

Scattering in the presence of electroweak phase transition bubble walls

Alejandro Ayala, Jamal Jalilian-Marian, and Larry McLerran

School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455

Axel P. Vischer

Oregon State University, Corvallis, Oregon 97331

and School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455

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We investigate the motion of fermions in the presence of an electroweak phase transition bubble wall. We derive and solve the Dirac equation for such fermions, and compute the transmission and reflection coefficients for fermions traveling from the symmetric to the asymmetric phases separated by the domain wall.

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I. INTRODUCTION

There has been much recent work which suggests that the baryon asymmetry of the Universe might have been produced at the electroweak phase transition [1]. If the equations of motion of electroweak theory are quantized, then anomalies arise which are responsible for the Chern-Simons, or baryon number changing, currents [2]. The rate of baryon number violation is rather small for low temperatures but it becomes large at a temperature scale on the order of the electroweak scale of about 100 GeV [3–5]. Since baryon number is violated, a net baryon number will be generated if there is a mechanism that biases the rate at the electroweak phase transition in such a way that, as the baryon number violation shuts off in the low temperature phase, an asymmetry remains frozen into the system. This can happen in practice if the electroweak phase transition is of first order, and if there is sufficient CP violation at the temperature of the phase transition.

It is expected that the electroweak phase transition is of first order [6–9]. In a first order phase transition, the conversion from one phase to another occurs through nucleation of the true phase in the false phase. This happens when the system is either supercooled or superheated. The bubbles of the true phase expand rapidly, absorbing the region of the false phase. For the electroweak phase transition, this true phase eventually fills the entire volume with no intermediate mixed phase [10]. At the bubble surface, there is a thin wall of microscopic dimensions which separates the phases. It is at this bubble wall that matter is strongly out of equilibrium, and here the baryon asymmetry is generated [11–18].

To quantitatively understand the generation of the baryon asymmetry in the bubble wall, the effect of the bubble wall on the propagation of fermions should be understood. Fermions passing through the bubble or domain wall acquire mass, generated by the Yukawa coupling, which is proportional to the finite temperature vacuum expectation value (VEV) of the Higgs field. This VEV is determined from the equations of motion of the

finite temperature effective action of the bubble.

To simplify the problem, we will work in the approximation where the energy densities of the two phases are degenerate. In practice this is a good approximation, since in cosmology, the Universe is expanding so slowly that the nucleation always begins at a temperature where the amount of supercooling is very small, and the energies of the two phases are degenerate. In this approximation, there is a one-dimensional kink solution which separates the phases, and the bubble wall is a domain wall which separates the domains of different energy. This kink wall can propagate at any velocity. In practice, the wall velocity is determined by a complicated analysis which involves computing the effects of dissipative processes, and is in the range 0.1–0.9 of the speed of light [9,19–21]. In our analysis, we will consider fermion propagation in the presence of the wall at rest. The case for a moving wall can be determined by Lorentz boosting to the moving wall frame.

A plot of the domain wall is shown in Fig. 1. At $x = -\infty$, the system is in the symmetric phase, that is, outside the bubble. At $x = +\infty$, the system is in the symmetry broken phase, that is, inside the bubble. The approximation of the bubble as a planar interface should be valid for bubbles which are large compared to a microscopic size scale. This is true for most of the evolution of bubbles produced in the electroweak phase transition [10].

The fermions moving through the bubble wall will interact not only with the wall but also with the particles in the surrounding plasma. If the mean free path λ_{MFP} of the fermions is much larger than the wall thickness L , so that $\lambda_{\text{MFP}} \frac{v_z}{v} \gg L$, then the collisions with the plasma particles will have a negligible effect. The ratio $\frac{v_z}{v}$ represents the projection on the axis perpendicular to the wall. On average though we can estimate the width of the wall [20] to be $L \sim \frac{\sqrt{2}\lambda}{\delta T} \sim (15 - 20)T^{-1}$ and the fermion mean free path as $\lambda_{\text{MFP}} \sim (1 - 10)/T$. The mean free path in the direction perpendicular to the wall is therefore in most cases on the order of or less than the wall thickness and the collisions with the plasma be-

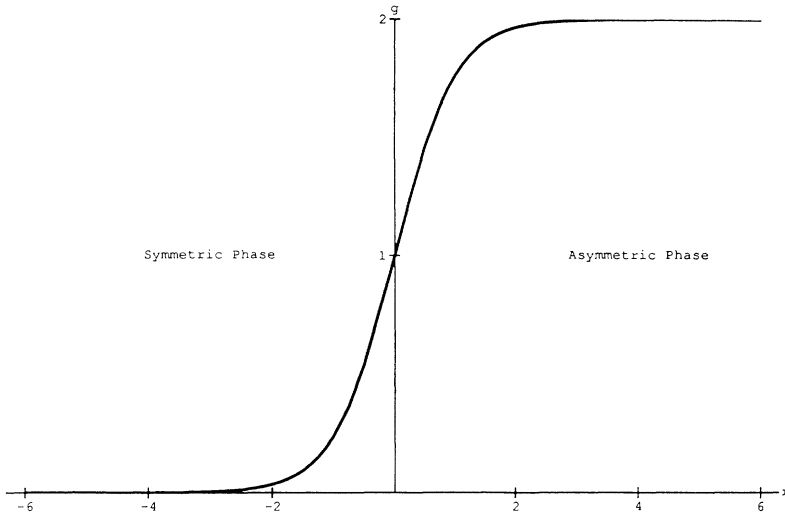


FIG. 1. The bubble profile. $x = -\infty$ corresponds to the bubble's outside. The domain wall is the region where the VEV of the Higgs field $g(x)$ changes rapidly.

come quite important. In this limit the propagation of fermions through the wall has to be treated as a transport problem. The investigation of this limit is subject of a later analysis. The object of such an analysis is to compute the diffusion of particles scattered from the wall and the resulting asymmetry. This diffusion problem is rather complicated and involves solving the Fokker-Planck equation. The solution has then to be convoluted with baryon number violating rates.

Although for the reasons discussed in the previous paragraph, the analysis we present has a limited scope, it is still useful in its own right, and there may be special theories where in fact the diffusion corrections are a relatively minor correction to the scattering from the wall. In most realistic cases, we expect that the diffusion corrections will be large, and that it is probably more accurate to use an approximate adiabatic limit. Until the diffusion computations are performed, it is difficult to say something concrete and quantitative.

The outline of this paper is as follows: In Sec. II we describe how to obtain the equation of motion for fermions in the presence of a domain wall in the minimal standard model. We obtain the Dirac equation with an effective mass proportional to the VEV of the Higgs field. We solve this equation in Sec. III, and compute the fermion wave function we obtain transmission and reflection coefficients. In Sec. IV, we present the normalized solutions of the Dirac equation. We finish by summarizing our results in Sec. V.

II. MOTION OF FERMIONS IN THE PRESENCE OF DOMAIN WALLS

In this section we show how to obtain simplified classical equations of motion for fermions in the standard model scattering from and interacting with the electroweak domain walls. We use the classical mean-field approximation in which the bosonic field operators are replaced by their classical expectation values.

We will briefly review the notation which we will use throughout the paper. The Lagrangian in the standard

model is invariant under $SU(2) \times U(1)$ transformations and the fields are therefore eigenstates of weak isospin and hypercharge Y . The $SU(2) \times U(1)$ invariant vacuum state of the Lagrangian exists only in a high temperature phase above the electroweak phase transition. The symmetry is broken spontaneously once we cool the system below the transition temperature T_c .

The Lagrangian is

$$\mathcal{L} = \mathcal{L}_{\text{gauge field}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{fermion}} + \mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{\text{GF}}. \quad (1)$$

The gauge field, fermion, scalar kinetic energy, and gauge-fixing terms are all written in the standard way [24]. For the potential energy of the Higgs field, we take

$$V(\Phi) = \lambda(\Phi^\dagger\Phi - v^2/2)^2, \quad (2)$$

so that the vacuum expectation value of the Higgs field is

$$\langle\Phi\rangle = \frac{v}{\sqrt{2}}. \quad (3)$$

To describe kink solutions and a first order phase transition, one has to include the modifications of the effective potential for the Higgs field due to interactions with the heat bath. Including effects of both one-loop diagrams and the sum of ring diagrams, the effective potential can be written to a good approximation as

$$V_{\text{eff}}(\phi, T) = \frac{\gamma}{2}(T^2 - T_c^2)\phi^2 - \delta T\phi^3 + \frac{\lambda}{4}\phi^4, \quad (4)$$

where $\phi = \sqrt{2}(\Phi^\dagger\Phi)^{1/2}$. The dimensionless parameters γ and δ are approximately given by

$$\begin{aligned} \gamma &\sim \frac{5}{16}g^2, \\ \delta &\sim \frac{1}{16\pi}g^3, \end{aligned} \quad (5)$$

when $M_W \sim M_Z \sim M_t$, where these are the masses of the W bosons, the Z boson, and the top quark, respectively. δ is small and is responsible for generating the first order phase transition. At some value of T , there

are two degenerate minima of this potential and a kink solution exists.

Of course the above evaluation of the effective potential may not be such a good approximation for realistic values of the Higgs mass. If the effective potential is greatly modified, then the kink solution, estimates for bubble wall velocity, and the considerations we present will all have to be modified. Nevertheless, our procedure should be instructive.

As the scalar field evolves through the kink solution, no current is generated for the vector field, so that no expectation value of the vector fields is generated. We are therefore justified in truncating the system to only the Higgs field degrees of freedom. The equation of motion is just the classical Higgs field equation of motion with zero background vector field.

To get the effective potential for the Higgs field into more transparent form [19], we introduce the dimensionless temperature $\zeta = \frac{\lambda\gamma}{\delta^2} [1 - (\frac{T_c}{T})^2]$ and the dimensionless field strength $g = \frac{\delta}{\delta T} \phi$ and obtain

$$V_{\text{eff}}(g) = \delta T \left(\frac{\delta T}{\lambda} \right)^3 \left(\frac{\zeta}{2} g^2 - g^3 + \frac{1}{4} g^4 \right). \quad (6)$$

The potential $V_{\text{eff}}(g)$ is plotted in Fig. 2 for different values of ζ . For large positive values of ζ we are in the high temperature phase with the VEV of the Higgs field being zero. Decreasing ζ we develop at $\zeta = 2.25$ a second relative minimum which for $\zeta = 2$ becomes degenerate with the first one. For smaller values of ζ , the high temperature phase at $g = 0$ becomes unstable with respect to the new absolute minimum at $g \neq 0$, the first order phase transition sets in, and bubbles of the broken phase start to nucleate. If we supercool to $\zeta = 0$, the system spinodally decomposes.

Since the expansion rate of the Universe ($\approx 10^{-11}$ s) is rather slow compared to the electroweak timescale ($\approx 10^{-26}$ s), we cannot strongly supercool and the phase transition will roughly proceed at $\zeta \approx 2$ where the minima of the effective potential are degenerate. The steady state solution describing the domain wall is thus given

approximately by the solution of the Higgs field g for a transition between the degenerate minima at temperature $\zeta = 2$.

The equation of motion for the kink is mathematically identical to treating the amplitude of the Higgs field g as a spatial coordinate for the inverted potential of the previous equation and taking the spatial coordinate of the kink as the time variable [23]. The mechanical analogue to this system is a frictionless ball rolling from the top of one hill through a valley to the top of another hill. For degenerate minima the tops of the two hills have the same height and there exists a solution where the ball starts at the top of one hill from rest and ends on the top of the other hill again at rest. Such a solution is the kink. Energy conservation for the kink reads

$$E = \frac{1}{2} \left(\frac{dg}{dr} \right)^2 - V_{\text{eff}}(g(r)), \quad (7)$$

which for a transition between degenerate minima ($E = 0$) and a kink bubble with dimensionless position $x = \frac{\delta}{\sqrt{2\lambda}} \text{Tr}$ can be rewritten as

$$\frac{dg}{dx} = -g(g - 2). \quad (8)$$

We can integrate the above equation implementing the boundary condition that at the center of the nucleated bubble ($x \rightarrow +\infty$) the system is in the broken phase with a finite VEV while outside the domain wall ($x \rightarrow -\infty$) it approaches a zero VEV in the high temperature phase, treating the domain wall as effectively flat and separating two semi-infinite domains. We obtain

$$g = 1 + \tanh(x). \quad (9)$$

This solution, commonly called the ‘‘kink,’’ is depicted in Fig. 1. It represents the classical, steady state, finite temperature solution to the equations of motion.

Given a background scalar field, we can solve the Dirac equation in the presence of this field. Rewriting the coordinates in terms of ordinary dimensionful length scales, the Dirac equation is

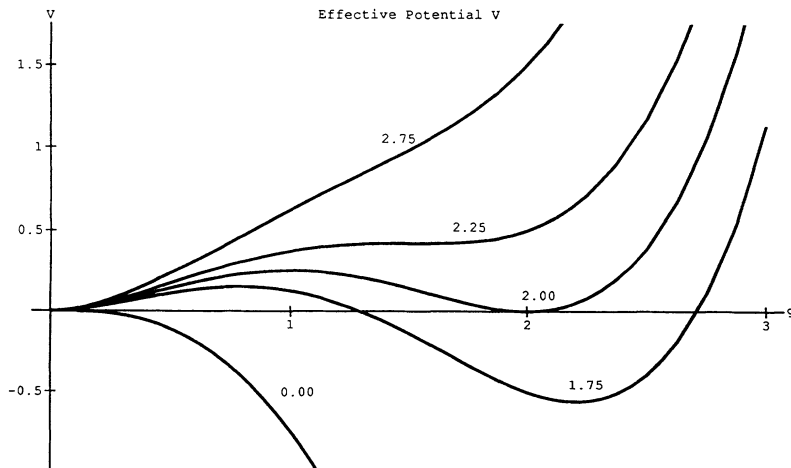


FIG. 2. The dimensionless effective potential for the Higgs field $V(g, \zeta)$ in units of $\delta T (\frac{\delta T}{\lambda})^3$ for different values of $\zeta = 0, 2, 1.75, 2.25, 2.75$.

$$\left(\not{p} - \frac{\delta T}{\sqrt{2\lambda}} \xi g(x) \right) \Psi(x) = 0, \quad (10)$$

where Ψ is any fermion field, and ξ is twice the ratio of quark to Higgs mass at zero temperature. The solution to this equation describes the motion of a fermion in the presence of a kink. To this order of approximation for the standard model, CP violation plays no essential role in this equation.

III. SOLVING THE DIRAC EQUATION

In this section we study the solutions to the Dirac equation (10). We start by explicitly showing how to solve this equation analytically for the fermion wave function. The second part is devoted to deriving the transmission and reflection coefficients from the wave function.

A. Wave function

We will solve the Dirac equation by looking for eigenstates of momentum in the plane of the wall, p_t , that is, the momentum perpendicular to a normal vector on the surface of the wall. The Dirac equation (10) is then a differential equation in one dimension and the only relevant variable (r) is the one along the direction normal to the domain wall. The degrees of freedom in the plane of the domain wall effectively decouple. This can be used by substituting the ansatz

$$\Psi(r, x_t, t) = \left(\not{p} + \frac{\delta T}{\sqrt{2\lambda}} \xi g(r) \right) e^{\pm i(p_t x_t - Et)} \Phi(r) \quad (11)$$

into Eq. (10), finding

$$\left\{ p^2 - \left(\frac{\delta T}{\sqrt{2\lambda}} \xi g(r) \right)^2 + \left[\not{p}, \frac{\delta T}{\sqrt{2\lambda}} \xi g(r) \right] \right\} \Phi(r) = 0. \quad (12)$$

The $+$ and $-$ signs in Eq. (11) correspond to the positive and negative energy solutions, respectively.

The commutator in Eq. (12) picks only a contribution from the direction normal to the wall and therefore it is proportional to just one of the γ matrices and we choose this to be γ_3 . We will use the ordinary representation for γ matrices where

$$\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}. \quad (13)$$

The eigenstates of γ_3 are

$$\gamma_3 \mathbf{u}_\pm^s = \pm i \mathbf{u}_\pm^s, \quad (14)$$

where $s = 1, 2$ and \mathbf{u}_\pm^s are

$$\mathbf{u}_\pm^1 = \begin{pmatrix} 1 \\ 0 \\ \pm i \\ 0 \end{pmatrix} \quad \text{and} \quad \mathbf{u}_\pm^2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ \mp i \end{pmatrix}. \quad (15)$$

Recalling that the dimensionless radius x is $x = \frac{\delta}{\sqrt{2\lambda}} Tr$, we obtain from (12), in dimensionless variables,

$$\left\{ \frac{d^2}{dx^2} + i\gamma_3 \xi \frac{d}{dx} g(x) - \xi^2 g^2(x) + \epsilon^2 \right\} \Phi(x) = 0. \quad (16)$$

The dimensionless energy parameter ϵ is given in terms of the energy E and the momentum parallel to the bubble wall p_t by

$$\epsilon = (\sqrt{2\lambda}/\delta T) \sqrt{E^2 - p_t^2}. \quad (17)$$

We write the spinor $\Phi(x)$ in the basis (15),

$$\Phi(x) = \sum \phi_\pm^s(x) \mathbf{u}_\pm^s. \quad (18)$$

The spinors (15) are linearly independent and thus writing (18) into Eq. (16) we get two differential equations for the functions ϕ_\pm^s ,

$$\left\{ \frac{d^2}{dx^2} \mp \xi \frac{d}{dx} g(x) - \xi^2 g^2(x) + \epsilon^2 \right\} \phi_\pm(x) = 0. \quad (19)$$

To further simplify Eqs. (19), we factorize the singularities. With the change of variable $z = \frac{1}{2}[1 - \tanh(x)]$ we substitute

$$\phi_\pm(x) = z^\alpha (1-z)^\beta \chi_\pm(z) \quad (20)$$

and examine the behavior of the resulting differential equations near the singular points $z = 0$ and $z = 1$. Assuming that the functions χ_\pm are slowly varying near the singularities, this yields two algebraic conditions from which we get

$$\begin{aligned} \alpha &= +\frac{i}{2} \sqrt{\epsilon^2 - 4\xi^2}, \\ \beta &= +\frac{i}{2} \epsilon \end{aligned} \quad (21)$$

(the choice of signs for α and β is discussed later). With the values of α and β at hand, the differential equation for the functions χ_\pm is the hypergeometric differential equation

$$\left\{ z(1-z) \frac{d^2}{dx^2} + [c - (1 + a_\mp + b_\mp)z] \frac{d}{dx} - a_\mp b_\mp \right\} \chi_\pm = 0, \quad (22)$$

with the parameters a_\mp , b_\mp , and c given by

$$\begin{aligned} a_\mp &= \alpha + \beta + \frac{1}{2} - \left| \xi \mp \frac{1}{2} \right|, \\ b_\mp &= \alpha + \beta + \frac{1}{2} + \left| \xi \mp \frac{1}{2} \right|, \\ c &= 2\alpha + 1. \end{aligned} \quad (23)$$

Each of the equations (22) has two independent solutions [22]

$$\begin{aligned}\chi_{\pm}^{(-\alpha)}(z) &= {}_2F_1(a_{\mp}, b_{\mp}, c; z), \\ \chi_{\pm}^{(+\alpha)}(z) &= z^{1-c} {}_2F_1(a_{\mp} + 1 - c, b_{\mp} + 1 - c, 2 - c; z).\end{aligned}\quad (24)$$

The hypergeometric equations are expressed here as expansions around $z = 0$. The superscripts $(\pm\alpha)$ are explained below.

The general solutions to Eqs. (19) are

$$\phi_{\pm}^s = (A^s)_{\pm}^{(-\alpha)} \phi_{\pm}^{(-\alpha)} + (A^s)_{\pm}^{(+\alpha)} \phi_{\pm}^{(+\alpha)}, \quad (25)$$

with

$$\begin{aligned}\phi_{\pm}^{(-\alpha)} &= z^{\alpha}(1-z)^{\beta} {}_2F_1(a_{\mp}, b_{\mp}, c; z), \\ \phi_{\pm}^{(+\alpha)} &= z^{-\alpha}(1-z)^{\beta} {}_2F_1(a_{\mp} \\ &\quad + 1 - c, b_{\mp} + 1 - c, 2 - c; z),\end{aligned}\quad (26)$$

where we have used (20). The superscripts $(\pm\alpha)$ indicate the behavior of these solutions at $x \rightarrow +\infty$ or correspondingly at $z = 0$ since

$$z^{\pm\alpha}(1-z)^{\beta} \stackrel{z \rightarrow +\infty}{\sim} e^{\mp 2\alpha x}, \quad (27)$$

$$z^{\alpha}(1-z)^{\pm\beta} \stackrel{z \rightarrow 0}{\sim} e^{\pm 2\beta x}, \quad (28)$$

and ${}_2F_1(a, b, c; 0) = 1$. The behavior of the solution in these limits will be needed in the next section to implement boundary conditions.

We now proceed to find the fermion wave function. We begin by rewriting Eq. (11) (in units of $\frac{\delta}{\sqrt{2\lambda}}T$) in a slightly different form

$$\Psi(x) = \left\{ \epsilon_r \tilde{\gamma} + i\gamma_3 \frac{d}{dx} + \xi g(x) \right\} \Phi(x), \quad (29)$$

where we have set

$$\begin{aligned}\tilde{\gamma} &= \gamma_0 \cosh \theta - \gamma_t \sinh \theta, \\ E &= \frac{\delta T}{\sqrt{2\lambda}} \epsilon_r \cosh \theta, \\ p_t &= \frac{\delta T}{\sqrt{2\lambda}} \epsilon_r \sinh \theta,\end{aligned}\quad (30)$$

$r = 1, 2$ and ϵ_r is given by

$$\epsilon_r = \begin{cases} \epsilon, & r = 1, \\ -\epsilon, & r = 2, \end{cases} \quad (31)$$

with ϵ given by Eq. (17). The parameter θ is related to the particle velocity in the direction parallel to the wall, v_t by $v_t = \tanh \theta$.

From now on, we set $p_t = 0$, that is, $\epsilon = E$ (in units of $\frac{\delta}{\sqrt{2\lambda}}T$) so that the fermion moves along the direction normal to the domain wall. From (30) we also see that $\tilde{\gamma}$ becomes γ_0 . The general case for which the fermion has a nonvanishing p_t can be obtained by Lorentz-boosting the solution along the direction of the wall by means of the transformation

$$S(\theta) = \exp(-\gamma_0 \gamma_t \theta). \quad (32)$$

To finish this section, let us work with the ϵ -independent part of Eq. (29) and evaluate

$$\left\{ +i\gamma_3 \frac{d}{dx} + \xi g(x) \right\} \Phi(x), \quad (33)$$

using Φ given by (18) and (25); with \mathbf{u}_{\pm}^s given by (15), the above equation becomes

$$\sum \varphi_{\pm}^s \mathbf{u}_{\pm}^s, \quad (34)$$

with

$$\varphi_{\pm}^s = (A_{\pm}^s)^{(-\alpha)} \varphi_{\pm}^{(-\alpha)} + (A_{\pm}^s)^{(+\alpha)} \varphi_{\pm}^{(+\alpha)}, \quad (35)$$

where we have defined, using ϕ_{\pm} given by (25) and (26),

$$\varphi_{\pm} \equiv \left\{ \mp \frac{d}{dx} + \xi g(x) \right\} \phi_{\pm}(x). \quad (36)$$

To construct the full wave function we set $\epsilon = \frac{\delta T}{\sqrt{2\lambda}}E$ and $\tilde{\gamma} = \gamma_0$ in (29) and boost then the so found solution in the direction along the domain wall. We need to know therefore how γ_0 operates on the spinors \mathbf{u} . In our representation,

$$\gamma_0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix},$$

and thus

$$\gamma_0 \mathbf{u}_{\pm} = \mathbf{u}_{\mp}. \quad (37)$$

Recall that each of the four terms in Eq. (18) is by itself a solution to Eq. (16) and so for a given sign of the energy, Eq. (29), with $\Phi(x)$ replaced by any of its four components, represents four solutions from which only two should be independent. In the Appendix we show that φ_{\pm} are related to ϕ_{\mp} by

$$\begin{aligned}\varphi_{\pm}^{(-\alpha)} &= 2(\xi \pm \alpha) \phi_{\mp}^{(-\alpha)}, \\ \varphi_{\pm}^{(+\alpha)} &= 2(\xi \mp \alpha) \phi_{\mp}^{(+\alpha)}.\end{aligned}\quad (38)$$

We also notice that $\phi_{\pm}^{(+\alpha)}(x)$ can be obtained from $\phi_{\pm}^{(-\alpha)}(x)$ by exchanging α with $-\alpha$. From Eq. (29) together with (35), (37), and (38), we get the four independent solutions to the Dirac equation labeled by the indices s and r ,

$$\begin{aligned}\Psi_r^s &= (A^s)^{(-\alpha)} [\epsilon_r \phi_+^{(-\alpha)} \mathbf{u}_-^s + 2(\xi + \alpha) \phi_-^{(-\alpha)} \mathbf{u}_+^s] \\ &\quad + (A^s)^{(+\alpha)} [\epsilon_r \phi_+^{(+\alpha)} \mathbf{u}_-^s + 2(\xi - \alpha) \phi_-^{(+\alpha)} \mathbf{u}_+^s],\end{aligned}\quad (39)$$

where the constants $(A^s)^{(-\alpha)}$ and $(A^s)^{(+\alpha)}$ replace $(A^s)_{\pm}^{(-\alpha)}$ and $(A^s)_{\pm}^{(+\alpha)}$, respectively, and have to be fixed by normalization and the choice of initial conditions. Again the general case for which the fermion has a nonvanishing p_t can be obtained by Lorentz boosting (39) along the direction of the wall according to (32). Starting from the solution (39) we will discuss in the next section the scattering states of fermions traversing the domain wall.

B. Scattering states

At the electroweak phase transition the temperature is larger than the typical particle masses such as that of the Z boson or light mass quarks and although it can be of the order of the top quark mass, most of the fermions will pass by the domain wall with energies above its height. To understand the interaction of fermions of such energies with the wall we proceed to study the scattering states of the fermion wave function and to derive reflection and transmission coefficients.

The two physical processes of interest here are either a fermion leaving the bubble or a fermion falling into the bubble. We call the solutions corresponding to the boundary conditions of these processes the type I and type II solutions, respectively. Both processes are depicted in Fig. 3.

For the type I solution we have an incident fermion from the right ($x \rightarrow +\infty$) with an energy parameter ϵ which is always larger than the height of the wall ($\epsilon^2 \geq 4\xi^2$). At the wall this fermion is scattered into an reflected wave going to the right and a transmitted

wave going to the left ($x \rightarrow -\infty$). Overall the fermion is therefore represented by an incoming and reflected wave to the right of the wall and by a transmitted wave to the left.

If a fermion is incident from the left (type II solution), it can approach the wall with energies above its height ($\epsilon^2 \geq 4\xi^2$) or below its height ($\epsilon^2 \leq 4\xi^2$). In any case there will be again the reflected wave running now to the left and the transmitted wave running to the right. But for energies below the barrier ($\epsilon \leq 4\xi^2$) the transmitted wave has to tunnel through the wall leading to a decaying instead of an oscillating wave.

In this section we start out by constructing the wave function corresponding to the boundary conditions of the type II solution and determine then the transmission and reflection coefficients for this case. We show then that we need to rewrite the hypergeometric functions as expansions around $z = 1$ to obtain the wave function for the boundary conditions of type I. It turns out that the transmission and reflection coefficients for this case are identical to the ones obtained for type II.

For a given pair of indices s and r , Eq. (39) consists of two terms with different asymptotic behaviors as $x \rightarrow +\infty$. In this limit, according to Eq. (27), the first term proportional to $\phi_{\pm}^{(-\alpha)}$ behaves like $\exp(-2\alpha x)$ whereas the second term proportional to $\phi_{\pm}^{(+\alpha)}$ behaves like $\exp(2\alpha x)$. The boundary conditions appropriate to the description of particles crossing the wall from the symmetric to the asymmetric phase are as follows: At $x = -\infty$, corresponding to $z = 1$ (outside the bubble), we require the solutions (39) to describe two plane waves, one moving towards (incoming wave Ψ^{inc}) and the other away from the wall (reflected wave Ψ^{ref}). At $x = +\infty$, corresponding to $z = 0$ (inside the bubble), we impose that for energies such that $\epsilon^2 > 4\xi^2$ there is only a single plane wave moving away from the wall (transmitted wave Ψ^{trans}), while for $\epsilon^2 \leq 4\xi^2$ the solution dies out exponentially.

The above conditions require that $(A^s)^{(-\alpha)} = 0$ in (39) and the wave function for case II looks therefore like

$$(\Psi_r^s)_{\text{II}} = A_{\text{II}}[\epsilon_r \phi_+^{(+\alpha)} \mathbf{u}_-^s + 2(\xi - \alpha) \phi_-^{(+\alpha)} \mathbf{u}_+^s]. \quad (40)$$

The normalization constant A_{II} is determined in the next section and turns out to be independent of the quantum numbers s and r .

To compute the transmission and reflection coefficients, we need first to look at the behavior of the solutions as $x \rightarrow \pm\infty$.

If we take the limit $x \rightarrow +\infty$ of Eq. (40), we find

$$(\Psi_r^s)_{\text{II}} \xrightarrow{x \rightarrow +\infty} (\Psi_r^s)_{\text{II}}^{\text{trans}} = A_{\text{II}}[\epsilon_r \mathbf{u}_-^s + 2(\xi - \alpha) \mathbf{u}_+^s] e^{2\alpha x}. \quad (41)$$

For $x \rightarrow -\infty$, we have first to evaluate the hypergeometric functions in the second equation of (26) at $z = 1$. Since these functions are defined as expansions around $z = 0$, they are ill defined at $z = 1$ and we need to use the identity [22]

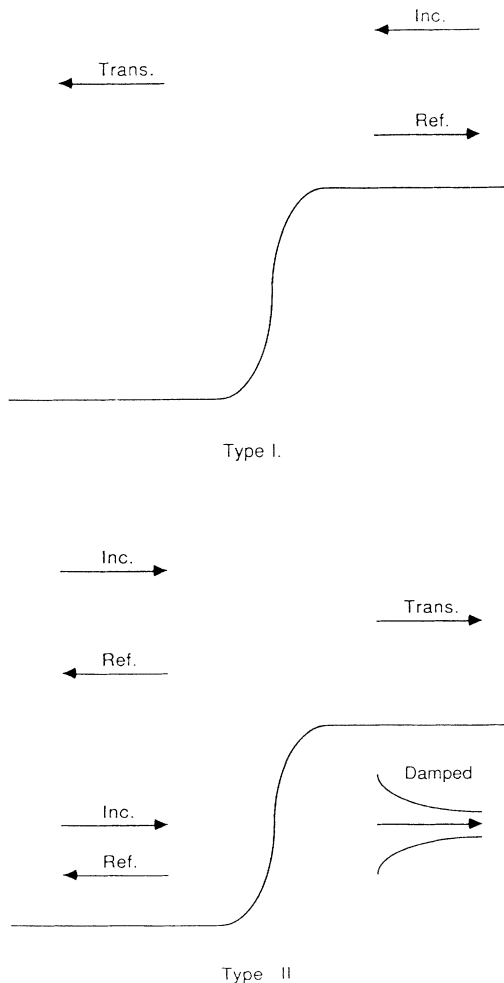


FIG. 3. Sketch of the asymptotic behavior of the solutions of types I and II.

$$\begin{aligned}
{}_2F_1(a, b, c; z) &= \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} {}_2F_1(a, b, a+b-c+1; 1-z) \\
&\quad + (1-z)^{c-a-b} \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)} {}_2F_1(c-a, c-b, c-a-b+1; 1-z).
\end{aligned} \tag{42}$$

Therefore, using Eq. (42) in (26) and considering the limit $x \rightarrow -\infty$ we get from (40), after some algebra,

$$\begin{aligned}
(\Psi_r^s)_{\text{II}} &\xrightarrow{x \rightarrow -\infty} (\Psi_r^s)_{\text{II}}^{\text{inc}} + (\Psi_r^s)_{\text{II}}^{\text{ref}}, \\
(\Psi_r^s)_{\text{II}}^{\text{inc}} &= \frac{A_{\text{II}}\Gamma(1-2\alpha)\Gamma(-2\beta)}{\Gamma(-\alpha-\beta+\xi)\Gamma(-\alpha-\beta-\xi)} \left(\frac{\epsilon_r \mathbf{u}_-^s}{-\alpha-\beta-\xi} + \frac{2(\xi-\alpha)\mathbf{u}_+^s}{-\alpha-\beta+\xi} \right) e^{2\beta x}, \\
(\Psi_r^s)_{\text{II}}^{\text{ref}} &= \frac{A_{\text{II}}\Gamma(1-2\alpha)\Gamma(2\beta)}{\Gamma(-\alpha+\beta+\xi)\Gamma(-\alpha+\beta-\xi)} \left(\frac{\epsilon_r \mathbf{u}_-^s}{-\alpha+\beta-\xi} + \frac{2(\xi-\alpha)\mathbf{u}_+^s}{-\alpha+\beta+\xi} \right) e^{-2\beta x}.
\end{aligned} \tag{43}$$

We can check that the reflected wave can be obtained from the incident by exchanging β with $-\beta$.

To compute the reflection and transmission coefficients, we have to compute the ratio of the reflected and transmitted fluxes to the incoming one. It is thus sufficient to calculate the ratios of the normal components of the corresponding vector currents.

The normal component of the currents associated with the plane waves that we have found are

$$j_3 = \bar{\Psi} \gamma_3 \Psi, \tag{44}$$

where Ψ is any of Ψ^{trans} , Ψ^{inc} , or Ψ^{ref} . From our solution, Eq. (40), the normal components of the incident, reflected, and transmitted currents are, for type II,

$$\begin{aligned}
(j_{\text{II}}^{\text{inc}})_3 &= 4|A_{\text{II}}|^2 \epsilon \epsilon_r \left| \frac{\Gamma(1+2\alpha)\Gamma(2\beta)}{\Gamma(1+\alpha+\beta+\xi)\Gamma(\alpha+\beta-\xi)} \right|^2, \\
(j_{\text{II}}^{\text{ref}})_3 &= -4|A_{\text{II}}|^2 \epsilon \epsilon_r \left| \frac{\Gamma(1+2\alpha)\Gamma(2\beta)}{\Gamma(1+\alpha-\beta+\xi)\Gamma(\alpha-\beta-\xi)} \right|^2, \\
(j_{\text{II}}^{\text{trans}})_3 &= 8|A_{\text{II}}|^2 \epsilon_r |\alpha|.
\end{aligned} \tag{45}$$

The reflection and transmission coefficients R and T are now the ratios of the reflected and transmitted normal

currents to the incident one, respectively, projected along a unit vector normal to the domain wall. Using Eqs. (45) and the identity [22]

$$\Gamma(x)\Gamma(-x) = \frac{-\pi}{x \sin(\pi x)}, \tag{46}$$

we find, after some simplification,

$$\begin{aligned}
T &= \frac{-\sin 2\pi\alpha \sin 2\pi\beta}{\sin \pi(\xi-\alpha-\beta) \sin \pi(\xi+\alpha+\beta)}, \\
R &= \frac{\sin \pi(\xi+\alpha-\beta) \sin \pi(\xi-\alpha+\beta)}{\sin \pi(\xi-\alpha-\beta) \sin \pi(\xi+\alpha+\beta)}.
\end{aligned} \tag{47}$$

From the above equations we see that for states with energy parameter $\epsilon^2 = 4\xi^2$, the transmission and reflection coefficients become 0 and 1. For states with $\epsilon^2 < 4\xi^2$, α becomes negative and real. From Eq. (41) we see that j^{trans} is identically zero and therefore the transmission coefficient T is also zero. Correspondingly, the reflection coefficient R for fermions with $\epsilon^2 < 4\xi^2$ is 1.

Both reflection and transmission coefficients are the same for positive and negative energy solutions and thus for fermions and antifermions. They are depicted in Fig. 4. In this and the following figures we choose two representative values for the parameter ξ . The heavier

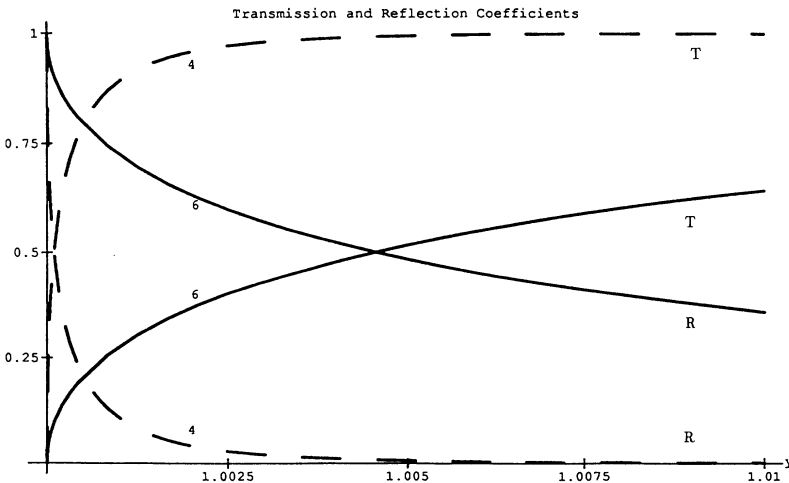


FIG. 4. Transmission and reflection coefficients for $\xi = 0.6, 4$ which correspond to a ratio of fermion to Higgs mass of 0.3 and 2, respectively. The reduced energy parameter is $y = \frac{\epsilon}{2\xi}$, so that the top of the barrier is at $y = 1$.

fermions are represented by $\xi = 4$ which corresponds to a ratio of fermion to Higgs mass of 2. It is expected that the mass of the top quark is lying in this mass region, so that we can consider the results for $\xi = 4$ to represent a top quark interacting with the domain wall. The other fermions, on the other hand, are much lighter than the Higgs. The corresponding value of ξ will be therefore very small. We choose a value of $\xi = 0.6$ to represent these particles which corresponds to a ratio of fermion to Higgs mass of 0.3.

We plot in Fig. 4 the transmission and reflection coefficients as a function of a reduced energy parameter $y = \frac{\epsilon}{2\xi}$. The top of the barrier is then at $y = 1$. We see that the interactions between the heavy fermions and the wall quickly die out if we increase the energy parameter. The light fermions on the other hand are still feeling the presence of the wall high above the top of barrier.

The result for the reflection and transmission coefficient is unchanged if we boost the solution along the direction of the wall to go to nonzero p_t . Since the boost operator defined in (32) commutes with γ_3 ,

$$[\gamma_3, S(\theta)] = 0,$$

we see that the currents defined in (44) are invariant under S . This shows that also (47) is unchanged.

We proceed now to the solution of type I. Here the transmitted wave has to behave like $e^{-2\beta x}$ for $x \rightarrow -\infty$. But the solution (39) is not regular in this limit—the hypergeometric functions diverge. To resolve this problem we have to rewrite (39) in terms of hypergeometric functions defined as expansions around $z = 1$.

There are two possible ways to obtain this modification of (39). One possibility is to restart from (24) and write the two independent solutions of the hypergeometric equation as hypergeometric functions defined as expansions around $z = 1$. Equation (24) becomes then [25]

$$\begin{aligned} \chi_{\pm}^{(+\beta)}(z) &= {}_2F_1(a_{\mp}, b_{\mp}, a_{\mp} + b_{mp} - c + 1; 1 - z), \\ \chi_{\pm}^{(-\beta)}(z) &= (1 - z)_2^{c - a_{\mp} - b_{\mp}} \\ &\quad \times F_1(c - a_{\mp}, c - b_{\mp}, c - a_{\mp} - b_{\mp} + 1; 1 - z), \end{aligned} \tag{48}$$

and (26) is now

$$\begin{aligned} \phi_{\pm}^{(+\beta)} &= z^{\alpha}(1 - z)^{\beta} {}_2F_1(a_{\mp}, b_{\mp}, a_{\mp} + b_{\mp} - c + 1; 1 - z), \\ \phi_{\pm}^{(-\beta)} &= z^{\alpha}(1 - z)^{-\beta} {}_2 \\ &\quad \times F_1(c - a_{\mp}, c - b_{\mp}, c - a_{\mp} - b_{\mp} + 1; 1 - z). \end{aligned} \tag{49}$$

Finally we repeat all steps leading to (39). The superscripts in the equations above indicate the behavior of these functions at $x \rightarrow -\infty$ corresponding to $1 - z = 0$. We can deduce this behavior by comparing (49) with (28).

A second approach for rewriting (39) is to start out with the four independent solutions we found in (39) and use then (42) and the identity [25]

$${}_2F_1(a, b, c; 1 - z) = z^{c-a-b} {}_2F_1(c - a, c - b, c; 1 - z) \tag{50}$$

to rewrite the four solutions in terms of hypergeometric functions of the variable $(1 - z)$ instead of z .

Both approaches yield

$$\begin{aligned} \Psi_r^s &= (A^s)^{(+\beta)}(\epsilon_r \phi_+^{(+\beta)} \mathbf{u}_-^s - 2\beta \phi_-^{(+\beta)} \mathbf{u}_+^s) \\ &\quad + (A^s)^{(-\beta)}(\epsilon_r \phi_+^{(-\beta)} \mathbf{u}_-^s + 2\beta \phi_-^{(-\beta)} \mathbf{u}_+^s). \end{aligned} \tag{51}$$

To impose the boundary conditions for the solution of type I we have to require that for $x \rightarrow -\infty$ only terms oscillating like $e^{-2\beta x}$ survive. We thus set $(A^s)^{(+\beta)} = 0$. The solution of type I is therefore

$$(\Psi_r^s)_I = A_I(\epsilon_r \phi_+^{(-\beta)} \mathbf{u}_-^s + 2\beta \phi_-^{(-\beta)} \mathbf{u}_+^s). \tag{52}$$

Continuing now analogously to Eqs. (41)–(47) we find the transmitted wave

$$(\Psi_r^s)_I \xrightarrow{x \rightarrow -\infty} (\Psi_r^s)_I^{\text{trans}} = A_I(\epsilon_r \mathbf{u}_-^s + 2\beta \mathbf{u}_+^s) e^{-2\beta x} \tag{53}$$

and the incident and reflected waves, for this case,

$$\begin{aligned} \Psi_r^s \xrightarrow{x \rightarrow +\infty} & (\Psi_r^s)_I^{\text{inc}} + (\Psi_r^s)_I^{\text{ref}}, \\ (\Psi_r^s)_I^{\text{inc}} &= \frac{A_I \Gamma(1 - 2\beta) \Gamma(-2\alpha)}{\Gamma(-\alpha - \beta + \xi) \Gamma(-\alpha - \beta - \xi)} \left(\frac{\epsilon_r \mathbf{u}_-^s}{-\alpha - \beta - \xi} + \frac{2\beta \mathbf{u}_+^s}{-\alpha - \beta + \xi} \right) e^{-2\alpha x}, \\ (\Psi_r^s)_I^{\text{ref}} &= \frac{A_I \Gamma(1 - 2\beta) \Gamma(2\alpha)}{\Gamma(\alpha - \beta + \xi) \Gamma(\alpha - \beta - \xi)} \left(\frac{\epsilon_r \mathbf{u}_-^s}{\alpha - \beta - \xi} + \frac{2\beta \mathbf{u}_+^s}{\alpha - \beta + \xi} \right) e^{+2\alpha x}. \end{aligned} \tag{54}$$

Here we see that we can obtain the reflected wave from the incident wave by exchanging α with $-\alpha$.

As before we can calculate the currents and find

$$\begin{aligned} (j_1^{\text{inc}})_3 &= 8|A_I|^2 |\alpha| \epsilon_r \left| \frac{\Gamma(1 + 2\beta) \Gamma(2\alpha)}{\Gamma(1 + \alpha + \beta + \xi) \Gamma(\alpha + \beta - \xi)} \right|^2, \\ (j_1^{\text{ref}})_3 &= -8|A_I|^2 |\alpha| \epsilon_r \left| \frac{\Gamma(1 + 2\beta) \Gamma(2\alpha)}{\Gamma(1 - \alpha + \beta + \xi) \Gamma(-\alpha + \beta - \xi)} \right|^2, \\ (j_1^{\text{trans}})_3 &= 4|A_I|^2 \epsilon_r \epsilon. \end{aligned} \tag{55}$$

If we compare these currents to the ones obtained in type II [Eq. (45)], we see that we obtain (55) from (45) by exchanging α with β . Since the transmission and reflection coefficients in (47) are unchanged under exchange of α with β , we find that the reflection and transmission coefficients for a fermion falling into the asymmetric phase or a fermion escaping the asymmetric phase are the same for energies above the barrier and are given by (47).

IV. NORMALIZATION AND ORTHOGONALITY

In this section we want to construct a general solution to the Dirac equation using the type I solution given in (52) and the type II solution given in (40). The general solution should have the form

$$\Psi_r^s = a_r^s(\Psi_r^s)_I + b_r^s(\Psi_r^s)_II, \quad (56)$$

where a, b are arbitrary coefficients normalized to 1,

$$1 = (a_r^s)^2 + (b_r^s)^2. \quad (57)$$

In (56), Ψ_I and Ψ_{II} should represent then the normal modes of the system as depicted in Fig. 3.

To obtain such a solution we have to normalize first Ψ_I and Ψ_{II} themselves and then assure that the two solutions form an orthogonal set. It will turn out that they actually are not orthogonal and we finally will show how to orthogonalize them to obtain a solution of the form (56).

To normalize the type I and II solutions given in (52) and in (40) we first choose an open interval $(+l, -l)$ on the x axis of Fig. 1. We define now the normalization condition for a state Ψ on this interval as follows:

$$\begin{aligned} 1 &= \lim_{l \rightarrow \infty} \int_{-l}^{+l} dx \bar{\Psi} \gamma_0 \Psi \\ &= \lim_{l \rightarrow \infty} \int_{-l}^{+l} dx \Psi^\dagger \Psi. \end{aligned} \quad (58)$$

For $l \rightarrow \infty$ we recover the usual normalization condition from (58).

To evaluate the integral in Eq. (58) we introduce an additional length scale δ on $(-l, +l)$ such that $\delta \ll l$. The scale δ defines an symmetric interval around $x = 0$ such that inside $[-\delta, +\delta]$ the Higgs potential $g(x)$ changes rapidly from its value close to zero, to its value close to 2,

$$\int_{-l}^l dx \Psi^\dagger \Psi = \left(\int_{-l}^{-\delta} + \int_{-\delta}^{+\delta} + \int_{+\delta}^{+l} \right) dx \Psi^\dagger \Psi.$$

If the scale l becomes very large, we can replace the wave function Ψ on the intervals $(-l, -\delta)$ and $(+\delta, +l)$ with its asymptotic limits at $x \rightarrow \mp\infty$, respectively, and obtain

$$\begin{aligned} \int_{-l}^l dx \Psi^\dagger \Psi &= \int_{-l}^{-\delta} dx \Psi_{(-\infty)}^\dagger \Psi_{(-\infty)} \\ &+ \int_{+\delta}^{+l} dx \Psi_{(+\infty)}^\dagger \Psi_{(+\infty)} + O(\delta/l). \end{aligned} \quad (59)$$

Here $\Psi_{\pm\infty}$ represents the asymptotic limit of the wave function at $x \rightarrow \pm\infty$, respectively, and the integrals have to be evaluated only at the indicated limits $\pm l$. If we let $l \rightarrow \infty$, the corrections of order $O(\delta/l)$ vanishes and expression (59) becomes exact. The result diverges in this limit linearly in l analogous to the divergence in the volume of the plane wave normalization.

To solve Eq. (59) for the constants A_I and A_{II} of Ψ_I and Ψ_{II} we use their asymptotic expressions given by (53), (54), (41), and (43), respectively. Using the orthogonality of the spinors \mathbf{u} in (15) and the fact that integrals over oscillating functions average to zero $\int dx e^{ikx} = 0$ we obtain, after some algebra,

$$\begin{aligned} |A_I|^2 &= \frac{\alpha T}{4l\epsilon^2\beta} \frac{1}{1 + R + \frac{\beta}{\alpha}T}, \\ |A_{II}|^2 &= \frac{\beta T}{4l\epsilon^2\alpha} \frac{1}{1 + R + \frac{\alpha}{\beta}T}. \end{aligned} \quad (60)$$

As before we can obtain A_I from A_{II} by exchanging α with β .

To prove the orthogonality of Ψ_I and Ψ_{II} we need to evaluate their overlap integral given by the scalar product

$$I = \int_{-l}^{+l} dx \Psi_I^\dagger \Psi_{II}. \quad (61)$$

The integral is defined as before on an interval $(-l, +l)$. If the overlap integral I in (61) is zero, then Ψ_I and Ψ_{II} are orthogonal; if it is nonzero, we still have to search for orthogonal solutions.

Proceeding analogously to Eqs. (58)–(59) we obtain

$$\begin{aligned} I &= \int_{-l}^{+l} dx (\Psi_I^\dagger)_{(+\infty)} (\Psi_{II})_{(+\infty)} \\ &+ \int_{-l}^{+l} dx (\Psi_I^\dagger)_{(-\infty)} (\Psi_{II})_{(-\infty)}. \end{aligned} \quad (62)$$

Using again the expressions for the asymptotic wave functions we find

$$\begin{aligned} I &= 8l A_{II} A_I^* \epsilon^2 (\beta - \alpha) \frac{\Gamma(2\beta)\Gamma(-2\alpha)}{\Gamma(\beta - \alpha + \xi)(\beta - \alpha - \xi + 1)} \\ &\neq 0. \end{aligned} \quad (63)$$

The overlap integral is therefore nonzero; Ψ_I and Ψ_{II} are not orthogonal. To obtain a solution containing normal modes as it was envisioned in (56) requires therefore to find a set of orthogonal states. This can be done using the Schmidt orthogonalization [26].

In the Schmidt orthogonalization we start out with a set of nonorthogonal states and construct out of these one by one an orthonormal basis. The set of nonorthogonal states is in our case Ψ_I and Ψ_{II} . Let us choose now Ψ_I as our first normalized basis state. The second orthonormal basis state corresponding to Ψ_I is then obtained through the ansatz

$$\Psi_{\text{ortho}} = N(a\Psi_I + \Psi_{II}). \quad (64)$$

Here N and a are constants to be fixed by the require-

ment that Ψ_{ortho} is normalized to 1 and is orthogonal to Ψ_{I} , respectively.

The orthogonality condition is evaluated by forming the scalar product of Ψ_{ortho} in (64) and Ψ_{I} ,

$$\int dx \Psi_{\text{I}}^\dagger \Psi_{\text{ortho}} = N \left(a \int dx \Psi_{\text{I}}^\dagger \Psi_{\text{I}} + \int dx \Psi_{\text{I}}^\dagger \Psi_{\text{II}} \right) \equiv 0. \quad (65)$$

The first integral on the right-hand side is 1 since Ψ_{I} is normalized and we obtain therefore

$$a = - \int dx \Psi_{\text{I}}^\dagger \Psi_{\text{II}} \equiv -I. \quad (66)$$

The constant a is therefore identical to the overlap integral I evaluated in (63). It is now straightforward to normalize Ψ_{ortho} . The normalization condition reads

$$1 = \int dx \Psi_{\text{ortho}}^\dagger \Psi_{\text{ortho}}, \quad (67)$$

which can be solved for

$$|N|^2 = \frac{1}{1 + |I|^2}. \quad (68)$$

We therefore found a general solution of the form (56) with Ψ_{II} replaced by Ψ_{ortho} . Of course, we could have started in (64) with the orthonormal basis state to Ψ_{II} ,

$$\Psi'_{\text{ortho}} = N'(a' \Psi_{\text{II}} + \Psi_{\text{I}}). \quad (69)$$

It is straightforward to see that in this case $a' = -I^*$ and $|N'|^2 = |N|^2$, so that up to a phase $\Psi'_{\text{ortho}} = \Psi_{\text{ortho}}^*$. Again we can construct the general solution of normal modes, this time replacing Ψ_{I} in (56) with Ψ_{ortho}^* .

V. CONCLUSION

In this paper we discussed the motion of fermions under the influence of electroweak domain walls. We showed that such fermions are well described by a Dirac equation with an effective mass term. This mass term was obtained via the Yukawa coupling in the Lagrangian applying the classical mean-field approximation. CP violation plays no role in this order of approximation. We substituted for the finite temperature VEV of the Higgs field the classical solution to the equations of motion of a Higgs field in a finite temperature effective potential.

We investigated the Dirac equation analytically and found the wave function of the fermion. Transmission and reflection coefficients were derived and found to be the same for both fermions and antifermions. It turned out that the interaction between the wall and a heavy fermion such as the top quark die out quickly if we increase the energy of the fermion. On the other hand the light fermions still feel the presence of the wall high above the top of the barrier.

These results are valid in the limit where the mean free path of the fermion is large in comparison to the dimensions of the wall. It remains to study the opposite

limit where due to the smallness of the mean free path the fermions scatter from the surrounding plasma particles and therefore have to diffuse through the wall.

Before proceeding to any quantitative questions such as the magnitude of the generated baryon asymmetry, the results of an asymmetry generated by scattering from the wall must be convoluted with the results of a diffusion calculation. Thus diffusion is a needed extra ingredient here. In the limit where the mean free path is greater than or of the order of the bubble wall size, such a treatment may be justified. In the opposite, probably more realistic case, an adiabatic approximation is probably a better starting point, and in this case the problem of scattering from the wall is probably physically irrelevant.

The goal of this paper was to study the scattering off of a phase transition bubble wall in the case where the phase transition occurs close to the point where the minima of the effective potential are approximately degenerate. In this case, the bubble wall profile is close to that of a kink. This problem may have more general application than electroweak baryogenesis, and in some circumstances may have application to the electroweak case as well.

The main result of this calculation was the computation of the reflection and transmission coefficients for this scattering. The transmission coefficient is

$$T = \frac{-\sin 2\pi\alpha \sin 2\pi\beta}{\sin \pi(\xi - \alpha - \beta) \sin \pi(\xi + \alpha + \beta)}, \quad (70)$$

where ξ is twice the zero temperature ratio of quark to Higgs mass. With

$$\epsilon = (2\sqrt{\lambda}/\delta T) \sqrt{E^2 - p_t^2}, \quad (71)$$

with δ and λ defined as parameters in the Higgs potential in the second section and E the fermion energy and p_t its transverse momentum, we have

$$\alpha = \frac{i}{2} \sqrt{\epsilon^2 - 4\xi^2} \quad (72)$$

and

$$\beta = \frac{i}{2} \epsilon. \quad (73)$$

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APPENDIX

Here, we would like to show that the φ 's defined as

$$\varphi_{\pm} = \left[\mp \frac{d}{dx} + \xi g(x) \right] \phi_{\pm} \quad (\text{A1})$$

in Eq. (36) are proportional to the ϕ 's given by Eq. (26) where $g(x) = 1 + \tanh x$. We will prove it for one case, show that it can be done for the others, and write the specific relations for all cases.

Let us start by looking at one case, for example,

$$\varphi_+^{(-\alpha)} = \left[-\frac{d}{dx} + \xi g(x) \right] \phi_+^{(-\alpha)}, \quad (\text{A2})$$

where

$$\begin{aligned} \phi_+^{(-\alpha)} &= z^\alpha (1-z)^\beta {}_2F_1(\alpha + \beta - \xi + 1, \alpha + \beta + \xi, 2\alpha + 1, z) \\ &\times F_1(\alpha + \beta - \xi + 1, \alpha + \beta + \xi, 2\alpha + 1, z) \end{aligned} \quad (\text{A3})$$

and $z = \frac{1}{2}(1 - \tanh x)$. Then it follows that

$$\left[-\frac{d}{dx} + \xi g(x) \right] = 2(1-z) \left[z \frac{d}{dz} + \xi \right], \quad (\text{A4})$$

and so we can rewrite $\varphi_+^{(-\alpha)}$ as

$$\varphi_+^{(-\alpha)} = 2(1-z) \left[z \frac{d}{dz} + \xi \right] \phi_+^{(-\alpha)}. \quad (\text{A5})$$

Let us consider

$$\frac{d}{dz} [z^\xi \phi_+^{(-\alpha)}] = \xi z^{\xi-1} \phi_+^{(-\alpha)} + z^\xi \frac{d}{dz} \phi_+^{(-\alpha)}.$$

Multiplying both sides of this equation by $2(1-z)z^{-\xi+1}$, we obtain

$$2(1-z)z^{-\xi+1} \frac{d}{dz} [z^\xi \phi_+^{(-\alpha)}] = 2(1-z) \left[z \frac{d}{dz} + \xi \right] \phi_+^{(-\alpha)}. \quad (\text{A6})$$

Comparing this with Eq. (26) and using the explicit form for ϕ , we have

$$\begin{aligned} \varphi_+^{(-\alpha)} &= 2(1-z)z^{-\xi+1} \frac{d}{dz} [z^{\xi+\alpha} (1-z)^\beta {}_2F_1(\alpha + \beta - \xi + 1, \alpha + \beta + \xi, 2\alpha + 1, z)] \\ &= 2(1-z)z^{\alpha+1} \left[\frac{d}{dz} (1-z)^\beta \right] {}_2F_1(\alpha + \beta - \xi + 1, \alpha + \beta + \xi, 2\alpha + 1, z) \\ &\quad + 2(1-z)^{\beta+1} z^{-\xi+1} \frac{d}{dz} [z^{\xi+\alpha} {}_2F_1(\alpha + \beta - \xi + 1, \alpha + \beta + \xi, 2\alpha + 1, z)]. \end{aligned}$$

We now add and subtract β to the exponent of z inside the bracket in the second term above and differentiate to get

$$\begin{aligned} \frac{d}{dz} [z^{\alpha+\xi} {}_2F_1(\alpha + \beta - \xi + 1, \alpha + \beta + \xi, 2\alpha + 1, z)] &= z^{-\beta} \frac{d}{dz} [z^{\alpha+\beta+\xi} {}_2F_1(\alpha + \beta - \xi + 1, \alpha + \beta + \xi, 2\alpha + 1, z)] \\ &\quad - \beta z^{\alpha+\xi-1} {}_2F_1(\alpha + \beta - \xi + 1, \alpha + \beta + \xi, 2\alpha + 1, z). \end{aligned}$$

Now we use the identity [25]

$$\frac{d}{dz} [z^b {}_2F_1(a, b, c, z)] = bz^{b-1} {}_2F_1(a, b+1, c, z),$$

and putting everything together, we find

$$\begin{aligned} \varphi_+^{(-\alpha)} &= -2\beta z^\alpha (1-z)^\beta {}_2F_1(\alpha + \beta - \xi + 1, \alpha + \beta + \xi, 2\alpha + 1, z) \\ &\quad + 2(\alpha + \beta + \xi) z^\alpha (1-z)^{\beta+1} {}_2F_1(\alpha + \beta - \xi + 1, \alpha + \beta + \xi + 1, 2\alpha + 1, z). \end{aligned}$$

In the second term of this equation we can use the identity [25]

$${}_2F_1(a, b, c, z) = \frac{c-a}{(b-a)(1-z)} {}_2F_1(a-1, b, c, z) - \frac{c-b}{(b-a)(1-z)} {}_2F_1(a, b-1, c, z)$$

and then combine the terms such that

$$\begin{aligned} \varphi_+^{(-\alpha)} &= \left[-2\beta - \frac{1}{\xi} [\alpha^2 - (\beta + \xi)^2] \right] z^\alpha (1-z)^\beta {}_2F_1(\alpha + \beta - \xi + 1, \alpha + \beta + \xi, 2\alpha + 1, z) \\ &\quad + \frac{1}{\xi} [(\alpha + \xi)^2 - \beta^2] z^\alpha (1-z)^\beta {}_2F_1(\alpha + \beta - \xi, \alpha + \beta + \xi + 1, 2\alpha + 1, z). \end{aligned}$$

Using the fact that $\alpha^2 = \beta^2 + \xi^2$, we see that the coefficient of the first term is identically zero and that

$$\frac{1}{\xi}[(\alpha + \xi)^2 - \beta^2] = 2(\xi + \alpha).$$

Finally, comparing to Eq. (26),

$$\begin{aligned} \phi_-^{(-\alpha)} &= z^\alpha (1-z)^\beta {}_2F_1(\alpha + \beta - \xi, \alpha + \beta + \xi + 1, 2\alpha + 1, z), \end{aligned}$$

we see that

$$\varphi_+^{(-\alpha)} = 2(\xi + \alpha)\phi_-^{(-\alpha)}. \quad (\text{A7})$$

Similarly we can show that

$$\begin{aligned} \varphi_-^{(-\alpha)} &= 2(\xi - \alpha)\phi_+^{(-\alpha)}, \\ \varphi_+^{(+\alpha)} &= 2(\xi - \alpha)\phi_-^{(+\alpha)}, \\ \varphi_-^{(+\alpha)} &= 2(\xi + \alpha)\phi_+^{(+\alpha)}. \end{aligned} \quad (\text{A8})$$

For the case when the φ 's and ϕ 's are expanded around $(1-z)$ instead of z , we can apply the same method to derive similar relationships. They are

$$\begin{aligned} \varphi_+^{(-\beta)} &= 2\beta\phi_-^{(-\beta)}, \\ \varphi_+^{(+\beta)} &= -2\beta\phi_-^{(+\beta)}. \end{aligned} \quad (\text{A9})$$

These relations are used in the calculation of the fermionic wave function to eliminate the linearly dependent solution.

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