,fm}^{-3}}\right]\times\left[\frac{\gamma}{18\ \mathrm{MeV\,fm}^{-1}}\right]\left[\frac{\sigma}{(75\,\mathrm{MeV})^3}\right]^{-2} , \tag{6}$$

where we have scaled to a large value of σ [19] attempting to address the minimum effect to be expected from the curvature term (see also [20] for a different approach to the calculation of σ). Even though the actual σ is not accurately known, our results can be easily scaled for any value of this parameter. What is clear from Eqs. (5) and (6) is that the curvature term is indeed very important for the nucleation of SQM in dense environments. The idea now is to set a lower bound on the temperature (i.e., the threshold for SQM nucleation) to see when and if that process is possible.

In the thermodynamic approach of Refs. [8,14,21] the nucleation rate is essentially dominated by a Boltzmann factor $\exp(-W_c/T)$, therefore, in principle, the knowledge of the thermodynamic quantities in Eq. (6) (all functions of μ_i) for a given state of the system is possible. Since the information contained in the growth of SQM bubbles is not thus included we have alternatively [5] estimated the lower bound on T we are interested in by using the Fokker-Planck-Zel'dovich approach based on the kinetic equation [22]

$$\frac{\partial f}{\partial t}=-\frac{\partial\xi}{\partial t} , \tag{7}$$

where $f(r,t)$ is the time-dependent size distribution of SQM bubbles (assumed to have a large number of particles, see below) and ξ is the nucleation rate (i.e., the number of SQM bubbles above the critical size per unit time and volume). In that theory, the integration of Eq. (7) must be performed imposing appropriate boundary conditions and involves the evaluation of the equilibrium size distribution function

$$f_o(r_c)=r_c^2 n_i n_e \exp(-4\pi\sigma r_c^2/3T) \tag{8}$$

in which we have followed Ref. [22] to give an estimate of the prefactor. The important point here is that, since we are interested in the SQM bubbles in the neighborhood of the critical radius r_c (and hence corresponding to a narrow range around the maximum W_c), a linear term such as the curvature one does not contribute and ξ can be calculated in a Gaussian approximation, namely,

$$\frac{1}{\xi}=\int_0^\infty \frac{dr\, e^{[-4\pi\sigma(r-r_c)^2/T]}}{B(r)f_o(r_c)} , \tag{9}$$

where $B(r)$ is the size diffusion coefficient. After integrating Eq. (9) and imposing a physically motivated form for $B(r_c)=T/8\pi\tau_w$, with $\tau_w\sim 10^{-9}$ s being the weak-interaction time-scale in the medium (see Refs. [3,5,6,11]), we may write ξ as

$$\xi=\frac{r_c^2 n_i n_e}{4\pi\tau_w}\left[\frac{T}{\sigma}\right]^{1/2}\exp(-4\pi\sigma r_c^2/3T) \tag{10}$$

in which all the effects of the curvature enter only through the value of r_c given in Eq. (4). In order to determine the total number of bubbles that nucleate we shall, as in Ref. [5], multiply the rate ξ by the time-scale in which favorable conditions for nucleation exists Δt and by the volume available for the process V_o. The former is determined by the initial cooling properties of the protoneutron star and is generally accepted to be $\sim$ seconds [23], therefore we shall use the fiducial value $\Delta t=1$ s. The latter is more difficult to address because it refers to the region above an uncertain density ρ_o in which the matter is sufficiently compressed to undergo the transition to SQM, which is intrinsically time dependent since the star compacts while its lepton content decreases due to neutrino emission [23]. We use $V_o=(10^5\ \mathrm{cm})^3$ corresponding to the inner sphere with $R<0.1R_n$ (where R_n is the radius of the object) for any instant of its evolution. Fortunately, Δt and V_o are not critical for our argument since the nucleation rate is largely dominated by the exponential factor in Eq. (10).

Since the condition for a neutron$\rightarrow$strange conversion [2,6,7,9–12] is that at least one SQM bubble is nucleated to start it, we impose $\xi\Delta t V_o\geq 1$, or

$$\frac{r_c^2 n_i n_e}{4\pi}\left[\frac{T}{\sigma}\right]^{1/2}\left[\frac{\Delta t}{\tau_w}\right]V_o\exp(-4\pi\sigma r_c^2/3T)\geq 1 , \tag{11}$$

which shows explicitly the dependence on the various factors, particularly the appearance of the quotient of the two relevant time-scales of the problem and the implicit dependence on β through r_c given by Eq. (4). After defining $x=T/\sigma$ and replacing the numerical values Eq. (11) can be solved for x to yield the threshold temperatures (those satisfying the imposed condition) T_γ shown in Table I. We also show for comparison the threshold temperatures corresponding to the same set of parameters and $\gamma=0$, denoted as T_o. Thus, the effects of the curvature term on the nucleation rate can be quantitatively appreciated in this kinetic approach. As expected, the curvature term makes the minimum temperature for nucleation to be higher, i.e., suppresses the nucleation for a given temperature.

The numbers in Table I may be compared to those

TABLE I. Threshold temperatures for SQM nucleation (see text for details).

σ (MeV^3)	r_c (fm)	T_γ (MeV)	r_{co} (fm)	T_o (MeV)
$(65)^3$	1.74	0.63	0.71	0.10
$(75)^3$	1.98	1.26	1.09	0.38

from our former work [5], where instead of imposing r_{co} given as the critical radius of W we have tried to express it in terms of the minimum (critical) numbers of quarks in the bubble N_{qc}. Although it was clear that the use of a baglike relationship overestimated the critical size r_{co} (the external pressure and chemical difference term F did not appear in such an estimate, so that we had assumed essentially a free configuration while the actual one is under a large compression), it is necessary to state that the estimate was too rough and the derived r_{co} resulted too large by a factor of ~ 3. Therefore, and due to the sensitiveness of the threshold temperatures on that parameter, they happen to be too high by an order of magnitude and should be corrected (see Table I). However, the conclusions of Ref. [5] are essentially the same as those from the present work: even though the curvature term acts against the SQM nucleation, the physical temperature of a just-born protoneutron star [23] is in any model more than enough to drive an efficient "boiling" of the neutron material. Moreover, observations of the neutrino flux from SN1987A are consistent with an effective temperature $T_{\rm eff} \sim 4$ MeV, which may be taken as an experimental lower limit to the actual core temperature. Therefore, and even if several improvements on the present estimate may and should be attempted, the conclusion seems to be robust in the SQM hypothesis framework and points to a prompt conversion of all neutron stars into strange stars.

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