Process of fermion level crossing in the electroweak instanton

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The whole process of the fermion level crossing phenomenon in the background fields of the electroweak instanton is demonstrated by numerically determining the fermion eigenvalues along Euclidean time. We assume that the fermions of a doublet are degenerate in mass and there are small-size instantons in the Weinberg-Salam model. The difference in the critical values of the Chern-Simons number at which the fermion level enters the positive and negative continuum in the level crossing process is obtained and we find that the fermion level crossing appears only in some part of the instanton configuration space.

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I. INTRODUCTION

Modern theories of elementary particles are based on the principles of local gauge symmetry for the Weyl fermion. It is now clear that the dynamical content of gauge theories is not exhausted by perturbation theory even in models with weak coupling. The nonperturbative aspects of gauge theories, such as the complex structure of a vacuum and the consequent nonconservation of fermion quantum number and the existence of solitons, become very important for further study.

In the standard model the baryon and lepton number are not absolutely conserved due to the quantum anomaly [1]. The process for baryon and lepton number nonconversation is spontaneous fermion number violation due to instanton-induced transitions between topologically distinct vacua. Strictly speaking, there is no instanton solution in the Weinberg-Salam model, because the Lagrangian of the Higgs field breaks the scale invariance of this theory. However, the small-sized instantons (or constrained instantons) may still exist in this model, which give the effects for baryon and lepton number violation. Direct observation of the processes with baryon and lepton number violation in electroweak theory is impossible at low energies, but there are a number of situations in which the rate of anomalous processes with nonconservation of the baryon and lepton number may not be small [2], such as at high temperatures [3] and high energies [4].

How can baryon and lepton number change by the tunneling of an instanton? It will be directly shown in the fermion level crossing picture. Let us now consider a system in an external gauge field $A(\mathbf{x},t)$ which changes adiabatically from $A(\mathbf{x},t_1)$ to $A(\mathbf{x},t_2)$. At each intermediate time t, we can calculate the fermion spectrum as a set of eigenvalues of the Dirac Hamiltonian in the external field $A(\mathbf{x},t)$ for fixed t. The spectrum varies in the course of time, and some of the levels cross zero from above and some from below. The difference $n_+ - n_-$ between the number of levels crossing zero from above (n_+) and the number of levels crossing zero from below (n_-) is, in general, nonzero. For each value of $A(\mathbf{x},t)$, the ground state of the fermion system is a state in which all negative-energy states are filled, whereas positive energy states are unoccupied. A real fermion corresponds to a filled positive level, and an antifermion corresponds to a free negative level. The net effect of the level crossing phenomenon is that the number of real fermions will change. The difference $(n_+ - n_-)$ is due to the difference between the Chern-Simons numbers of the gauge field.

The level crossing phenomenon in the background fields of an instanton have been intensively investigated after the works of 't Hooft. The zero mode of a massless Euclidean Dirac operator in the presence of an instanton of Yang-Mills fields was first given by 't Hooft [1]. A more intuitive discussion of the physics involved has been given by Callan *et al.* and Kiskis [5]. The zero modes of fermions in the instanton of Weinberg-Salam theory are considered by Krasnikov *et al.* [6] and the explicit expressions are given by Rubakov [7]. Recently, the general zero modes were discussed by Kastening [8] and Anselm *et al.* [9] when there is no custodial global SU(2) symmetry in Weinberg-Salam theory to ensure degenerate fermion masses. However, all of these works only consider the zero modes of a fermion in the background field of an instanton. By following the zero mode, the continuity and chiral symmetry imply that the level crossing phenomenon will exist.

To understand fully the mechanism of baryon and lepton number violations in the standard model, it is necessary to investigate the detailed process of fermion level crossing. In this paper we will demonstrate the whole process of fermion level crossing in the background field of an electroweak instanton by numerical calculation. We work in the $A_0(x)=0$ gauge and focus upon the Euclidean time parameter x_0. As x_0 changes from $-\infty$ to $+\infty$, the one-instanton field evolves from one pure gauge configuration to another topologically distinct pure gauge. The three-dimensional Dirac Hamiltonian in the presence of such a field depends parametrically on x_0 through $A_i(x_0,\mathbf{x})$. The spectrum of the eigenenergy of a fermion as a function of x_0 gives information about the behavior of the quantized Dirac field in the adiabatic approximation. We assume that the fermions of a doublet

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are degenerate in mass which allows for a spherically symmetric ansatz for all of the fields when the Weinberg mixing angle dependence is neglected. Recently, Kunz and Brihaye [10] have presented the level crossing phenomenon for fermions in the background field of the sphaleron barrier along the minimal energy path from one vacuum to another [11].

The remainder of this paper is organized as follows. Section II introduces the Weinberg-Salam model Lagrangian with the approximations employed and the anomalous current equation for fermion number violation. The instanton solution and the form of the gauge transformation are given in Sec. III. There the Chern-Simons number and the static energy of the instanton are presented as a function of Euclidean time. In Sec. IV we derive the radial equations for the fermions and rescale the instanton solution in this equation. We present the fermion level crossing picture in the background of small size instantons by numerical calculation in Sec. V. Finally, in Sec. VI a brief discussion is given.

II. LAGRANGIAN OF WEINBERG-SALAM MODEL

Let us consider the bosonic sector of the Weinberg-Salam model in the limit of the vanishing mixing angle. In this limit the U(1) field decouples and can consistently be set to zero:

$$\mathcal{L}_b=-\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu,a}+(D_\mu\Phi)^\dagger(D^\mu\Phi)-\lambda\left(\Phi^\dagger\Phi-\frac{v^2}{2}\right)^2 \tag{2.1}$$

with the $SU(2)_L$ field strength tensor

$$F^a_{\mu\nu}=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu+g\epsilon^{abc}A^b_\mu A^c_\nu\ , \tag{2.2}$$

and the covariant derivative for the Higgs field,

$$D_\mu\Phi=(\partial_\mu-\frac{i}{2}g\tau^a A^a_\mu)\Phi\ , \tag{2.3}$$

where g is the gauge coupling constant and in electroweak theory we employ the value $g=0.67$. $A^a_\mu(x)(a=1,2,3,)$ are real vector fields and can be described as a matrix field $A_\mu(x)=\frac{1}{2}g\tau^a A^a_\mu(x)$, τ^a being the isospin Pauli matrices.

The $SU(2)_L$ gauge symmetry is spontaneously broken due to the nonvanishing vacuum expectation value v of the Higgs field,

$$\langle\Phi\rangle=\frac{v}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix} \tag{2.4}$$

leading to the boson masses

$$M_W=M_z=\tfrac{1}{2}gv\ ,\quad M_H=v\sqrt{2\lambda}\ . \tag{2.5}$$

For a vanishing mixing angle, considering only fermion doublets degenerate in mass for simplicity, the fermion Lagrangian in the chiral representation reads

$$\begin{aligned}\mathcal{L}_f=&\bar{q}_L i\gamma^\mu D_\mu q_L+\bar{u}_R i\gamma^\mu\partial_\mu u_R+\bar{d}_R i\gamma^\mu\partial_\mu d_R\\&-f_q\bar{q}_L(\tilde{\Phi}u_R+\Phi d_R)-f_q(\bar{d}_R\Phi^\dagger+\bar{u}_R\tilde{\Phi}^\dagger)q_L\ ,\end{aligned} \tag{2.6}$$

where q_L denotes the left-handed doublet (u_L,d_L), u_R and d_R are the right-handed singlets, with the covariant derivative

$$D_\mu q_L=(\partial_\mu-\frac{i}{2}g\tau^a A^a_\mu)q_L\ , \tag{2.7}$$

and $\tilde{\Phi}=i\tau_2\Phi^*$. The fermion mass is given by

$$m_u=m_d=m_f=\frac{1}{\sqrt{2}}f_q v\ . \tag{2.8}$$

The gauge-invariant current of the doublet $J^\mu=\bar{q}_L\gamma^\mu q_L$ is conserved at the classical level, but is anomalous at the quantum level [12].

$$\partial_\mu J^\mu=\frac{g^2}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}\ . \tag{2.9}$$

The integration of the right-hand side of Eq. (2.9) is an expression of the topological charge of a gauge field configuration:

$$Q=\frac{g^2}{16\pi^2}\int d^4x\,\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}\ . \tag{2.10}$$

The topological current is

$$K^\mu=\frac{g^2}{8\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}(A_\nu\partial_\rho A_\sigma-i\tfrac{2}{3}gA_\nu A_\rho A_\sigma)\ . \tag{2.11}$$

Equation (2.9) indicates that the number of fermions may not be conserved, the changes of baryon number B and lepton number L are given as

$$\Delta B=\Delta L=n_f Q\ , \tag{2.12}$$

where n_f is the number of generations. All gauge field configurations can be classified by the Chern-Simons number given by

$$N_{\mathrm{CS}}=\int d^3x K^0\ . \tag{2.13}$$

The Chern-Simons number N_{CS} may be regarded as a coordinate in gauge-orbit space which measures the position of topologically inequivalent vacua. N_{CS} is invariant under all "proper" gauge transformations (transformations continuously connected to the identity), but changes by an integer under topologically nontrivial gauge transformations. For the vacua the Chern-Simons number is identical to the integer; for a nonvacuum it may take on arbitrary real values.

III. INSTANTON AND GAUGE TRANSFORMATION

The SU(2) pure Yang-Mills fields have solitonlike solutions of the classical Euclidean equation of motion that are localized in time as well as space and dubbed instantons [13], which connect two topologically distinct vacua. The instanton may be viewed as a solution of the Euclidean gauge field equations in which a vacuum at $x_0=-\infty$ evolves by propagation in imaginary time to a different vacuum at $x_0=+\infty$. For an SU(2) gauge field, an explicit solution with a topological charge $Q=1$ in the regular gauge is given by

$$A_0(x)=-\frac{i\tau\cdot\mathbf{x}}{x^2+\rho^2}\ ,\qquad \mathbf{A}(x)=-\frac{i(\tau x_0+\tau\times\mathbf{x})}{x^2+\rho^2}\ , \tag{3.1}$$

where $x^2=x_0^2+\mathbf{x}^2$ and ρ is some arbitrary scale parameter, often referred to as the instanton size.

In the Weinberg-Salam model, because of the presence of the Higgs field, strictly speaking, for $v\neq 0$, there does not exist a finite action solution of the classical Euclidean equations of motion, since there are no combined solutions of the Higgs and gauge field equations of motion. However, as long as the size of the instanton ρ is not too large compared to the inverse size of the order parameter v in the Weinberg-Salam model, as $\rho v \ll 1$, one can find a good approximate solution in the Weinberg-Salam model. This approximate solution still has the gauge field configurations given by the SU(2) instanton of (3.1), while the scalar doublet field takes the form [1]

$$\Phi=\frac{x_0-i\tau\cdot\mathbf{x}}{\sqrt{x^2+\rho^2}}\frac{v}{\sqrt{2}}\Phi_0\ , \tag{3.2}$$

where Φ_0 is a constant SU(2) spinor and we can take $\Phi_0=\binom{0}{1}$ and $\tilde{\Phi}_0=\binom{1}{0}$.

To cast the vector potential $A_\mu(x)$ given in (3.1) to the form of the $A_0(x)=0$, we make a gauge transformation $V(x)$ on $A_\mu(x)$ into the temporal gauge,

$$A_0'(x)=V^{-1}(x)A_0(x)V(x)+V^{-1}(x)\partial_0 V(x)=0\ , \tag{3.3}$$

and define

$$\mathbf{A}'(x)=V^{-1}(x)\mathbf{A}(x)V(x)+V^{-1}(x)\nabla V(x)\ . \tag{3.4}$$

Under this gauge transformation, the Higgs field Φ can be transformed accordingly as

$$\Phi'(x)=V^{-1}(x)\Phi(x)\ . \tag{3.5}$$

From Eq. (3.3), we can find the solution for $V(x)$ by integration as

$$V(x)=\exp[i\tau\cdot\mathbf{x}f(x)] \tag{3.6}$$

with

$$f(x)=\frac{1}{\sqrt{\mathbf{x}^2+\rho^2}}\left[\arctan\left[\frac{x_0}{\sqrt{\mathbf{x}^2+\rho^2}}\right]+\theta(\mathbf{x})\right]\ , \tag{3.7}$$

where $\theta(\mathbf{x})$ is a time-independent residual gauge freedom with respect to the $A_0(x)=0$ gauge. We can choose it to be constant:

$$\theta=(n+\tfrac{1}{2})\pi\ ,\quad n=0,1,2,\ldots\ . \tag{3.8}$$

By substituting (3.1) and (3.2) into (3.4) and (3.5), respectively, we obtain the general spherically symmetric form for the gauge field $\mathbf{A}'(x)$ and Higgs field $\Phi'(x)$ as

$$\mathbf{A}'(x)=\frac{1}{g}\{a(r)(\tau\times\hat{\mathbf{x}})+b(r)[\tau-(\tau\cdot\hat{\mathbf{x}})\hat{\mathbf{x}}]+c(r)(\tau\cdot\hat{\mathbf{x}})\hat{\mathbf{x}}\}\ ,\qquad \Phi'(x)=\frac{v}{\sqrt{2}}[h(r)+i\tau\cdot\hat{\mathbf{x}}k(r)]\Phi_0 \tag{3.9}$$

with

$$\begin{aligned}
a(r)&=-\frac{1}{x^2+\rho^2}(r\cos 2f+x_0\sin 2f)-\frac{\sin^2 f}{r}\ ,\\
b(r)&=-\frac{1}{x^2+\rho^2}(x_0\cos 2f-r\sin 2f)-\frac{\sin 2f}{2r}\ ,\\
c(r)&=-\frac{x_0}{x^2+\rho^2}-\frac{df}{dr}\ ,\\
h(r)&=\frac{1}{\sqrt{x^2+\rho^2}}(-r\sin f+x_0\cos f)\ ,\\
k(r)&=\frac{1}{\sqrt{x^2+\rho^2}}(-r\cos f-x_0\sin f)\ ,
\end{aligned} \tag{3.10}$$

where $x^2=x_0^2+r^2$, $r=\sqrt{\mathbf{x}^2}$ and $\hat{\mathbf{x}}$ is a unit three-vector in the radial direction given by $\hat{\mathbf{x}}=\mathbf{x}/r$.

By using (3.9), the Chern-Simons number N_{CS} in (2.13) can be given by

$$N_{CS}(x_0)=\frac{2}{\pi}\int_0^\infty r^2dr\left[2c\left[a^2+b^2+\frac{1}{r}a\right]+(ba'-ab')\right] \tag{3.11}$$

where the prime means differentiation with respect to r. Because of the x_0 dependence of the functions $a(r)$, $b(r)$, and $c(r)$, the N_{CS} is the function of x_0. We do not now attribute any physical significance to the variable x_0 and regard $\mathbf{A}(\mathbf{x},x_0)$ and $\Phi(\mathbf{x},x_0)$ as a path in the configuration space, x_0 being simply a parameter along this path. In this way, x_0 can describe the configuration space path as the same as the N_{CS}.

The total static energy of gauge and Higgs fields for the configurations at some fixed value of the parameter x_0 can be obtained by using (3.9):

$$\begin{aligned}
E(x_0)=\frac{4\pi}{g^2}\int_0^\infty dr\Bigg\{&2\left[a^2+b^2+\frac{1}{r}a\right]^2+\left[a'+\frac{1}{r}a+2bc\right]^2+\left[b'+\frac{1}{r}b-\frac{1}{r}(1+2ra)c\right]^2\\
&+2(k^2+h^2)[1+2ra+r^2(2a^2+2b^2+c^2)]\\
&+2(1+2ra)(k^2-h^2)-8rbhk+2r^2(h'^2+k'^2)-4r^2c(k'h-kh')+\frac{4\lambda r^2}{g^2}(h^2+k^2-1)^2\Bigg\}\ ,
\end{aligned} \tag{3.12}$$

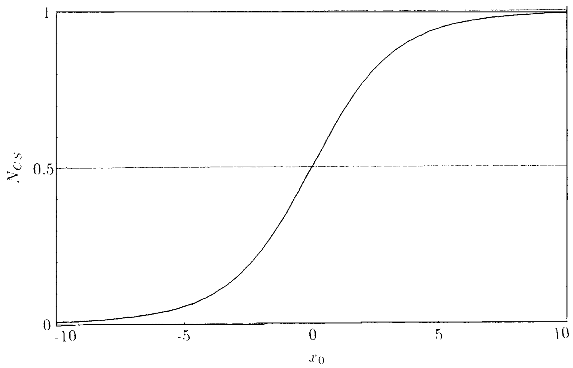

FIG. 1. x_0 dependence of Chern-Simons number $N_{\rm CS}$ for the size of instanton $\rho=5$ (in arbitrary unit) and $n=0$ for θ.

where the prime means differentiation with respect to r. E is the function of x_0. We can define parametrically the energy E as a function of $N_{\rm CS}$.

By use of the functions in (3.10), we can calculate numerically the Chern-Simons number $N_{\rm CS}$ and the static energy E where we take the size of the instanton $\rho=5$ (in arbitrary units) and $n=0$ for θ. The Euclidean time x_0 dependence of $N_{\rm CS}$ is shown in Fig. 1; we see that $N_{\rm CS}$ changes from 0 to 1 when x_0 varies from $-\infty$ to $+\infty$. The topological charge is the difference in the Chern-Simons number of the gauge field at $x_0=\pm\infty$, as $Q=N_{\rm CS}(+\infty)-N_{\rm CS}(-\infty)=1$, which confirms the value of the topological charge of an instanton. The static energy E as a function of x_0 is shown in Fig. 2. There exists a symmetric potential barrier, while $E(x_0=\pm\infty)=0$ since the gauge and Higgs fields tend to a pure gauge configuration as $x_0\to\pm\infty$ and $E(x_0=0)$ is at the top of the barrier. This picture suggests the interpretation that the instanton configuration corresponds to tunneling between different vacuum states.

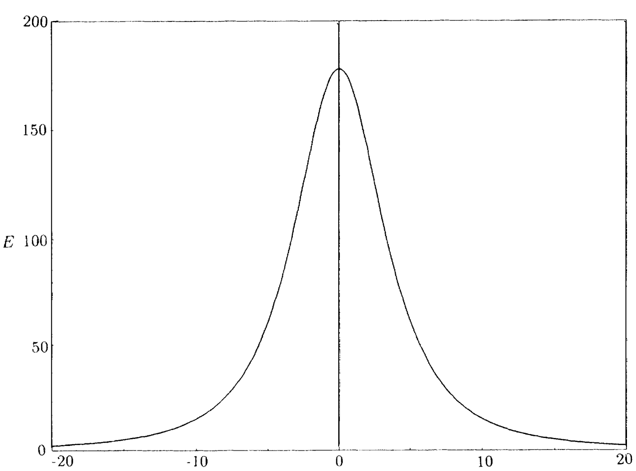

FIG. 2. The total static energy E (in arbitrary unit) of gauge and Higgs fields is shown as a function of Euclidean time x_0 for the size of instanton $\rho=5$ (in arbitrary unit) and $n=0$ for θ. Here, we have made an approximation for $\lambda=0$.

IV. THE EQUATION FOR FERMIONS

Let us now consider the fermions in the background fields of the electroweak instanton with the form of (3.9). To retain spherical symmetry we consider only fermion doublets degenerate in mass. From the fermion Lagrangian (2.6), for each value of Euclidean time x_0, we obtain the time-independent Dirac equations for the left-handed doublet,

$$iD_0 q_L + i\sigma^i D_i q_L - f_q(\tilde{\Phi}u_R+\Phi d_R)=0 \tag{4.1}$$

and for the right-handed singlets,

$$\begin{aligned} &i\partial_0 u_R - i\sigma^i\partial_i u_R - f_q\tilde{\Phi}^\dagger q_L=0\ ,\\ &i\partial_0 d_R - i\sigma^i\partial_i d_R - f_q\Phi^\dagger q_L=0\ , \end{aligned} \tag{4.2}$$

where σ^i are Pauli spin matrices. Wave functions q_L, u_R, and d_R depend on x_0 which enters only as a parameter.

Spherically symmetric fermion fields are described by the ansatz

$$\begin{aligned} &q_L(r)=e^{-i\epsilon t}[G_L(r)+i\boldsymbol{\sigma}\cdot\mathbf{x}F_L(r)]\chi_h\ ,\\ &u_R(r)=e^{-i\epsilon t}[G_R(r)+i\boldsymbol{\sigma}\cdot\mathbf{x}F_R(r)]\chi_1\ ,\\ &d_R(r)=e^{-i\epsilon t}[G_R(r)+i\boldsymbol{\sigma}\cdot\mathbf{x}F_R(r)]\chi_2\ , \end{aligned} \tag{4.3}$$

with

$$\begin{aligned} &\chi_h=\frac{1}{\sqrt{2}}\left[\begin{pmatrix}1\\0\end{pmatrix}_S\begin{pmatrix}0\\1\end{pmatrix}_I-\begin{pmatrix}0\\1\end{pmatrix}_S\begin{pmatrix}1\\0\end{pmatrix}_I\right]\ ,\\ &\chi_1=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\end{pmatrix}_S\ ,\quad \chi_2=-\frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix}_S\ , \end{aligned} \tag{4.4}$$

where S refers to spin, I to isospin. χ_h is the hedgehog spinor satisfying the spin-isospin relation $\sigma\chi_h+\tau\chi_h=0$. χ_1, χ_2 are the constant spin spinors which can construct the hedgehog spinor χ_h with isospin spinors Φ_0 and $\tilde{\Phi}_0$.

By using those ansatz for Eqs. (4.1) and (4.2), we obtain the following set of four coupled first-order differential equations:

$$\begin{aligned} &F_L'+\left[\frac{2}{\tilde{r}}+2\tilde{a}\right]F_L+(2\tilde{b}+\tilde{c})G_L-\tilde{\epsilon}G_L=-(\tilde{h}G_R+\tilde{k}F_R)\ ,\\ &G_L'-2\tilde{a}G_L+(2\tilde{b}-\tilde{c})F_L+\tilde{\epsilon}F_L=(\tilde{h}F_R-\tilde{k}G_R)\ ,\\ &F_R'+\frac{2}{\tilde{r}}F_R+\tilde{\epsilon}G_R=(\tilde{h}G_L-\tilde{k}F_L)\ ,\\ &G_R'-\tilde{\epsilon}F_R=-(\tilde{h}F_L+\tilde{k}G_L)\ , \end{aligned} \tag{4.5}$$

where $\tilde{r}=m_f r$, $\tilde{\epsilon}=\epsilon/m_f$ and the prime means differentiation with respect to $\tilde{r}$. The functions $\tilde{a}$, $\tilde{b}$, $\tilde{c}$, $\tilde{h}$, and $\tilde{k}$ come from the functions $a(r)$, $b(r)$, $c(r)$, $h(r)$, and $k(r)$ in (3.10), respectively, by variables replacement as $\tilde{x}_0=m_f x_0$ for x_0, $\tilde{r}=m_f r$ for r and $\tilde{\rho}=m_f\rho$ for ρ. There are two parameters $\tilde{x}_0$ and $\tilde{\rho}$ in these equations.

Solving the eigenvalue equations (4.5) for the fermions in the background field of an instanton requires certain boundary conditions for the fermion wave functions. Wave functions G_L and G_R are finite and F_L and F_R are zero at $\tilde{r}=0$ and all wavefunctions G_L, G_R, F_L, and F_R tend to zero in the limit $\tilde{r}\rightarrow\infty$.

V. FERMION LEVEL CROSSING

We solve numerically the fermion eigenvalue equations (4.5) under the boundary conditions for the bound state. Giving set values for parameters $\tilde{x}_0$ and $\tilde{\rho}$, which stand for the Euclidean time x_0 and the size of instanton ρ, respectively, for fixing the fermion mass m_f, we can solve the equations by computer and obtain the eigenvalue $\tilde{\epsilon}$ and eigenfunctions F_L, G_L, F_R, and G_R. The results of the calculations are given in the following sections.

A. Zero-mode solutions at $\tilde{x}_0=0$

For all the values of parameter $\tilde{\rho}$, there are only $\tilde{\epsilon}=0$ solutions in the case of $\tilde{x}_0=0$ at which the Chern-Simons number $N_{\rm CS}=1/2$. In this case, the functions F_R decouple and are identically equal to zero; the other normalizable zero-mode eigenfunctions are shown in Fig. 3; as a typical one, we have taken $\tilde{\rho}=0.5$. In the following, we will find that these normalizable eigenstates with zero eigenvalues are the zero mode of the fermion in the background fields of an instanton that have been discussed by many authors [5,7].

To see that, we make a gauge transformation for the left-hand zero-mode eigenfunctions

$$q_L'(x)=V(x)q_L(x)\ , \tag{5.1}$$

by using the expression (3.6) for $V(x)$ and (4.3) for q_L. We obtain

$$\begin{aligned} G_L' &= G_L\cos f+F_L\sin f\ , \\ F_L' &= -G_L\sin f+F_L\cos f\ . \end{aligned} \tag{5.2}$$

Comparing the functions G_L' and F_L' with the corresponding analytical solutions given by Rubakov [7] for the fermion zero mode, one found that they coincide with each other, respectively. When the mass of the fermions vanishes, the function F_L' decouples and is identically equal to zero. The function G_L' also coincides with the corresponding solution in the massless case obtained in Ref. [5].

B. The solutions of $\tilde{x}_0\neq0$

Giving a value of parameter $\tilde{\rho}$, we solve the fermion eigenvalue equations (4.5) for nonzero values of the parameter $\tilde{x}_0$ which can vary from $-\infty$ to $+\infty$. Since the configurations of the instanton are symmetric about $x_0=0$ at which the zero mode appears, the fermion eigenvalue $\tilde{\epsilon}$ should be antisymmetric with respect to the $x_0=0$ configuration.

In Fig. 4 we represent the fermion eigenvalues $\tilde{\epsilon}$ for the dependence of the Chern-Simons number $N_{\rm CS}$ as the parameter $\tilde{\rho}=0.1$, 0.5, and 1.0. Figure 4 shows the whole process of fermion level crossing that is from the positive-energy continuous state to the negative-energy continuous state. Because the $\tilde{\rho}$ is the product of instanton size and fermion mass, for the fixing value of mass m_f the lines in Fig. 4 present the behavior of level crossing for different sizes of the instanton; for fixing the size of instanton, it presents the behavior of level crossing for fermions with different masses. Since $N_{\rm CS}$ is a function of x_0, Fig. 4 also represents the eigenvalue $\tilde{\epsilon}$ dependence of x_0 as $N_{\rm CS}$ varies from 0 to 1 corresponding to x_0 varying from $-\infty$ to $+\infty$ and the gauge field configurations changing from one vacuum to another. From Fig. 4 we observe that the level crossing appears only in some part of the configuration space of an instanton. For small values of $\tilde{\rho}$ the level crossing occurs in a close region to the axis of $N_{\rm CS}=1/2$, for large values of $\tilde{\rho}$ it occurs in almost the full range of $N_{\rm CS}$.

In Fig. 4, for every value of the parameter $\tilde{\rho}$, the eigenvalue $\tilde{\epsilon}$ has two symmetric critical values of $N_{\rm CS}$; at one point the fermion bound state enters the continuum of positive energy and at another point it enters the continuum of negative energy. The difference $\Delta N_{\rm CS}$ of the Chern-Simons number $N_{\rm CS}$ for the two points depending

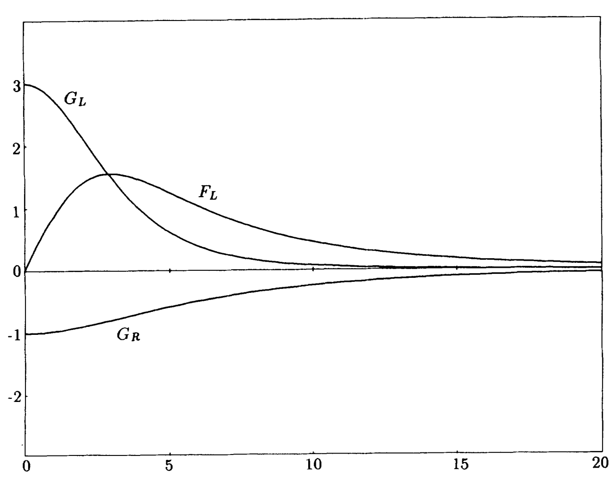

FIG. 3. The fermion zero-mode wave function components G_L, F_L, and G_R (in arbitrary scale) are shown as a function of $\tilde{r}=m_f r$ for the parameter $\rho=0.5$ and $\tilde{x}_0=0$.

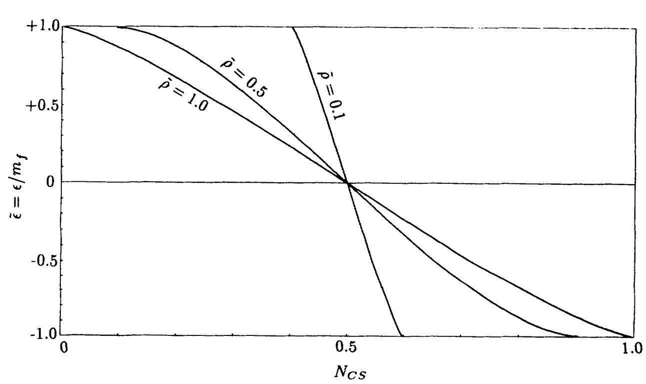

FIG. 4. The normalized fermion eigenvalue $\tilde{\epsilon}=\epsilon/m_f$ is shown as a function of the Chern-Simons number $N_{\rm CS}$ which changes from zero to one for $\tilde{\rho}=0.1$, $\tilde{\rho}=0.5$, and $\tilde{\rho}=1.0$.

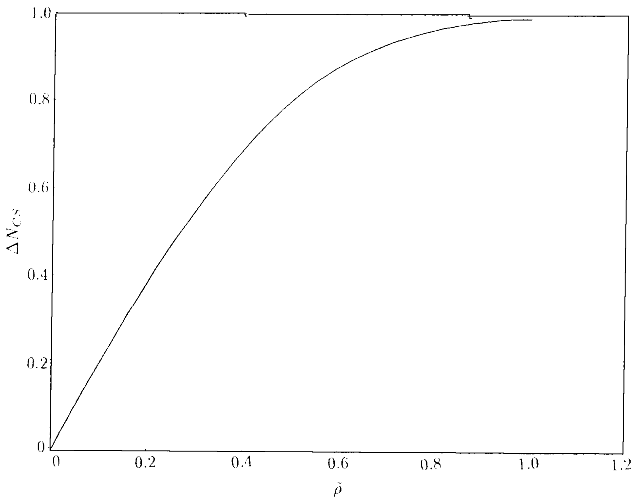

FIG. 5. The difference $\Delta N_{\rm CS}$ of two critical values of Chern-Simons number $N_{\rm CS}$ at which the bound state enters the continuum is shown as a function of the parameter $\tilde{\rho}=m_f\rho$.

on $\tilde{\rho}$ are shown in Fig. 5. We found that the value of $\Delta N_{\rm CS}$ for level crossing tends to zero for small values of $\tilde{\rho}$ and tends to one for large values of $\tilde{\rho}$. Considering $\tilde{\rho}=m_f\rho$, for one size instanton (fixing the size ρ), the curve in Fig. 5 presents the $\Delta N_{\rm CS}$ distribution for the fermion mass m_f; for a certain fermion (fixing the mass m_f), it presents the $\Delta N_{\rm CS}$ distribution for the size of the instanton. The difference $\Delta N_{\rm CS}$ tends to zero when $m_f \to 0$ or $\rho \to 0$. So, in the zero-mass limit or zero-size limit of instanton fermions are only bound at the configuration of $N_{\rm CS}=1/2$ and the level crossing takes place only in one configuration of the instanton with $\Delta N_{\rm CS}=0$.

VI. DISCUSSION

The whole process of the fermion level crossing phenomenon in the background fields of the electroweak instanton have been demonstrated by a numerical method. The detailed behavior of level crossing for various sizes of the instanton and various masses of the fermion are presented. However, our ultimate aim is to relate this process to physical baryon and lepton number violation in the electroweak theory.

In the preceding section, we found a significant fact that the fermion level crossing appears only in some part of instanton configuration space. In Sec. II we found that there are only small size instantons in the Weinberg-Salam model, $\rho \ll 1/v \approx 4$ TeV^{-1}. The masses of quarks and leptons in the standard model are under several GeV except for the undiscovered top quark. Comparing with the scale of the inverse of instanton size $1/\rho$ which may be larger than several TeV, except for the top quark, all of the other quarks and leptons may be considered as massless fermions. According to Fig. 5, the level crossing process or the baryon and lepton number violation process take place just in the much smaller range of configurations around $N_{\rm CS}=1/2$ that is the top of the energy barrier, as shown in Fig. 2. So, the configuration of $x_0=0$ in the instanton gives the essential effect for the baryon and lepton number violation.

The Minkowski space version for the instanton describes the tunneling process in potential barrier which connects one vacuum to another vacuum. If high temperature or high energy is involved, instead of tunneling through the barrier, the field configuration may pass over it. From the discussions above, only the configurations close to $x_0=0$ give the effect to baryon and lepton number violation for the light quarks and leptons. In the collision of high-energy particles, it may only need to create the $x_0=0$ configuration of electroweak instanton to make the baryon and lepton number violation appear.

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