WT identities of gauge theories from broken OSp(3,1 | 2) symmetry in a superspace formulation

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We show that Ward-Takahashi identities of gauge theories can be derived in a compact and elegant form $\partial\overline{W}/\partial\theta=0$ from partial OSp(3,1|2) invariance of the action in a superspace formulation of gauge theories.

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I. INTRODUCTION

The proof of the renormalizability of gauge theories requires the use of Ward-Takahashi (WT) identities, the derivation of which is greatly facilitated by Becchi-Rouet-Stora (BRS) symmetry [1]. As BRS transformations contain an anticommuting parameter, this has naturally led to superfield and/or superspace formulations of gauge theories [2,3] in which the underlying BRS structure becomes evident in a simple way.

With a view to cast the WT identities of gauge theories in a very compact and elegant form, a new superspace formulation of gauge theories was proposed [3]. This formulation uses a six-dimensional superspace and a vector and scalar superfield (to be detailed below). All superfields are *a priori* unrestricted (unlike earlier formulations [2]), and the BRS transformations are effectively generated from within. The main advantage of this formulation arose from the fact that the source terms for the composite BRS variations, so crucial to a simple formulation of WT identities, were also generated from within. Moreover, the sources for such operators and for all fields could be accommodated in source supermultiplets (a vector and a scalar). This formulation is shown to be related simply to the usual gauge theories [see the Eq. (24) below]. In terms of the generating functional $\overline{W}$ of superspace theory [see Eq. (5) below], the WT identities of gauge theories could be cast in an extremely simple and elegant form $\partial\overline{W}/\partial\theta=0$ [4]. The symmetries of the renormalization transformations of sources for composite operators and all fields have been shown recently to arise from the supermultiplet structure of sources [5].

The derivation of the WT identity in Ref. [4] relied heavily upon the relation between $\overline{W}$ and W, the generating functional of the usual gauge theories, and then showed that the equation $\partial\overline{W}/\partial\theta=0$ is implied by the WT identities arising from the BRS invariance of the action in W. In a sense, the above derivation is a bit artificial. The purpose of this paper is to derive the WT identities for $\overline{W}$ directly from its partial OSp(3,1|2) invariance (i.e., without recourse to its simplified form obtained after functional integrations of auxiliary fields).

We shall briefly present the plan of the paper. In Sec. II we summarize the notations and results of superspace formulation of Ref. [3] and introduce the OSp(3,1|2) transformations. In Sec. III we briefly discuss the symmetries of the superspace-generating functional $\overline{W}$. In Sec. IV we shall derive the superspace WT identities using the partial OSp invariance of $\overline{W}$.

II. PRELIMINARY

We shall work in a (somewhat modified) superspace formulation of Yang-Mills theory given in Ref. [3], which we briefly review as we progress. The formulation uses an underlying six-dimensional superspace (x^μ,λ,θ) with λ,θ as real Grassmannian variables. The superspace is endowed with a metric g_{ij} with nonzero components

$$g_{00}=-g_{11}=-g_{22}=-g_{33}=-g_{45}=g_{54}=1\ .$$

The infinitesimal orthosymplectic coordinate transformations are given by six Lorentz transformations (which leave λ and θ invariant), three symplectic transformations characterized by the infinitesimal parameters $\omega_1,\omega_2,\omega_3$,

$$\begin{aligned}&J_1:\lambda'=\lambda-\omega_1\theta\ ,\quad J_2:\lambda'=\lambda+\omega_2\theta\ ,\quad J_3:\lambda'=(1-\omega_3)\lambda\ ,\\ &\theta'=\theta-\omega_1\lambda\ ,\qquad \theta'=\theta-\omega_2\lambda\ ,\qquad \theta'=(1+\omega_3)\theta\ ,\end{aligned} \tag{1}$$

and eight supersymmetry (SUSY) transformations (last of Ref. [2]),

$$\begin{aligned}x'^\mu&=x^\mu+\varepsilon^\mu a\lambda+\delta^\mu b\theta\ ,\\ \lambda'&=\lambda+\delta^\mu x_\mu b\ ,\\ \theta'&=\theta-\varepsilon^\mu x_\mu a\ ,\end{aligned} \tag{2}$$

where ε^μ and δ^μ are four-vectors and a,b are real infinitesimal Grassmannians. The above 17 transformations all leave $g_{ij}\bar{x}^i\bar{x}^j=x^2-2\lambda\theta$ invariant to first order in infinitesimals. Equations (1) and (2) define the transformation rules for an OSp(3,1|2) "vector" (x^μ,λ,θ) under infinitesimal symplectic and SUSY transformations, respectively.

The superspace formulation of Ref. [3] utilizes an anticommuting antighost scalar superfield $\zeta^\alpha(\bar{x})$, a covariant vector superfield $\overline{A}_i^\alpha(\bar{x})\equiv(A_\mu^\alpha(\bar{x}),c_4^\alpha(\bar{x}),c_5^\alpha(\bar{x}))$, and

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a scalar supersource $t^\alpha(\bar{x})$ and an anticommuting vector supersource $\bar{K}^{\alpha i}(\bar{x})=(K^{\alpha\mu}(\bar{x}),K^{\alpha 4}(\bar{x}),K^{\alpha 5}(\bar{x}))$. As an illustration, we consider

$$\begin{aligned}\bar{A}^\alpha_\mu(\bar{x})&=\bar{A}^\alpha_\mu(x,\lambda,\theta)\\&\equiv A^\alpha_\mu(x)+\lambda\tilde{A}^\alpha_{\mu,\lambda}+\theta\tilde{A}^\alpha_{\mu,\theta}+\lambda\theta\tilde{A}^\alpha_{\mu,\lambda\theta}\ ,\end{aligned}$$

which contains four ordinary Lorentz vector fields of which $A^\alpha_\mu(x)$ are the usual Yang-Mills fields, $\tilde{A}^\alpha_{\mu,\lambda}$ and $\tilde{A}^\alpha_{\mu,\theta}$ are anticommuting Lorentz vector fields, and $\tilde{A}^\alpha_{\mu,\lambda\theta}$ is not a dynamical field at all. As for OSp(3,1|2) transformations, $\bar{A}^\alpha_i(\bar{x})$ transformations as a covariant vector, i.e., like $\partial/\partial x^i$. For example, since, under SUSY,

$$\begin{aligned}\frac{\partial}{\partial x'^\mu}&=\frac{\partial}{\partial x^\mu}+\varepsilon_\mu a\frac{\partial}{\partial\theta}-\delta_\mu b\frac{\partial}{\partial\lambda}\ ,\\\frac{\partial}{\partial\theta'}&=\frac{\partial}{\partial\theta}+\delta^\mu b\frac{\partial}{\partial x^\mu}\ ,\end{aligned}\tag{3}$$

etc. So we have, under SUSY,

$$\begin{aligned}A'^\alpha_\mu(\bar{x}')&=A^\alpha_\mu(\bar{x})+\varepsilon_\mu ac^\alpha_5(\bar{x})-\delta_\mu bc^\alpha_4(\bar{x})\ ,\\c'^\alpha_5(\bar{x}')&=c^\alpha_5(\bar{x})+\delta^\mu bA^\alpha_\mu(\bar{x})\ ,\end{aligned}\tag{4}$$

etc. The superspace formulation of Ref. [3] (slightly modified as explained) constructs the generating functional

$$\bar{W}[\bar{K}(\bar{x}),t(\bar{x})]\equiv\int\{d\bar{A}\}\{d\zeta\}\exp(iS[\bar{A},\zeta,\bar{K},t])\ ,\tag{5}$$

with

$$\begin{aligned}\bar{S}&=\int d^4x[-\tfrac{1}{4}g^{ik}g^{jl}\bar{F}_{ij}\bar{F}_{kl}]+\int d^4x\frac{\partial}{\partial\theta}\left\{\bar{K}^i(\bar{x})\bar{A}_i(\bar{x})+\zeta^\alpha(\bar{x})\left[\partial_i\bar{A}_j(\bar{x})g^{ij}+\frac{1}{2\eta_0}\zeta^a{}_{,\theta}+t^\alpha\right]\right\}\\&\equiv\int d^4x\mathcal{L}_0[\bar{A}]+\int d^4x\mathcal{L}_1[\bar{A}]\equiv\bar{S}_0+\bar{S}_1\end{aligned}\tag{6}$$

and

$$\{d\bar{A}\}=\{dA_\mu\}\{dc_5\}\{dc_4\}\ ,\tag{7}$$

with a standard definition that for a superfield $B(x,\lambda,\theta)$ [3],

$$\{dB(\bar{x})\}=\prod_x dB(\bar{x})d\tilde{B}_{,\lambda}(\bar{x})d\tilde{B}_{,\theta}(\bar{x})\ .\tag{8}$$

One of the differences between $\bar{W}$ of Ref. [3] and that of Eq. (5) above is that the latter uses, in the term that generates gauge fixing, ∂^iA_i $(i=0,\dots,5)$ rather than $\partial^\mu A_\mu$. This modifies $\bar{W}$ somewhat, the difference being explained later [see the paragraph above Eq. (24)].

III. SYMMETRIES

The following invariance properties of various quantities in $\bar{W}$ are immediately verified; viz., the following quantities are invariant under infinitesimal OSp(3,1|2) transformations: (i) d^4x, (ii) $g^{ik}g^{jl}\bar{F}^\alpha_{ij}\bar{F}^\alpha_{kl}$, (iii) $\bar{K}^{i\alpha}\bar{A}^\alpha_i$, (iv) $g^{ij}\partial_i\bar{A}^\alpha_j(\bar{x})$, (v) $\{d\zeta\}$, (vi) $\{dA^\alpha_\mu(\bar{x})\}\{dc^\alpha_4(\bar{x})\}\{dc^\alpha_5(\bar{x})\}$, and of course (vii) ζ^α and t^α. (A subtlety involved regarding the inequivalence of $\int d^4x'$ and $\int d^4x$ is clarified later, however.)

The results for (v) and (vi) are obtained by noting that the superdeterminants involved in the field transformations are unity to first order in the infinitesimals since (i) anticommuting blocks of the superdeterminants are of first order in the infinitesimals, while the diagonal elements of commuting blocks are 1 up to first order (see Ref. [1] for superdeterminants).

As a result of this, the functional measures and $\mathcal{L}_0$ are OSp invariants. The $\mathcal{L}_1$ is not, however, invariant under *all* OSp(3,1|2) transformations. Noting the invariants (i), (iii), (iv), and (vii) listed above, $\mathcal{L}_1$ will be invariant under those transformations which preserve $\partial/\partial\theta$. Such transformations are (i) Lorentz transformations $M_{\mu\nu}$, (ii) those generated by (J_1+J_2) (viz., $\lambda'=\lambda$, $\theta'=\theta-\omega\lambda$), and (iii) four SUSY transformations [with $b=0$; see Eq. (2)] $S_{4\mu}$. These 11 transformations are an exact symmetry of $\bar{W}[\bar{K},\bar{t}]$ (modulo the subtlety about $\int d^4x\neq\int d^4x'$) and lead to identities for $\bar{W}$.

IV. DERIVATION OF WT IDENTITIES

In this section we shall derive the main result of this work, viz., the derivation of WT identities of gauge theories in a compact form $\partial\bar{W}/\partial\theta=0$ using the broken OSp(3,1|2) symmetry. This derivation proceeds quite like any derivation of identities for a generating functional from symmetry of the action. We shall perform a coordinate transformation determined by the generator $S_{4\mu}$ and find its effect on the action. We shall see that S is not quite invariant under this transformation (principally because $\int d^4x$ is not, as explained at the end of Sec. III). However, we shall, by making effective use of OSp rotations on fields at $\bar{x}=(x,\lambda,\theta)$ and $\bar{x}_\pi=(-x,\lambda,\theta)$, show that $\bar{W}$ is, however, invariant under this transformation, leading to the identity (19) below. We shall then, using the translational invariance of $\bar{W}$ under space translations, show that (19) leads to the WT identity $\partial\bar{W}/\partial\theta=0$ in the simple form first arrived at in Ref. [4]. We shall then proceed to show that this simple equation, indeed, embodies the WT identities of gauge theories. This we do by recalling the relation between $\bar{W}$ and W of the usual gauge theories [Eq. (24)] below.

Let, for a field (or a source), $A_\pi(x,\lambda,\theta)\equiv A(-x,\lambda,\theta)\equiv A(x_\pi)$. Let f be a functional of the fields (and having no explicit x dependence) [6]. Let

$$f_\pi \equiv f \Big|_{\substack{A \to A_\pi \\ \zeta \to \zeta_\pi}} .$$

We define

$$\langle\langle f \rangle\rangle_{\bar{K},t} \equiv \int \{d\bar{A}\}\{d\zeta\} f \exp(i\bar{S}[\bar{A},\zeta,\bar{K},t]) . \quad (9)$$

Then by changing sources $(\bar{K},t) \to (\bar{K}_\pi, t_\pi)$ everywhere and the change of integration variables $A \to A_\pi$, $\zeta \to \zeta_\pi$ on the right-hand side of Eq. (9), one finds, using the invariance of $\bar{S}$ and of the measure under such an operation,

$$\langle\langle f \rangle\rangle_{\bar{K}_\pi,t_\pi} = \int \{d\bar{A}\}\{d\zeta\} f_\pi \exp(i\bar{S}[\bar{A},\zeta,\bar{K},t]) . \quad (10)$$

In particular, for a function with the property $f = -f_\pi$, one has

$$\langle\langle f \rangle\rangle_{\bar{K},t} + \langle\langle f \rangle\rangle_{\bar{K}_\pi,t_\pi} = 0 . \quad (11)$$

Also, setting $f=1$, in Eqs. (9) and (10), one has

$$\bar{W}[\bar{K}_\pi,t_\pi] = \bar{W}[\bar{K},t] . \quad (12)$$

Now we go back to $\bar{W}$ of Eq. (5) which contains $\bar{S}$. $\bar{S}$ has in it $\int d^4x$, which implies integrating over x, keeping λ and θ fixed. Even when we perform an OSp transformation, this interpretation of $\int d^4x$ cannot be changed, and so to obtain the consequence of broken OSp invariance, we proceed as follows:

$$\bar{S}[\bar{A},\zeta,\bar{K},t] = \tfrac{1}{2}\int d^4x \{\mathcal{L}_0[\bar{A}(\bar{x})] + \mathcal{L}_0[\bar{A}(\bar{x}_\pi)]\} + \bar{S}_1 . \quad (13)$$

We now perform an infinitesimal SUSY transformation $S_{4\mu}$: $\bar{x}' = \Lambda\bar{x}$. Then, using $\bar{A}(\bar{x}) = \Lambda^{-1}\bar{A}'(\bar{x}') = \Lambda^{-1}\bar{A}'(\Lambda\bar{x})$, etc., and the invariance of $\mathcal{L}_0$ and source terms under $S_{4\mu'}$, we obtain

$$\bar{S}[\bar{A},\zeta,\bar{K},t] = \tfrac{1}{2}\int d^4x \left\{\mathcal{L}_0[\bar{A}'(\Lambda\bar{x})] + \mathcal{L}_0[\bar{A}'(\Lambda\bar{x}_\pi)]\right\} + \int d^4x \frac{\partial}{\partial\theta}\left\{\bar{K}'^i(\Lambda\bar{x})\bar{A}'_i(\Lambda\bar{x}) + \cdots\right\} . \quad (14)$$

Now, noting that $\Lambda\bar{x} = (x + \varepsilon a\lambda, \lambda, \theta - \varepsilon\cdot xa)$ and $\Lambda\bar{x}_\pi = (-x + \varepsilon a\lambda, \lambda, \theta + \varepsilon\cdot xa)$, we obtain

$$\tfrac{1}{2}\int d^4x \{\mathcal{L}_0[\bar{A}'(x+\varepsilon a\lambda,\lambda,\theta-\varepsilon\cdot xa)] + \mathcal{L}_0[\bar{A}'(-x+\varepsilon a\lambda,\lambda,\theta+\varepsilon\cdot xa)]\}$$
$$= \tfrac{1}{2}\int d^4x \{\mathcal{L}_0[\bar{A}'(x+\varepsilon a\lambda,\lambda,\theta)] + \mathcal{L}_0[\bar{A}'(-x+\varepsilon a\lambda,\lambda,\theta)]\} + \tfrac{1}{2}\int d^4x \left\{\frac{\partial\mathcal{L}_0}{\partial\theta}[\bar{A}'] - \frac{\partial\mathcal{L}_0}{\partial\theta}[\bar{A}'_\pi]\right\}\varepsilon\cdot xa$$
$$\equiv \tfrac{1}{2}\int d^4x \left\{\mathcal{L}_0[\bar{A}'(\bar{x})] + \mathcal{L}_0[\bar{A}'(\bar{x}_\pi)]\right\} + \int d^4x\, \varepsilon\cdot xa f[\bar{A}] , \quad (15)$$

where translational invariance has been used in the first term and $\bar{A}' \to \bar{A}$ in the second term as it is of $O(\varepsilon)$ already. We also have $f_\pi[\bar{A}] = -f[\bar{A}]$. Also, the source term in (14) is

$$\int d^4x \frac{\partial}{\partial\theta}\left\{\bar{K}'^i(\Lambda\bar{x})\bar{A}'_i(\Lambda\bar{x}) + \cdots\right\} = \int d^4x \frac{\partial}{\partial\theta}\left\{\bar{K}'^i(\bar{x})\bar{A}'_i(\bar{x}) + \cdots\right\} \quad (16)$$

on account of translational invariance first in θ and then in x. Using Eqs. (13)–(16) in (5) and using the invariance of the measure, one obtains, to the first order in ε,

$$\bar{W}[\bar{K}(\bar{x}),t(\bar{x})] = \bar{W}[\bar{K}'(\bar{x}),t'(\bar{x})] + \int d^4x\, \varepsilon\cdot xa \langle\langle f[A]\rangle\rangle_{\bar{K},t} . \quad (17)$$

Now, repeating the procedure with $\bar{K}_\pi$ and t_π as sources, we obtain a very similar relation,

$$\bar{W}[\bar{K}_\pi(\bar{x}),t_\pi(\bar{x})] = \bar{W}[\bar{K}'_\pi(\bar{x}),t'_\pi(\bar{x})] + \int d^4x\, \varepsilon\cdot xa \langle\langle f[A]\rangle\rangle_{\bar{K}_\pi,t_\pi} . \quad (18)$$

Adding Eqs. (17) and (18) and noting Eqs. (11) and (12), we obtain

$$\bar{W}[\bar{K}(\bar{x}),t(\bar{x})] = \bar{W}[\bar{K}'(\bar{x}),t'(\bar{x})] = \bar{W}[\Lambda^{-1}\bar{K}'(\Lambda\bar{x}),t'(\Lambda\bar{x})] . \quad (19)$$

Equation (19) embodies the result of the $S_{4\mu}$ "invariance" of $\bar{S}$. We next proceed to show that Eq. (19) contains the WT identities of gauge theory by using the invariance of $\bar{W}$ under x_μ translations. In Eq. (19) above, we substitute

$$\bar{K}'(\bar{x}) \equiv \Lambda\bar{K}(\Lambda^{-1}\bar{x}) = \bar{K}(\bar{x}) - \varepsilon^\mu a\lambda\partial_\mu\bar{K}(\bar{x}) + \varepsilon\cdot xa\bar{K}_{,\theta}(\bar{x}) + (\Lambda-1)\bar{K}(\bar{x}) + O(\varepsilon^2) \quad (20)$$

and a similar relation for t'. We then compare both sides of Eq. (19), to obtain

$$\int \varepsilon\cdot xa \left[K^{\bar{i}}_{,\theta}(\bar{x})\frac{\delta\bar{W}}{\delta K^i(x)} + t_{,\theta}(\bar{x})\frac{\delta\bar{W}}{\delta t(x)}\right] d^4x = \delta_1\bar{W}[\bar{K}(\bar{x}),t(\bar{x})] + O(\varepsilon^2) . \quad (21)$$

Here $\delta_1\bar{W}$ is the change in $\bar{W}$ on account of the change

$(\Lambda-1)\bar{K}$ in $\bar{K}$ and in (21) we have dropped a piece proportional to $\varepsilon^{\mu}a\lambda$ on account of invariance of $\overline{W}$ under x_{μ} translations. Now the functional derivatives of $\delta_1\overline{W}$, which involves functional derivatives of $\overline{W}$, have the property of translational invariance in space-time coordinates. The left-hand side of (21), however, involves an explicit reference to x_{μ}. It is easy to show that (say, by taking an arbitrary functional derivative) the only way the left-hand side of (21) can lead to translationally invariant Green's functions is to have the integral of the square bracket in (21) be zero (this can happen when, say, it is a total derivative in x_{μ}), i.e.,

$$\int\left[K^{i}{}_{,\theta}(\bar{x})\frac{\delta\overline{W}}{\delta K^{i}(x)}+t_{,\theta}(\bar{x})\frac{\delta\overline{W}}{\delta t(x)}\right]d^4x=0\ . \qquad (22)$$

Noting that the dependence of $\overline{W}$ on θ is through its dependence on $\bar{K}$ and t only [3], this leads to

$$\frac{\partial}{\partial\theta}\overline{W}[\bar{K}(\bar{x}),t(\bar{x})]=0\ . \qquad (23)$$

This equation, obtained by using the broken OSp(3,1|2) invariance of $\overline{W}$, will now be briefly shown to contain the WT identities of gauge theories along the lines of Ref. [4]. While this has already been shown in Ref. [4], certain steps are reproduced because of the difference between $\overline{W}$ of Eq. (5) and that of Ref. [3] and also for completeness. The gauge-fixing term in S_1 of Eq. (6) contains $g^{ij}\partial_i A_j(\bar{x})$, as opposed to $g^{\mu\nu}\partial_\mu A_\nu=\partial\cdot A$ as in Ref. [4], and this is a full OSp invariant. A straightforward evaluation along the lines of Ref. [3] shows that (i) $\overline{W}$ is modified, but that (ii) it contains all information on ordinary gauge theories and (iii) this is still correctly given by the slightly modified relation, with $c^{\alpha}_{\xi,\lambda\theta}$ set to zero,

$$\int[dK^4][dK^4{}_{,\theta}]\overline{W}[\bar{K},t]=W[K^{\alpha\mu}{}_{,\theta}(\bar{x});K^{\alpha5}{}_{,\theta}(\bar{x}),-t^{\alpha}{}_{,\theta}(\bar{x});K^{\alpha\mu}(\bar{x});K^{\alpha5}(\bar{x}),t^{\alpha}(\bar{x});\beta_0]\ , \qquad (24)$$

where $\beta_0=\eta_0/(1-\eta_0)$ and W is the standard gauge theory generating functional [1] with sources for BRS variations of $A,c,\bar{c}$ introduced, viz.,

$$W[j^{\alpha\mu},\bar{\xi}^{\alpha},\xi^{\alpha},\kappa^{\alpha\mu},l^{\alpha},t^{\alpha},\beta_0]=\int DA\,Dc\,D\bar{c}\,\exp(i\Sigma) \qquad (25)$$

and

$$\Sigma=S_0[A]+\int d^4x\left\{-\frac{\beta_0}{2}[\partial\cdot A^{\alpha}+t^{\alpha}]^2-\partial^{\mu}\bar{c}^{\alpha}D^{\alpha\beta}_{\mu}c^{\beta}+\kappa^{\alpha\mu}D^{\alpha\beta}_{\mu}c^{\beta}-\tfrac{1}{2}g_0 l^{\alpha}f^{\alpha\beta\gamma}c^{\beta}c^{\gamma}+j^{\alpha\mu}A^{\alpha}_{\mu}+\bar{\xi}^{\alpha}c^{\alpha}+\bar{c}^{\alpha}\xi^{\alpha}\right\}\ . \qquad (26)$$

Integrating Eq. (23) over $K^{\alpha4}(x)$ and $K^{\alpha4}{}_{,\theta}(x)$ and noting that $\partial/\partial\theta$ commutes with these operations, we obtain

$$\frac{\partial}{\partial\theta}W[K^{\alpha\mu}{}_{,\theta}(\bar{x}),K^{\alpha5}{}_{,\theta}(\bar{x}),-t^{\alpha}{}_{,\theta}(\bar{x}),K^{\alpha\mu}(\bar{x}),K^{\alpha5}(\bar{x}),t^{\alpha}(\bar{x})]=0\ , \qquad (27)$$

i.e.,

$$0=K^{\alpha\mu}{}_{,\theta}\frac{\delta W}{\delta K^{\alpha\mu}}+K^{\alpha5}{}_{,\theta}\frac{\delta W}{\delta K^{\alpha\mu}}+t^{\alpha}{}_{,\theta}\frac{\delta W}{\delta t^{\alpha}}\ . \qquad (28)$$

Using the definition of W of Eq. (25), this reduces to

$$0=\int DA\,Dc\,D\bar{c}\,\exp(i\Sigma)\left\{K^{\alpha\mu}{}_{,\theta}D^{\alpha\beta}_{\mu}c^{\beta}+K^{\alpha5}{}_{,\theta}(-\tfrac{1}{2}g_0 f^{\alpha\beta\gamma}c^{\beta}c^{\gamma})-\beta_0[\partial\cdot A+t^{\alpha}]t^{\alpha}{}_{,\theta}\right\}\ . \qquad (29)$$

This is precisely the statement of invariance of W under BRS transformations:

$$\delta A^{\alpha}{}_{\mu}=D^{\alpha\beta}_{\mu}c^{\beta}\delta\Lambda\ ,\quad \delta c^{\alpha}=-\tfrac{1}{2}g_0 f^{\alpha\beta\gamma}c^{\beta}c^{\gamma}\delta\Lambda\ ,$$
$$\delta\bar{c}^{\alpha}=-\beta_0[\partial\cdot A^{\alpha}+t^{\alpha}]\delta\Lambda\ . \qquad (30)$$

Thus the BRS invariance and/or gauge invariance of gauge theories which leads to WT identities, all summarized in a single equation $\partial\overline{W}/\partial\theta=0$, has been shown to arise from partial OSp invariance, a coordinate invariance in superspace.

We finally comment on the commutation relation that has bearing upon the derivation. Writing $P_\mu=i\partial_\mu$, $S_{4\nu}=i[a\lambda\partial_\nu-x_\nu a\partial/\partial\theta]$, one easily sees that $[P_\mu,S_{4\nu}]=g_{\mu\nu}a\partial/\partial\theta=ig_{\mu\nu}aP_\theta$, $P_\theta=i\partial/\partial\theta$ being the generator of θ translations. $\overline{W}$ has invariance under P_μ and a peculiar "invariance" under $S_{4\nu}$ [modulo the subtlety associated with $\int_{\lambda,\theta}d^4x\neq\int_{\lambda',\theta'}d^4x'$]. Combining these two, we have been able to show $P_\theta\overline{W}=0$. One could calculate $\partial\overline{W}/\partial\theta$ directly and evaluate it using the equations of motion of auxiliary fields, but this turns out to be a very cumbersome way of proving Eq. (23).

V. CONCLUSIONS

In conclusion, we have replaced the BRS invariance of gauge theories by a true coordinate invariance in six-dimensional superspace. As a result, the complicated WT identities found a simple expression in this formulation. More importantly, one would expect that the solution of WT identities, especially for the insertion of a gauge-invariant operator [7], could be enormously simplified in this approach where a field transformation invariance is replaced by a very simple coordinate rotation invariance. We hope to communicate this elsewhere.

[1] See, for example, B. W. Lee, in *Methods in Field Theory,* Proceedings of Les Houches Summer School of Theoretical Physics, Les Houches, France, 1975, edited by R. Balian and J. Zinn-Justin (North-Holland, Amsterdam, 1976).

[2] For example, see, K Fujikawa, Prog. Theor. Phys. **59**, 2045 (1978); S. Ferrara, O. Piguet, and M. Schweda, Nucl. Phys. **B119**, 493 (1977); L. Bonora and M. Tonin, Phys. Lett. **98B**, 48 (1981); A. C. Hirshfeld and H. Leschke, *ibid.* **101B**, 48 (1981); R. Delbourgo and P. D. Jarvis, J. Phys. A **15**, 611 (1982); L. Baulieu and J. Thierry-Meig, Nucl. Phys. **B197**, 477 (1982); B. McClain, A. Niemi, C. Taylor, and L. C. R. Wijewardhana, *ibid.* **B217**, 430 (1983).

[3] S. D. Joglekar (unpublished); Phys. Rev. D **43**, 1307 (1991); **48**, 1878(E) (1993).

[4] S. D. Joglekar, Phys. Rev. D **44**, 3879 (1991); **48**, 1879(E) (1993), which includes a slightly more general statement of the WT identities of Eq. (23).

[5] S. D. Joglekar and B. P. Mandal, this issue, Phys. Rev. D **49**, 5617 (1994).

[6] In the transformation $f \rightarrow f_\pi$ below, it is understood that not only $A \rightarrow A_\pi$, but also $\partial \cdot A \rightarrow \partial^\pi \cdot A_\pi$, etc.

[7] S. D. Joglekar and B. W. Lee, Ann. Phys. (N.Y.) **97**, 160 (1976).