

Primordial nucleosynthesis with a decaying τ neutrino

Scott Dodelson,¹ Geza Gyuk,² and Michael S. Turner¹⁻³

¹*NASA/Fermilab Astrophysics Center, Fermi National Accelerator Laboratory, Batavia, Illinois 60510-0500*

²*Department of Physics, The University of Chicago, Chicago, Illinois 60637-1433*

³*Department of Astronomy and Astrophysics, Enrico Fermi Institute, The University of Chicago, Chicago, Illinois 60637-1433*

(Received 30 December 1993)

A comprehensive study of the effect of an unstable τ neutrino on primordial nucleosynthesis is presented. The standard code for nucleosynthesis is modified to allow for a massive decaying τ neutrino whose daughter products include neutrinos, photons, e^\pm pairs, and/or noninteracting (sterile) daughter products. τ -neutrino decays influence primordial nucleosynthesis in three distinct ways: (i) the energy density of the decaying τ neutrino and its daughter products affect the expansion rate tending to increase ${}^4\text{He}$, D, and ${}^3\text{He}$ production; (ii) electromagnetic (EM) decay products heat the EM plasma and dilute the baryon-to-photon ratio tending to decrease ${}^4\text{He}$ production and increase D and ${}^3\text{He}$ production; and (iii) electron neutrinos and antineutrinos produced by τ -neutrino decays increase the weak rates that govern the neutrino-to-proton ratio, leading to decreased ${}^4\text{He}$ production for short lifetimes ($\lesssim 30$ sec) and masses less than about 10 MeV and increased ${}^4\text{He}$ production for long lifetimes or large masses. The precise effect of a decaying τ neutrino on the yields of primordial nucleosynthesis and the mass-lifetime limits that follow depend crucially upon decay mode. We identify four generic decay modes that serve to bracket the wider range of possibilities: τ neutrino decays to (1) sterile daughter products (e.g., $\nu_\tau \rightarrow \nu_\mu + \phi$; ϕ is a very weakly interacting scalar particle); (2) sterile daughter product(s) + daughter product(s) that interacts electromagnetically (e.g., $\nu_\tau \rightarrow \nu_\mu + \gamma$); (3) electron neutrino + sterile daughter product(s) (e.g., $\nu_\tau \rightarrow \nu_e + \phi$); and (4) electron neutrino + daughter product(s) that interact electromagnetically ($\nu_\tau \rightarrow \nu_e + e^\pm$). Mass-lifetime limits are derived for the four generic decay modes assuming that the abundance of the massive τ neutrino is determined by its electroweak annihilations. In general, nucleosynthesis excludes a τ neutrino of mass 0.4 MeV–30 MeV for lifetimes greater than about 300 sec. These nucleosynthesis bounds are timely since the current laboratory upper bounds to the τ -neutrino mass are around 30 MeV, and together the two bounds very nearly exclude a long-lived τ neutrino more massive than about 0.4 MeV. Further, our nucleosynthesis bounds together with other astrophysical and laboratory bounds exclude a τ neutrino of mass 0.4 MeV–30 MeV of any lifetime that decays with EM daughter product(s). We use our results to constrain the mass times relic abundance of a hypothetical, unstable species with similar decay modes. Finally, we note that a τ neutrino of mass 1 MeV to 10 MeV and lifetime 0.1 sec–10 sec whose decay products include an electron neutrino can reduce the ${}^4\text{He}$ yield to less than that for two massless neutrino species. This fact could be relevant if the primordial mass fraction of ${}^4\text{He}$ is found to be less than about 0.23 and can also lead to a modification of the nucleosynthesis bound to the number of light ($\ll 1$ MeV) neutrino (and other) particle species.

PACS number(s): 98.80.Ft, 13.35.-r, 14.60.Lm, 98.80.Cq

I. INTRODUCTION

Primordial nucleosynthesis is one of the cornerstones of the hot big-bang cosmology. The agreement between the predictions for the abundances of D, ${}^3\text{He}$, ${}^4\text{He}$, and ${}^7\text{Li}$ and their inferred primordial abundances provides the big-bang cosmology's earliest, and perhaps most, stringent test. Further, big-bang nucleosynthesis has been used to provide the best determination of the baryon density [1,2] and to provide crucial tests of particle-physics theories, e.g., the stringent bound to the number of light neutrino species [3,4].

Over the years various aspects of the effect of a decaying τ neutrino on primordial nucleosynthesis have been considered [5–12]. Each previous study focused on a specific decay mode and incorporated different microphysics. To be sure, no one study was complete or exhaustive. Our purpose here is to consider all the effects of a

decaying τ neutrino on nucleosynthesis in a comprehensive and coherent manner. In particular, for the first time interactions of decay-produced electron neutrinos and antineutrinos, which can be important for lifetimes shorter than 100 sec or so, are taken into account.

The nucleosynthesis limits to the mass of an unstable τ neutrino are currently of great interest as the best laboratory upper mass limits [13], 31 MeV by the ARGUS Collaboration and 32.6 MeV by the CLEO Collaboration,¹ are tantalizingly close to the mass range excluded by nucleosynthesis, approximately 0.4–30 MeV for lifetimes

¹Both are 95% C.L. mass limits based upon end-point analyses of τ decays to final states containing five pions. The CLEO data set contains 113 such decays and the ARGUS data set contains 20 such decays [13].

greater than about 300 sec. If the upper range of the cosmologically excluded band can be convincingly shown to be greater than the upper bound to the mass from laboratory experiments, the two bounds together imply that a long-lived τ neutrino must be less massive than about 0.4 MeV. This was a major motivation for our study.

The effects of a massive, decaying τ neutrino on primordial nucleosynthesis fall into three broad categories: (i) the energy density of the τ neutrino and its daughter product(s) increase the expansion rate, tending to increase ${}^4\text{He}$, D, and ${}^3\text{He}$ production; (ii) the electromagnetic (EM) plasma is heated by the daughter product(s) that interact electromagnetically (photons and e^\pm pairs), diluting the baryon-to-photon ratio and decreasing ${}^4\text{He}$ production and increasing D and ${}^3\text{He}$ production; and (iii) electron neutrino (and antineutrino) daughters increase the weak interaction rates that govern the neutron-to-proton ratio, leading to decreased ${}^4\text{He}$ production for short lifetimes ($\lesssim 30$ sec) and masses less than about 10 MeV and increased ${}^4\text{He}$ production for long lifetimes. Decays that take place long after nucleosynthesis ($\tau_\nu \sim 10^5 - 10^6$ sec) can lead to the destruction of the light elements through fission reactions and additional constraints [14], neither of which are considered here.

In terms of the effects on primordial nucleosynthesis there are, broadly speaking, four generic decay modes.

(1) τ neutrino decays to daughter products that are all sterile, e.g., $\nu_\tau \rightarrow \nu_\mu + \phi$ (ϕ is a very weakly interacting boson). Here, only effect (i) comes into play. Aspects of this case were treated in Refs. [5,7,10–12]; the very recent work in Ref. [12] is the most complete study of this mode.

(2) τ neutrino decays to a sterile daughter product(s) plus a daughter product(s) that interacts electromagnetically, e.g., $\nu_\tau \rightarrow \nu_\mu + \gamma$. Here, effects (i) and (ii) come into play. This case was treated in Ref. [6], though not specifically for a decaying τ neutrino.

(3) τ neutrino decays into an electron neutrino and sterile daughter product(s), e.g., $\nu_\tau \rightarrow \nu_e + \phi$. Here, effects (i) and (iii) come into play. This case was treated in Ref. [8]; however, the interactions of electron neutrinos and antineutrinos with the ambient thermal plasma were not taken into account. They can be important: The interaction rate of a high-energy electron neutrino produced by the decay of a massive τ neutrino relative to the expansion rate $\Gamma/H \sim (m_\nu/\text{MeV})(\text{sec}/t)$.

(4) τ neutrino decays into an electron neutrino and daughter product(s) that interact electromagnetically, e.g., $\nu_\tau \rightarrow \nu_e + e^\pm$. Here, all three effects come into play. Aspects of this case were treated in Ref. [9], though interactions of electron neutrinos and antineutrinos with the ambient thermal plasma were neglected and the ν_e spectrum was taken to be a δ function.

As we shall emphasize more than once, the effect of a τ neutrino of a given mass and lifetime—and therefore limits to its mass/lifetime—depends very much upon decay mode.

While these four generic decay modes serve to bracket the possibilities, the situation is actually somewhat more complicated. Muon neutrinos are not completely sterile,

as they are strongly coupled to the electromagnetic plasma down to temperatures of order a few MeV (times of order a fraction of a second), and thus can transfer energy to the electromagnetic plasma and electron neutrinos (we briefly address this at the end of Sec. III A). However, for lifetimes longer than a few seconds, their interactions with the electromagnetic plasma are not very significant (see Ref. [15]), and so to a reasonable approximation muon-neutrino daughter products can be considered sterile. Precisely how much electromagnetic entropy is produced and the effect of high-energy neutrinos on the proton-neutron interconversion rates depend upon the energy distribution of the daughter products and their interactions with the ambient plasma (photons, e^\pm pairs, and neutrinos), which in turn depends upon the number of daughter products and the decay matrix element.

Without going to extremes, one can easily identify more than ten possible decay modes. However, we believe the four generic decay modes serve well to illustrate how the nucleosynthesis mass-lifetime limits depend upon the decay mode and provide reasonable estimates thereof. In that regard, input assumptions, e.g., the acceptable range for the primordial abundances and the relic neutrino abundance² probably lead to comparable, if not greater uncertainties in the precise limits.

Finally, a brief summary of our treatment of the microphysics: (1) The relic abundance of the τ neutrino is determined by standard electroweak annihilations and is assumed to be frozen out at its asymptotic value during the epoch of nucleosynthesis, thereafter decreasing due to decays only. Because we assume that the relic abundance of the τ neutrino has frozen out, we cannot accurately treat the case of short lifetimes, $\tau_\nu \lesssim (m_\nu/\text{MeV})^{-2}$ sec, where inverse decays can significantly affect the τ neutrino abundance and that of its daughter products [16].³ (2) Sterile daughter products, other than neutrinos, are assumed to have a negligible predecay abundance (if this is not true, the nucleosynthesis limits become even more stringent). (3) The electromagnetic energy produced by tau-neutrino decays is assumed to be quickly thermalized and to increase the entropy in the electromagnetic plasma according to the first law of thermodynamics. (4) The perturbations to the phase-space distributions of electron and muon neutrinos due to τ neutrino decays and partial coupling to the electromagnetic plasma are computed. (5) The change in the weak rates that interconvert neutrons and protons due to the distorted electron-neutrino distribution are calculated. (6) The total energy of the Universe includes that of photons, e^\pm pairs, neutrinos, and sterile daughter product(s).

²The variation between different calculations of the τ neutrino abundance are of the order of 10–20%; they arise to different treatments of thermal averaging, particle statistics, and so on. Since we use the asymptotic value of the τ neutrino abundance, our abundances are in general smaller, making our limits more conservative.

³For generic decay mode (1) the effect of inverse decays for short lifetimes was considered in Ref. [12]; it leads to additional mass constraints for short lifetimes.

The paper is organized as follows: In the next section we discuss the modifications that we have made to the standard nucleosynthesis code. In Sec. III we present our results, discussing how a decaying τ neutrino affects the yields of nucleosynthesis and deriving the mass/lifetime limits for the four generic decay modes. In Sec. IV we discuss other astrophysical and laboratory limits to the mass/lifetime of the τ neutrino, and finish in Sec. V with a brief summary and concluding remarks.

II. MODIFICATIONS TO THE STANDARD CODE

In the standard treatment of nucleosynthesis [17–19], it is assumed that there are three massless neutrino species that completely decouple from the electromagnetic plasma at a temperature well above that of the electron mass ($T \sim 10 \text{ MeV} \gg m_e$). Thus, the neutrino species do not share in the “heating” of the photons when the e^\pm pairs disappear.

In order to treat the most general case of a decaying τ neutrino, we have made a number of modifications to the standard code. These modifications are of four kinds: (1) change the total-energy density to account for the massive τ neutrino and its daughter products, (2) change the first law of thermodynamics for the electromagnetic plasma to account for the injection of energy by τ decays and interactions with the other two neutrino seas, (3) follow the Boltzmann equations for the phase-space distributions for electron and muon neutrinos, accounting for their interactions with one another and the electromagnetic plasma, and (4) modify the weak interaction rates that interconvert neutrons and protons to take account of the perturbations to the electron-neutrino spectrum.

These modifications required tracking five quantities as a function of $T \equiv R^{-1}$, the neutrino temperature in the fully decoupled limit ($R =$ the cosmic-scale factor). They are ρ_{ν_τ} , ρ_ϕ (where ϕ is any sterile, relativistic decay product), T_γ , and Δ_e and Δ_μ , the perturbations to the electron-neutrino and μ neutrino phase-space distributions.

Our calculations were done with two separate codes. The first code tracks ρ_{ν_τ} , ρ_ϕ , T_γ , Δ_e , and Δ_μ as a function of T , for simplicity, using Boltzmann statistics.⁴ These five quantities were then converted to functions of the photon temperature using the $T(T_\gamma)$ relationship calculated, and their values were then passed to the second code, a modified version of the standard nucleosynthesis code [17]. We now discuss in more detail the four modifications.

A. Energy density

There are four contributions to the energy density: massive τ neutrinos, sterile decay products, two massless

neutrino species, and the EM plasma. Let us consider each in turn.

As mentioned earlier, we fix the relic abundance of τ neutrinos assuming that freeze out occurs before nucleosynthesis commences ($t \ll 1 \text{ sec}$). We follow Ref. [10] in writing

$$\rho_{\nu_\tau} = r \left[\frac{\sqrt{(3.151T)^2 + m_\nu^2}}{3.151T} \right] \rho_\nu(m_\nu=0) \exp(-t/\tau_\nu), \quad (1)$$

where r is the ratio of the number density of massive neutrinos to a massless neutrino species, the $(3.151T)^2$ term takes account of the kinetic energy of the neutrino, and the exponential factor takes account of decays. The relic abundance is taken from Ref. [10]; for a Dirac neutrino it is assumed that all four degrees of freedom are populated for masses greater than 0.3 MeV (see Ref. [10] for further discussion).

Note that for temperatures much less than the mass of the τ neutrino, $\rho_{\nu_\tau}/\rho_\nu(m_\nu=0) = rm_\nu e^{-t/\tau_\nu}/3.151T$, which increases as the scale factor until the τ neutrinos decay; further, rm_ν determines the energy density contributed by massive τ neutrinos and, hence, essentially all of their effects on nucleosynthesis. The relic neutrino abundance times mass (rm_ν) is shown in Fig. 1.

The energy density of the sterile decay products is slightly more complicated. Since the ϕ 's are massless, their energy density is governed by

$$\frac{d\rho_\phi}{dT} = \frac{4\rho_\phi}{T} - \frac{f_\phi}{T} \frac{\rho_{\nu_\tau}}{H\tau_\nu}, \quad (2)$$

where the first term accounts for the effect of the expansion of the Universe and the second accounts for the energy dumped into the sterile sector by τ neutrino decays. The quantity f_ϕ is the fraction of the τ neutrino decay energy that goes into sterile daughters: for $\nu_\tau \rightarrow$ all-sterile daughter products, $f_\phi = 1$; for $\nu_\tau \rightarrow \phi + \nu_e$ or $\nu_e + \text{EM}$, $f_\phi = 0.5$; and for all other modes $f_\phi = 0$. Equation (2) was integrated numerically, and ρ_ϕ was passed to the nu-

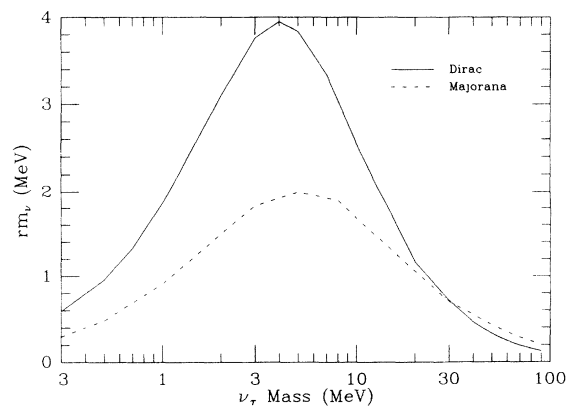


FIG. 1. The relic neutrino abundance used in our calculations as a function of neutrino mass: Dirac neutrino, including both helicity states (solid curve), and Majorana neutrino (broken curve); results from Ref. [10].

⁴The correct statistics for all species are of course used in the nucleosynthesis code; the errors made by using Boltzmann statistics in the first code should be at most of order 10% in the perturbed quantities.

leosynthesis code by means of a look-up table.

The neutrino seas were the most complicated to treat. The contribution of the neutrino seas was divided into two parts, the standard, unperturbed thermal contribution and the perturbation due to the slight coupling of neutrinos to the EM plasma and τ neutrino decays,

$$\rho_\nu = \rho_{\nu 0} + \delta\rho_\nu. \quad (3)$$

The thermal contribution is simply $6T^4/\pi^2$ per massless neutrino species (two in our case). The second term is given as an integral over the perturbation to the neutrino phase-space distribution,

$$\delta\rho_\nu = \sum_{i=e,\mu} \frac{2}{(2\pi)^3} \int p d^3p \Delta_i(p, t), \quad (4)$$

where the factor of 2 accounts for neutrinos and antineutrinos.

Finally, there is the energy density of the EM plasma. Since the electromagnetic plasma is in thermal equilibrium it only depends upon T_γ :

$$\frac{dT_\gamma}{dt} = \frac{-3H(\rho_{\text{TOT}} + p_{\text{TOT}} - 4\rho_{\nu 0}/3) - d(\delta\rho_\nu + \rho_\phi + \rho_{\nu_\tau})/dt}{d\rho_{\text{EM}}/dT_\gamma}. \quad (8)$$

The quantity $d\rho_{\text{EM}}/dT_\gamma$ is easily calculated, and the time derivatives of the densities can either be solved for analytically, or taken from the previous time step.

C. Neutrino phase-space distribution functions

The Boltzmann equations governing the neutrino phase-space distribution functions in the standard case were derived and solved in Ref. [15]. We briefly summarize that treatment here, focusing on the modifications required to include massive τ neutrino decays.

We start with the Boltzmann equation for the phase-space distribution of neutrino species a in the absence of decays:

$$\frac{\partial f_a}{\partial t} - \frac{H|p|^2}{E_a} \frac{\partial f_a}{\partial E} = -\frac{1}{2E_a} \sum_{\text{processes}} \int d\Pi_1 d\Pi_2 d\Pi_3 (2\pi)^4 \delta^4(p_a + p_1 - p_2 - p_3) |\mathcal{M}_{a+1 \leftrightarrow 2+3}|^2 [f_a f_1 - f_2 f_3], \quad (9)$$

where the processes summed over include all the standard electroweak $2 \leftrightarrow 2$ interactions of neutrinos with themselves and the electromagnetic plasma, and Boltzmann statistics have been used throughout.

We write the distribution functions for the electron and muon neutrinos as an unperturbed part plus a small perturbation,

$$f_i(p, t) = \exp(-p/T) + \Delta_i(p, t), \quad (10)$$

where we have assumed that both species are effectively massless. During nucleosynthesis the photon temperature begins to deviate from the neutrino temperature T , and we define

$$\delta(t) = T_\gamma/T - 1.$$

Equation (9) is expanded to lowest order in Δ_i and $\delta(t)$, leading to master equations of the form

$$\rho_{\text{EM}} = \frac{6T_\gamma^4}{\pi^2} + \frac{2m_e^3 T_\gamma}{\pi^2} \left[K_1(m_e/T_\gamma) + \frac{3K_2(m_e/T_\gamma)}{m_e/T_\gamma} \right], \quad (5)$$

where K_1 and K_2 are modified Bessel functions. We numerically compute T_γ as a function of T by using the first law of thermodynamics.

B. First law of thermodynamics

Energy conservation in the expanding Universe is governed by the first law of thermodynamics,

$$d[\rho_{\text{TOT}} R^3] = -p_{\text{TOT}} dR^3, \quad (6)$$

where in our case $\rho_{\text{TOT}} = \rho_{\text{EM}} + \rho_{\nu 0} + \delta\rho_\nu + \rho_\phi + \rho_{\nu_\tau}$, $p_{\text{TOT}} = p_{\text{EM}} + p_{\nu 0} + \delta p_\nu + p_\phi + p_{\nu_\tau}$, $\delta p_\nu = \delta\rho_\nu/3$, $p_\phi = \rho_\phi/3$, and

$$p_{\text{EM}} = \frac{2T_\gamma^4}{\pi^2} + \frac{2m_e^2 T_\gamma^2}{\pi^2} K_2(m_e/T_\gamma). \quad (7)$$

Equation (6) can be rewritten in a more useful form,

$$\begin{aligned} \frac{p}{T} \dot{\Delta}_i(p, t) = & 4G_F^2 T^3 [-A_i(p, t)\Delta_i(p, t) + B_i(p, t)\delta(t) \\ & + C_i(p, t) + C'_i(p, t)], \end{aligned} \quad (11)$$

where $i=e, \mu$ and the expressions for A_i , B_i , C_i , and C'_i are given in Ref. [15] [in Eq. (2.11d) for C_μ the coefficient $(c+8)$ should be $(c+7)$].

In context of τ neutrino decays we treat decay-produced muon neutrinos as a sterile species, and thus we are only interested in modifying the master equation for electron neutrinos to allow for decays. In the case of two-body decays (e.g., $\nu_\tau \rightarrow \nu_e + \phi$ or $\nu_\tau \rightarrow \nu_e + \text{EM}$) the additional term that arises on the right-hand side of Eq. (11) is

$$\frac{p}{T} \dot{\Delta}_e(p, T) = \dots + \frac{n_{\nu_\tau}}{\tau_\nu} \frac{2\pi^2}{pT} \delta(p - m_\nu/2), \quad (12)$$

where n_{ν_τ} is the number density of massive τ neutrinos.⁵

The decay mode $\nu_\tau \rightarrow \nu_e + e^\pm$ has a three-body final state, so that the energy distribution of electron neutrinos is no longer a δ function. In this case, the source term is

$$\frac{p}{T} \dot{\Delta}_e(p, T) = \dots + \frac{32\pi^2 n_{\nu_\tau} p (3 - 4p/m_{\nu_\tau})}{\tau_{\nu_\tau} m_{\nu_\tau}^3 T} \theta(p - m_{\nu_\tau}/2), \quad (13)$$

where for simplicity we have assumed that all particles except the massive τ neutrino are ultrarelativistic.

D. Weak-interaction rates

Given Δ_e , it is simple to calculate the perturbations to the weak interaction rates that convert protons to neutrons and vice versa (see Ref. [15] for details). The perturbations to the weak rates are obtained by substituting $\exp(-p/T) + \Delta_e(p, t)$ for the electron phase-space distribution function in the usual expressions for the rates [18,19] and then expanding to lowest order. The perturbations to the rates for proton-to-neutron conversion and neutron-to-neutron conversion (per nucleon) are, respectively,

$$\delta\lambda_{pn} = \frac{1}{\lambda_0 \tau_n} \int_{m_e}^{\infty} E dE (E^2 - m_e^2)^{1/2} (E + Q)^2 \Delta_e(E + Q), \quad (14)$$

$$\delta\lambda_{np} = \frac{1}{\lambda_0 \tau_n} \int_{m_e}^{\infty} E dE (E^2 - m_e^2)^{1/2} (E - Q)^2 \Delta_e(E - Q), \quad (15)$$

where Boltzmann statistics have been used for all species, τ_n is the neutron mean lifetime, $Q = 1.293$ MeV is the neutron-proton mass difference, and

$$\lambda_0 \equiv \int_{m_e}^Q E dE (E^2 - m_e^2)^{1/2} (E - Q)^2.$$

The perturbations to the weak rates are computed in the first code and passed to the nucleosynthesis code by means of a look-up table. The unperturbed part of the weak rates are computed by numerical integration in the nucleosynthesis code; for all calculations we took the neutron mean lifetime to be 889 sec.

III. RESULTS

In this section we present our results for the four generic decay modes. Mode by mode we discuss how the light-element abundances depend upon the mass and lifetime of the τ neutrino and derive mass/lifetime limits. We exclude a mass and lifetime if, for no value of the baryon-to-photon ratio, the light-element abundances can satisfy

$$Y_p \leq 0.24, \quad (16)$$

⁵For m_{ν_τ} we actually use our expression for the total τ neutrino energy $E_{\nu_\tau} = \sqrt{m_{\nu_\tau}^2 + (3.151T)^2}$. Except for very short lifetimes and small masses, $E_{\nu_\tau} \approx m_{\nu_\tau}$.

$$D/H \geq 10^{-5}, \quad (17)$$

$$(D + {}^3\text{He})/H \leq 1.1 \times 10^{-4}, \quad (18)$$

$$\text{Li}/H \leq 1.4 \times 10^{-10}. \quad (19)$$

For further discussion of this choice of constraints to the light-element abundances we refer the reader to Ref. [2].

The ${}^4\text{He}$ and $D + {}^3\text{He}$ abundances play the most important role in determining the excluded regions. The mass/lifetime limits that follow necessarily depend upon the range of acceptable primordial abundances that one adopts, a fact that should be kept in mind when comparing the work of different authors and assessing confidence levels. Further, the relic abundances used by different authors differ by 10–20%. Lastly, the precise limit for a specific decay mode will of course differ slightly from that derived for its “generic class.”

In illustrating how the effects of a decaying τ neutrino depend upon lifetime and in comparing different decay modes, we use as a standard case an initial (i.e., before decay and e^\pm annihilations) baryon-to-photon ratio $\eta_i = 8.25 \times 10^{-10}$. In the absence of entropy production (no decaying τ neutrino or decay modes 1 and 3, which produce no EM entropy) the final baryon-to-photon ratio $\eta_0 = 4\eta_i/11 = 3 \times 10^{-10}$, where 4/11 is the usual factor that arises due to the entropy transfer from e^\pm pairs to photons [18,19]. In the case of decay modes 2 and 4 there can be significant EM entropy production, and the final baryon-to-photon ratio $\eta = \eta_0/(S_f/S_i) \leq \eta_0$ (S_f/S_i is the ratio of the EM entropy per comoving volume after decays to that before decays). Even though η_0 does not correspond to the present baryon-to-photon ratio if there has been entropy production, we believe that comparisons for fixed η_0 are best for isolating the three different effects of a decaying τ neutrino on nucleosynthesis. For reference, in the absence of a decaying τ neutrino the ${}^4\text{He}$ mass fraction for our standard case is $Y_p = 0.2228$ (two massless neutrino species) and 0.2371 (three massless neutrino species).

A. $\nu_\tau \rightarrow$ sterile daughter products

Since we are considering lifetimes greater than 0.1 sec, by which time muon neutrinos are essentially decoupled, the muon neutrino is by our definition effectively sterile, and examples of this decay mode include $\nu_\tau \rightarrow \nu_\mu + \phi$, where ϕ is a very weakly interacting scalar particle (e.g., majoron) or $\nu_\tau \rightarrow \nu_\mu + \nu_\mu + \bar{\nu}_\mu$.

For this decay mode the only effect of the unstable τ neutrino on nucleosynthesis involves the energy density it and its daughter products contribute. Thus, it is the simplest case, and we use it as “benchmark” for comparison to the other decay modes. The light-element abundances as a function of τ neutrino lifetime are shown in Figs. 2–4 for a Dirac neutrino of mass 20 MeV.

The energy density of the massive τ neutrino grows relative to a massless neutrino species as $rm_{\nu_\tau}/3T$ until the τ neutrino decays, after which the ratio of energy density in its relativistic daughter products to a massless neutrino species remains constant. For τ neutrino masses in the 0.3–30 MeV mass range and lifetimes greater than about a second the energy density of the massive τ neutrino

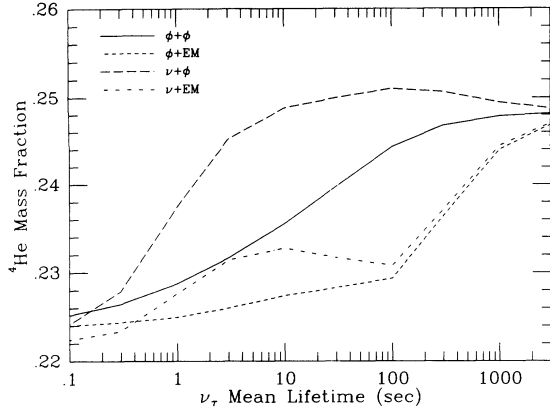


FIG. 2. The ${}^4\text{He}$ yield as a function of τ neutrino lifetime for the four generic decay modes, a 20-MeV Dirac neutrino, and a baryon-to-photon ratio that in the absence of entropy production leads to a present value $\eta_0 = 3 \times 10^{-10}$. For reference, the ${}^4\text{He}$ yield in the absence of a decaying τ neutrino is $Y_p = 0.2228$ (two massless neutrinos) and 0.2371 (three massless neutrinos).

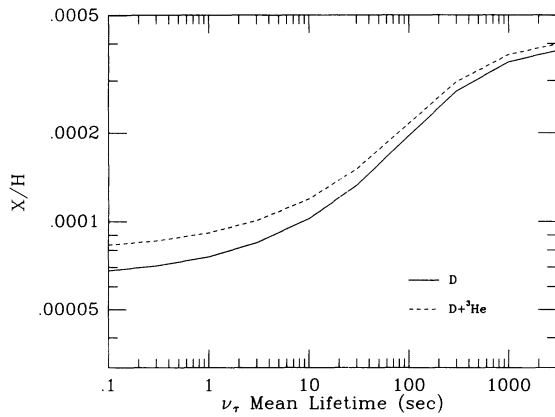


FIG. 3. The D (solid) and $\text{D} + {}^3\text{He}$ (broken) yields as a function of τ neutrino lifetime for the all-sterile decay mode, $\eta = 3 \times 10^{-10}$, and a 20-MeV Dirac neutrino.

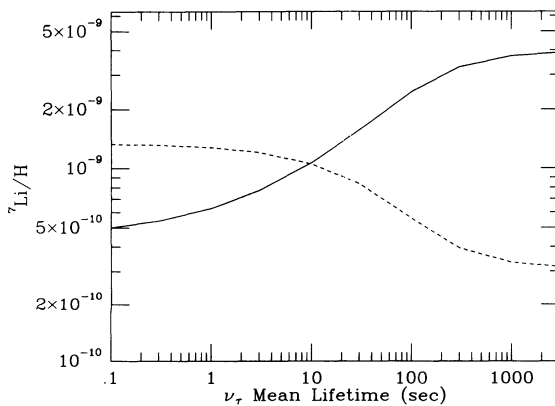


FIG. 4. The ${}^7\text{Li}$ yield as a function of τ neutrino lifetime for the all-sterile decay mode, $\eta = 10^{-10}$ (solid) and 10^{-9} (broken), and a 20-MeV Dirac neutrino.

exceeds that of a massless neutrino species before it decays, in spite of its smaller abundance (i.e., $r \ll 1$). The higher-energy density increases the expansion rate and ultimately ${}^4\text{He}$ production because it causes the neutron-to-proton ratio to freeze out earlier and at a higher value and because fewer neutrons decay before nucleosynthesis begins. Since the neutron-to-proton ratio freezes out around 1 sec and nucleosynthesis occurs at around a few hundred seconds, the ${}^4\text{He}$ abundance is only sensitive to the expansion rate between one and a few hundred seconds.

In Fig. 2 we see that for short lifetimes ($\tau_\nu \ll 1$ sec) the ${}^4\text{He}$ mass fraction approaches that for two massless neutrinos (τ neutrinos decay before their energy density becomes significant). As expected, the ${}^4\text{He}$ mass fraction increases with lifetime leveling off at a few hundred seconds at a value that is significantly greater than that for three massless neutrino species.

The yields of D and ${}^3\text{He}$ depend upon how much of these isotopes are not burnt to ${}^4\text{He}$. This in turn depends upon competition between the expansion rate and nuclear reaction rates: Faster expansion results in more unburnt D and ${}^3\text{He}$. Thus the yields of D and ${}^3\text{He}$ increase with τ neutrino lifetime, and begin to level off for lifetimes of a few hundred seconds as this is when nucleosynthesis is taking place (see Fig. 3).

The effect on the yield of ${}^7\text{Li}$ is a bit more complicated. Lithium production decreases with increasing η for $\eta \lesssim 3 \times 10^{-10}$ because the final abundance is determined by competition between the expansion rate and nuclear processes that destroy ${}^7\text{Li}$, and increases with increasing η for $\eta \gtrsim 3 \times 10^{-10}$ because the final abundance is determined by competition between the expansion rate and nuclear processes that produce ${}^7\text{Li}$. Thus, an increase in expansion rate leads to increased ${}^7\text{Li}$ production for $\eta \lesssim 3 \times 10^{-10}$ and decreased ${}^7\text{Li}$ production for $\eta \gtrsim 3 \times 10^{-10}$; this is shown in Fig. 4. Put another way the valley in the ${}^7\text{Li}$ production curve shifts to larger η with increasing τ neutrino lifetime (see Fig. 3 of Ref. [10]).

We show in Figs. 5 and 6 the excluded region of the

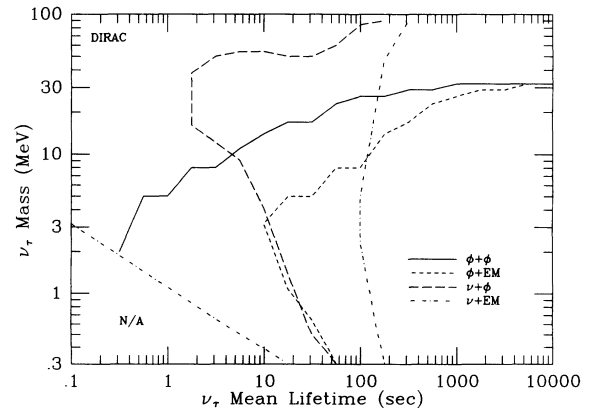


FIG. 5. Excluded regions of the mass-lifetime for a Dirac neutrino and the four generic decay modes. The excluded regions are to the right of the curves; our results are not applicable to the region labeled N/A as τ neutrino inverse decays can be important and have not been included (see Ref. [12]).

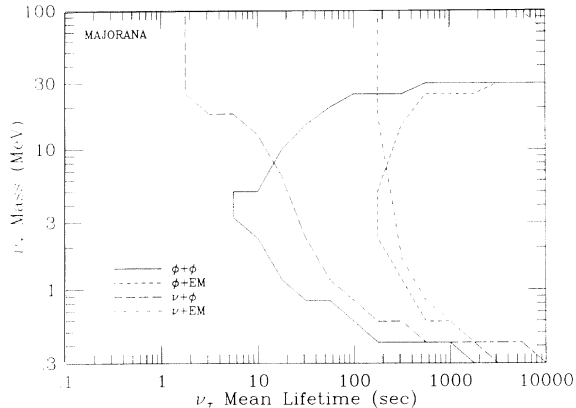


FIG. 6. Excluded regions of the mass-lifetime for a Majorana neutrino and the four generic decay modes. The excluded regions are to the right of the curves. An additional portion of the lower left part of the mass-lifetime plane can be excluded when inverse decays are taken into account (see Ref. [12]).

mass/lifetime plane for a Dirac and Majorana τ neutrino respectively. As expected, the excluded mass range grows with lifetime, asymptotically approaching 0.3–33 MeV (Dirac) and 0.4–30 MeV (Majorana). We note the significant dependence of the excluded region on lifetime; our results are in good agreement with the one other work where comparison is straightforward [10], and in general agreement with Refs. [7,12].

Muon neutrinos are not truly sterile as they can interact with electron neutrinos and/or the EM plasma, and thus in principle the mode $\nu_\tau \rightarrow \nu_\mu + \phi$ is not identical to $\nu_\tau \rightarrow \phi + \phi$. The most important effect is the heating of electron neutrinos, which affects ${}^4\text{He}$ production (see below). For lifetimes greater than 10 sec the effect is negligible, and so the $\nu_\tau \rightarrow \nu_\mu + \phi$ mode is identical to the all-sterile mode. For lifetimes of 1 sec or less ${}^4\text{He}$ production is reduced by as much as $\Delta Y = -0.02$, and so the excluded region for $\nu_\tau \rightarrow \nu_\mu + \phi$ is slightly smaller than for $\nu_\tau \rightarrow \phi + \phi$.

B. $\nu_\tau \rightarrow$ sterile + electromagnetic daughter products

Again, based upon our definition of sterility, the sterile daughter could be a muon neutrino; thus, examples of this generic decay mode include $\nu_\tau \rightarrow \nu_\mu + \gamma$ or $\nu_\tau \rightarrow \nu_\mu + e^\pm$. Our results here are based upon a two-body decay (e.g., $\nu_\tau \rightarrow \nu_\mu + \gamma$), and change only slightly in the case of a three-body decay (e.g., $\nu_\tau \rightarrow \nu_\mu + e^\pm$), where a larger fraction of the τ neutrino mass goes into electromagnetic entropy.

Two effects now come into play: the energy density of the massive τ neutrino and its daughter products speed up the expansion rate, tending to increase ${}^4\text{He}$, ${}^3\text{He}$, and D production; and EM entropy production due to τ neutrino decays reduce the baryon-to-photon ratio (at the time of nucleosynthesis), tending to decrease ${}^4\text{He}$ production and to increase D and ${}^3\text{He}$ production. Both effects tend to shift the ${}^7\text{Li}$ valley (as a function of η_0) to larger η_0 .

While the two effects have the “same sign” for D, ${}^3\text{He}$, and ${}^7\text{Li}$, they have opposite signs for ${}^4\text{He}$. It is instruc-

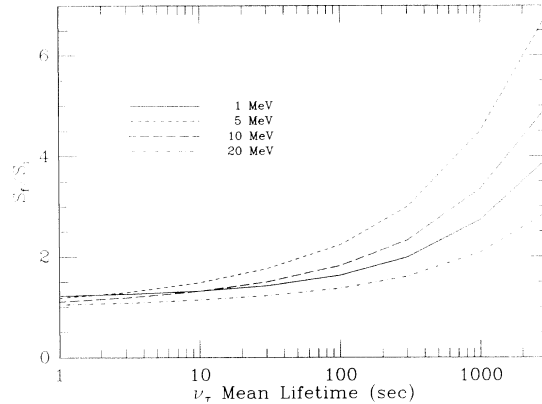


FIG. 7. Entropy production as a function of τ neutrino lifetime for a Dirac neutrino of mass 1,5,10,20 MeV and the $\nu_\tau \rightarrow \phi + \text{EM}$ decay mode. S_f/S_i is the ratio of the entropy per comoving volume after τ neutrino decays to that before.

tive to compare ${}^4\text{He}$ production as a function of lifetime to the previous “all-sterile” decay mode. Because of the effect of entropy production, there is little increase in ${}^4\text{He}$ production until a lifetime greater than 1000 sec or so. For lifetimes greater than 1000 sec the bulk of the entropy release takes place after nucleosynthesis, and therefore does not affect the value of η during nucleosynthesis.

Because of the competing effects on ${}^4\text{He}$ production, the impact of an unstable, massive τ neutrino on nucleosynthesis is significantly less than that in the all-sterile decay mode for lifetimes less than about 1000 sec. The excluded region of the mass/lifetime plane is shown in Figs. 5 and 6. For lifetimes greater than about 1000 sec the excluded mass interval is essentially the same as that for the all-sterile decay mode; for shorter lifetimes it is significantly smaller.

Finally, because of entropy production, the final value of the baryon-to-photon ratio is smaller for fixed initial baryon-to-photon ratio: It is reduced by the factor by which the entropy per comoving volume is increased. In the limit of significant entropy production ($S_f/S_i \gg 1$), this factor is given by Eq. (5.73) of Ref. [19],

$$S_f/S_i \simeq 0.13 r m_\nu \sqrt{\tau_\nu / m_{\text{pl}}} \simeq 1.5 \frac{r m_\nu}{\text{MeV}} \left[\frac{\tau_\nu}{1000 \text{ sec}} \right]^{1/2}. \quad (20)$$

A precise calculation of entropy production for this decay mode is shown in Fig. 7. As can be seen in the figure or from Eq. (20), entropy production becomes significant for lifetimes longer than about 100 sec.

C. $\nu_\tau \rightarrow \nu_e +$ sterile daughter products

Once again, by our definition of sterility this includes decay modes such as $\nu_\tau \rightarrow \nu_e + \phi$ or $\nu_\tau \rightarrow \nu_e + \nu_\mu \bar{\nu}_\mu$. Here, we specifically considered the two-body decay mode $\nu_\tau \rightarrow \nu_e + \phi$, though the results for the three-body mode are very similar.

Two effects come into play: the energy density of the massive τ neutrino and its daughter products and the interaction of daughter electron neutrinos with the nu-

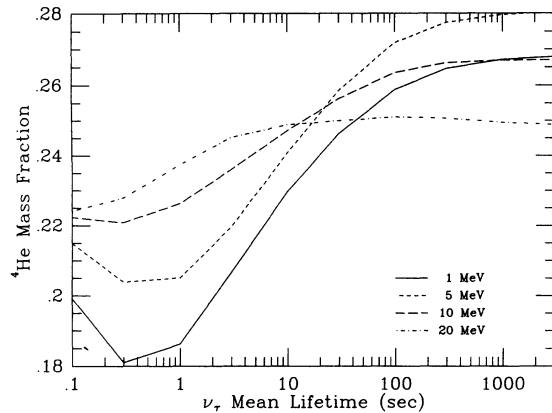


FIG. 8. ${}^4\text{He}$ yield as a function of τ neutrino lifetime for the $\nu_\tau \rightarrow \nu_e + \phi$ decay mode, $\eta = 3 \times 10^{-10}$, and Dirac masses of 1, 5, 10, 20 MeV. For reference, the ${}^4\text{He}$ yield in the absence of a decaying τ neutrino is $Y_p = 0.2228$ (two massless neutrinos) and 0.2371 (three massless neutrinos). For the mode $\nu_\tau \rightarrow \nu_e + \nu_e \nu_e$, ${}^4\text{He}$ production can be reduced to as low as $Y = 0.12$ (for lifetimes between 0.1 sec and 1 sec).

cleons and the ambient plasma. The first effect has been discussed above. The second effect leads to some interesting new effects.

Electron neutrinos and antineutrinos produced by τ neutrino decays increase the weak rates that govern the neutron-to-proton ratio. For short lifetimes ($\lesssim 30$ sec) and masses less than about 10 MeV the main effect is to delay slightly the “freeze-out” of the neutron-to-proton ratio, thereby decreasing the neutron fraction at the time of nucleosynthesis and ultimately ${}^4\text{He}$ production. For long lifetimes, or short lifetimes and large masses, the perturbations to the $n \rightarrow p$ and $p \rightarrow n$ rates (per nucleon) are comparable; since after freeze out of the neutron-to-proton ratio there are about six times as many protons as neutrons, this has the effect of increasing the neutron fraction and ${}^4\text{He}$ production. This is illustrated in Fig. 8. The slight shift in the neutron fraction does not affect the other light-element abundances significantly.

The excluded portion of the mass/lifetime plane is shown in Figs. 5 and 6. It agrees qualitatively with the results of Ref. [8].⁶ Comparing the limits for this decay mode with the all-sterile mode, the effects of electron-neutrino daughter products are clear: for long lifetimes much higher mass τ neutrinos are excluded and for short lifetimes low-mass τ neutrinos are allowed.

D. $\nu_\tau \rightarrow \nu_e + \text{electromagnetic daughter products}$

Now we consider the most complex of the decay modes, where none of the daughter products is sterile. Specifically, we consider the decay mode $\nu_\tau \rightarrow \nu_e + e^\pm$, though our results change very little for the two-body decay $\nu_\tau \rightarrow \nu_e + \gamma$.

In this case all three effects previously discussed come

⁶The authors of Ref. [8] use a less stringent constraint to ${}^4\text{He}$ production, $Y_p \leq 0.26$; in spite of this, in some regions of the m_ν - τ_ν plane their bounds are as, or even more, stringent. This is presumably due to the neglect of electron-neutrino interactions with the ambient plasma.

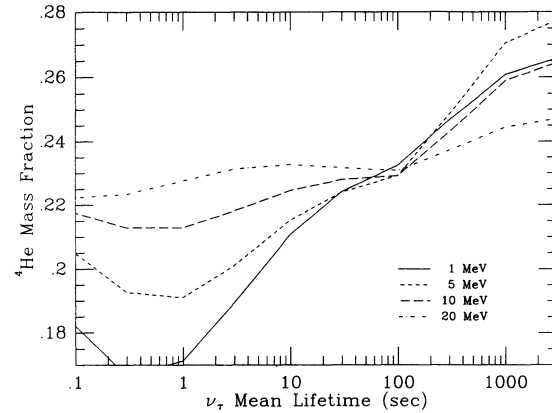


FIG. 9. ${}^4\text{He}$ yield as a function of τ neutrino lifetime for the $\nu_\tau \rightarrow \nu_e + \text{EM}$ decay mode, $\eta_0 = 3 \times 10^{-10}$, and Dirac masses of 1, 5, 10, 20 MeV. For reference, the ${}^4\text{He}$ yield in the absence of a decaying τ neutrino is $Y_p = 0.2228$ (two massless neutrinos) and 0.2371 (three massless neutrinos).

into play: The energy density of the massive τ neutrino and its daughter products speed up the expansion rate; the entropy released dilutes the baryon-to-photon ratio; and daughter electron neutrinos increase the weak-interaction rates that control the neutron fraction. The net effect on ${}^4\text{He}$ production is shown in Fig. 9 for a variety of τ neutrino masses. The main difference between this decay mode and the previous one, $\nu_\tau \rightarrow \nu_e + \text{sterile}$, is for lifetimes between 30 and 300 sec, where the increase in ${}^4\text{He}$ production is less due to the entropy production, which reduces the baryon-to-photon ratio at the time of nucleosynthesis.

The excluded region of the mass/lifetime plane is shown in Figs. 5 and 6. It agrees qualitatively with the results of Ref. [9].⁷ The excluded region for this decay mode is similar to that of the previous decay mode, except that lifetimes less than about 100 sec are not excluded as entropy production has diminished ${}^4\text{He}$ production in this lifetime interval.

E. Limits to a generic light species

We can apply the arguments for the four decay modes discussed above to a hypothetical species whose relic abundance has frozen out at a value r relative to a massless neutrino species before the epoch of primordial nucleosynthesis (also see Refs. [6,7]). The previous limits become limits to rm as a function of lifetime τ and mass m , which are difficult to display. With the exception of the effect that involves daughter electron neutrinos, all other effects only depend upon rm , which sets the energy density of the massive particle and its daughter products. In Fig. 10, we show that for lifetimes greater than about 100 sec and masses greater than about 10 MeV, the ${}^4\text{He}$ production is relatively insensitive to the mass of the decaying particle. This means that for lifetimes greater

⁷The authors of Ref. [9] use a less stringent constraint to ${}^4\text{He}$ production, $Y_p \leq 0.26$; in spite of this, in some regions of the m_ν - τ_ν plane their bounds are as, or even more, stringent. This is presumably due to the neglect of electron-neutrino interactions with the ambient plasma.

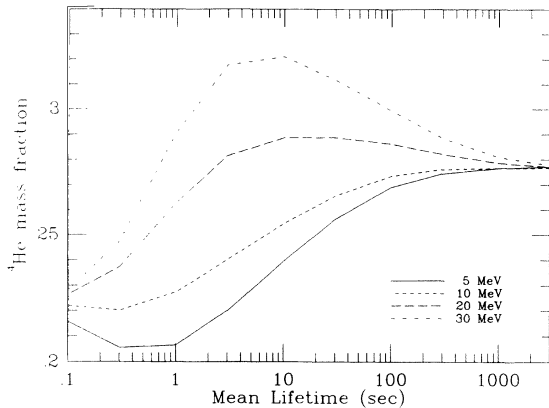


FIG. 10. ${}^4\text{He}$ yield as a function of τ neutrino lifetime for the $\nu_e + \phi$ decay mode, $\eta_0 = 3 \times 10^{-10}$, and Dirac masses of 1, 5, 10, 20 MeV. For reference, the ${}^4\text{He}$ yield in the absence of a decaying τ neutrino is $Y_p = 0.2228$ (two massless neutrinos) and 0.2371 (three massless neutrinos).

than about 100 sec the limit to rm should be relatively insensitive to particle mass.

We show in Fig. 11 the excluded regions of the rm - τ plane for a 20 MeV decaying particle. In deriving these limits we used the same criteria for acceptable light-element abundances and assumed three massless neutrino species. The limits to rm for decay modes without electron-neutrino daughter products are strictly independent of mass; the two other should be relatively insensitive to the particle mass for $\tau \gtrsim 100$ sec (and the actual limits are more stringent for $m > 20$ MeV).

IV. LABORATORY AND OTHER LIMITS

There are a host of other constraints to the mass and lifetime of the τ neutrino [20]. As a general rule, cosmological arguments, such as the one presented above, pose *upper* limits to the τ neutrino lifetime for a given mass: Cosmology has nothing to say about a particle that decays very early, since it would not have affected the

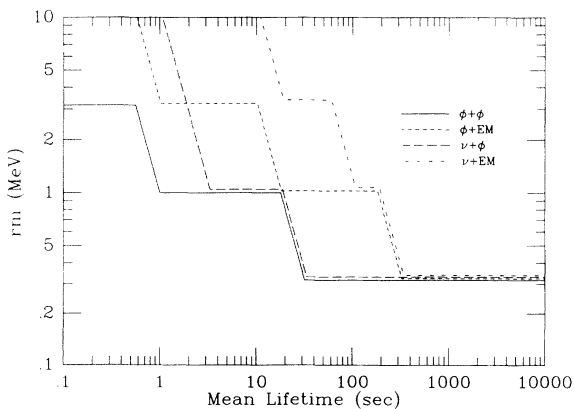


FIG. 11. Excluded regions of the rm - τ plane for the four different decay modes and a 20-MeV mass particle. The limits for the first two decay modes are strictly independent of mass; for the last two decay modes should be relatively insensitive to mass for $\tau \gtrsim 100$ sec (and actually more stringent than those shown here for $m > 20$ MeV). Excluded regions are above the curves.

“known cosmological history.” Laboratory experiments on the other hand pose *lower* limits to the lifetime because nothing happens inside a detector if the lifetime of the decaying particle is too long. Finally, astrophysical considerations generally rule out bands of lifetime, since “signals” can only be detected if (a) the τ neutrinos escape the object of interest before decaying, and (b) they decay before they pass by earthly detectors.

A. Laboratory

The most important limits of course are the direct limits to the τ neutrino mass. These have come down steadily over the past few years. The current upper limits are 31 and 32.6 MeV [13].

If the τ neutrino has a mass greater than $2m_e = 1.02$ MeV, then the decay $\nu_\tau \rightarrow \nu_e + e^\pm$ takes place through ordinary electroweak interactions at a rate

$$\Gamma = \frac{G_F^2 m_\nu^5}{192\pi^3} |U_{e\tau}|^2 |U_{ee}|^2 \simeq \frac{(m_\nu/\text{MeV})^5 |U_{e\tau}|^2}{2.9 \times 10^4 \text{ sec}}, \quad (21)$$

where $U_{e\tau}$ and U_{ee} are elements of the unitary matrix that relates mass eigenstates to weak eigenstates, the leptonic equivalent of the Cabibbo-Kobayashi-Maskawa matrix. We note that the rate could be larger (or even perhaps smaller) in models where the decay proceeds through new interactions. Thus, limits to $U_{e\tau}$ give rise to model-dependent limits to the τ neutrino lifetime.

A number of experiments have set limits to $U_{e\tau}$. The most sensitive experiment in the mass range $1.5 \text{ MeV} < m_\nu < 4 \text{ MeV}$ was performed at the power reactor in Gosgen, Switzerland [21], which produces τ neutrinos at a rate proportional to $|U_{e\tau}|^2$ through decay of heavy nuclei and $\nu_e - \nu_\tau$ mixing. Above this mass range, experiments that search for additional peaks in the positron spectrum of the $\pi^+ \rightarrow e^+ \nu$ decay (due to $\nu_e - \nu_\tau$ mixing) provide the strictest limits. In the mass range $4 \text{ MeV} < m_\nu < 20 \text{ MeV}$, Bryman *et al.* [22] set the limits shown in Fig. 12; for larger masses the best limits come from Ref. [23].

There are also direct accelerator bounds to the lifetime of an unstable τ neutrino that produces a photon or e^\pm pair. In particular, as has been recently emphasized by Babu, Gould, and Rothstein [24], the BEBC (Big European Bubble Chamber) beam dump experiment [25] provides model-independent limits based upon the direct search for the EM decay products. These limits, while not quite as strict as those mentioned above, are of interest, since they apply to the photon mode and to the e^\pm mode even if the decay proceeds through new interactions. The limit,

$$\tau_\nu > 0.18 (m_\nu/\text{MeV}) \text{ sec}, \quad (22)$$

is shown in Fig. 12.

B. Astrophysical

The standard picture of type-II supernovae has the binding energy of the newly born neutron star (about 3×10^{53} erg) shared equally by neutrinos of all species, which are emitted from a neutrinosphere of temperature

of about 4 MeV. There are two types of limits based upon SN 1987A, and combined they rule out a large region of m_ν - τ_ν plane.

First, if the τ neutrino decayed after it left the progenitor supergiant, which has a radius $R \approx 3 \times 10^{12}$ cm, the high-energy daughter photons could have been detected [26–28]. The Solar Maximum Mission Gamma-ray Spectrometer set an upper limit to the fluence of γ rays during the 10 sec in which neutrinos were detected:

$$f_\gamma < 0.9 \text{ cm}^{-2}, \quad 4.1 \text{ MeV} < E_\gamma < 6.4 \text{ MeV}. \quad (23)$$

As we will see shortly, if only one in 10^{10} of the τ neutrinos leaving the supernova produced a photon, this limit would have been saturated. In the mass regime of interest there are two ways out of this constraint: The lifetime can be so long that the arrival time was more than 10 sec after the electron antineutrinos arrived, or the lifetime can be so short that the daughter photons were produced inside the progenitor. We can take account of both of these possibilities in the following formula for the expected fluence of γ rays:

$$f_{\gamma,10} = f_{\nu\bar{\nu}} W_\gamma B_\gamma \langle F_1 F_2 \rangle, \quad (24)$$

where the subscript 10 reminds us that we are only interested in the first 10 sec, $f_{\nu\bar{\nu}} \approx 1.4 \times 10^{10} \text{ cm}^{-2}$ is the fluence of a massless neutrino species, $W_\gamma \approx 1/4$ is the fraction of decay photons produced with energies between 4.1 and 6.4 MeV, F_1 is the fraction of τ neutrinos that decay outside the progenitor, and F_2 is the fraction of these that decay early enough so that the decay products were delayed by less than 10 sec. The quantity B_γ is the branching ratio to a decay mode that includes a photon. For $m_\nu \gtrsim 1$ MeV one expects the $\nu_e + e^\pm$ mode to be dominant; however, ordinary radiative corrections should lead to $B_\gamma \approx 10^{-3}$ [29]. Finally angular brackets denote an average over the Fermi-Dirac distribution of neutrino momenta,

$$\langle A \rangle \equiv \frac{1}{1.5\xi(3)T^3} \int_0^\infty \frac{A dp p^2}{e^{E/T} + 1}, \quad (25)$$

where $T \approx 4$ MeV is the temperature of the neutrinosphere and $E = (p^2 + m_\nu^2)^{1/2}$.

To evaluate the fluence of γ rays we need to know F_1 and F_2 . The fraction F_1 that decay outside the progenitor is simply e^{-t_1/τ_L} , where the time it takes to escape the progenitor $t_1 = R/v = RE/p$ and the “lab” lifetime $\tau_L = \tau E/m_\nu$. Of these, the fraction whose decay products arrive *after* 10 sec is $e^{-t_2/\tau_L}/e^{-t_1/\tau_L}$, where $t_2 = 10 \text{ sec}/(1-v/c)$; thus, $F_2 = 1 - e^{-(t_1-t_2)/\tau_L}$. Figure 12 shows this constraint assuming a branching ratio $B_\gamma = 10^{-3}$.

The second constraint comes from observing that if τ neutrinos decayed within the progenitor supergiant, the energy deposited (up to about 10^{53} erg) would have “heated up” the progenitor so much as to conflict with the observed optical luminosity of SN 1987A (and other type-II supernovae) [29,30]. We require

$$E_{\text{input}} = \langle (1 - F_1) \rangle E_\nu \lesssim 10^{51} \text{ erg}, \quad (26)$$

where $E_\nu \sim 10^{53}$ erg is the energy carried off by a massless neutrino species, and $1 - F_1$ is the fraction of τ neutrinos

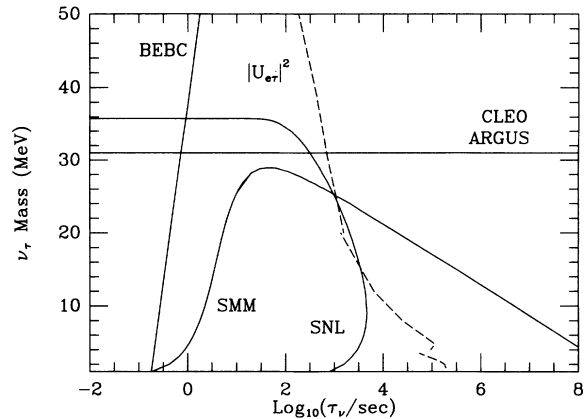


FIG. 12. Regions of the τ neutrino mass-lifetime plane excluded by laboratory experiments and astrophysical arguments. The excluded regions are to the side of the curve on which the label appears. The dashed curve summarizes a host of different laboratory limits to $|U_{e\tau}|^2$, translated to a model-dependent bound to $\tau_\nu(\nu_\tau \rightarrow \nu_e e^\pm)$, cf. Eq. (21). SNL denotes the supernova-light constraint, which extends to lifetimes shorter than those shown here.

that decay within the progenitor. This constraint is mode independent, since decay-produced photons or e^\pm pairs will equally well “overheat” the progenitor. As Fig. 12 shows, the “supernova-light” bound is extremely powerful.

Finally, a note regarding our SN 1987A constraints. We have assumed that a massive τ neutrino species has a Fermi-Dirac distribution with the same temperature as a massless ($m_\nu \ll 10$ MeV) neutrino species. This is almost certainly false. Massive ($m_\nu \gtrsim 10$ MeV or so) τ neutrinos will drop out of chemical equilibrium (maintained by pair creation/annihilations and possibly decays/inverse decays) interior to the usual neutrinosphere as the Boltzmann factor suppresses annihilation and pair creation rates relative to scattering rates. This leads us to believe that we have actually *underestimated* the fluence of massive neutrinos. While the problem has yet to be treated rigorously, we are confident that, if anything, our simplified treatment results in limits that are overly conservative. Accurate limits await a more detailed analysis [31].

C. Cosmological

The most stringent cosmological constraint for masses $0.1 \text{ MeV} \lesssim m \lesssim 100 \text{ MeV}$ is the nucleosynthesis bound discussed in this paper. Nonetheless, it is worthwhile to mention some of the other cosmological limits since they are based upon independent arguments. A stable τ neutrino with mass in the MeV range contributes much more energy density than is consistent with the age of the Universe. Such a neutrino must be unstable, with a lifetime short enough for its decay products to lose enough of their energy by “redshifting” [32]. The lifetime limit is mass dependent; a neutrino with a mass of about 1 MeV must have a lifetime shorter than about 10^9 sec, and the constraint gets less severe for larger or smaller masses. There is an even more stringent bound based on the necessity of the Universe being matter dominated by a

red shift of about 10^4 in order to produce the observed large-scale structure [33]. Finally, there are other nucleosynthesis bounds based upon the dissociation of the light elements by decay-produced photons or electron-neutrinos [14] and by e^\pm pairs produced by the continuing annihilations of τ neutrinos [34].

V. SUMMARY AND DISCUSSION

We have presented a comprehensive study of the effect of an unstable τ neutrino on primordial nucleosynthesis. The effects on the primordial abundances and the mass/lifetime limits that follow depend crucially upon the decay mode. In the context of primordial nucleosynthesis we have identified four generic decay modes that bracket the larger range of possibilities: (1) all-sterile daughter products; (2) sterile daughter product(s) + EM daughter product(s); (3) ν_e + sterile daughter product(s); and (4) ν_e + EM daughter product(s). The excluded regions of the τ neutrino mass/lifetime plane for these four decay modes are shown in Figs. 5 (Dirac) and 6 (Majorana).

In the limit of long lifetime ($\tau_\nu \gg 100$ sec), the excluded mass range is 0.3–33 MeV (Dirac) and 0.4–30 MeV (Majorana). Together with current laboratory upper mass limits, 31 MeV (ARGUS) and 32.6 MeV (CLEO), our results very nearly exclude a long-lived, τ neutrino more massive than about 0.4 MeV. Moreover, other astrophysical and laboratory data exclude a τ neutrino in the 0.3–50 MeV mass range if its decay product(s) include a photon or e^\pm pair. Thus, if the mass of the τ neutrino is the range 0.4 to 30 MeV, then its decay products cannot include a photon or an e^\pm pair and its lifetime must be shorter than a few hundred seconds.

We note that the results of Ref. [12] for the all-sterile decay mode are more restrictive than ours, excluding masses from about 0.1 MeV to about 50 MeV for $\tau_\nu \gg 100$ sec. This traces in almost equal parts to (i) small ($\Delta Y \simeq +0.003$), but significant, corrections to the ${}^4\text{He}$ mass fraction synthesized due to finite nucleon mass effects and differences in the numerical techniques used and (ii) slightly larger relic neutrino abundance. With regard to the first difference, this illustrates the sensitivity to the third significant figure of the ${}^4\text{He}$ mass fraction. With regard to the second difference, it is probably correct that within the assumptions made the τ neutrino abundance during nucleosynthesis is larger than what we used. However, other effects that have been neglected probably lead to differences in the τ neutrino abundance of the same magnitude. For example, for τ neutrino masses around the upper range of excluded masses, 50–100 MeV, finite-temperature corrections, hadronic final states (e.g., a single pion), and τ neutrino mixing have not been included in the annihilation cross section and are likely to be important at the 10% level.

So is a τ neutrino with lifetime greater than a few hundred seconds and mass greater than a fraction of an MeV

ruled out or not? Unlike a limit based upon a laboratory experiment, it is impossible to place standard error flags on an astrophysical or cosmological bound. This is because of assumptions that must be made and modeling that must be done. For example, the precise limits that one derives depend upon the adopted range of acceptable light-element abundances. To be specific, in Ref. [12] the upper limit of the excluded mass range drops from 50 MeV to around 38 MeV and the lower limit increases from 0.1 MeV to about 0.4 MeV when the primordial ${}^4\text{He}$ mass fraction is allowed to be as large as 0.245 (rather than 0.240). In our opinion, a very strong case has been made against a τ neutrino mass in the mass range 0.4–30 MeV with lifetime *much greater* than 100 sec; together with the laboratory limits this very nearly *excludes* a long-lived, τ neutrino of mass greater than 0.4 MeV.

Perhaps the most interesting thing found in our study is the fact that a τ neutrino of mass 1–10 MeV and lifetime 0.1–10 sec that decays to an electron neutrino and a sterile daughter product can very significantly decrease the ${}^4\text{He}$ mass fraction (to as low as $Y=0.12$ or so). It has long been realized that the standard picture of nucleosynthesis would be in trouble if the primordial ${}^4\text{He}$ mass fraction were found to be smaller than about 0.23; within the standard framework we have found one way out: an unstable τ neutrino.⁸ In principle, the possibility of an unstable τ neutrino also loosens the primordial-nucleosynthesis bound to the number of light species, which is largely based on the overproduction of ${}^4\text{He}$. This, and other interesting implications, of a 1–30 MeV τ neutrino whose decay products include electron neutrinos are explored elsewhere [35,36]. In particular, in Ref. [36] it is shown that a 20–30 MeV τ neutrino whose abundance is slightly less than that assumed here can lead to a significant relaxation in the nucleosynthesis bound to the baryon density.

Finally, we translated our results for the τ neutrino into limits to the relic abundance of a hypothetical particle species that decays into one of the four generic decay models discussed. Those very stringent limits are shown in Fig. 11.

ACKNOWLEDGMENTS

We thank Robert Scherrer, David Schramm, Gary Steigman, and Terry Walker for useful comments. This work was supported in part by the DOE (at Chicago and Fermilab) and by the NASA through NAGW-2381 (at Fermilab). G.G. was supported in part by an NSF grant. MST thanks the Aspen Center for Physics for its hospitality where some of this work was carried out.

⁸Based upon dimensional considerations the lifetime for the mode $\nu_\tau \rightarrow \nu_e + \phi$ is expected to be $\tau_\nu \sim 8\pi f^2 / m_\nu^3$, where f is the energy scale of the superweak interactions that mediate the decay. For $\tau_\nu \sim 10$ sec and $m_\nu \sim 10$ MeV, $f \sim 10^9$ GeV.

- [1] J. Yang, M. S. Turner, G. Steigman, D. N. Schramm, and K. A. Olive, *Astrophys. J.* **281**, 493 (1984).
- [2] T. P. Walker, G. Steigman, D. N. Schramm, K. Olive, and H.-S. Kang, *Astrophys. J.* **376**, 51 (1991).
- [3] G. Steigman, D. N. Schramm, and J. Gunn, *Phys. Lett.* **66B**, 202 (1977); also see V. F. Shvartsman, *Pis'ma Zh. Eksp. Teor. Fiz.* **9**, 315 (1969) [*JETP Lett.* **9**, 184 (1969)].
- [4] For a recent review of cosmological limits based upon primordial nucleosynthesis see, e.g., R. A. Malaney and G. J. Mathews, *Phys. Rep.* **229**, 145 (1993).
- [5] E. W. Kolb and R. J. Scherrer, *Phys. Rev. D* **25**, 1481 (1982).
- [6] R. J. Scherrer and M. S. Turner, *Astrophys. J.* **331**, 19 (1988).
- [7] R. J. Scherrer and M. S. Turner, *Astrophys. J.* **331**, 33 (1988).
- [8] N. Terasawa and K. Sato, *Phys. Lett. B* **185**, 412 (1987); for earlier work, see R. J. Scherrer, *Mon. Not. R. Astron. Soc.* **210**, 359 (1984).
- [9] N. Terasawa, M. Kawasaki, and K. Sato, *Nucl. Phys.* **B302**, 697 (1988).
- [10] E. W. Kolb, M. S. Turner, A. Chakravorty, and D. N. Schramm, *Phys. Rev. Lett.* **67**, 533 (1991).
- [11] A. Dolgov and I. Rothstein, *Phys. Rev. Lett.* **71**, 476 (1993).
- [12] M. Kawasaki *et al.*, *Nucl. Phys. B* (to be published).
- [13] CLEO Collaboration, D. Cinabro *et al.*, *Phys. Rev. Lett.* **70**, 3700 (1993); ARGUS Collaboration, H. Albrecht *et al.*, *Phys. Lett. B* **292**, 221 (1992). Also see H. Albrecht *et al.*, *Phys. Lett. B* **202**, 149 (1988); Particle Data Group, K. Hikasa *et al.*, *Phys. Rev. D* **45**, S1 (1992); D. F. Cowen, in *Second Workshop on Tau-lepton Physics*, edited by K. K. Gan (World Scientific, Singapore, 1993), p. 445; S. E. Csorna *et al.*, *Phys. Rev. D* **35**, 2747 (1987).
- [14] D. N. Lindley, *Mon. Not. R. Astron. Soc.* **188**, 15 (1979); *Astrophys. J.* **294**, 1 (1985); J. Audouze, D. N. Lindley, and J. Silk, *ibid.* **293**, L53 (1985); N. Terasawa, M. Kawasaki, and K. Sato, *Nucl. Phys.* **B302**, 697 (1988); R. J. Scherrer, *Mon. Not. R. Astron. Soc.* **210**, 359 (1984); J. Gratsias *et al.*, *Phys. Lett. B* **262**, 298 (1991); A. A. de Laix and R. J. Scherrer, *Phys. Rev. D* **48**, 562 (1993).
- [15] S. Dodelson and M. S. Turner, *Phys. Rev. D* **46**, 3372 (1992).
- [16] H.-S. Kang, M. Kawasaki, and G. Steigman, *Nucl. Phys.* **B402**, 323 (1993).
- [17] M. Smith, L. H. Kawano, and R. A. Malaney, *Astrophys. J. Suppl.* **85**, 219 (1993); L. Kawano, *FERMILAB-Pub-92/04-A*, 1992.
- [18] S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972), Chap. 15.
- [19] E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley, Redwood City, CA, 1990), Chap. 4.
- [20] A nice review of many of these constraints can be found in S. Sarkar and A. M. Cooper, *Phys. Lett.* **148B**, 347 (1984).
- [21] L. Oberauer, F. Von Feilitzsch, and R. L. Mossbauer, *Phys. Lett. B* **198**, 113 (1987).
- [22] D. A. Bryman *et al.*, *Phys. Rev. Lett.* **50**, 1546 (1983).
- [23] N. De Leener-Rosier *et al.*, *Phys. Lett. B* **177**, 228 (1986).
- [24] K. S. Babu, T. M. Gould, and I. Z. Rothstein, *Phys. Lett. B* **321**, 140 (1994).
- [25] A. M. Cooper-Sarkar *et al.*, *Phys. Lett.* **160B**, 207 (1985).
- [26] D. J. Forrest *et al.*, *Sol. Phys.* **65**, 15 (1980); E. L. Chupp, W. T. Vestrand, and C. Reppin, *Phys. Rev. Lett.* **62**, 505 (1989).
- [27] E. W. Kolb and M. S. Turner, *Phys. Rev. Lett.* **62**, 509 (1989).
- [28] Another, probably more stringent limit based upon SN 1987A follows the analysis of data from a γ -ray detector on the late Pioneer Venus Orbiter; A. Jaffe and M. S. Turner (unpublished).
- [29] R. N. Mohapatra, S. Nussinov, and X. Zhang, *Phys. Rev. D* **49**, 3434 (1994). These authors also suggest other ways, besides the expected branching ratio of 10^{-3} due to radiative corrections, in which decay-produced e^\pm might convert into photons.
- [30] R. Cowsik, D. N. Schramm, and P. Hoffich, *Phys. Lett. B* **218**, 91 (1989). This is an application of the original argument made by S. W. Falk and D. N. Schramm, *Phys. Lett.* **79B**, 511 (1978).
- [31] G. Sigl and M. S. Turner (unpublished).
- [32] D. Dicus, E. W. Kolb, and V. Teplitz, *Phys. Rev. Lett.* **39**, 168 (1977).
- [33] G. Steigman and M. S. Turner, *Nucl. Phys.* **B253**, 375 (1985).
- [34] J. A. Frieman, E. W. Kolb, and M. S. Turner, *Phys. Rev. D* **41**, 3080 (1990).
- [35] S. Dodelson, G. Gyuk, and M. S. Turner, *Phys. Rev. Lett.* (to be published).
- [36] G. Gyuk and M. S. Turner, *astro-ph/9403054*, 1994 (unpublished).