PHYSICAL REVIEW D VOLUME 49, NUMBER 10 15 MAY 1994

New look at QED$_4$: The photon as a Goldstone boson and the topological interpretation of electric charge

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(Received 15 July 1993)

We develop the dual picture for quantum electrodynamics in 3+1 dimensions. It is shown that the photon is massless in the Coulomb phase due to spontaneous breaking of the magnetic symmetry group. The generators of this group are the magnetic fluxes through any infinite surface Φ_S. The order parameter for this symmetry breaking is the operator $V(C)$, which creates an infinitely long magnetic vortex. We show that although the order parameter is a stringlike rather than a local operator, the Goldstone theorem is applicable if $\langle V(C)\rangle \neq 0$. If the system is properly regularized in the infrared, we find $\langle V(C)\rangle \neq 0$ in the Coulomb phase and $\langle V(C)\rangle = 0$ in the Higgs phase. The Higgs-Coulomb phase transition is therefore understood as a condensation of magnetic vortices. The electric charge in terms of $V(C)$ is topological and is equal to the winding number of the mapping from a circle at spatial infinity into the manifold of possible vacuum expectation values of a magnetic vortex in a given direction. Since the vortex operator takes values in S^1 and $\Pi_1(S^1)=Z$, the electric charge is quantized topologically.

PACS number(s): 12.20.Ds, 11.15.Ex, 11.30.−j

I. INTRODUCTION

Gauge theories play a dominant role in modern elementary particle physics. It is clear beyond a reasonable doubt that all the interactions of elementary particles known to date are described by a gauge theory. As a consequence, in some physicists' minds, gauge symmetry attained a status of a philosophical principle. It must be noted, however, that the reason for this is purely empirical. The "gauge principle" does not have the same deep philosophical roots as, say, the equivalence principle of general relativity or the uncertainty principle of quantum mechanics. Mainly this is because it pertains to the form of the description, the "language" in which one describes a physical law, rather than to the physical law itself. This language proved to be indispensable when formulating renormalizable theories of interacting vector particles. In many instances it is also very convenient for actual calculations, since the degrees of freedom used in this description are almost free and the perturbation theory can be easily applied. Such is the case in QED and the ultraviolet region of QCD.

In some cases, however, although a neat mathematical construction, the gauge symmetry in fact obscures rather than highlights the underlying nonperturbative physics. The main conceptual difficulty with the gauge description is that it makes use of redundant nonphysical quantities, which often makes interpreting a calculation almost as difficult as performing the calculation itself.

One example of such a situation is the understanding of the (constituent) quark degrees of freedom in QCD. On the large nonphysical Hilbert space, those appear as multiplets of the "global color" SU(3) group. However, this group acts trivially on the physical gauge-invariant states of the theory, hence the problem of understanding in physical terms what is precisely meant by the color and its confinement.

It is of course possible in principle to fix the gauge completely. However, in a completely gauge-fixed formulation the fields that appear in the Lagrangian are as a rule nonlocal. For example, in QED, fixing the axial gauge turns the matter fields into variables localized on a line rather than at a point,

$$\phi_{\text{axial}}(x)=\phi(x)\exp\left\{ie\int_x^\infty dy^3 A^3(y)\right\}, \tag{1}$$

where the initial fields $\phi(x)$ and $A_\mu(x)$ are "local" but on the nonphysical Hilbert space. In the Coulomb gauge the matter field

$$\phi_{\text{Coulomb}}(x)=\phi(x)\exp\left\{ie\int d^3y\ A^i(y)\frac{x^i-y^i}{|x-y|^3}\right\} \tag{2}$$

creates the electric field of a point charge and has therefore a nonvanishing support everywhere in space. When written in terms of these fields, the Lagrangian is nonlocal and the theory looks very different from a local field theory.

Because of this unfortunate feature, there are several interesting physical questions already in the simplest Abelian gauge theories which either do not arise naturally or tend to be ignored in the framework of the standard

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gauge description. Here are several of these, which motivated us in the present research.

(1) Exact masslessness of a photon. In the standard formulation the masslessness of a photon is almost a consequence of kinematics. However, we know that the photon can become massive in some circumstances, e.g., in the Higgs phase, and that this is in fact a profound dynamical effect. When one discovers a massless particle, the natural question is, what keeps it from acquiring a mass? The simplest possible explanation is that it is the Goldstone theorem. So there is a question of whether the photon in QED is a Goldstone boson and, if yes, of what symmetry.

(2) The nature of the Higgs-Coulomb phase transition. The Higgs-Coulomb phase transition is usually described as due to spontaneous breaking of the electric charge in the Higgs phase. This description is, however, not without a flaw. Electric charge, being equal to a surface integral of the electric field at spatial infinity, does not have a local order parameter. (This is the reason why the Goldstone theorem is not applicable in the Higgs phase.) Consequently, the Coulomb and Higgs phases differ only in expectation values of *nonlocal* fields. In this situation, however, there is no physical argument that tells us that the two phases must be separated by a phase transition. In fact, in the similar situation in Z_N gauge theories it is known that the phases are analytically connected [1]. In QED, however, the two phases are separated by a genuine phase transition which is second or first order depending on the values of parameters. The question is whether one can give a different, complementary characterization of the Higgs and Coulomb phases in QED which will make clear that those are really distinct phases. Usually, such an explanation involves spontaneous breaking of some *global* symmetry.

(3) Topological nature of the electric charge. The electric current in QED is trivially conserved: $\partial_\mu J^\mu = \partial_\mu(\partial_\nu F^{\mu\nu}) = 0$. In quantum theory the charge is also quantized.[1] Both these features would automatically follow if the electric charge could be represented as a topological charge associated with a nontrivial homotopy of a vacuum manifold. The possibility that the electric charge could be topological is not so unnatural. One can measure the charge by making local measurements of the electric field far from its location, making use of Gauss' law. For a nontopological charge this would be impossible. Maybe it is possible to find in QED a set of variables in terms of which the degeneracy of the vacuum and the topological nature of the electric charge become explicit.

It would be very interesting to find an alternative formulation of a gauge theory or, at least (since the exact reformulation turns out to be very difficult), an alternative basis in which these questions become natural and the answers to them are relatively straightforward.

In fact, in $2+1$ dimensions there exists a "dual" representation that allows one to answer all of these questions in the affirmative. There one is able to define such a variable: the complex vortex operator $V(x)$ [2,3]. Although it is defined in terms of an exponential of a line integral of the electric field, it can be shown to be a local scalar field [4]. It is an eigenoperator of the conserved charge: the magnetic flux through the plane [5]. In the Coulomb phase the field $V(x)$ has a nonvanishing expectation value and the flux symmetry is spontaneously broken [3,6]. This results in the appearance in the spectrum of a massless Goldstone boson: the photon. The electric charge, when expressed in terms of the vortex field, has the form of a topological charge associated with the homotopy group $\Pi_1(S^1)$: $Q \propto \int d^2x\, \epsilon_{ij}\partial_i(iV^*\partial_j V + \text{c.c.})$ [7]. In the Higgs phase $\langle V \rangle = 0$. Consequently, the charges are completely screened and there is no massless particle in the spectrum.

In this picture it is clear that the Higgs and Coulomb phases must be separated by a genuine phase transition line. The relevant symmetry, the magnetic flux symmetry, is stored in the Higgs phase. The Nielsen-Olesen (NO) vortices exist in this phase as particles that carry the corresponding charge. The Coulomb-Higgs phase transition can be thought of as condensation of the NO vortices in the Coulomb phase. Moreover, on the basis of universality, one concludes that whenever it is second order the Higgs-Coulomb phase transition must be in the universality class of the XY model.

In $(1+1)$-dimensional QED the dual representation also exists. Since in $1+1$ dimensions there is no massless photon and no Coulomb-Hibbs phase transition, only the third question can be asked. In this case the electric charge can be represented as topological in terms of the dual field σ: $Q = \int dx\, \partial\sigma = \sigma(+\infty) - \sigma(-\infty)$. For the massless and massive Schwinger models, the standard bosonization procedure leads to an exact reformulation of the theory in terms of the field σ only [8] and thereby to the *exact* dual Lagrangian. In the scalar Higgs model, the dual transformation can be only performed approximately [9], but the topological interpretation of the electric charge is nevertheless achieved.

The aim of this paper is to develop a similar picture for the $(3+1)$-dimensional Higgs model. In Sec. II we discuss the analogue of the flux symmetry in $3+1$ dimensions. The conserved currents of this magnetic symmetry are the components of the dual field strength tensor $\tilde{F}_{\mu\nu}$. Because of the constraint $\partial_i B_i = 0$, no local order parameter can be found. The $(3+1)$-dimensional analogue of $V(x)$ are stringlike operators $V(C)$ which create infinitely long magnetic vortex lines along a curve C.

In Sec. III we show that although these operators are not local, they are still good order parameters, in the sense that the Goldstone theorem is applicable in the phase where they have a nonzero expectation value. We also show that in this phase the electric charge is topolog-

[1]Here we are interested only in QED with matter fields carrying integer multiples of an elementary charge. In principle, one can formulate a theory with incommensurate charges, and in this case the electric charge will have no topological interpretation. However, since the quantization of electric charge is experimentally well established, these theories are only of a marginal interest. We will return to this point in the last section of the paper.

ical in terms of $V(C)$ and is thereby quantized. An electrically charged state carries a unit wind of the phase of $V(C_{\mathbf{u}})$ where $V(C_{\mathbf{u}})$ is the set of all magnetic vortices associated with straight lines in the direction $\mathbf{u}$. In the classical approximation, indeed, $\langle V(C)\rangle \neq 0$ in the Coulomb phase.

Because of the infinite length of the vortex line, the operator $V(C)$ cannot have a finite expectation value beyond the classical approximation. Infrared divergences due to phase fluctuations of $V(C)$ leads to a vanishing of the vacuum expectation value (VEV) even in the Coulomb phase, not unlike the vanishing of an order parameter in $(1+1)$-dimensional theories with a "classically broken" continuous symmetry. In the present case, however, one can define a regularized model in which one of the spatial dimensions is compact. Vortex lines parallel to this direction will then have a finite expectation value in the Coulomb phase and the Goldstone theorem, and the topological interpretation of the electric charge will be retained. This is discussed in Sec. IV.

Section V is devoted to a brief discussion of the dual picture and possible extension of this approach to non-Abelian theories.

In the Appendix we point to similarities and differences between the realization of magnetic symmetry in QED$_4$ and continuous symmetries in $1+1$ dimensions.

This paper is an attempt to generalize our previous work on QED$_3$ [3,4,7] to four dimensions. The reader may find the terminology and some of the concepts we use somewhat unfamiliar. Most of these, however, have been dwelled upon extensively in [3], and reading [3] should be helpful for easier understanding of this paper.

II. MAGNETIC SYMMETRY, THE COULOMB-HIGGS PHASE TRANSITION, AND THE VORTEX OPERATOR IN 3+1 DIMENSIONS

The approaches to all three questions mentioned in the Introduction, which we are addressing in this paper, have one common element. They all require the construction of a sufficiently local (gauge-invariant) order parameter. Let us briefly recall how this was constructed in $2+1$ dimensions.

The symmetry which is broken in the Coulomb phase of the $(2+1)$-dimensional Higgs model is the magnetic flux symmetry generated by

$$\Phi=\int d^2x\, B(x)\ , \tag{3}$$

with the conserved current $\tilde{F}_{\mu}$. The defining relation for the vortex operator $V(x)$ therefore is the commutation relation with the magnetic field

$$[B(x),V(y)]=-g\delta^2(x-y)V(y)\ . \tag{4}$$

One also insists on the locality of $V(x)$: It has to commute with all gauge-invariant local fields at spacelike separations:

$$[V(x),O(y)]=0,\quad x\neq y\ . \tag{5}$$

These conditions determine $V(x)$ up to a multiplicative local gauge-invariant factor as

$$V(x)=C\exp\left\{\frac{i}{e}\int d^2y\left[\epsilon_{ij}\frac{(x-y)_j}{(x-y)^2}E_i(y)+\Theta(x-y)J_0(y)\right]\right\}, \tag{6}$$

where $\Theta(x)$ is an angle between the vector x_i and the x_1 axis, $0<\Theta<2\pi$. The requirement of locality leads in particular to the quantization condition on possible eigenvalues of the magnetic flux g: $g=2\pi n/e$.

The operator $V(x)$ has a simple physical meaning: It creates a pointlike magnetic vortex of the strength g. In the Higgs phase the magnetic flux symmetry is not broken and the NO vortices behave like particles. In the Coulomb phase they condense. This breaks the flux symmetry spontaneously and leads to the appearance of the massless photon.

A. Vortex operator

Let us now try to implement the same program in the $(3+1)$-dimensional Higgs model.

The analogue of the conserved flux current in $3+1$ dimensions is the dual magnetic field strength tensor $\tilde{F}_{\mu\nu}$. Its conservation equation is again just the homogeneous Maxwell's equation of electrodynamics. It was shown in [10] that the matrix element of $\tilde{F}_{\mu\nu}$ between the vacuum and the one-photon state in the Coulomb phase has the characteristic form of a matrix element of a spontaneously broken current between the vacuum and a state with one Goldstone boson. In the circular polarization basis,

$$\langle 0|\tilde{F}_{0i}(0)|e^{\lambda}_{\pm},\mathbf{p}\rangle = \mp\frac{i}{(2\pi)^{3/2}}\left[\frac{p_0}{2}\right]^{1/2} e_i^{\pm}\lim_{p^2\to 0}\frac{1}{1-\Pi(p^2)}\ , \tag{7}$$

$$\langle 0|\tilde{F}_{ij}(0)|e^{\lambda}_{\pm},\mathbf{p}\rangle = -\frac{i}{(2\pi)^{3/2}}\left[\frac{p_0}{2}\right]^{1/2}\epsilon_{ijk}e^{k}_{\pm}\lim_{p^2\to 0}\frac{1}{1-\Pi(p^2)}\ , \tag{8}$$

where $\Pi(p^2)$ is the vacuum polarization.

One can define many conserved charges associated with the currents $\tilde{F}_{\mu\nu}$. In particular, the magnetic flux through any infinite surface S,

$$\Phi_S=\int_S dS^i B_i\ , \tag{9}$$

is time independent. Since the magnetic flux through any closed surface vanishes, the set of independent Φ_S is given by the set of boundaries (at spatial infinity) rather than the set of surfaces themselves. It will suffice for our purposes to consider only the planes perpendicular to the three coordinate axes. We define the magnetic charge Φ_i as the average magnetic flux through a plane perpendicular to the ith axis,

$$\Phi_i\equiv\lim_{L\to\infty}\frac{1}{2L}\int_{-L}^{L}dx^i\int_{S(x^i)}d\mathbf{S}\cdot\mathbf{B}(x^i)\ , \tag{10}$$

where $S(x^i)$ is the plane perpendicular to the ith axis with the ith coordinate x^i.

We now construct an order parameter for $\widetilde{F}_{0i}=B$. This is an operator which creates magnetic vortices. Clearly, in $3+1$ dimensions no local operator of this kind can be constructed. Since the magnetic field is divergenceless, the magnetic flux must either form closed loops or infinitely long lines. Closed loops, however, do not carry any global charge. The best one can do therefore is to construct an operator creating an infinite vortex line. This operator should be "line local."

Any gauge-invariant local field should commute with $V(C)$ at all points but on the line C. In particular,

$$[B_i(x),V(C)]=gl_i(x,C)V(C)\ ,\qquad l_i(x,C)=\int d\tau\,\delta(x-\bar{x}(\tau))\frac{d\bar{x}_i(\tau)}{d\tau}\ . \tag{11}$$

The commutator of $V(C)$ and $J_i(x)$ should also vanish for $x\notin C$. Analogously to $2+1$ dimensions, these two conditions determine $V(C)$ as [11]

$$V_n(C)=\exp\left\{i\frac{n}{e}\int d^3y[a_i(y-x)E_i(y)+b(x-y)J_0(y)]\right\}, \tag{12}$$

where $a_i(x)$ is a vector potential of an infinitesimally thin magnetic vortex along C, $\epsilon_{ijk}\partial_j a_k(x)=l_i(x,C)$, and the function $b(x)$ satisfies $\partial_i[b(x)]_{\mathrm{mod}2\pi}=a_i(x)$. Since $a_i(x)$ has a nonvanishing curl, the function $b(x)$ must have a surface of singularities ending at the curve C. For example, for a straight line C running along the x_3 axis one has $a_i(x)=\epsilon_{ij}x_i/x_1^2+x_2^2$, $i=1,2$; $a_3(x)=0$ and $b(x)=\Theta(x)$, with Θ the polar angle in the x_1-x_2 plane (Fig. 1). As in $2+1$ dimensions, the operator $V(C)$ is an operator of a singular gauge transformation with the gauge function $nb(x)$. This ensures the commutativity of $V(C)$ with any local gauge-invariant operator outside the line of singularities C. The single valuedness of the gauge transformation imposes the quantization condition on possible fluxes in a vortex: $g=2\pi n/e$. This of course corresponds to the well-known fact that the Abrikosov vortices in the Higgs phase carry quantized flux. From now on we will concentrate on the elementary vortex operator $n=1$.

Choosing C_i as a straight line parallel to the ith axis, we have

$$[V(C_i),\Phi_j]=\delta_{ij}\frac{2\pi}{e}V(C_i)\ . \tag{13}$$

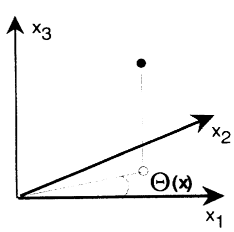

FIG. 1. Function $\Theta(x)$.

Using Gauss's law and integrating by parts, one can recast the vortex operator into the form

$$V(C)=\exp\left\{i\frac{2\pi}{e}\int_{S:\partial S=C}dS^iE^i\right\}, \tag{14}$$

where the integration is over the half plane bounded by C (Fig. 2).

B. Vortex operator and the dual vector potential

Let us now make a slight digression and show how the vortex operator can be represented in terms of the dual vector potential. This is particularly simple in the case of a free photon. Gauss's law in this case reduces to $\partial_iE_i=0$ and can be solved by introducing the dual vector potential χ_i via

$$E_i=\epsilon_{ijk}\partial_j\chi_k\ . \tag{15}$$

To reproduce the correct equal time commutation relations, we must also have

$$B_i(x)=\pi_i(x)\ , \tag{16}$$

where π_i is canonically conjugate to χ_i. Of course, the field χ_i is determined by Eq. (15) only up to a gradient of a scalar function. The transformation

$$\chi_i\rightarrow\chi_i+\partial_i\lambda \tag{17}$$

is generated by ∂_iB_i and is in fact a magnetic gauge symmetry associated with the constraint

$$\partial_iB_i=0\ . \tag{18}$$

As in the case of the electric gauge symmetry, one should be careful when identifying the functions λ for which the dual vector potentials connected by Eq. (17) are equivalent. The transformations Eq. (17) with gauge functions λ that satisfy $\lim_{x\rightarrow\infty}\lambda(x)\rightarrow 0$ are indeed generated by $\exp\{i\int\lambda(x)\partial_iB_i(x)\}$ and are therefore gauge symmetries. However, if the function λ does not vanish at the spatial infinity, the transformation Eq. (17) is generated by $\exp\{i\int\partial_i(\lambda B_i)\}$. The operator of the transformation is not a unit operator on the constraint Eq. (18), and the transformation therefore is a true physical symmetry. So, for example, if $\lambda(x)=a\delta^2(x-X(V_S))$, where $x(V_S)$ are points inside a half space bounded by the surface S, one has a global transformation generated by Φ_S

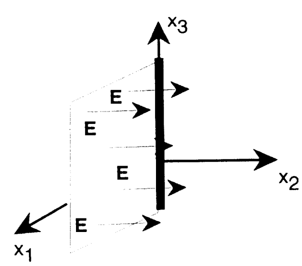

FIG. 2. Vortex operator $V(C_3)$ which creates the magnetic flux tube parallel to the third axis.

of Eq. (9). The magnetically gauge-invariant operators are the 't Hooft loops [12] (the dual analogue of Wilson loops) or infinite 't Hooft lines

$$V(C)=\exp\left\{ig\int_C dl_i\chi_i\right\} . \tag{19}$$

In a theory of a free photon, the constant g is not quantized.

In the interacting theory Gauss's law is

$$\partial_i E_i - J_0 = 0 . \tag{20}$$

We can now define the dual potential in the following way. Since the electric current is conserved, it can be potentiated:

$$J_\mu = \epsilon_{\mu\nu\lambda\rho}\partial_\nu K_{\lambda\rho} . \tag{21}$$

The tensor potential $K_{\mu\nu}$ is defined up to a Kalb-Ramond gauge transformation

$$K_{\mu\nu} \to K_{\mu\nu} + \partial_{[\mu}M_{\nu]} . \tag{22}$$

Let us fix this Kalb-Ramond gauge symmetry by requiring that, for any surface S,

$$\int_S dS^i \epsilon_{ijk}K_{jk} = en(S) , \tag{23}$$

where e is the electric coupling constant and $n(S)$ is an integer which depends on the surface. In QED this is an admissible gauge fixing, since the divergenceless part of $\epsilon_{ijk}K_{ij}$ can be changed arbitrarily by a choice of M_i and the charge inside any closed surface is an integer of e. The dual vector potential is then defined by

$$E_i - \epsilon_{ijk}K_{jk} = \epsilon_{ijk}\partial_j\chi_k . \tag{24}$$

With this definition one has

$$\exp\left\{i\frac{2\pi n}{e}\int_C dl_i\chi_i\right\} = \exp\left\{i\frac{2\pi n}{e}\int_{S:\partial S=C} dS^i E^i\right\} . \tag{25}$$

Comparing this with Eq. (14), we find

$$V(C)=\exp\left\{i\frac{2\pi}{e}\int_C dl_i\chi_i\right\} . \tag{26}$$

III. GOLDSTONE THEOREM AND THE TOPOLOGICAL INTERPRETATION OF THE ELECTRIC CHARGE: THE CLASSICAL APPROXIMATION

A. Goldstone theorem

We have now constructed eigenoperators of the magnetic symmetry in $3+1$ dimensions. The first question to ask is whether their vacuum expectation value vanishes. First, let us consider the classical approximation. We will calculate the quantum corrections in the next section. Although in the infinite system they change the results in a very important way, we will see in the next section that in the IR-regularized system, where some of the dimensions are compact, the classical result is indeed qualitatively correct.

In the classical approximation the electric field and the electric charge density in the vacuum vanish. Therefore the dual vector has a "pure gauge" form

$$\chi_i = \partial_i\lambda . \tag{27}$$

As discussed earlier, the dual potentials χ_i that are given by the functions λ with different values at spatial infinity are not gauge equivalent.

Therefore in this approximation the QED vacuum is infinitely degenerate with a degeneracy that corresponds to global transformations generated by Φ_S. The expectation value of a vortex operator in any of these vacua also does not vanish and is given by

$$\langle V(C)\rangle = \exp\left\{i\frac{2\pi}{e}\int dl_i\chi_i\right\} . \tag{28}$$

This still does not answer the question of whether the Goldstone theorem applies, even if $V(C)$ has a nonvanishing expectation value. The problem is that $V(C)$ is a nonlocal operator and no local order parameters of Φ_i exist. This at first sight seems to be similar to the situation with the electric charge in the Higgs phase. There the Goldstone theorem was not applicable and no massless particle existed for the following reason. Suppose one has a spontaneously broken charge Q. For the Goldstone theorem to hold [13], there must exist an operator O which satisfies the two conditions

$$\lim_{V\to\infty}\langle[Q_V,O]\rangle \neq 0 , \tag{29}$$

$$\lim_{V\to\infty}\langle[\dot{Q}_V(t),O]\rangle = 0 . \tag{30}$$

Here $Q_V \equiv \int_V d^D x J_0(x)$ is the spontaneously broken charge in the volume V. To satisfy Eq. (29) it is sufficient to find any order parameter of Q with a nonvanishing expectation value. It is less trivial to satisfy the second equation. If the operator O is local, Eq. (30) is satisfied automatically:

$$\begin{aligned}\lim_{V\to\infty}&\langle[\dot{Q}_V(t),O(x)]\rangle \\ &= \lim_{V\to\infty}\int_{S:\partial V=S} dS^i\langle[J^i(y,t),O(x)]\rangle \\ &= 0 ,\end{aligned} \tag{31}$$

since in the limit $V\to\infty$ the points x and y are infinitely far apart and the commutator vanishes for any finite time t. However, if O is nonlocal, in general it need not commute with J_i at spatial infinity and Eq. (30) need not be satisfied. This is indeed the reason why the spontaneous breaking of electric charge is not accompanied by an appearance of a Goldstone particle.

In the case of magnetic symmetry, it turns out, however, that the Goldstone theorem is indeed applicable even though the order parameter is nonlocal. To see this let us consider the charge Φ_3, the magnetic flux in the z direction. The corresponding order parameter is $V(C_3)$ of Eq. (13). The regularized flux operator $\Phi_3(L)$ is defined as in

Eq. (10), but without taking the limit $L \to \infty$.[2] For any finite L, $V(C_3)$ is still an order parameter. Therefore, if $\langle V(C_3)\rangle \neq 0$, Eq. (29) is satisfied. Furthermore, only the boundary of the volume V in which $\Phi_3(L)$ is defined which is perpendicular to the axis x_3 is crossed by the fluxon created by $V(C_3)$. Therefore the only vanishing contribution to Eq. (30) can arise from the commutator of $V(C_3)$ with the third component of the magnetic current. However, the third component of the current is $\tilde{F}_{33}$ and vanishes identically at all times due to the antisymmetry of $\tilde{F}_{\mu\nu}$. Therefore Eq. (30) is also satisfied and the Goldstone theorem is applicable. The corresponding Goldstone boson is the linearly polarized photon with the magnetic field in the x_3 direction.

Clearly, the same argument applies to all the changes Φ, if one chooses $V(C_i)$ as the corresponding order parameters. In this way photons with any direction of the wave vector and polarization should have a gapless dispersion relation, e.g., to be massless.

B. Topological interpretation of the electric charge

Let us now show that the electric charge has an explicit topological interpretation in terms of the vortex operators. First, let us explain what we mean by this. As we mentioned in the Introduction, the electric current is trivially conserved. In the usual representation of the electrodynamics via gauge fields, the charge, however, is not explicitly given as some "winding number," but rather as a surface integral of the electric field.

Consider a state with a pointlike charged particle at the origin. We know that if we place an infinite magnetic vortex somewhere in space and move it adiabatically around the charge, no matter how large the distance between the vortex and charge is, the Aharonov-Bohm (or rather the Aharonov-Casher) [14] phase will be accumulated. In order for that to happen, the vortex must complete the rotation around the charge. This means then that, although locally the charged state at spatial infinity is indistinguishable from the vacuum (for example, $F_{\mu\nu}$, J_μ, and $T_{\mu\nu}$ all vanish), there exists some global characteristics which do distinguish between them.

Remembering that the QED vacuum is in fact degenerate, the natural possibility is that locally at every point at infinity the charged state is similar to *one* of the vacua, but moving from point to point we actually move from one vacuum to another. If this is the case, then when the rotation is complete, one should, of course, come back to the same vacuum. If this closed path in the manifold of the vacuum states is not contractible, there should be a topological winding number associated with it. Given the fact that the electric charge in QED has features characteristic of a topological charge (trivially conserved and quantized), it is natural to expect that it is identical to this winding number.

[2]One can also restrict the integration in the perpendicular directions x_1 and x_2 to a finite domain, but this is irrelevant to our argument.

Note that, although the notion of topology of the manifold of the vacua originates in the classical field theory, it has a precise quantum meaning. Suppose one has a vacuum degeneracy in the quantum theory due to spontaneous breaking of some symmetry group G. The different vacuum states will differ not only in the expectation value of the order parameter O, but also all its correlators and, in fact, all operators which are nontrivial representations of G. However, since the vacuum degeneracy is only due to the spontaneous breaking of G, the VEV of any operator O_i, on which the action of G is free unambiguously determines the values of all the other correlators. Therefore the possible values of this order parameter O can be taken to parametrize the manifold of the quantum vacua and the topology is identical to the classical one.

Analogously, one defines a notion of a topological soliton which "interpolates between different vacuum states" at spatial infinity. Usually (when the broken symmetry has a local order parameter), this is a state with the following properties. Consider a chunk of space T of linear dimension a at a distance R from the soliton core, so that $a/R \to 0$. Then the expectation value of any local operator $O(x)$, $x \in T$, and any correlator of local operators $O_1(x_1)\cdots O_n(x_n)$, $x_1,\ldots,x_n \in T$, will be equal to their vacuum expectation values in one of the vacua. In another chunk T_1, which is also very far from the soliton core but also far from T so that $|x_T - x_{T_1}| = o(R)$, the values of these correlators are given by their expectation values in another vacuum state. So in this soliton state at each "point" at infinity one has a vacuum state in the sense that all local and quasilocal operators (operators with finite support) have expectation values equal to their VEV's in a vacuum. These vacua are, however, different at different points at infinity, and the mapping from the spatial boundary into the vacuum manifold is not homotopic to a trivial map. The soliton charge in this case is equal to the winding number corresponding to the homotopy group $\Pi_{D-1}(M)$, where $D-1$ is the dimension of a spatial boundary and M is the manifold of the vacua.

This was precisely the picture in QED in $2+1$ dimensions. The vacuum manifold was S_1 corresponding to the phase of the VEV of the vortex operator $V(x)$. In a charged state with charge n, the configuration of the vortex field looked asymptotically like a hedgehog: $V(x) \to_{x\to\infty} e^{in\Theta}$, and the electric charge was equal to the winding number corresponding to the homotopy group $\Pi_1(S^1)=Z$.

The situation in QED_{3+1} is slightly different. The vacuum (at least in the classical approximation) is still degenerate. However, the broken symmetry group is represented trivially on all local operators. The only operators that carry the broken charges and whose VEV's therefore distinguish between different vacua are the infinitely long magnetic vortex lines $V(C)$. It is clear, therefore, that in any solitonlike state (if it exists) all quasilocal operators will have the same VEV at all points at spatial infinity. The soliton is not characterized by $\Pi_2(M)$ or rather $\Pi_2(M)=0$. To see the difference between different regions of space far from the soliton core, one has to calculate the VEV of $V(C)$. One therefore has to divide the spatial infinity not into quasipointlike regions, but rather into quasistringlike regions, and compare the VEV's of $V(C)$ and their correlators (which fit into one such region).

The set of all the vortex operators is overcomplete. As in the case when a local order parameter exists, it is enough to pick the minimal set of operators so that every group element of the spontaneously broken group by represented nontrivially. In our case the set of broken charges is $\{\Phi_S\}$. The most convenient choice for $\{V(C)\}$ is the set of all straight lines.

The operators whose VEV's one compares should be transformable into each other by *translation*. The operation of translation does not change either the orientation or the form of a string. Moreover, all the points on a string should be far from the soliton core. Therefore for a given straight vortex line the set of operators to which it should be compared can be chosen as the set of all straight vortices having the same direction and the same distance from the soliton core in the limit where this distance becomes infinite. We see therefore that the relevant homotopy is the first rather than the second homotopy group $\Pi_1(M)$. As we have discussed earlier, the vacuum manifold is infinitely dimensional, corresponding to an infinite number of broken charges Φ_S, and therefore this homotopy group is huge. However, if we only consider rotationally symmetric solitons, things simplify considerably. Since the straight lines in different directions can be all transformed into each other by a rotation, for the rotationally symmetric soliton the winding numbers for all sets of straight lines are the same. Since the vortex operator creating a straight line in a given direction takes values in S^1, we see that for rotationally symmetric configurations the soliton charges must take values in $\Pi_1(S^1)=Z$.

Let us now calculate expectation values of the magnetic vortex lines in the third direction in a state with an electric charge at the origin. We again do this in the classical approximation. The vortex operator which creates a fluxon parallel to the third axis with coordinates (X_1,X_2) is

$$V(X_1,X_2)=\exp\left\{i\frac{2\pi}{e}\int_{-\infty}^{\infty}dx_3\int_{X_1}^{\infty}dx_1E_2(x_1,X_2,x_3)\right\} . \tag{32}$$

In the classical approximation the phase factor is proportional to the electric flux through the half plane $(x_2=X_2, x_1>X_1)$. For the spherically symmetric configuration of a pointlike electric charge, this is proportional to the plane angle Θ between the vector (X_1,X_2) and the axis x_1. Since the total flux is equal to e, we find

$$\langle V(R,\Theta)\rangle=\exp\{i\Theta\} . \tag{33}$$

For a pointlike electric charge, this expression is valid for any R. If the charged state has some charge distribution, the expression (33) will be still valid asymptotically for $R\to\infty$. In the state with an electric charge eN, one clearly has

$$\langle V(R,\Theta)\rangle=\exp\{iN\Theta\} . \tag{34}$$

We see, therefore, that electrically charged states realize the nontrivial windings of the vortex operators. The electric charge is equal to the winding number.

It is quite easy to construct states with winding numbers corresponding to more general elements of $\Pi_1(M)$ and not only $\Pi_1(S^1)$. For example, consider a charged state which is not spherically symmetric, but has all the electric flux lines asymptotically parallel to the (x_1x_3) plane. In this case all the operators considered earlier will have a unit expectation value, since no flux crosses the (x_1x_3) plane. However, the fluxons in the direction x_2, for example, will still have a winding number 1. So this state has a nonzero winding with respect to transformations generated by Φ_2, but is trivial with respect to transformations generated by Φ_3.

However, the mere fact that one can construct a state with a given topological charge does not mean it is necessarily realized in the theory. It must also pass the test of having a finite energy. Electrically charged states which are not asymptotically rotationally invariant have infinite energy and are of no interest in QED_4.

As a final comment in this section, we note that in the classical approximation the Higgs phase cannot be studied. Since the vortex operator is defined as a unitary operator, classically its VEV cannot be zero, and therefore we are always in the Coulomb phase. This is similar to the nonlinear σ model, where in the classical approximation one does not see the unbroken phase. Quantum corrections, of course, induce the phase transition there. In the present case, as we will see in the following section, the same phenomenon occurs.

IV. QUANTUM CORRECTIONS AND THE INFRARED REGULARIZATION

A. Quantum corrections to $\langle V(C)\rangle$

We will now calculate $\langle V(C_3)\rangle$ taking into account the lowest-order quantum fluctuations.

Let us start with the Coulomb phase. The lowest order in the e correction to the classical result is[3]

$$\langle V(C_3)\rangle=\exp\left\{-\frac{1}{2}\left[\frac{2\pi}{e}\right]^2\int d^4k\, a_i(k)a_j(-k)\times G_{ij}(k)\right\} , \tag{35}$$

where

$$a_i(k)=\delta_{i2}\frac{1}{k_1}\delta(k_3) \tag{36}$$

and $G_{ij}(k)$ is the propagator of the electric field,

$$G_{ij}(k)=i\left\{\frac{k_0^2}{k^2}\left[\delta_{ij}-\frac{k_ik_j}{k_0^2}\right]-\delta_{ij}\right\} . \tag{37}$$

[3]Since the calculation is analogous to the corresponding one in 2+1 dimensions, we skip the details, which can be found in [3].

The integral in Eq. (35) is both ultraviolet and infrared divergent. Introducing the ultraviolet cutoff Λ and the infrared cutoff (in real space) L, we find

$$\langle V(C_3)\rangle = \exp\left\{-\left[\frac{2\pi}{e}\right]^2 \Lambda L\right\} . \tag{38}$$

The reason for both divergences is intuitively clear. The ultraviolet divergence appears since the vortex line created by $V(C)$ has zero thickness. It can be dealt with by either regularizing the vortex itself (making it finite in cross section) or by multiplicative renormalization [11]. The infrared divergence comes about because of the infinite length of the vortex line.

So we find that in one very important respect the quantum corrections change the classical result. Now in the limit $L\to\infty$ we have $\langle V(C)\rangle\to 0$. The situation is similar in a certain sense to $(1+1)$-dimensional field theories with a continuous global symmetry. There, too, in the classical approximation one can have a nonvanishing order parameter, which, however, is found to vanish when quantum fluctuations are taken into account. As a result, a continuous symmetry is never broken in $1+1$ dimensions. We further discuss the similarities as well as differences between the realization of the magnetic symmetry in QED_4 and continuous symmetries in $1+1$ dimensions in the Appendix.

In the next section we will perform an infrared regularization of the theory which, without explicit breaking of the magnetic symmetry, yields a finite expectation value for the vortex operator. In this regularized theory the arguments concerning the Goldstone theorem and the topological interpretation of the electric charge presented in the previous section in the classical approximation will be valid also on the quantum level.

Before doing that, let us calculate $\langle V\rangle$ in the Higgs vacuum. The most convenient way to do this is using the Euclidean path integral formalism. The expectation value $\langle V(C_3)\rangle$ can be written in the following form [15]:

$$\langle V(C_3)\rangle = \int \mathcal{D}A_\mu \mathcal{D}\phi \exp\left\{-\left[\frac{1}{4e^2}(\tilde{F}_{\mu\nu}-\tilde{f}_{\mu\nu})^2 + |D_\mu\phi|^2 + U(\phi^*\phi)\right]\right\} , \tag{39}$$

where

$$\tilde{f}_{\mu\nu}(x) = \delta_{1[\mu}\delta_{\nu]3}\delta(x_0)\delta(x_2)\theta(x_1) ,$$

with $\theta(z)$ a step function. For any given x_3 the field $\tilde{f}$ satisfies

$$\partial_\nu \tilde{f}_{\mu\nu} = \delta_{3\mu}\delta^3(x) . \tag{40}$$

If we now view x_3 as the Euclidean time, $\tilde{f}$ is the magnetic field of the Dirac string of a static magnetic monopole propagating in time. At the tree level therefore the VEV is given by a Euclidean action of a static magnetic monopole in the Higgs phase. In the Higgs phase magnetic monopoles are linearly confined and the energy of a single monopole diverges linearly with the dimension of a system. The action therefore diverges quadratically, and we obtain

$$\langle V(C_3)\rangle = e^{-\alpha L_1 L_3} , \tag{41}$$

where α is a dimensional constant and L_1 and L_3 are infrared cutoffs on the first and third directions, respectively.

We see that the VEV vanishes much faster in the infrared than in the Coulomb phase. In fact, even if we make the system finite in the direction of the vortex line, the VEV still vanishes in the limit $L_1\to\infty$. So even though in the infinite system the VEV of the vortex operator vanishes in both phases, there is a qualitative difference in the dependence on the infrared cutoff. This difference will be reflected in the VEV of a closed vortex loop ('t Hooft loop). Evidently, the large loops in the Coulomb phase will have a perimeter law behavior, while in the Higgs phase, the area law. This result of course coincides with 't Hooft's discussion of the expected behavior of vortex loops [12]. This means that in the Coulomb phase there is a condensate of 't Hooft loops of arbitrarily large radius, whereas in the Higgs phase there is no such condensate.[4]

B. Infrared regularization

Let us now describe the simplest infrared regularized theory which has a finite VEV of the vortex operator in the Coulomb phase. Consider QED defined on a spatial manifold which is compact in the direction of the x_3 axis. This means that all the gauge-invariant fields (B_i, E_i, J_0, etc.) must at all times obey the periodic boundary condition

$$O(x_1,x_2,x_3) = O(x_1,x_2,x_3+L) . \tag{42}$$

In this theory the magnetic flux Φ_3 is still a conserved charge. We will concentrate on it and on its order parameter $V(C_3)$. As previously, $V(C_3)$ is a well-defined operator, except that now the vortex line it creates has a finite length L. The calculation of $\langle V(C_3)\rangle$ is the same as previously. The only alteration is that the photon's propagator must be modified according to the new boundary condition, so that k_3 in Eq. (37) takes discrete values $K_3=2\pi n/L$. The exact form of the propagator, however, does not matter as we have seen earlier, and we obtain

$$\langle V(C_3)\rangle = e^{-\alpha\Lambda L} . \tag{43}$$

The proof of the Goldstone theorem goes through in precisely the same way as in the unbounded case since the dual field strength tensor remains antisymmetric. The

[4]This consideration applies only to loops with integer flux 't Hooft loops that carry noninteger flux do not condense even in the Coulomb phase because of their nonlocal properties (see the Appendix).

Goldstone bosons that appear due to spontaneous breaking of Φ_3 are linearly polarized photons with a magnetic field pointing in the direction x_3.

Note that in the Higgs phase $\langle V(C_3)\rangle=0$ because of the infinite extent of our system in the direction x_1. The Higgs-Coulomb phase transition therefore is attributed to the spontaneous breaking of Φ_3.

One also immediately realizes that an electrically charged state has a nonzero winding of $V(C_3)$. The configuration of the electric field of a point charge near the location of a charge is the same as in the unbounded case. However, since the electric flux cannot escape through the boundary due to periodic boundary conditions, near the boundary the electric flux lines get squeezed and become parallel to the x_1x_2 plane, so that all the flux escapes to infinity (Fig. 3).

Repeating now the calculation of a previous section, we find that the surface associated with $V(C_3)$ collects the electric flux proportional to the planar angle, and therefore Eq. (33) is still valid.

For free (as opposed to periodic) boundary conditions, part of the electric flux would have escaped through the boundary and the wind of $V(C_3)$ would not be complete. However, in this case the electric charge would also not be conserved, since charged particles would be able to leave the system freely through the boundary. And, of course, if a charge is not conserved, it cannot be topological.

Other infrared regularizations are possible. For example, one could take the system to be finite in two dimensions x_3 and x_2. Then both $\langle V(C_3)\rangle$ and $\langle V(C_2)\rangle$ would be nonvanishing in the Coulomb phase. The masslessness of photons with two linear polarizations would then follow by Goldstone's theorem. If the boundary conditions preserve the $\pi/2$ rotations around the first axis, the finite energy electrically charged states will carry a unit winding of both $V(C_2)$ and $V(C_3)$. Note, however, that the regularization in which all three directions are made compact is illegal since in this case $V(C)$ cannot be defined. The reason is that the surface of singularities associated with $V(C)$ must be infinite and there are no such surfaces in a completely finite system. The same is true in $2+1$ dimensions where one cannot define the local vortex field $V(x)$ in a finite system with periodical boundary conditions.

We see, therefore, that any sensible infrared regularization leads to a nonvanishing VEV of the vortex operators. Goldstone's theorem for the photon and identity of the electric charge with the winding number holds for any finite value of the infrared cutoff. In this sense both results are also true in the unbounded theory, although the actual VEV of the order parameter vanishes.

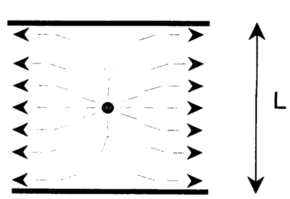

FIG. 3. Schematic distribution of the electric flux lines of the field of a pointlike charge in a box of a finite height L with periodic boundary conditions.

Note that QED_4 differs from a typical field theory in the following respect. Usually, the tendency of a smaller system is toward a restoration of any broken symmetries, since the potential barrier between the degenerate vacua becomes smaller. In a certain sense this is why the high-temperature phase is usually the one where all the symmetries are restored. In QED_4, as we have seen, the opposite happens. Because of the nonlocality of the order parameter, its expectation value is actually larger for a smaller system. This might be connected to the fact that in QED_4 the high-temperature phase is the Coulomb phase, in which the flux symmetry is broken, whereas the low-temperature Higgs (superconducting) phase has the symmetry restored.

V. DISCUSSION

In this paper we approached QED from an unconventional point of view. Instead of concentrating our attention on the standard degrees of freedom such as photons and charged particles, we have analyzed the behavior of the dual variables: the magnetic vortex lines. The picture that transpires from this point of view is somewhat similar to $(2+1)$-dimensional electrodynamics.

In the Coulomb phase the operators creating infinitely long vortex lines $V(C)$ have "finite expectation value per unit length." What this means is that the expectation value of such an operator in a system with a finite infrared cutoff in the direction of the vortex line behaves as $e^{-\alpha L}$ in the Coulomb phase. The operators $V(C)$ are eigenoperators of the magnetic symmetry generators Φ_S. Therefore in the Coulomb phase the magnetic symmetry group is spontaneously broken. Although the only order parameters for Φ_S are the nonlocal vortex operators, the Goldstone theorem is still applicable and the spontaneous breaking of this symmetry leads to exact masslessness of the photon.

The electric charge in this picture is topological and corresponds to the homotopy group $\Pi_1(S^1)$ of possible string configurations.

In the Higgs phase, the VEV $\langle V(C)\rangle$ vanishes. The magnetic symmetry is restored and no massless excitations are present. The Higgs-Coulomb phase transition is driven by condensation of the magnetic vortices.

According to the standard lore, a theory near a phase transition (and also away from the phase transition but at low energies) should be describable in terms of a Landau-Ginzburg-type Lagrangian for the order parameter. In the case at hand, this would not be a standard field theory, but rather a string theory of the vortex lines $V(C)$. An approximate derivation of this string theory is given in [16]. In fact, the dual field strength tensor $\tilde{F}_{\mu\nu}$ can be expressed via $V(C)$ in the same way as $F_{\mu\nu}$ is expressed via the Wilson line [17],

$$\tilde{F}_{\mu\nu}(x)=V^{\dagger}(C)\frac{\delta}{\delta S_{\mu\nu}(x)}V(C)\,, \qquad (44)$$

where $\delta/\delta S_{\mu\nu}(x)$ is the area derivative at the point x.

The kinetic term therefore can be rewritten in terms of the vortex creation operator as

$$\frac{\delta}{\delta S_{\mu\nu}} V^{\dagger}(C) \frac{\delta}{\delta S_{\mu\nu}} V(C) . \tag{45}$$

There is an interesting possibility that the exact dual reformulation of QED_4 exists. Then QED should be exactly equivalent to an interacting string theory. Clearly, the weakly interacting QED will be described by a strongly interacting dual string theory. This must be so, since the spectrum of a free string theory contains an infinite number of particles, whereas the spectrum of QED contains just the familiar excitations: the photon and charged particles. It should also be noted that a massless photon will arise in this string theory in a way very different from massless gauge particles in a free string theory, since it is massless in the phase in which the strings are condensed.

It is clear from the discussion in Sec. III that the topological mechanism of quantization of electric charge is intimately related to the existence of a "line-local" vortex operator. If a line-local vortex operator does not exist, of course, the whole discussion of topology in Sec. III would be meaningless. Conversely, a theory with nonquantized electric charges does not admit a line-local vortex field. A simple example is furnished by QED_4 with incommensurate charges. Here the Dirac condition necessary for the existence of a line-local operator V cannot be satisfied. On the other hand, the quantization of the electric charge in nature is a well-established experimental fact, and therefore any phenomenologically relevant theory must admit a topological interpretation of the electric charge. One could thus entertain the idea that the topological nature is a primary quality of the electric charge, and if accepted as a basic principle, it provides a natural mechanism for charge quantization.[5] It would also imply that a more natural setup for the generalization of electrodynamics is the dual string formulation (which ensures the existence of the line-local vortex field and the correct topology) rather than the original gauge-field theory formulation. This, however, technically is beyond our present abilities.

There are many further questions which have been asked in the context of $(2+1)$-dimensional gauge theories which we did not address in this paper. For example, what elements of the dual picture should be modified if the matter fields are fermionic? But the most interesting one is perhaps, can this picture be generalized to non-Abelian theories? It would be very rewarding to have a simple qualitative picture of confinement based on a topological interpretation of the electric charge similar to the one available in $2+1$ dimensions [18]. There constituent quarks can be understood as topological defects like electric charges in QED, but the flux symmetry is broken explicitly (by the nonperturbative monopole instanton effects). As a result of this explicit breaking, the vacuum is nondegenerate (or has a finite degeneracy) and the topological defects are linearly confined. In non-Abelian theories in $3+1$ dimensions, there are also nonperturbative effects due to magnetic monopoles (which now are particles rather than instantons). The appearance of the monopoles again breaks explicitly the magnetic symmetry since the dual field strength is not conserved anymore. It is interesting to see whether this explicit breaking leads to linear confinement of the topological defects as in $2+1$ dimensions, although the defects now are of quite a different nature. It is also worth noting that in $SU(N)$ theories with adjoint matter fields only the monopoles carry N units of the elementary Dirac quantum. Therefore the discreet subgroup of the magnetic group will still survive (just as in $2+1$ dimensions). The phase transition between the "completely broken" Higgs phase and a confinement phase can then be attributed to the spontaneous breaking of this discreet symmetry.

APPENDIX

As mentioned in Sec. IV, there are certain similarities between the behavior of the order parameter in QED_4 and in $U(1)$-invariant $(1+1)$-dimensional models. In both cases the order parameter in the massless phase, although nonzero in the classical approximation, vanishes when the quantum corrections are taken into account.

There is, however, a very important physical difference between the two cases. In the $(1+1)$-dimensional models, the order parameter is local. The vanishing of its expectation value therefore persists also for a finite infrared cutoff as long as the cutoff theory preserves the symmetry. Technically, the system is disordered by the zero mode. If the infrared regularization is performed in such a way that the "spontaneously broken" symmetry is not broken explicitly, the zero mode is still present and it still leads to the vanishing of the VEV of the order parameter. If the regularization is such that the zero mode is given a finite mass, the VEV can be nonzero, but the symmetry is then broken explicitly.

In the case at hand, though, the order parameter is nonlocal and this nonlocality, rather than the zero-mode contribution, is the factor which leads to the vanishing of the VEV. This can be seen explicitly by taking a massive rather than a massless propagator in Eq. (37). The result is still linearly infrared divergent.

Another basic qualitative difference is the absence of the so-called symmetry enhancement in QED_4. In $U(1)$-invariant $(1+1)$-dimensional models, the symmetry in the massless phase is actually larger than $U(1)$ [19,20]. Because of the extreme softness of the interaction of massless "almost Goldstone bosons," Hilbert space of these models contains finite-energy states with arbitrary real (noninteger) charges and the symmetry group actually becomes R. In contrast, in QED_4 the symmetry enhancement does not occur. Even in the Coulomb phase the energy of a state with a noninteger magnetic flux is infinite relative to that of an integer flux.

Let us calculate the energy of the state with a fluxon of

[5]Curiously, this explanation of charge quantization as well as the one based on a possibility of the existence of magnetic monopoles also use the Dirac quantization condition, but now in the guise of quantization of the vorticity of a fluxon.

strength γ/e with arbitrary γ created from the vacuum by an operator $V_\gamma(C_3)$ (see Fig. 3):

$$|\gamma\rangle \equiv V_\gamma(C_3)|0\rangle \ . \tag{A1}$$

As was discussed in Sec. II, for $\gamma=2\pi n$ the operator $V_\gamma(C)$ is line local and therefore does not perturb the vacuum state outside the line C_3. On the other hand, for arbitrary γ, $V_\gamma(C_3)$ has a support on a half plane S_3: $x_1>0$, $x_2=0$. Accordingly, one expects that the energy of the state $|n\rangle$ has finite energy per unit length ($E_n \propto L$), whereas the linear energy density in $|\gamma\rangle$ is expected to diverge. Indeed,

$$E_\gamma = \langle 0|V_\gamma^\dagger(C_3) H V_\gamma(C_3)|0\rangle \ . \tag{A2}$$

Since V_γ is an operator of a singular range transformation, we have

$$\begin{aligned} V_\gamma^\dagger(C_3)B_i(x)V_\gamma(C_3) &= B_i(x)+\frac{\gamma}{e}\delta_{i3}\delta(x-C_3) \ , \\ V_\gamma^\dagger(C_3)J_i(x)V_\gamma(C_3) &= J_i(x)+\gamma O_i(x)\delta(x-C_3) \\ &\quad +\gamma_{\mathrm{mod}2\pi}\phi(x)\delta_{i2}\delta^2(x-S_3) \ . \end{aligned} \tag{A3}$$

The local operator $O_i(x)$ depends on the UV regularization of the vortex operator V along the line C_3, but its exact form is unimportant. The energy of the state is therefore

$$\begin{aligned} E_\gamma = E_0 &+ \left[\frac{\gamma}{e}\right]^2 L_3\Lambda^2+\gamma^2 L_3\Lambda^2\langle O_i^2\rangle \\ &+2\gamma L_3\langle J_i(x)O_i(x)\rangle+(\gamma_{\mathrm{mod}2\pi})^2 L_3 L_1\langle\phi^*\phi\rangle \ , \end{aligned} \tag{A4}$$

where Λ is a UV cutoff. For $\gamma_{\mathrm{mod}2\pi}=0$ we have $E_\gamma - E_0 \propto L_3$, while for noninteger fluxons the last term gives a leading contribution which is divergent quadratically if $\langle\phi^*\phi\rangle\neq 0$. Stated differently, this means that an introduction of a noninteger fluxon into the vacuum should lead to the appearance of vacuum currents which screen the noninteger part of the flux, so that the vacuum can only support the existence of integer fluxons. This phenomenon indeed occurs in QED$_4$ with both bosonic and fermionic matter [21].

In the Higgs phase $\langle\phi\rangle\neq 0$ already on the classical level. In the Coulomb phase classically $\langle\phi\rangle=0$, but quantum mechanically of course $\langle\phi^*\phi\rangle\neq 0$. Consequently, in both phases states with noninteger fluxons have divergent linear energy density, and are therefore not present in the physical Hilbert space.[6] The symmetry enhancement in this model therefore does not take place.[7]

[6]We have considered a particular state with a noninteger fluxon. However, any state which carries a noninteger flux has the form $K(C_3)V_\gamma(C_3)|0\rangle$ with $K(C)$ an operator which has a support on C_3. It is clear from the previous discussion that this modification of the state does not change the dependence of the energy on the dimensions of the system.

[7]Very far from the phase transition, the VEV $\langle\phi^*\phi\rangle$ vanishes since the charged particles become very heavy and for heavy particles $\langle\phi^*\phi\rangle\propto 1/M^2$. In this limit the magnetic symmetry group "decompactifies." This behavior is the same as in any theory with spontaneously broken compact global symmetry infinitely far from the phase transition inside a broken phase. This, however, has nothing to do with symmetry enhancement.

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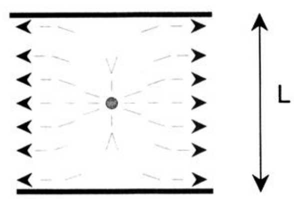

FIG. 3. Schematic distribution of the electric flux lines of the field of a pointlike charge in a box of a finite height L with periodic boundary conditions.