Heavy quark fragmentation to baryons containing two heavy quarks

Adam F. Falk,[1,*] Michael Luke,[2,†] Martin J. Savage,[2,‡] and Mark B. Wise[3]
[1]*Stanford Linear Accelerator Center, Stanford, California 94309*
[2]*Department of Physics, University of California at San Diego, 9500 Gilman Drive, La Jolla, California 92093*
[3]*California Institute of Technology, Pasadena, California 91125*
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We discuss the fragmentation of a heavy quark to a baryon containing two heavy quarks of mass $m_Q \gg \Lambda_{\rm QCD}$. In this limit the heavy quarks first combine perturbatively into a compact diquark with a radius small compared to $1/\Lambda_{\rm QCD}$, which interacts with the light hadronic degrees of freedom exactly as does a heavy antiquark. The subsequent evolution of this QQ diquark to a QQq baryon is identical to the fragmentation of a heavy antiquark to a meson. We apply this analysis to the production of baryons of the form ccq, bbq, and bcq.

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The spectroscopy and interactions of baryons consisting of two heavy quarks and one light quark simplify in the limit that the heavy quark masses m_Q tend to infinity. This is because the heavy quarks are bound into a diquark whose radius r_{QQ} is much smaller than the typical length scale $1/\Lambda_{\rm QCD}$ of nonperturbative QCD interactions. In the limit $r_{QQ} \ll 1/\Lambda_{\rm QCD}$ the heavy diquark has interactions with the light quark and other light degrees of freedom which are identical to those of a heavy antiquark. Hence as far as these light degrees of freedom are concerned, the diquark is nothing more than the pointlike, static, color antitriplet source of the confining color field in which they are bound [1–3].

One immediate result of this limit is that the spectrum of such "doubly heavy" baryons is related to the spectrum of mesons containing a single heavy antiquark [1]. It also follows that the form factors describing their semileptonic decays may be related to the Isgur-Wise function, which arises in the semileptonic decay of heavy mesons [2]. In this paper we will apply the same symmetries to the nonperturbative dynamics which governs the production of such states via fragmentation processes. We will use our results to estimate the production rates for baryons of the form ccq, bbq, and bcq; however, we note that, especially in the cc system, the heavy diquarks are not particularly small relative to $1/\Lambda_{\rm QCD}$, so there may well be sizable corrections to our results.

The fragmentation of a heavy quark Q into a QQq (or $QQ'q$) baryon factorizes into short- and long-distance contributions. The heavy quark first fragments into a heavy diquark via a process which is perturbatively calculable. In fact, the amplitude may be trivially related to that for the fragmentation of Q into quarkonium $Q\bar{Q}$. The subsequent fragmentation of the diquark QQ to a baryon is identical to the fragmentation of a $\bar{Q}$ to a meson $\bar{Q}q$; this information may be obtained from experimental data on production of heavy mesons.

We will begin with the case of baryons of the form QQq, in which the two heavy quarks have the same flavor. For concreteness we will discuss the production of baryons with two charm quarks ccq; the extension to bottom baryons bbq will be trivial. As the color wave function of the charm quarks is antisymmetric and the quarks are taken to be in the ground state S wave, the spin wave function must be symmetric. Hence the cc can only form a spin-1 diquark, which we shall denote by (cc). The (cc) can then fragment either to a spin-$\frac{1}{2}$ baryon, which we shall denote by Σ_{cc}, or to a spin-$\frac{3}{2}$ baryon, which we shall call Σ_{cc}^*.

The initial short-distance fragmentation process $c \to (cc)\bar{c}$ is analogous to that for the fragmentation into charmonium, $c \to \psi c$, which has been shown by Braaten, Cheung, and Yuan [4] to be calculable in QCD perturbation theory. The Feynman diagrams responsible for (cc) production are shown in Fig. 1. The computation, which we shall not repeat, follows directly that outlined for

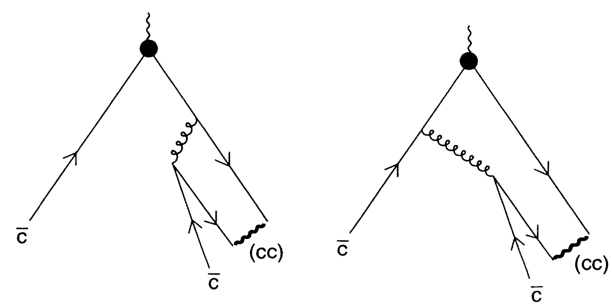

FIG. 1. Feynman diagrams responsible for the fragmentation $c \to (cc)\bar{c}$.

*Present address: Dept. of Physics, U.C. San Diego, La Jolla, CA 92093.

†Present address: Dept. of Physics, University of Toronto, Toronto, Ontario, Canada M5S 1A7.

‡Present address: Dept. of Physics, Carnegie Mellon University, Pittsburgh, PA 15213.

$c \to \psi c$ in Ref. [4]. In fact, after some rearrangement the form of the fragmentation function $D_{c\to(cc)}(z)$ is exactly the same as that for $D_{c\to\psi}(z)$, except for an overall normalization factor. Since the charm quarks in the (cc) are in an overall color $\bar{3}$ rather than a singlet, there is a color factor of $\frac{2}{3}$ instead of $\frac{4}{3}$ in the amplitude. The color wave function of the diquark is

$$|(cc)_a\rangle = \tfrac{1}{2}\epsilon_{abc}|c^b\rangle|c^c\rangle ,$$

where a, b, and c are color indices. There is a two in the matrix element from Fermi statistics. Collecting this with the color factors there is an overall $\sqrt{3}/2$ in the amplitude relative to that for production of a color singlet. Squaring and summing over the two possible final colors then gives the desired fragmentation function, renormalized at the scale $\mu=2m_c$:

$$D_{c\to(cc)}(z,\mu=2m_c) = \frac{4}{9\pi}\frac{|R_{(cc)}(0)|^2}{m_c^3}\alpha_s(2m_c)^2 F(z) , \quad (1)$$

where

$$F(z) = \frac{z(1-z)^2}{(2-z)^6}(16-32z+72z^2-32z^3+5z^4) . \quad (2)$$

Here $R_{(cc)}(0)$ is the nonrelativistic radial wave function at the origin for the perturbatively bound diquark. Unlike the case of charmonium, there is no physical "decay constant" $f_{(cc)}$ to which it may be related. However, we may naively scale $R_{(cc)}(0)$ from charmonium by noting that in a hydrogenlike potential $R(0)$ is proportional to $(C_F\alpha_s)^{3/2}$ where the color factor C_F is $\frac{4}{3}$ for $c\bar{c}$ and $\frac{2}{3}$ for cc. Hence we expect that

$$|R_{(cc)}(0)|^2 \approx |R_\psi(0)|^2/8 \approx (0.41\ \text{GeV})^3 .$$

The complete fragmentation function $D_{c\to\Sigma}(z)$ for $c\to(cc)\bar{c}\to(\Sigma_{cc},\Sigma_{cc}^*)\bar{c}$ is given by convolving the function $D_{c\to(cc)}(z)$ with the amplitude for the heavy antitriplet diquark to fragment to a ground-state baryon with a single light quark. This latter function, which we shall denote $D_{\bar{Q}\to M}(z)$, may be determined by data on charm and bottom antiquark fragmentation to heavy mesons. Summing over the Σ_{cc} and Σ_{cc}^* baryons, we then find

$$D_{c\to\Sigma}(z) = \int_z^1 \frac{dy}{y} D_{c\to(cc)}(z/y) D_{\bar{Q}\to M}(y) . \quad (3)$$

However, in the limit $m_c \gg \Lambda_{\text{QCD}}$ the heavy diquark carries all of the momentum of the baryon, and

$$D_{\bar{Q}\to M}(y) = \delta(1-y) P_{\bar{Q}\to M} ,$$

where $P_{\bar{Q}\to M}$ is the integrated probability for a heavy antiquark to fragment to a ground-state meson. Then (3) takes the simpler form

$$D_{c\to\Sigma}(z) = P_{\bar{Q}\to M} D_{c\to(cc)}(z) . \quad (4)$$

Since in the limit we are considering all excited states will decay strongly to the ground state, we may replace this probability by unity. (There is a small correction from the fragmentation probability to baryons, $P_{\bar{Q}\to\bar{\Lambda},\bar{\Sigma}}$; here this would lead to an exotic $cc\bar{q}\bar{q}$ final state.) Then the integral of (1) yields the final result

$$\int_0^1 dz\, D_{c\to\Sigma}(z) = \frac{4}{9\pi}\alpha_s(2m_c)^2 \frac{|R_{(cc)}(0)|^2}{m_c^3}\left(\tfrac{1189}{30} - 57\ln 2\right) . \quad (5)$$

The Σ_{cc}^* and Σ_{cc} baryons will be produced in the ratio 2:1. However, we may use the polarized fragmentation functions $D_{c\to(cc)}(z)$ derived in Ref. [5] to compute individually the populations of the various helicity states of the Σ_{cc} and Σ_{cc}^*. Let P be the net polarization of the initial charm quarks. Then the populations of the baryon helicity states can be determined from P and the fraction ζ of the produced diquarks which are transversely rather than longitudinally aligned. For example, a charm quark with energy E and helicity $\frac{1}{2}$ can fragment to a (cc) diquark with helicity 0 or 1, but not to one with helicity -1, to leading order in m_c/E. Hence the net polarization of the diquarks is degraded to ζP, where $\zeta = 0.69$ [5]. To make the baryon, the diquark must then be combined with a light quark from the nonperturbative part of the fragmentation. In the limit $m_c \gg \Lambda_{\text{QCD}}$, the helicity of the diquark is irrelevant to this soft process; the parity invariance of QCD then requires that the light quarks populate equally the helicities $\pm\frac{1}{2}$. Since the Σ_{cc} and Σ_{cc}^* are produced incoherently, we may compute independently the probabilities that the light quark and the diquark will combine into the two possible angular momentum states, $\frac{1}{2}$ and $\frac{3}{2}$. We then find the various helicity states to be populated in the ratios

$$\Sigma_{cc}^*(\pm\tfrac{3}{2}):\Sigma_{cc}^*(\pm\tfrac{1}{2}):\Sigma_{cc}(\pm\tfrac{1}{2}) = \tfrac{1}{4}\zeta(1\pm P):\tfrac{1}{3}+\tfrac{1}{4}\zeta(-1\pm\tfrac{1}{3}P):\tfrac{1}{6}(1\pm\zeta P) .$$

The quark fragmentation function (3) has been computed at the renormalization scale $\mu=2m_c$. The Altarelli-Parisi equations must then be used to evolve it up to a high scale $\mu=M$ typical of collider energies, a procedure which sums large logarithms of the form $\ln(m_c/M)$. This evolution has two effects. First, it softens the z distributions, but because the relevant splitting functions $P_{c\to cg}(z)$ and $P_{c\to gc}(z)$ integrate to zero, the fragmentation probability $\int_0^1 dz\, D_{c\to\Sigma}(z)$ remains unchanged. Second, a gluon fragmentation function $D_{g\to\Sigma}(z,M)$ is induced via the gluon splitting function $P_{g\to c\bar{c}}(z)$. In leading logarithmic approximation, this effect, of order $\alpha_s^3\ln(M/m_c)$, dominates over any direct contribution to gluon fragmentation, which would be of order α_s^3 without the $\ln(M/m_c)$ enhancement.

We now turn to the production of baryons of the form bcq, in which the heavy quarks are not identical. In this case the bc diquark may be in either a spin-0 state, which we shall denote (bc), or a spin-1 state, which we shall denote $(bc)^*$. In addition to the formation of Σ_{bc} and Σ_{bc}^* baryons from the fragmentation of the $(bc)^*$ diquark, we now have the possibility of the formation of Λ_{bc} baryons from the fragmentation of the (bc) diquark.

Just as in the case of diquarks consisting of two identical heavy quarks, it is straightforward to relate the fragmentation functions for $b\to(bc)^{(*)}\bar{c}$ and $c\to(bc)^{(*)}\bar{b}$ to those for production of the physical states B_c and B_c^*. These latter functions have been calculated by Braaten, Cheung, and Yuan [6], and we use their computation to obtain our results. Unlike the (cc) case, the color wave function of the diquark is

$$|(bc)_a\rangle = \frac{1}{\sqrt{2}}\epsilon_{abc}|b^b\rangle|c^c\rangle \ ,$$

and there is no factor of 2 from Fermi statistics, so the spin-0 diquark fragmentation function with an initial bottom quark is given by

$$D_{b\to(bc)}(z) = \frac{2}{9\pi}\frac{|R_{(bc)}(0)|^2}{m_c^3}\alpha_s(2m_c)^2 F(z,r) \ , \tag{6}$$

where

$$F(z,r) = \frac{rz(1-z)^2}{12[1-(1-r)z]^6}[6-18(1-2r)z+(21-74r+68r^2)z^2-2(1-r)(6-19r+18r^2)z^3+3(1-r)^2(1-2r+2r^2)z^4] \ , \tag{7}$$

and $r=m_c/(m_c+m_b)$. The analogous function for an initial charm quark, $D_{c\to(bc)}(z)$, is given by (6) with the replacements $F(z,r)\to(m_c/m_b)^3F(z,1-r)$ and with the argument of α_s set to $2m_b$. Finally, the fragmentation function $D_{b\to(bc)^*}(z)$ to the spin-1 diquark is given by (6) with $F(z,r)\to F^*(z,r)$ where

$$F^*(z,r) = \frac{rz(1-z)^2}{4[1-(1-r)z]^6}[2-2(3-2r)z+3(3-2r+4r^2)z^2-2(1-r)(4-r+2r^2)z^3+(1-r)^2(3-2r+2r^2)z^4] \ . \tag{8}$$

Then the same replacements

$$F^*(z,r)\to(m_c/m_b)^3F^*(z,1-r)$$

and $\alpha_s(2m_c)\to\alpha_s(2m_b)$ yield $D_{c\to(bc)^*}(z)$.

A calculation similar to that in Ref. [5] gives $\zeta=0.69$ for the net alignment of the $(bc)^*$. Hence the populations of the various helicity states of the Σ_{bc} and Σ_{bc}^* will be in the same ratios as for the Σ_{cc} and Σ_{cc}^*.

The diquark distributions (6)–(8) must be convolved as in (3) with experimentally determined meson fragmentation functions $D_{\bar{Q}\to M}(z)$ to obtain fragmentation functions to Σ_{bc}, Σ_{bc}^*, and Λ_{bc}. These distributions are then subject to Altarelli-Parisi evolution up to collider energies. As in the case of (cc) production, the quark fragmentation functions are softened by this evolution, and gluon fragmentation functions are induced. The detailed effect of the Altarelli-Parisi equations on the fragmentation functions to B_c and B_c^* is presented in Ref. [6], and the discussion given there applies here as well. Of course, the fragmentation functions reduce directly to integrated probabilities as in (4) and (5), quantities which do not evolve and are more accessible experimentally.

It is interesting to note that at high-energy colliders the rates for production of these doubly heavy baryons are comparable to those for the more familiar quarkonium systems. Relating our fragmentation probabilities to those for $c\to\psi$[4] and $b\to B_c,B_c^*$[6] we find the probability for $c\to\Sigma_{cc},\Sigma_{cc}^*$ to be $\sim 2\times10^{-5}$, for $b\to\Lambda_{bc}$ to be $\sim 4\times10^{-5}$, and for $b\to\Sigma_{bc},\Sigma_{bc}^*$ to be $\sim 5\times10^{-5}$. The probabilities for $b\to\Sigma_{bb}$, Σ_{bb}^*, $c\to\Lambda_{bc}$ and $c\to\Sigma_{bc}$, Σ_{bc}^* are down by roughly $(m_c/m_b)^3$, or two orders of magnitude. Hence it may be possible to observe the doubly heavy baryons $\Sigma_{cc}^{(*)}$, Λ_{bc}, and $\Sigma_{bc}^{(*)}$ at the Tevatron.

There will be additional contributions to the fragmentation to doubly heavy baryons from radially excited diquark states which subsequently decay to the ground state. Equations (5) and (6) also hold for these processes, using the appropriate value for the diquark wave function at the origin. Since for the ψ system, $(R_{\psi(2S)}/R_\psi)^2\simeq 0.4$, we expect that $(R_{[cc(2S)]}/R_{(cc)})^2$ is also not particularly small and that excited diquarks will contribute significantly to the production of doubly heavy baryons.

The largest uncertainty in our calculation arises from our lack of knowledge of the diquark wave function at the origin. We have naïvely scaled the values for quarkonium systems by a color factor of $(1/2)^3$, which is valid only for wave functions living entirely in the Coulombic region of the potential. This is certainly a poor approximation for cc, bc, and bb bound states. However, we note that in deriving (5) and (6) we have assumed nothing about the potential except that the diquark is sufficiently tightly bound to have an $\sim 100\%$ probability of fragmenting to baryons:

$$\int_0^1 dy\, D_{(QQ)\to QQq}(y) = \int_0^1 dy\, D_{\bar{Q}\to M}(y) = 1 \ .$$

In reality, this integrated probability is somewhat less than one, since the diquark may dissociate as it hadronizes, for example $(cc)\to DD+X$. Thus it is likely that (5) and (6) somewhat overestimate the true fragmentation probabilities.

In summary, we have calculated the fragmentation functions of a heavy quark Q to a doubly heavy baryon of the form QQq or $QQ'q$. The perturbative part of the calculation can be related simply to the fragmentation function to quarkonium, and is of a similar magnitude. The nonperturbative part may be related to the fragmentation of heavy quarks to heavy mesons, which may be measured experimentally. We have used our results to estimate fragmentation probabilities of b and c quarks to doubly heavy baryons of the form ccq, bcq, and bbq. We find the production rates for doubly heavy baryons are large enough that some of these states may eventually be observable.

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