Large-N analysis of the $(2+1)$-dimensional Thirring model

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We analyze the $(2+1)$-dimensional vector-vector type four-Fermi interaction (Thirring) model in the framework of the $1/N$ expansion. By solving the Dyson-Schwinger equation in the large-N limit, we show that in the two-component formalism the fermions acquire parity-violating mass dynamically in the range of the dimensionless coupling α, $0 \le \alpha \le \alpha_c \equiv \frac{1}{16}\exp(-N\pi^2/16)$. The symmetry breaking pattern is, however, in a way to conserve the overall parity of the theory such that the Chern-Simons term is not induced at any orders in $1/N$. α_c turns out to be a nonperturbative UV-fixed point in $1/N$. The β function is calculated to be $\beta(\alpha) = -2(\alpha - \alpha_c)$ near the fixed point, and the UV-fixed point and the β function are shown to be exact in the $1/N$ expansion.

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Recently, there has been a resurgence of interest in the four-Fermi interaction partly due to the extraordinary heaviness of the top quark, compared to other quarks and leptons [1]. One of the key ideas in this approach is that the four-Fermi interaction, introduced in the standard electroweak theory as a low energy effective interaction, becomes a relevant operator as the ultraviolet cutoff, Λ, goes to ∞, due to a strong interaction among fermions. When the four-Fermi coupling is larger than a critical value, the four-Fermi interaction induces the condensation of the top quark, as shown in the original Nambu-Jona-Lasinio model [2]. Thus the top quark gets a large mass, and the electroweak symmetry breaks dynamically. As described below, similar dynamical behavior occurs in the $(2+1)$-dimensional Thirring model.

The $(2+1)$-dimensional Thirring model is given in the Euclidean version by

$$\mathcal{L} = i\bar{\psi}_i \not{\partial}\psi_i + \frac{g}{2N}\left(\bar{\psi}_i\gamma_\mu\psi_i\right)\left(\bar{\psi}_j\gamma^\mu\psi_j\right), \qquad (1)$$

where ψ_i are two-component spinors and i, j are summed over from 1 to N. The γ matrices are defined as

$$\gamma_3 = \sigma^3, \quad \gamma_1 = \sigma^1, \quad \gamma_2 = \sigma^2, \qquad (2)$$

where σ's are the Pauli matrices. Since the four-Fermi coupling g has a mass-inverse dimension, the model is not renormalizable in ordinary (weak) coupling expansion. But it has been shown to be renormalizable for $(2+1)$ dimensions in the large flavor (N) limit [3]. It is therefore sensible to analyze the three-dimensional (3D) Thirring model in the large-N expansion.

There are at least two ways of viewing the 3D Thirring model in treating the dimensional coupling constant g. One is taking g as a genuine dimensional parameter that sets the natural scale of the theory; for example, the dynamically generated fermions, if any, will be proportional to this scale, $m_{\rm dyn} \sim 1/g$. The other one is to take the dimensional parameter $1/g$ as the UV cutoff of the theory, $g \equiv \frac{1}{\alpha\Lambda}$, where Λ is the UV cutoff and α is a dimensionless coupling. Therefore, in this case, the only dimensional parameter in the model is the ultraviolet cutoff. In the continuum limit, the four-Fermi operator (together with the ultraviolet cutoff) $\frac{1}{\alpha\Lambda}\left(\bar{\psi}\gamma_\mu\psi\right)^2$ becomes a relevant operator in the large-N approximation. In this approach, if dynamical mass is generated, it will be independent of the ultraviolet cutoff Λ; it will be the one introduced in place of the dimensionless parameter α by the so-called dimensional transmutation, which happens in any renormalizable theories.

The first viewpoint is taken by several authors. For instance, it has been shown in [4] that the 3D Thirring model is UV finite at all orders of $1/N$ since the scale $1/g$ is negligible in the deep UV region. And also Gomes *et al.* [5] found in this viewpoint that the model behaves similarly to QED_{2+1} [6], which also has a dimensional parameter e, the electric charge; in both models, the fermion mass is generated when $\frac{1}{N} > \frac{1}{N_c}$. But, as we shall see later, it is in the second viewpoint that the 3D Thirring model is similar to the 3D Gross-Neveu model [7]. Namely, the 3D Thirring model has a two-phase structure, parity broken and parity unbroken, and the fermion acquires dynamical mass for strong coupling, $g > g_c$ (or $0 \le \alpha \le \alpha_c$). Though the model is still UV finite perturbatively in the $1/N$ expansion, there exists a nonperturbative (in $1/N$) renormalization for α. The coupling is running, $\beta(\alpha) = -2(\alpha - \alpha_c)$, for the same reason as in the Gross-Neveu model. The UV-fixed point α_c is found to be $\frac{1}{16}\exp(-\frac{N\pi^2}{16})$ in the $1/N$ expansion. The UV-fixed point and the β function do not change at all, even if one includes higher order corrections, due

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to the Ward-Takahashi identity and the UV structure of the theory. This is in contrast with the result in [4] presenting a vanishing β function. Therefore, we see that the two viewpoints are in many ways different from each other.

Now we start with the effective theory with a UV cutoff. Introducing an auxiliary field A_μ to facilitate the $1/N$ expansion, we can rewrite Eq. (1) as

$$\mathcal{L} = i\bar{\psi}_i \, \partial\!\!\!/ \psi_i - \frac{1}{\sqrt{N}} A_\mu \left(\bar{\psi}_i \gamma_\mu \psi_i\right) + \frac{1}{2}\alpha\Lambda A_\mu^2, \tag{3}$$

where $\alpha\Lambda = \frac{1}{g}$. As was mentioned in [5], the theory is consistent for positive α. As we shall see later, for negative α, the theory is unstable, showing tachyons in the four-point fermion Green's function. Equation (3) is not gauge invariant under the usual gauge transformation on ψ and A_μ. However, as was claimed in Ref. [5], Eq. (3) with a gauge fixing term has a restricted gauge symmetry. In this paper we choose to work in the Landau gauge. The Thirring model with N two-component complex spinors has U(N) global symmetry and parity also. Under U(N),

$$\psi_i \mapsto \psi_i' = g_i^j \psi_j, \quad \text{for } g \in \mathrm{U}(N), \tag{4}$$

and under parity P, $x = (x, y, t) \mapsto x' = (-x, y, t)$, the fermion fields transform as

$$\psi(x) \mapsto \psi'(x') = e^{i\delta}\sigma^1 \psi(x). \tag{5}$$

One can see that the fermion mass term is parity odd. When the number of fermion flavors is even, the model has another obvious discrete Z_2 symmetry, which interchanges half of the fermions with the other half: Z_2 mixes the fermion fields as, for $i = 1, \ldots, \frac{N}{2}$,

$$\begin{aligned} \psi_i(x) &\mapsto \psi_{\frac{N}{2}+i}(x), \\ \psi_{\frac{N}{2}+i} &\mapsto \psi_i(x). \end{aligned} \tag{6}$$

We define a new parity P_4 which combines the parity for the two-component spinor with Z_2, $P_4 \equiv PZ_2$ [8]. As described below, in the (2 + 1)-dimensional Thirring model, it is P (not P_4) that is spontaneously broken. The fermion mass is dynamically generated in such a way that P_4 is conserved. When P_4 is not broken, the Chern-Simons term is not induced.

Now we will examine the pattern of the spontaneous breaking of parity. An order parameter for the spontaneous breaking of parity is the vacuum condensate of the fermion bilinear, $\langle\bar{\psi}\psi(x)\rangle$, which will be determined once one finds the (asymptotic) behavior of the fermion propagator [9].

In the $1/N$ expansion one has the following Dyson-Schwinger gap equation:

$$-[Z(p) - 1]\, p\!\!\!/ + \Sigma(p) = \frac{1}{N}\int^\Lambda \frac{d^3k}{(2\pi)^3} D_{\mu\nu}(p-k)\gamma_\nu \frac{Z(k)\, k\!\!\!/ - \Sigma(k)}{Z^2(k)k^2 + \Sigma^2(k)} \Gamma_\mu(k, p-k; p), \tag{7}$$

where $D_{\mu\nu}$ is the photon propagator, Σ is the fermion self energy, Z is the fermion wave function renormalization constant, and Γ_μ is the vertex function. In the Landau gauge, the photon propagator is given by

$$D_{\mu\nu} = \frac{1}{p^2}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\Pi_1(p^2) + \epsilon_{\mu\nu\rho}\frac{p_\rho}{|p|^2}\Pi_2(p^2), \tag{8}$$

where Π_1 and Π_2 are given by

$$\Pi_1(p^2) = \frac{\alpha\Lambda + \Pi_e}{(\alpha\Lambda + \Pi_e)^2/p^2 + \Pi_o^2}, \tag{9}$$

$$\Pi_2(p^2) = \frac{\Pi_o}{(\alpha\Lambda + \Pi_e)^2/p^2 + \Pi_o^2}. \tag{10}$$

The resummation technique of the $1/N$ expansion results in the nontrivial photon propagator as given above. Π_e [Π_o] in Eq. (9) [(10)] is the even (odd) part of the vacuum polarization which will be determined once we solve the above coupled Dyson-Schwinger equations, Eqs. (7) and (8).

Since $Z(p) = 1 + O(1/N)$ and $\Gamma_\mu = \gamma_\mu + O(1/N)$, we may take, at leading order in $1/N$, $Z(p) = 1$ and $\Gamma_\mu = \gamma_\mu$ consistently in Eq. (7). Then, taking the trace over the γ matrix, we get

$$\Sigma(p) = \frac{1}{N}\int^\Lambda \frac{d^3k}{(2\pi)^3}\frac{2\Pi_1(p-k)}{(p-k)^2}\frac{\Sigma(k)}{k^2+\Sigma^2(k)} + \frac{1}{N}\int^\Lambda \frac{d^3k}{(2\pi)^3}\frac{(p-k)\cdot k}{|p-k|^3}\frac{\Pi_2(p-k)}{k^2+\Sigma^2(k)}. \tag{11}$$

The magnitude of dynamically generated mass must be small, compared to the cutoff Λ of the theory in the $1/N$ approximation [6]. We may therefore assume $\Sigma(p) \ll p \ll \Lambda$. The vacuum polarization tensor takes then a simple form

$$\Pi_e(p) = \frac{|p|}{16}, \tag{12}$$

$$\Pi_o(p) = \frac{1}{N}\sum_{i=1}^N M_i \frac{1}{4|p|}, \tag{13}$$

where $M_i \simeq \Sigma_i(0)$, the mass of the ith fermion. In general, it is hard to find M_i by solving the gap equations directly. But following the same argument of Coleman and Witten [10], one can easily show that the magnitude of M_i is independent of i in the large-N limit [11]. Therefore, it is reasonable to assume that $M_i = M$ for $i = 1, ..., N-L$ and $M_i = -M$ for $i = N-L+1, ..., N$, as is done in [6]. For momenta p such that $M \ll p \ll \Lambda$,

$$\frac{\Pi_1(p)}{p^2} = \frac{1}{\alpha\Lambda + \frac{|p|}{16}}, \tag{14}$$

$$\frac{\Pi_2(p)}{p^2} = \frac{M\theta}{4\,|p|}\frac{1}{(\alpha\Lambda + \frac{|p|}{16})^2}, \tag{15}$$

where $\theta = 1 - \frac{2L}{N}$, a parameter characterizing the parity (P_4) violation of the theory. Now we will show that $\theta = 0$ admits a consistent solution of the gap equation. Taking the fermion self-energy at zero momentum from Eq. (7) and letting $M_i \simeq \Sigma_i(0)$, we find that

$$M_i = \frac{2}{N}\int^\Lambda \frac{d^3k}{(2\pi)^3}\frac{M_i}{k^2+M^2}\frac{1}{\alpha\Lambda + \frac{|k|}{16}} - \frac{1}{N}\int^\Lambda \frac{d^3k}{(2\pi)^3}\frac{k^2}{k^2+M^2}\frac{M\theta}{4\,|k|}\frac{1}{(\alpha\Lambda + \frac{|k|}{16})^2}. \tag{16}$$

The above equations are consistent only if $\theta = 0 + O(1/N)$ for $M \neq 0$. Thus P is broken, but P_4 is not. One can understand this result on the basis of Vafa and Witten's argument [12]. The Euclidean fermionic determinant $\mathrm{Det}(\not\partial + M + i\not A)$ picks a parity-violating phase that depends at low momenta on the sign of the fermion mass. This phase factor increases the ground state energy, and so the model with the lowest ground state energy should be the one in which the overall phase is minimized. Since $\mathrm{Det}(\not\partial + M + i\not A)$ is the complex conjugate of $\mathrm{Det}(\not\partial - M + i\not A)$, the overall phase vanishes provided the number of positive mass fermions is equal to the number of negative ones, which is possible only when N is even.

Actually, one can show further that $\theta = 0$ at all orders of $1/N$ using the nonrenormalization theorem of Coleman and Hill [13]. The requirement of this nonrenormalization theorem is the gauge invariance and analyticity of the one-loop n-photon function, which is fulfilled in the $\alpha \leq \alpha_c$ in this model. In this range of the coupling α, the Chern-Simons term is not subject to the radiative corrections beyond one loop; thus the one-loop result is exact. One speculates, however, that there is a finite radiative correction to the Chern-Simons term in the other phase due to the lack of analyticity of the fermionic loops in the infrared region [14].

Then Vafa and Witten's argument ensures the cancellation of the Chern-Simons term generated from each fermion and this explains the absence of the corrections at all.

Following the Cornwall-Jackiw-Tomboulis formalism [15], we calculate the effective potential of the operator expectation value $\langle\bar\psi\psi(x)\rangle$. At the extrema it is found to be

$$V = \frac{N}{2\pi^2}\int^\Lambda dp p^2\left[\frac{\Sigma^2}{p^2+\Sigma^2} - \ln\left(1 + \frac{\Sigma^2}{p^2}\right)\right]. \tag{17}$$

This is the same expression as in QED_{2+1} in the $1/N$ expansion [6]. It can be easily seen from Eq. (17) that any nontrivial solution has a lower energy than the perturbative vacuum solution $\Sigma(p) = 0$. Therefore, once such a parity-breaking solution is found, it is always energetically favored over the symmetric one. Our solution to the gap equation has thus lower vacuum energy than the trivial solution.

Rewriting Eq. (16) when $\theta = 0$,

$$M = \frac{2}{N}\int^\Lambda \frac{d^3k}{(2\pi)^3}\frac{M}{k^2+M^2}\frac{1}{\alpha\Lambda + \frac{|k|}{16}}. \tag{18}$$

The above equation indicates that there is a two-phase structure. When $\alpha > \alpha_c$, the parity symmetry is manifest, where the critical value α_c is defined by the equation

$$1 = \frac{2}{N}\int^\Lambda \frac{d^3k}{(2\pi)^3}\frac{1}{k^2}\frac{1}{\alpha_c\Lambda + \frac{|k|}{16}}. \tag{19}$$

If $\alpha \leq \alpha_c$, nontrivial parity-violating fermion mass is generated in a way to preserve the total parity symmetry of the theory.

From Eq. (19), one obtains

$$\alpha_c = \frac{1}{16}\exp\left(-\frac{1}{16}N\pi^2\right). \tag{20}$$

From the above equation, one sees the nonperturbative nature in the phase transition point. The $1/N$ factor in the gap equation (18) has been traded for the nonanalytic factor N in α_c. A similar phenomenon can be seen in the exponential hierarchy between the dynamically generated fermion mass and the cutoff in QED_{2+1} [6]. The parity-violating region $0 \leq \alpha \leq \alpha_c$ is very small, and so the theory has parity symmetry for almost all the positive region of α.

The cutoff dependence of the bare coupling is determined by the requirement that the gap equation (18) be independent of the UV cutoff Λ, as taken to ∞. In the vicinity of α_c, we get the β function for α given by

$$\beta(\alpha) \equiv \Lambda\frac{\partial\alpha}{\partial\Lambda} = -2(\alpha - \alpha_c). \tag{21}$$

α increases (decreases) to α_c when $\alpha < \alpha_c$ ($\alpha > \alpha_c$). α_c is the UV-fixed point of both phases of the Thirring model. The above equation shows that α_c is a UV-fixed point which is again nonperturbative in $1/N$ (it has nonanalytic dependence on $1/N$). Therefore, it is not feasible to study the theory in the vicinity of α_c with the perturbative $1/N$ expansion. To observe the effects of the running of the coupling α as in Eq. (21), one needs to go beyond the perturbative expansion; one needs to solve, for example, the Dyson-Schwinger equation, just as is done in this paper.

As we shall see later, the theory is UV finite at any finite orders of $1/N$. Within the $1/N$ expansion the UV behavior of the theory is more improved than the one in a Gross-Neveu-type model. In the latter, the resummation technique of the $1/N$ expansion results in the UV dimension of the auxiliary field $\sigma = \bar{\psi}\psi$ being 1 (2 classically). This is a key point of the renormalizability of the theory. In the Thirring model the mass of the photon propagator in Eq. (8) should not be neglected at high momenta. It renders the entire integration UV finite [16].

This can be checked by a direct calculation. For instance, the fermion wave function renormalization Z at $O(1/N)$ is given by Eq. (7). It is not difficult to see that the integration is finite. This (perturbative) UV finiteness of the theory is consistent with the nonperturbative nature of the UV-fixed point α_c.

As we mentioned in the introductory part of this paper, the mass M is that it is *not* a value which can be determined as in QED_{2+1} [6] but a parameter as in a Gross-Neveu-type model [3]. M is a physical quantity; in fact it is the pole mass of the fermions, and therefore it should be independent of Λ. As in the case of the Gross-Neveu model, one may interpret Eq. (18) as *fine-tuning* the coupling g in order to have $M \ll \Lambda$. In other words, for $M \ll \Lambda$, the coupling is tuned to be very close, or equal, to the critical value Eq. (20).

The four-point fermionic Green's function, in leading order, is given by the photon propagator in Eq. (8). We see that there are no tachyons for $g > 0$, and so the theory is consistent in that region. For $g < 0$, the Green's function shows the existence of tachyons.

Finally, we show that the features of the 3D Thirring model we have found are exact in the $1/N$ expansion even if one includes higher order corrections. To go beyond leading order, one should solve the Dyson-Schwinger equation keeping the higher order corrections in the propagators and the vertex function. But by following general arguments, the Dyson-Schwinger equation takes a simpler form. First, since $\theta = 0$ in all orders in $1/N$ as we showed earlier, we can set $\Pi_o(p) = 0$ in the photon propagator. Second, beyond leading order, the magnitude of M_i is the same for all flavors, $|M_i| = M$, which can be seen easily by an argument similar to that of Coleman and Witten [10]. Therefore, the gap equation Eq. (18) becomes

$$M = \frac{2}{N}\int^{\Lambda}\frac{d^3k}{(2\pi)^3}\frac{M}{Z(k)k^2+M^2}\frac{1}{\alpha\Lambda+\Pi_e(k)}\Gamma(k,0;k), \tag{22}$$

where $\Gamma(k,0;k) = \frac{1}{4}g^{\mu\nu}\mathrm{Tr}\gamma_\mu\Gamma_\nu(k,0;k)$. Keeping terms up to $O(1/N)$, we find by explicit calculations that

$$Z(k) = 1 - \frac{16}{3N\pi^2}\ln\left(\frac{16\alpha+1}{16\alpha}\right) + O\left(\frac{1}{\Lambda}\right), \tag{23}$$

$$\Gamma_\mu(k,0;k) = \gamma_\mu\left[1 - \frac{16}{3N\pi^2}\ln\left(\frac{16\alpha+1}{16\alpha}\right)\right] + O\left(\frac{1}{\Lambda}\right). \tag{24}$$

Similarly, the even part of the vacuum polarization is, up to the terms in $1/N$,

$$\Pi_e(k) = \frac{|k|}{16} + \mathrm{const}\times\frac{1}{N}\frac{k^2}{\alpha\Lambda}\ln\frac{\Lambda}{M} + O\left(\frac{1}{\Lambda}\right). \tag{25}$$

where const is a pure number. The Feynman diagrams relevant to the corrections are shown in Fig. 1.

The above results, Eqs. (23)–(25), show that next-to-leading corrections are either finite or suppressed by $1/\Lambda$; the $1/N$ corrections are UV finite. By dimensional counting, one can easily show that the 3D Thirring model is in fact UV finite in all orders in the $1/N$ expansion [17].

This is the same result as in [4], but the reason is quite different. In our case, because of $\alpha\Lambda$ in the photon propagator, the loop integrations are UV finite.

The dangerous terms in deriving the UV-fixed point α_c will be the terms which are not suppressed as $\Lambda \to \infty$. But such terms do not occur in any orders in $1/N$ because of the UV finiteness of the theory. Since higher order corrections to the vacuum polarization are suppressed by $1/\Lambda$, the equation defining α_c becomes now

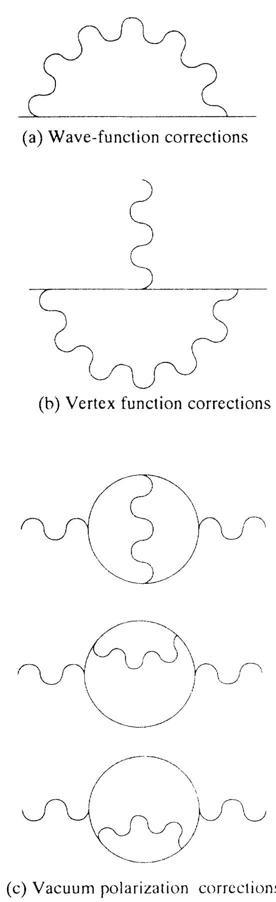

FIG. 1. The Feynman diagrams relevant to the $1/N$ corrections (wavy line—photon, solid line—fermion).

$$1 = \frac{2}{N}\int^{\Lambda}\frac{d^3k}{(2\pi)^3}\frac{1}{[1-F(\alpha_c)]k^2}\frac{1}{\alpha_c\Lambda+\frac{|k|}{16}}[1-F(\alpha_c)] +O\left(\frac{1}{\Lambda}\right), \quad (26)$$

where $F(\alpha_c) = \frac{16}{3N\pi^2}\ln\left(\frac{16\alpha_c+1}{16\alpha_c}\right) + O(1/N^2)$.

We see that in the above equation the contributions from the $1/N$ corrections in Eqs. (23) and (24) cancel each other exactly. This is due to the Ward-Takahashi identity of the gauge invariance in the 3D Thirring model. The equation defining α_c is therefore same for all orders in $1/N$, and the UV-fixed point and the β function we found earlier are in fact exact in the $1/N$ expansion.

In summary, we have analyzed the $(2+1)$-dimensional Thirring model in the $1/N$ expansion. It has been shown that the fermions acquire parity-violating mass in a way that conserves the total parity symmetry of the theory. The Chern-Simons term is not generated at any finite orders in $1/N$. We have also shown that the theory has a two-phase structure and a UV-fixed point as in the Gross-Neveu model, albeit at nonperturbative order in $1/N$. The UV-fixed point $\alpha_c = \frac{1}{16}\exp(-\frac{N\pi^2}{16})$ and the β function $\beta(\alpha) = -2(\alpha-\alpha_c)$, which we have found by solving the Dyson-Schwinger equation, are shown to be exact in the $1/N$ expansion.

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