Constraints on the strength of a primordial magnetic field from big bang nucleosynthesis

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The effects of magnetic fields on big bang nucleosynthesis (BBN) are calculated, and the impact on the abundances of the light elements are investigated numerically. An upper limit on the strength of primordial magnetic fields compatible with observations of light element abundances is thus obtained. In the framework of standard BBN theory, the maximum strength of the primordial magnetic fields, on scales greater than 10^4 cm but smaller than the event horizon at the BBN epoch (~ 1 min, $\sim 2\times 10^{12}$ cm), is $\leq 10^{11}$ G. This limit is shown to allow magnetic fields at the time of recombination no stronger than ~ 0.1 G on scales $\geq 10^{11}$ cm. Our results also strongly indicate that, at the BBN epoch, and for field strengths $B \leq 10^{13}$ G, the effects of magnetic fields on the primordial abundances of light elements are dominated by effects from reaction rates in the presence of primeval magnetic fields rather than by magnetic density effects on the expansion rate.

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I. INTRODUCTION

It is recognized that primordial nucleosynthesis provides a unique quantitative window on the early Universe [1,2]. Since the synthesis of the light elements is determined by events occurring in the epoch from ~ 1 s to ~ 1000 s in the history of the Universe, when the temperatures varied from $\sim 10^{10}$ K (≥ 1 MeV) to $\sim 10^9$ K (≤ 0.1 MeV), the observed abundances constitute a probe of the Universe at epochs far earlier than those directly probed by the cosmic microwave background radiation (CMBR) [$t \sim 10^5$ yr; $T \sim 10^4$ K (~ 1 eV)]. Thus, through a detailed comparison of the predicted abundances with the observational data, proposed cosmological models can be tested and their controlling parameters can be constrained. For example, big bang nucleosynthesis (BBN) was found to constrain the number of families of light neutrinos [3] prior to accurate accelerator measurements.

In this paper, we reexplore how the strength of certain primordial magnetic fields can be constrained by BBN. If magnetic fields of sufficient strength existed in the early Universe, particularly at or just before the epoch of primordial nucleosynthesis, they could have had direct influences on both the expansion rate of the Universe and the nuclear reaction rates [4,5]. These influences could, of course, affect the abundances of the light elements produced in this environment. In addition, if the scale of the primeval magnetic field were greater than the event horizon, the geometry of the Universe would also be affected and an anisotropic Universe might result. An analysis of nucleosynthesis in anisotropic Euclidean universes, in which the dependence of the primordial abundances of ^{4}He, ^{3}He, and D on the isotropy parameters was specified more precisely, has been presented by Thorne [4] and by Hawking and Tayler [6]. If a significant degree of anisotropy had persisted up to times $\geq 20\,000$ years, the primordial ^{4}He abundance would have been reduced to a few percent. On the contrary, if the anisotropy is important only during the early stages of the expansion, the ^{4}He abundance is about 30% while there is no (neligible) D or ^{3}He production, and one might hope to eventually reach agreement with the observational values by refining this model. This had been done by Juszkiewicz *et al.* [7] who studied the influence of the anisotropic momentum distribution of neutrinos neglected by Thorne. The resultant limit on the magnetic field at the BBN time, set by the condition of small anisotropy for $t > 1$ s, is about $B < 4.1\times 10^{12}$ G [8].

On the other hand, if the primeval magnetic field were sufficiently spread over distances small compared with the event horizon at that epoch, the geometry of the Universe would not be affected [9] and it would still be described by a Robertson-Walker metric. For this situation, it has been qualitatively pointed out by a number of authors [9,10] that, in the presence of a very intense magnetic field ($B \geq 10^{13}$ G), the neutron would decay more rapidly than in the field-free case; this could obviously affect light element synthesis in a dramatic way.

However, in previous studies, only the effects of a very strong primordial magnetic field ($B > 10^{13}$ G) on abundances of ^{4}He, D, and ^{3}He have been studied, not vice versa, and no critical limit on the primeval magnetic field was explicitly derived. The questions we address in this paper are the following: (1) What is the limit on the primordial magnetic field? (2) How does the magnetic field influence the emerging abundances of other light elements ($A > 9$), such as lithium, boron, etc.? We find that there are still constraints to be explored on the strength and coherence scale of primordial magnetic fields, using observational abundances and BBN.

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II. THE DIRECT EFFECTS OF THE PRIMEVAL FIELD ON BBN

In the early expansion of the Universe, the existence of a large scale primordial magnetic field may have both direct and indirect effects on BBN. In this regard, the two most sensitive and competing effects are (1) the effects on reaction rates and (2) the effects on the expansion rate. These two effects will further alter the resulting abundances of the elements.

For simplicity, we assume that our Universe is fully filled by randomly oriented and distributed thin-wall magnetic domains (or bubbles) [11]. The size of each domain is large enough so that the field inside the domain can be seen as a uniform field, but it is still small compared with the event horizon. Thus, the magnetic field will have similar effects on the motion of particles in each domain. Here, we will neglect the boundary effects since the wall is assumed to be thin.

As to the effects on reaction rates, we have recently derived the reaction rates as a function of a uniform magnetic field **B** in the presence of an arbitrary degeneracy and polarization [12]. As an application, if we assume that the magnetic field is nearly uniform in each domain, we can use our derived preliminary results (Ref. [12]) as a first-order approximation for our purpose here. However, we would like to point out that, if the magnetic field is not uniform through the whole Universe but varies with spatial variables, or if the scale of the magnetic field (or magnetic bubbles) is much smaller than the horizon scale and the magnetic domains are disconnected from each other, the nuclear reaction rates will become inhomogeneous; i.e., the reaction rates will differ from region to region even though the geometry of the Universe is still not affected. This would require that we introduce reaction rate fluctuations into the standard big bang code (similar to the introduction of the density fluctuations associated with the first order QCD phase transition [13]), and perform multizone calculations. Such exploration is beyond the scope of the present paper and will be addressed in future work.

Now let us explore the effects of the magnetic fields on the expansion rate of the Universe. According to our assumptions, the globally chaotic (but locally orderly) magnetic fields will have no effect on the geometry of the Universe. The geometry of the Universe is still described by a Robertson-Walker metric. For this metric, the work-energy equations can be expressed as

$$\frac{d}{dt}(\rho R^3)+\frac{p}{c^2}\frac{d}{dt}(R^3)=0\ , \tag{2.1}$$

where $R(t)$ is the distance measure, ρ is the total mass-energy density, and p is the total pressure.

In general, we consider that the Universe consists of three types of matter during the epoch of interest. These are (1) the strongly and electromagnetically interacting particles (e.g., nucleons, electrons, photons, etc.), which can be described as a perfect fluid; (2) the weakly interacting particles, which nevertheless affect the n-p ratio (e.g., electron neutrinos, etc.); and (3) the effectively noninteracting particles (e.g., ν_μ and so on) which only contribute to the energy density but do not enter into specific reactions. The total mass-energy density ρ and pressure p can be expressed as

$$\rho=\rho_\gamma+\rho_e+\rho_\nu+\rho_b+\rho_B\ ,\quad p=p_\gamma+p_e+p_\nu+p_b+p_B\ , \tag{2.2}$$

where

$$\rho_e=\rho_{e^-}+\rho_{e^+}\ ,\quad p_e=p_{e^-}+p_{e^+}\ ,$$

$$\rho_\nu=\rho_{\nu_{e^-}}+\rho_{\nu_{e^+}}+\rho_{\nu_\mu}+\rho_{\nu_\tau}+\rho_{\bar\nu_e}+\rho_{\bar\nu_\mu}+\rho_{\bar\nu_\tau}\ ,$$

$$p_\nu=p_{\nu_{e^-}}+p_{\nu_{e^+}}+p_{\nu_\mu}+p_{\nu_\tau}+p_{\bar\nu_e}+p_{\bar\nu_\mu}+p_{\bar\nu_\tau}\ ,$$

and the subscripts γ, e, ν_e, ν_μ, ν_τ, b, and B stand, respectively, for photons, electrons, e neutrinos, μ neutrinos, τ neutrinos, baryons, and magnetic field. Expressions for these thermodynamic quantities are given below, for the case of nondegenerate neutrinos [14]

$$\begin{aligned}&\rho_\gamma=8.42T_9^4\ \mathrm{g\,cm^{-3}}\ ,\quad p_\gamma=\tfrac{1}{3}\rho_\gamma c^2\ ,\quad \rho_\nu=6\rho_{\nu_i}=\frac{21}{8}\rho_\gamma\left[\frac{T_\nu}{T}\right]^4\ ,\\ &p_\nu=\tfrac{1}{3}\rho_\nu c^2\ ,\quad \rho_e=\tfrac{7}{4}\rho_\gamma(T_9\gg 6)\ ,\quad p_e=\tfrac{1}{3}\rho_e c^2(T_9\gg 6)\ ,\\ &\rho_b=7\times 10^{-6}T_9^3\ \mathrm{g\,cm^{-3}}\ ,\quad p_b=n_b kT\sum_i Y_i,\quad Y_i=\frac{X_i}{A_i}\ ,\quad \rho_B=\frac{B^2}{8\pi}\ ,\end{aligned} \tag{2.3}$$

where

$$T_9=T/(10^9\ \mathrm{K})\ ,$$

$$\rho_{\nu_i}(\nu_i=\nu_e,\bar\nu_e,\nu_\mu,\bar\nu_\mu,\nu_\tau,\bar\nu_\tau)=\frac{7}{16}\rho_\gamma\left[\frac{T_\nu}{T}\right]^4$$

is the mass density of each type of neutrino and antineutrino, T_ν is the neutrino temperature, n_b is the number density of baryons, and Y_i, A_i, and Z_i designate the mass fraction, mass number, and atomic number of the ith nucleus.

From the assumptions of flux conservation and the presence of a conducting medium (as appropriate for the Universe prior to recombination), we can obtain a simple temperature dependence for B:

$$B\propto R^{-2}\propto T^2\ .$$

Therefore, the energy density of the magnetic field has the same temperature dependence as the energy density of the leptons and that of the radiation field.

We now define

$$\rho=\rho_1+\rho_2\ ,\quad p=p_1+p_2\ ,$$
$$\rho_1\equiv\rho_\gamma+\rho_{e^-}+\rho_{e^+}+\rho_B\ ,\quad p_1\equiv p_\gamma+p_{e^-}+p_{e^+}+p_B\ ,$$
$$\rho_2\equiv\rho_\nu+\rho_b\ ,\quad p_2=p_\nu+p_b\ , \tag{2.4}$$
$$\chi\equiv\frac{\rho_B}{\rho_\gamma+\rho_e+\rho_\nu+\rho_b}\equiv\frac{\rho_B}{\rho_0}\ ,\quad \rho_0\equiv\rho(B=0)\ .$$

The relation between the magnetic field and radiation is thus

$$\rho_B/\rho_\gamma\simeq\tfrac{43}{8}\chi\ . \tag{2.5}$$

Substituting these into Eqs. (2.2) and (2.1), noticing the following approximations

$$\rho_b\ll\rho_\nu\rightarrow\rho_2\simeq\rho_\nu\ ,\quad p_b\ll p_\nu\rightarrow p_2\simeq p_\nu\ , \tag{2.6}$$

and using the fact that $\rho_\nu\propto R^{-4}$, we obtain

$$\frac{dR}{dT}=\frac{-R}{3[\rho_1(T)+p_1(T)/c^2]}\frac{d\rho_1}{dT}\ . \tag{2.7}$$

By using the expansion rate

$$\frac{1}{R}\frac{dR}{dt}=\pm\left[\frac{8\pi G}{3}\rho\right]^{1/2}, \tag{2.8}$$

where G is the gravitational constant, the relation between the photon temperature and the time is found to be

$$\frac{dT}{dt}=\mp\left[\frac{8\pi G}{3}\rho\right]^{1/2}\left[\rho_1(T)+\frac{p_1(T)}{c^2}\right]\left[\frac{d\rho_1}{dT}\right]^{-1}. \tag{2.9}$$

At high temperatures, $T_\nu=T\propto R^{-1}$, and

$$\rho=\rho_0(1+\chi)\simeq\tfrac{43}{8}\rho_\gamma(1+\chi)\ , \tag{2.10}$$

$$\rho_1=\tfrac{11}{4}\rho_\gamma+\rho_B=\frac{11}{4}\rho_\gamma\left[1+\frac{43}{22}\chi\right]\equiv\rho_{10}\left[1+\frac{43}{22}\right], \tag{2.11}$$
$$\rho_{10}\equiv\rho_1(B=0)\ ,$$

$$\frac{d\rho_1}{dT}=\left[1+\frac{43}{22}\chi\right]\frac{d\rho_{10}}{dT}+\frac{43}{22}\rho_{10}\frac{d\chi}{dT}\ , \tag{2.12}$$

$$\frac{d\rho_{10}}{dT}\simeq\beta T_9^3\ ,\quad \beta=9.262\times10^{-8}\ . \tag{2.13}$$

Incorporating Eq. (2.5) into Eq. (2.8), we obtain a final expression for the relationship between photon temperature and time

$$\frac{dT}{dt}=\mp\frac{\left[\frac{8\pi G}{3}\rho_0\right]^{1/2}(1+\chi)^{1/2}\rho_{10}\left[1+\frac{43}{22}\chi+\frac{1}{3}+\frac{p_B}{\rho_{10}c^2}\right]}{\left[1+\frac{43}{22}\chi\right]\beta T_9^3+\frac{43}{22}\rho_{10}\frac{d\chi}{dT}}\ . \tag{2.14}$$

Moreover, considering the fact that, $B\propto T^2$, $\rho_B\propto T^4$, and $\rho_0\simeq\frac{43}{8}\rho_\gamma\propto T^4$, we can introduce a convenient invariant measure of magnetic-field strength and assume that the ratio of the magnetic energy density (ρ_B) to the total other energy density (ρ_0) is nearly a constant during BBN. This gives $d\chi/dT=0$. We now consider two cases:

A. A global zero magnetic pressure ($p_B=0$, but $B\neq0$)

Physically, this corresponds to a situation where there exists a nonzero local uniform magnetic field (inside each bubble), but a zero total magnetic pressure (pressure free) due to the random distribution of the tangled magnetic bubbles. In this case, Eq. (2.14) becomes

$$\frac{dT}{dt}=\mp\frac{\left[\frac{8\pi G}{3}\rho_0\right]^{1/2}(1+\chi)^{1/2}\left[1+\frac{129}{88}\chi\right]}{3\left[1+\frac{43}{22}\chi\right]}T_9\ . \tag{2.15}$$

Integrating, we obtain

$$T_9=\kappa\frac{\left[1+\frac{43}{22}\chi\right]^{1/2}}{(1+\chi)^{1/4}\left[1+\frac{129}{88}\chi\right]^{1/2}}t^{-1/2}\ , \tag{2.16}$$

where

$$\kappa=\left[\frac{12\pi Ga g_{\text{eff}}}{2c^2}\right]^{-1/4}\simeq\begin{cases}10.4, & \text{if } N_\nu=2 \text{ and } g_{\text{eff}}=9\ ;\\ 4.7\ , & \text{if } N_\nu=3 \text{ and } g_{\text{eff}}=\frac{43}{4}\ ,\end{cases}$$

in cgs units, g_{eff} is the "effective" number of relativistic degrees of freedom (helicity states), and a is the Stefan blackbody constant.

B. A nonzero magnetic pressure ($p_B=\rho_B c^2$)

If the magnetic field inside each bubble and the distribution of the magnetic bubbles are not so chaotic, for example, if each magnetic bubble is dipolelike, we will have an averaged magnetic pressure $p_B\propto\rho_B c^2$. In this instance, Eq. (2.14) becomes

$$\frac{dT}{dt} = \mp \frac{\left[\frac{8\pi G}{3}\rho_0\right]^{1/2}(1+\chi)^{1/2}\left[\frac{1}{3}+\frac{43}{44}\chi\right]}{4\left[1+\frac{43}{22}\chi\right]} T_9 , \tag{2.17}$$

and integration yields

$$T_9 = \kappa \frac{\left[1+\frac{43}{22}\chi\right]^{1/2}}{(1+\chi)^{1/4}\left[1+\frac{129}{44}\chi\right]^{1/2}} t^{-1/2} . \tag{2.18}$$

Note that in the limit when the magnetic fields are absent or very weak ($\rho_B=0$; $\chi=0$, or $\chi \ll 1$), Eqs. (2.16) and (2.18) both reduce to

$$T_9 \simeq \kappa t^{-1/2} . \tag{2.19}$$

This is just the formula used in standard BBN calculations [14,15].

If the magnetic field is very strong, $\chi \gg 1$, then Eqs. (2.16) and (2.18) become, respectively,

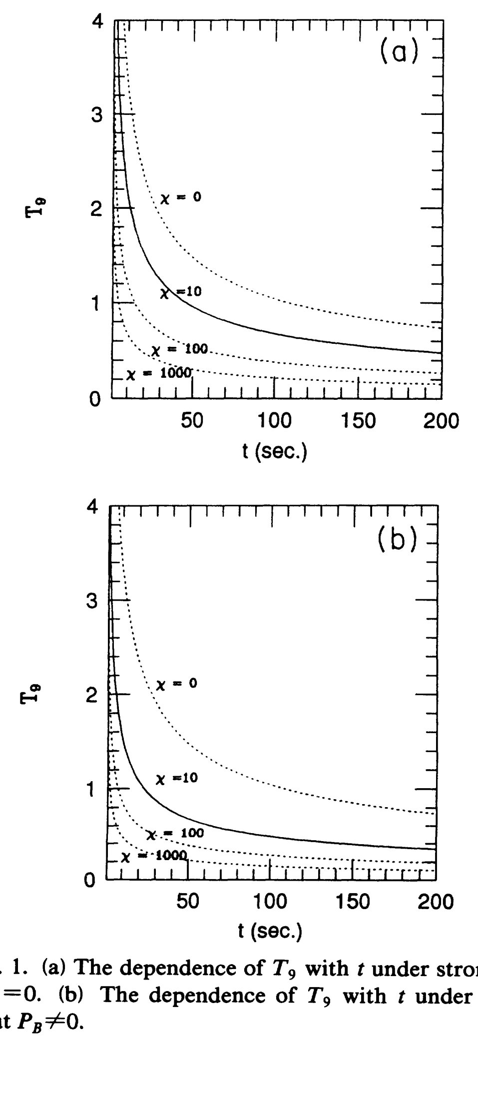

FIG. 1. (a) The dependence of T_9 with t under strong B field but $P_B=0$. (b) The dependence of T_9 with t under strong B field but $P_B \neq 0$.

$$T_9 = \sqrt{4/3}\chi^{-1/4}\kappa t^{-1/2} \text{ and } T_9 = \sqrt{2/3}\chi^{-1/4}\kappa t^{-1/2} . \tag{2.20}$$

The dependences of the temperature on the time t and the magnetic parameter χ, in the presence of a strong magnetic field, are shown in Figs. 1(a) and 1(b). These relations clearly indicate that the effect of the presence of a strong magnetic field on the expansion rate of the Universe is indeed significant.

Now we introduce another magnetic parameter $\gamma = B/(2B_c)$, where $B_c = m_e^2c^3/e\hbar = 4.414\times10^{13}$ G is the field strength where quantized cyclotron line effects begin to occur; we will refer to this as a quantum critical-field value [16] (see Appendix A). According to our definition of the factor χ, we have

$$\chi \equiv \frac{B^2/8\pi}{43\rho_\gamma/8} = \frac{B^2}{43\pi a T^4} \simeq 0.76\frac{\gamma^2}{T_{10}^4} , \tag{2.21}$$

where a is the blackbody constant and $T_{10} = T/(10^{10}\text{ K})$. If we use the "critical" temperature $T_c \sim 1.28\times10^{10}$ K (note: $nkT_c \sim B_c^2/8\pi$, k is the Boltzmann constant, $n \simeq 20\ \mathrm{T}^3$ is the number density of particles). Equation (2.21) can be reexpressed as

$$\chi \simeq 0.283(T_c/T)^4\gamma^2 . \tag{2.22}$$

III. NUMERICAL RESULTS AND A LIMIT ON THE FIELD STRENGTH AND FIELD COHERENCE LENGTH

We will now take into account the two independent effects of magnetic fields on reaction rates and on expansion rates calculated in Ref. [12] and Sec. II, respectively, in a reexamination of big bang nucleosynthesis. We use the new version of the Wagoner code developed by Kawano [16]. Specifically, we have replaced the old formulas in the code for both the reaction rates and the expansion rate, with the newly derived preliminary equations (3.8)–(3.11), in Ref. [12], and Eq. (2.18) above, and calculated the abundances. These are to be compared with observational data to determine the implied constraints on the strength and coherence scales of a primordial magnetic field. The observed abundances used are those summarized by Walker *et al.* in 1991 [2]. The main technique used is to adjust γ until the calculated abundances no longer match the observational data.

In order to obtain a limit on the strength of primordial magnetic fields and to focus on the effects on BBN explicitly, we have fixed all model parameters other than γ in our calculations: in particular, the neutron lifetime τ_n, the number of neutrino species N_ν, and the baryon to photon ratio $\eta \equiv n_b/n_\gamma$. For our purpose, we adopt the following values for these parameters [2]:

$$\tau_n = 889.6\pm2.9 \text{ s} , \quad N_\nu = 3 ,$$
$$2.8\times10^{-10} \le \eta \le 4.0\times10^{-10} . \tag{3.1}$$

Moreover, we have assumed nondegenerate neutrinos ($\phi_e=0$, $\phi_\nu=0$) (Thomas, Olive, and Schramm [17], Steigman and Kang [18]). For these choices, we then compute

TABLE I. Abundance of the elements ($n-{}^{8}$Li) in the presence of B fields but nondegenerate neutrinos.

Element				B Field			
$\phi_e=0$	$\gamma=10$	$\gamma=1$	$\gamma=0.5$	$\gamma=0.1$	$\gamma=0.05$	$\gamma=0.001$	$\gamma=0$
n	4.58×10^{-14}	2.22×10^{-14}	1.12×10^{-14}	9.15×10^{-15}	8.25×10^{-15}	5.81×10^{-15}	5.77×10^{-15}
^{1}H	0.441	0.495	0.624	0.716	0.751	0.778	0.777
^{2}H	1.52×10^{-3}	1.19×10^{-3}	7.5×10^{-4}	5.59×10^{-4}	5.02×10^{-4}	4.65×10^{-4}	4.65×10^{-4}
^{3}H	4.7×10^{-6}	3.64×10^{-6}	2.25×10^{-6}	1.67×10^{-6}	1.5×10^{-6}	1.39×10^{-6}	1.39×10^{-6}
^{3}He	6.78×10^{-5}	5.68×10^{-5}	4.15×10^{-5}	3.48×10^{-5}	3.28×10^{-5}	3.15×10^{-5}	3.15×10^{-5}
^{4}He	0.555	0.501	0.373	0.281	0.248	0.219	0.219
^{6}Li	4.55×10^{-13}	2.83×10^{-13}	1.03×10^{-13}	4.96×10^{-14}	3.7×10^{-14}	2.95×10^{-14}	2.93×10^{-14}
^{7}Li	1.41×10^{-8}	8.43×10^{-9}	2.66×10^{-9}	1.15×10^{-9}	8.23×10^{-10}	6.37×10^{-10}	6.37×10^{-10}
^{7}Be	2.03×10^{-13}	2.34×10^{-13}	2.89×10^{-13}	3.3×10^{-13}	3.43×10^{-13}	3.54×10^{-13}	3.54×10^{-13}
^{8}Li	1.45×10^{-13}	7.46×10^{-14}	1.54×10^{-14}	4.41×10^{-15}	2.58×10^{-15}	1.68×10^{-15}	1.68×10^{-15}

the primordial abundances numerically. Our computational results are displayed in both Tables I–III and Figs. 2–4. Each figure contains seven subfigures [(a)–(g)], which represent the abundances of the elements for different strengths of the primordial magnetic fields on a coherence scale of L(< the event horizon at that epoch $\sim 2\times10^{12}$ cm).

As expected, our calculations reveal that the abundances of the light elements can be dramatically affected by a strong magnetic field. For instance, if the magnetic fields on scales less than the horizon are as strong as $B\geq10^{13}$ G, the abundances of most elements (except for

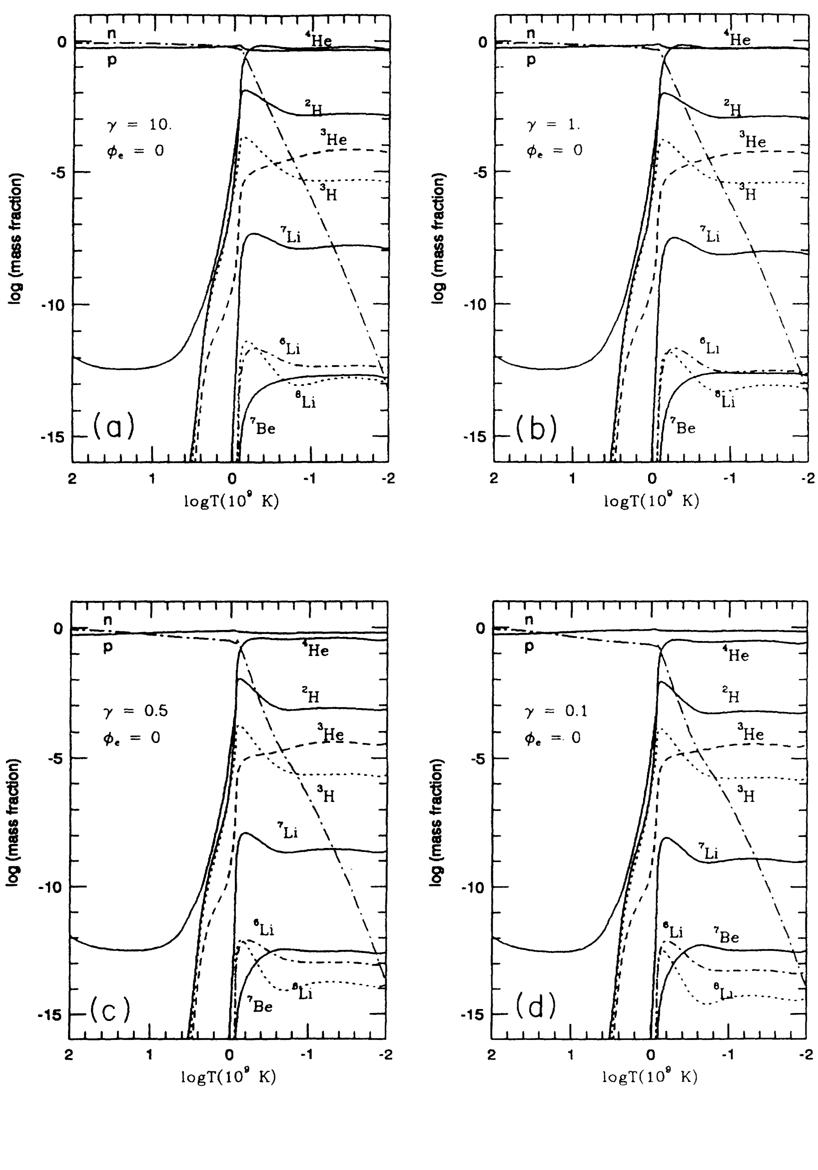

FIG. 2. Abundance of elements ($n-{}^{8}$Li) in the presence of B fields but nondegenerate neutrinos.

TABLE II. Abundance of the elements (^{8}B–^{12}C) in the presence of B fields but nondegenerate neutrinos.

Element	B Field						
$\phi_e=0$	$\gamma=10$	$\gamma=1$	$\gamma=0.5$	$\gamma=0.1$	$\gamma=0.05$	$\gamma=0.001$	$\gamma=0$
^{8}B	2.27×10^{-25}	2.02×10^{-25}	1.60×10^{-25}	1.40×10^{-25}	1.33×10^{-25}	1.29×10^{-25}	1.29×10^{-25}
^{9}Be	1.20×10^{-24}	3.11×10^{-25}	1.60×10^{-25}	1.40×10^{-25}	1.33×10^{-25}	1.29×10^{-25}	1.29×10^{-25}
^{10}B	1.05×10^{-20}	1.58×10^{-20}	2.65×10^{-20}	2.93×10^{-20}	2.79×10^{-20}	2.59×10^{-20}	2.55×10^{-20}
^{11}B	4.32×10^{-15}	1.91×10^{-15}	3.03×10^{-16}	7.79×10^{-17}	4.48×10^{-17}	2.88×10^{-17}	2.88×10^{-17}
^{11}C	2.27×10^{-25}	2.02×10^{-25}	1.60×10^{-25}	1.40×10^{-25}	1.33×10^{-25}	1.29×10^{-25}	1.29×10^{-25}
^{12}B	2.27×10^{-25}	2.02×10^{-25}	1.60×10^{-25}	1.40×10^{-25}	1.33×10^{-25}	1.29×10^{-25}	1.29×10^{-25}
^{12}C	1.39×10^{-13}	7.19×10^{-14}	1.48×10^{-14}	4.27×10^{-15}	2.50×10^{-15}	1.63×10^{-15}	1.60×10^{-15}

protons) increase manifestly. In particular, the concentrations of ^{2}H, ^{4}He, ^{6}Li, ^{7}Li, ^{9}Be, and ^{14}N are all enhanced. Some elements, for example, ^{4}He, ^{7}Li, ^{8}Li, ^{11}B, ^{12}C, ^{13}C, ^{14}C, and ^{15}N show sharp increases (i.e., $^4\mathrm{He}\geq 0.5$, $^7\mathrm{Li/H}\geq 8.43\times10^{-9}\sim10^{-8}$, $^8\mathrm{Li/H}\geq 7.5\times10^{-14}\sim10^{-13}$, $^{11}\mathrm{B/H}\sim10^{-15}$, etc.), as illustrated in Figs. I(a)–I(d) (I=2–4) and Tables I–III. On the contrary, if the magnetic fields are as weak as 10^{11} G, the emerging abundances of the light elements, according to our calculations, are only affected slightly and the variations become negligible. Figures I(e)–I(g) (I=2–4) display these outcomes.

For comparison, we have also computed the effects on the abundances resulting from the energy density of the magnetic field only (without any variations on the reaction rates). We find that the effects on the abundances of the light elements from reaction rates dominate the contributions from the energy density, unless the magnetic

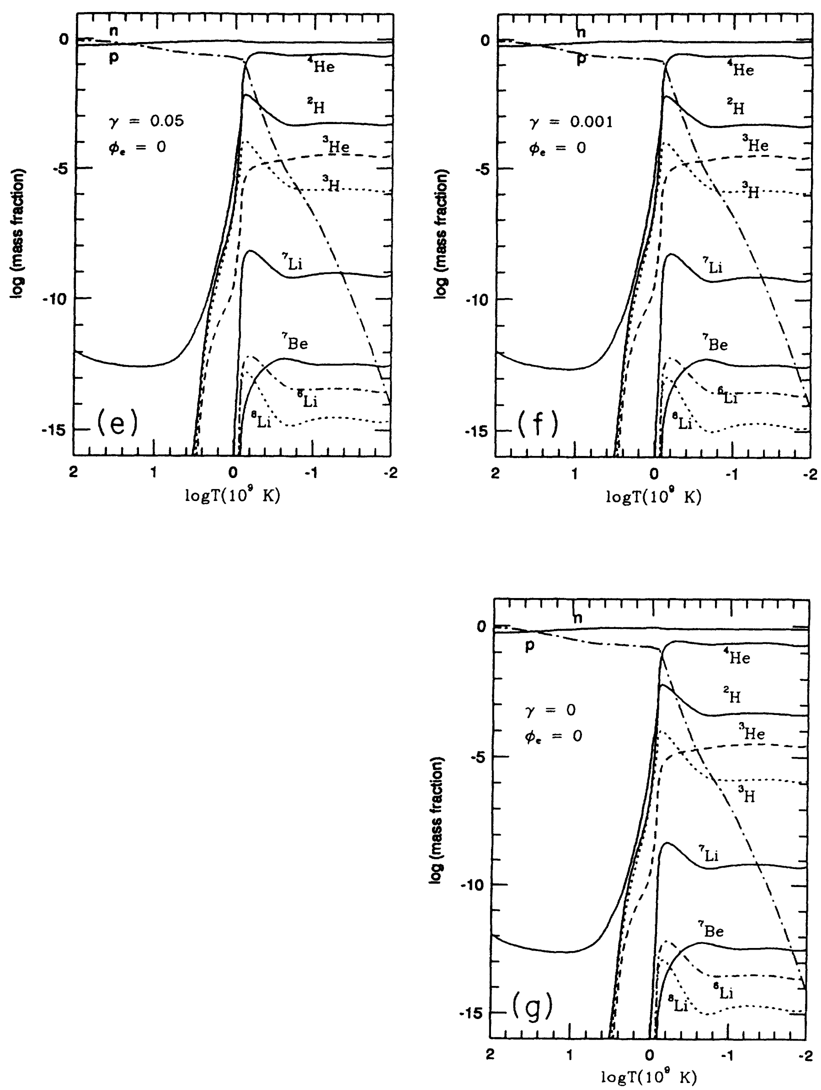

FIG. 2. (*Continued*).

TABLE III. Abundance of the elements (^{12}N$-^{15}$O) in the presence of B fields but nondegenerate neutrinos.

Element	B Field						
$\phi_e=0$	$\gamma=10$	$\gamma=1$	$\gamma=0.5$	$\gamma=0.1$	$\gamma=0.05$	$\gamma=0.001$	$\gamma=0$
^{12}N	2.27×10^{-25}	2.02×10^{-25}	1.60×10^{-25}	1.40×10^{-25}	1.33×10^{-25}	1.29×10^{-25}	1.29×10^{-25}
^{13}C	1.26×10^{-16}	7.30×10^{-16}	1.85×10^{-16}	5.96×10^{-17}	3.61×10^{-17}	2.38×10^{-17}	2.35×10^{-17}
^{13}N	2.27×10^{-25}	2.02×10^{-25}	1.60×10^{-25}	1.40×10^{-25}	1.33×10^{-25}	1.29×10^{-25}	1.29×10^{-25}
^{14}C	2.77×10^{-16}	1.31×10^{-16}	1.99×10^{-17}	4.15×10^{-18}	2.07×10^{-18}	1.17×10^{-18}	1.15×10^{-18}
^{14}N	4.76×10^{-18}	4.17×10^{-18}	2.06×10^{-18}	9.24×10^{-19}	6.18×10^{-19}	4.36×10^{-19}	4.30×10^{-19}
^{14}O	2.27×10^{-25}	2.02×10^{-25}	1.60×10^{-25}	1.40×10^{-25}	1.33×10^{-25}	1.29×10^{-25}	1.29×10^{-25}
^{15}N	1.94×10^{-20}	9.31×10^{-21}	1.55×10^{-21}	3.95×10^{-22}	2.26×10^{-22}	1.44×10^{-22}	1.44×10^{-22}
^{15}O	2.27×10^{-25}	2.02×10^{-25}	1.60×10^{-25}	1.40×10^{-25}	1.33×10^{-25}	1.29×10^{-25}	1.29×10^{-25}

field is very intense ($B > 10^{13}$ G). These results are shown in Figs. 5 and 6, respectively. Also, as is true for the standard BBN model, a high ratio n_b/n_γ will globally enhance the high A element abundances.

We note that the observed mass fraction of helium is approximately (Skillman 1993) [19]

$$Y_p^{\rm obs}=0.235\pm0.01 \ . \tag{3.2}$$

For other elements, the adopted primordial abundances are displayed in Table IV (Walker *et al.* 1991) [2]. By comparing the predicted abundances in the presence of B fields with the observational determinations (using particularly the abundance of ^{4}He), we can constrain the B field on scales less than the horizon. Figures I(a)–I(g) (I=2–4) and Tables I–III show the abundances of the elements in the presence of magnetic fields, with field

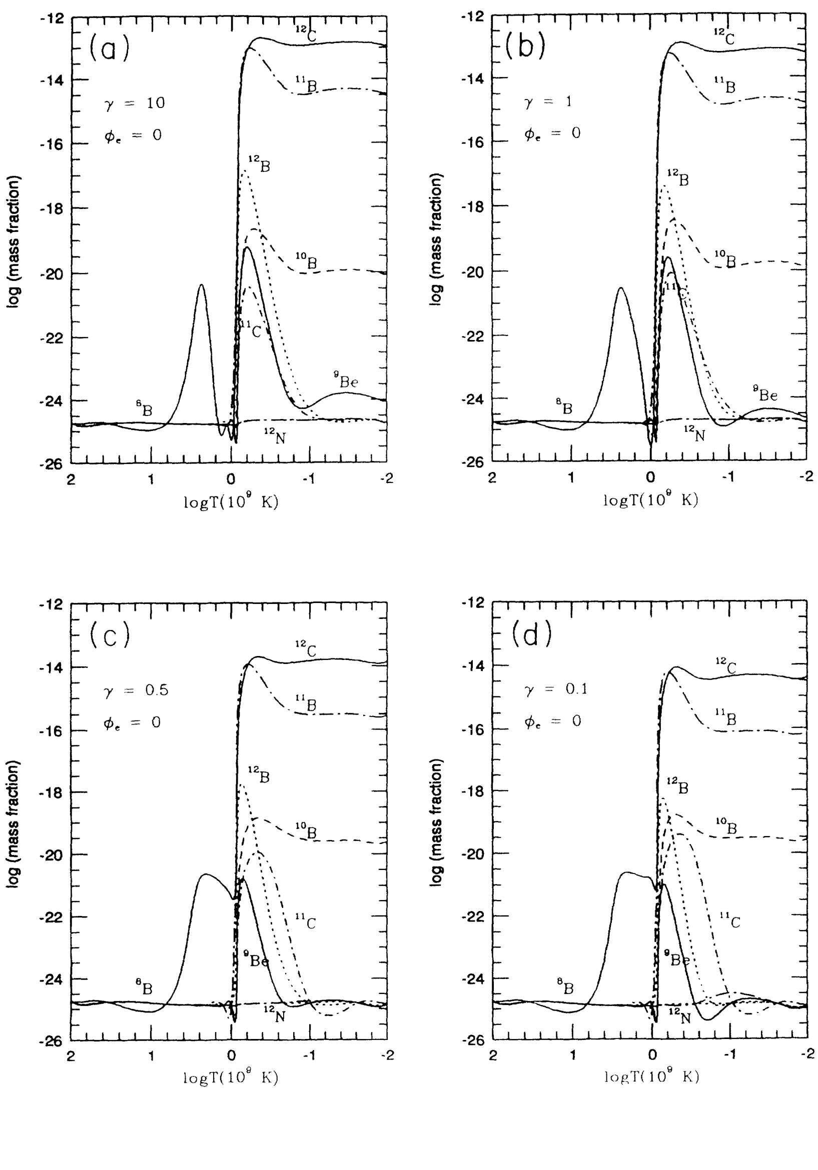

FIG. 3. Abundance of the elements (^{8}B$-^{12}$C) in the presence of B fields but nondegenerate neutrinos.

TABLE IV. Observed abundances.

Element	Where observed	Mass fraction
D	presolar	$1.8\times10^{-5}\leq y_2\leq 3.3\times10^{-5}(2\sigma)$
^{3}He	presolar	$1.3\times10^{-6}\leq y_3\leq 1.8\times10^{-5}(2\sigma)$
D+^{3}He	presolar	$3.3\times10^{-5}\leq y_{23}\leq 4.9\times10^{-5}(2\sigma)$
^{4}He	H II region	$Y_p=0.23\pm0.007$ ($\sigma_Y=0.009$)
^{7}Li	pre-population II	$12+\log(^7\mathrm{Li/H})\leq 2.15$ (95%CL)
^{6}Li	pre-population II	$\leq 0.1\,^7$Li
^{9}Be	Hyades	$\leq(1-2)\times10^{-11}$
$\geq{}^{12}X$		$\leq 10^{-12}$

strengths ranging from zero to $B=8.8\times10^{14}$ G. Through the comparison between our numerical calculations and the observational results, we ascertain that to keep the abundances of light elements compatible with the observations, the primordial magnetic fields at the BBN epoch ($\sim$1 min.) must satisfy the requirement

$$\gamma\leq 0.001\ ,\quad B\leq 10^{11}\ \mathrm{G}\ , \tag{3.3}$$

on scales less than the horizon. At this limit, the calculated abundances of light elements are shown in Table V.

Incorporating our above upper limit into Eqs. (2.21) and (2.5), we can further estimate the ratio of the energy

Fig. 3

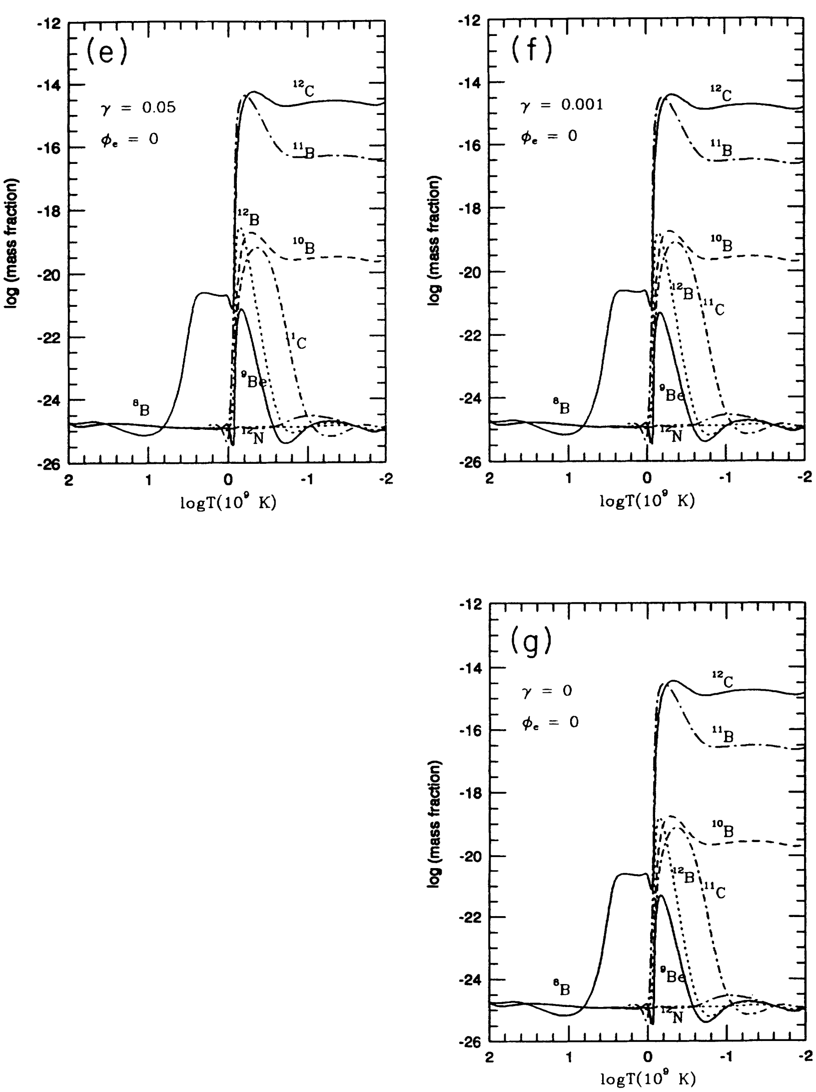

FIG. 3. (*Continued*).

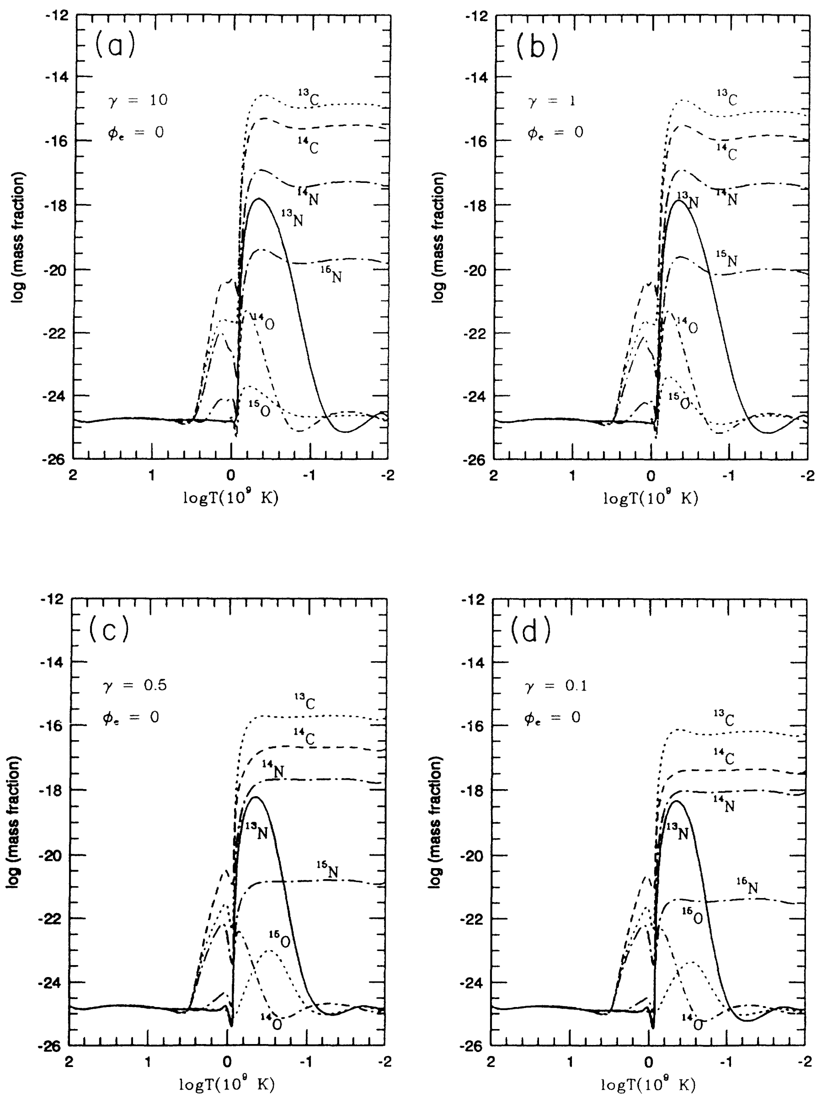

FIG. 4. Abundance of the elements (^{12}N$-^{15}$O) in the presence of B fields but nondegenerate neutrinos.

density between magnetic fields and cosmic radiations at the BBN epoch $T_{10} \sim 0.1$ as

$$\rho_B / \rho_\gamma \sim 4\% \ . \tag{3.4}$$

As to the evolution of the fields prior to recombination, a brief discussion is given in Appendix B, where an empirical estimate under certain conditions is presented.

IV. CONCLUSIONS AND FUTURE WORK

The effects of the magnetic fields on big bang nucleosynthesis and the cosmological expansion rate have been investigated generally in this paper for coherent and chaotic fields on scales smaller than the event horizon. An upper limit has been provided on the strength of the primordial magnetic field on scales smaller than the event horizon. Our results show that, in the framework of standard big bang nucleosynthesis, the maximum strength of the primordial magnetic field on scales greater than 10^4 cm but smaller than the horizon at the BBN epoch ($\sim 10^{12}$ cm), can only be 10^{11} G, which implies that the magnetic fields at recombination time would, in principle, be no stronger than 0.1 G. Moreover, in our calculations, we find that, of the two major effects of a primordial magnetic field, those arising from modification of the reaction rates will dominate those arising from modification of the expansion rate (or B-field energy density), unless the magnetic field is very intense ($B \gg 10^{13}$ G).

Finally, here we would like to make two comments:

A. Rate fluctuations (or inhomogeneous model)

If the magnetic field is not uniform or the size of the magnetic bubbles is much smaller than the horizon scale and the bubbles are disconnected from each other, the nuclear reaction rates will become inhomogeneous; i.e., the reaction rates inside a region will differ from those outside the region, even though the geometry of the Universe is still not affected. This would require that we introduce reaction rate fluctuations into the big bang calculation (similar to the introduction of the density fluctuations associated with the first-order QCD phase transition), and perform multizone calculations.

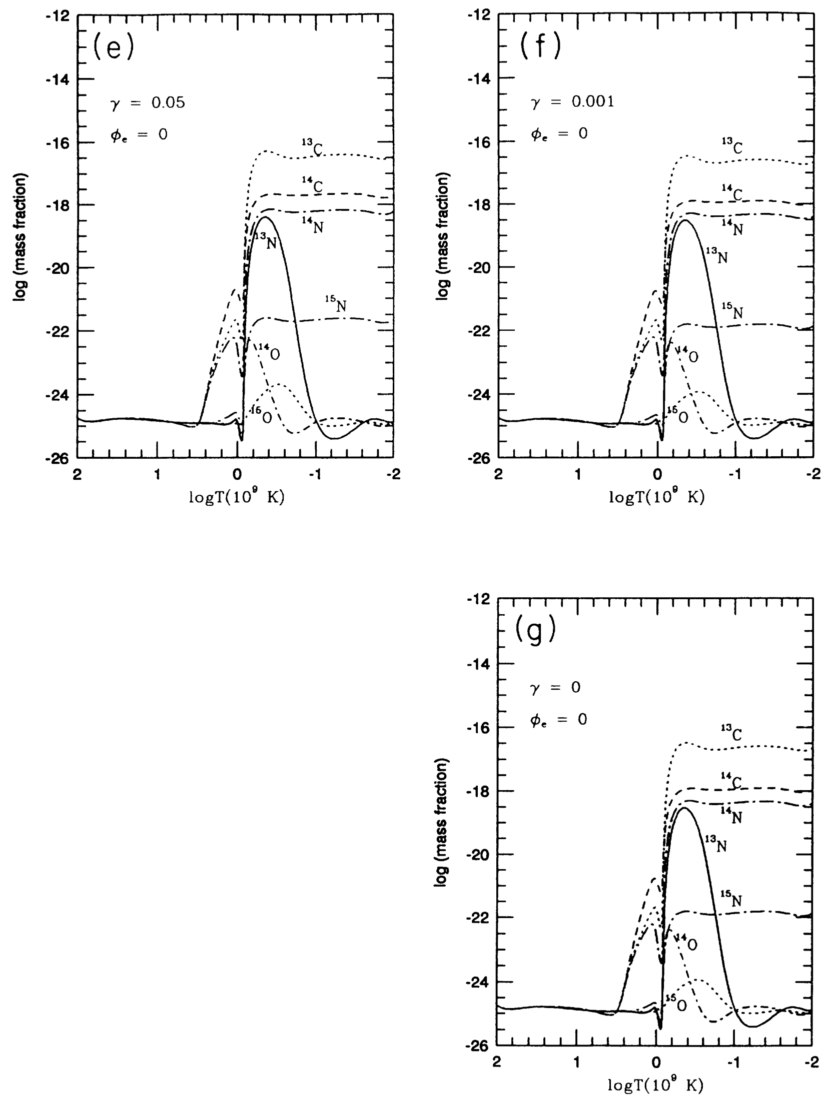

FIG. 4. (*Continued*).

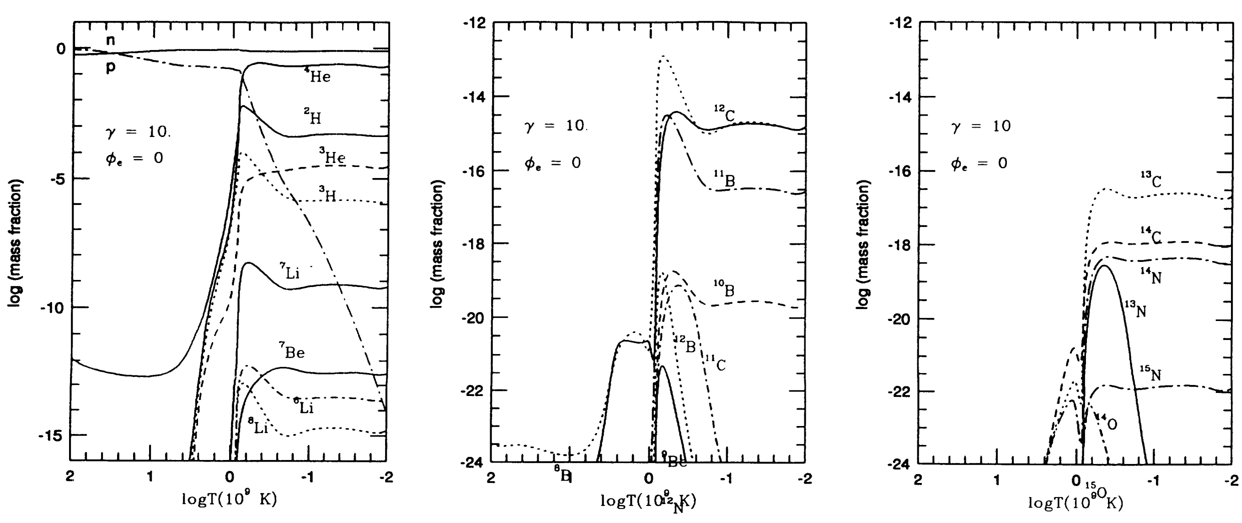

FIG. 5. Abundance of the elements affected by energy density of B field ($\gamma = 10$) only.

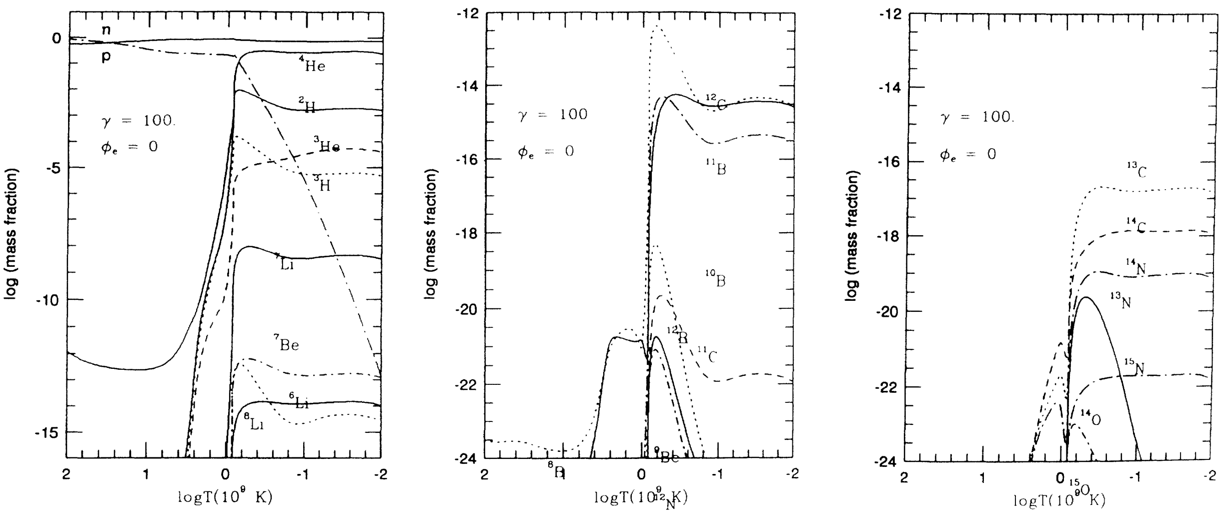

FIG. 6. Abundance of the elements affected by energy density of B field ($\gamma = 100$) only.

B. Anisotropy (or effects on geometry)

If the size of the magnetic bubbles were larger than the horizon scale, the effects on the geometry of the Universe would need to be examined, and the Robertson-Walker metric would need to be replaced by other metrics. In addition, an anisotropy of the Universe would result which might have important galactic consequences. (This has been explored by Thorne, 1967, but not with the more extensive network used here.)

A subsequent paper will examine these coherence scales.

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APPENDIX A: QUANTUM MECHANICAL CONSIDERATIONS

The applicability of classical electrodynamics to electrons requires that the wavelength of the synchrotron line in the presence of the magnetic field be much larger than the distances of order of $\hbar/m_e c$, for which classical electromagnetics will break down because of quantum effects [20]. Let us now derive the limit at which classical electrodynamics leads to internal contradictions [21].

Consider a system in which a charge e, with mass m, moves in a uniform magnetic field B. The synchrotron frequency ω of motion (angular frequency) can then be written as

$$\omega = ecB/E \quad \text{or} \quad \lambda \sim \frac{mc^2}{eB} , \tag{A1}$$

where E is the total energy of the charged particle and λ is the wavelength of the synchrotron line.

If quantum effects become important, we would have

$$E = mc^2 = \hbar\omega \quad \text{or} \quad \lambda \sim \hbar/mc . \tag{A2}$$

The magnetic field will then satisfy

$$B \sim B_c = \frac{\omega_c mc^2}{ec} = \frac{m^2 c^3}{e\hbar} = 4.4 \times 10^{13}\ \text{G} , \tag{A3}$$

which presents a limit where quantum electrodynamics plays a role. Therefore, Eq. (A3) could be considered in some sense as a quantum critical limit for an organized primordial magnetic field on large scales. For this critical value, we can estimate the corresponding temperature by letting

$$B_c^2/8\pi = nkT_c , \quad n \simeq 20\ \text{T}^3 K^{-3} , \tag{A4}$$

thus

$$T_c = \left[\frac{B_c^2}{16\pi k} \right]^{1/4} \simeq 1.28 \times 10^{10}\ \text{K} . \tag{A5}$$

Note that T_c is comparable to the temperature immediately prior to the BBN epoch. This means that, if the primordial magnetic field prior to the BBN epoch were to be as strong as $B > 10^{13}$ G, then its origin would probably be some quantum process in the early Universe. For such conditions, we would not have a field organized on a

TABLE V. The calculated abundances at limit $B \le 10^{11}$ G.

Light element	Abundance
D/H	$\le 4.65 \times 10^{-4}$
T/H	$\le 1.39 \times 10^{-6}$
^{3}He/H	$\le 3.17 \times 10^{-5}$
^{4}He	$= 0.22$
^{6}Li/H	$\le 2.95 \times 10^{-14}$
^{7}Li/H	$\le 6.374 \times 10^{-10}$
^{7}Be/H	$\le 3.54 \times 10^{-13}$
^{8}Li/H	$\le 1.68 \times 10^{-15}$

large scale as the Universe expanded out of the quantum domain, because most of the energy released would be converted into small-wavelength radiation rather than into an ordered magnetic field. It is interesting that our numerical calculations ($B < 10^{11}$ G) in Sec. III appear to have ruled this possibility out. Instead, it could be suggested that any primordial magnetic field must have been initially in the classical regime $B < 10^{11}$ G.

APPENDIX B: EMPIRICAL ESTIMATES UNDER OHMIC DISSIPATION

In the magnetohydrodynamic approximation, the magnetic-field lines either diffuse or are "frozen in" with the fluid motion, depending whether the magnetic Reynolds number is significantly smaller or larger than unity. If the magnetic stresses are smaller than the pressure in the system, ohmic dissipation through Spitzer conduction would dominate. Otherwise, the field lines would dissipate by magnetic stresses moving the matter around and reconnecting. In an expanding Universe, especially in the regime of radiation dominated, Alfven velocity is much smaller compared to the velocity of particles, then the ohmic dissipation seems more important than the stress dissipation. If this is the case (note: at BBN epoch, $p_B \ll p$), the cutoff scale separating diffusion or frozen for magnetic field, can be calculated as follows. The characteristic diffusion time for magnetic fields on a scale $l[\sim L(R/R_{\rm nuc})]$ is [22]

$$\tau_d \simeq 4\pi l^2 \sigma / c^2 \ . \tag{B1}$$

Here $R \equiv (1+z)^{-1}$ is the cosmological scale factor normalized to unity today, z is the redshift, L and $R_{\rm nuc}$ are, respectively, the coherence scale of the B field and the scale factor at the BBN epoch, $\sigma = (c^2/4\pi\eta)\ {\rm s}^{-1}$ is electrical conductivity of the Universe, where $\eta \simeq 4\times 10^{12} \ln\Lambda / T_e^{3/2}$ emu describes the Lorentz resistivity [23] (for electron-proton collisions) due to electron collisions with neutral hydrogen, T_e denotes the electron temperature in K, and Λ is a so-called Coulomb integral, which has a typical value around 15 ± 5 [23]. Taking account of the expansion time $\tau_{\rm exp}(\propto R^2)$ of the Universe (or Hubble time), the ratio of the characteristic diffusion time to the Hubble time can be calculated by

$$\frac{\tau_d}{t_{\rm exp}} \sim \frac{4\pi L^2}{c^2} \frac{\sigma}{t_{\rm nuc}} \ , \tag{B2}$$

where $t_{\rm nuc}$ is the time of the BBN epoch. Noting that scales that cannot dissipate at recombination ($\sim 10^{12}-10^{13}$ s) could not have dissipated earlier. Taking the calculation at recombination, we thus can estimate the ratio as

$$\frac{\tau_d}{\tau_{\rm exp}} \sim 17.5 \left[\frac{L}{\rm cm}\right]^2 \tau_{\rm exp}^{-3/4} \ , \tag{B3}$$

from which we find that if the coherence scale of the magnetic fields at the BBN epoch is larger than 10^4 cm ($\sim 10^{-6}$ of the horizon scale), the field will not be dissipated prior to recombination. We now take our constraints at the BBN epoch: $B \le 10^{11}$ G on scales of $L \ge 10^4$ cm (and $L \le 10^{12}$ cm), and evolve these to the recombination era by using conservation of the flux B ($\propto R^{-2}$), the implied magnetic field at the time of recombination, for ohmic dissipation only, would thus be

$$B_{\rm rec} \le 0.1 \ {\rm G} \ , \tag{B4}$$

coherent on scales of $L_{\rm domain} \ge 10^{10}$ cm (and $\le 10^{18}$ cm) (note: the result would be quite different if the dissipation is not dominated by *ohmic dissipation*). On scales much larger than the size of the magnetic domains (or bubbles), the physical mechanisms driving field generation are uncorrelated. To put this in current perspective, Hogan [24] has estimated with certain assumptions that such a field at recombination would correspond to an intergalactic field limit today of $\le 7\times 10^{-9}$ G.

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