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Doublet-triplet splitting in a supersymmetric SO(10) model without fine-tuning

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We propose a supersymmetric SO(10) model in which light Higgs doublets are obtained without fine-tuning of parameters, while giving grand-unification-scale masses to their colored partners. Unlike the previous attempts, the gauge coupling constant does not blow up below the Planck scale in the present model.

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The presence of a large hierarchy between two mass scales $M_{\rm GUT} \sim 10^{16}$ GeV and $m_W \sim 10^2$ GeV is a basic assumption in grand unified theories (GUT's) [1]. However, this requires an extremely precise adjustment of parameters in all orders of the perturbation theory to maintain such a large hierarchy between two scales. Supersymmetry (SUSY) is well known to provide a partial solution to this problem [2]. That is, it protects the hierarchy built at the tree level against radiative corrections. The tree-level hierarchy is, however, not explained by the SUSY itself. For example, one has to fine tune unrelated coupling constants and masses to produce light Higgs doublets in minimal SUSY GUT's [5]. Though there have been attempts to realize the triplet-doublet splitting without fine-tuning of parameters [6,7], all models proposed so far become strongly interacting below the Planck scale, and hence the perturbative description of GUT's is broken down before reaching supergravity. In this note we construct a SUSY SO(10) model, which the desired tree-level hierarchy is naturally built in, and the gauge coupling constant does not blow up below the Planck scale.

Let us start with the SUSY SO(10) model proposed by Dimopoulos and Wilczek [3] and Srednicki [4]. The Higgs sector consists of three kinds of chiral supermultiplets, A(45), S(54) and $\phi_1(10)$, $\phi_2(10)$ of SO(10). The superpotential is assumed to be

$$W_0 = m_A {\rm Tr} A^2 + m_S {\rm Tr} S^2 + \lambda_A {\rm Tr} A^2 S + \lambda_S {\rm Tr} S^3 + g_\phi\, {}^t\phi_1 A \phi_2, \qquad (1)$$

where $A(45)$ is represented as an antisymmetric 10×10 matrix, $S(54)$ as a traceless symmetric 10×10 matrix, and $\phi_1(10)$, $\phi_2(10)$ as 10-dimensional row vectors. One finds a vacuum state satisfying

$$\phi_1 = \phi_2 = 0,$$
$$A_{ij} = \begin{pmatrix} i\sigma_2 & & & & \\ & i\sigma_2 & & & \\ & & i\sigma_2 & & \\ & & & 0 & \\ & & & & 0 \end{pmatrix} a_0,$$
$$S_{ij} = \begin{pmatrix} 2\times 1\!\!1 & & & & \\ & 2\times 1\!\!1 & & & \\ & & 2\times 1\!\!1 & & \\ & & & -3\times 1\!\!1 & \\ & & & & -3\times 1\!\!1 \end{pmatrix} s_0, \qquad (2)$$

where

$$i\sigma_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad 1\!\!1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad (3)$$

and

$$a_0^2 = (5/\lambda_A)(2 m_S s_0 - 3\lambda_S s_0^2), \quad s_0 = -m_A/2\lambda_A. \qquad (4)$$

Here A_{ij} and S_{ij} are written in 2×2 block diagonal notation. A crucial point here is that the $SU(2)_L$-doublet components remain massless, while the last term in Eq. (1) gives large masses to the colored components of ϕ_1 and ϕ_2, avoiding the too-fast proton decay via dimension five operators [8].

Unfortunately, this model is unsatisfactory. First of all, there remains a gauge symmetry, $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ unbroken. Second, there are four massless chiral multiplets of Higgs $SU(2)_L$-doublets, which destroy the successful unification of three gauge coupling constants in the minimal SUSY standard model [9]. The second problem can be easily solved by putting a large mass $M\phi_2\phi_2$. To solve the first problem we introduce $\psi_H(16)$, $\bar\psi_H(16^*)$ and a singlet $\chi(1)$ Higgs chiral multiplets.

We add new terms to the original superpotential Eq. (1) as[1]

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[1] ψ_H and $\bar\psi_H$ cannot couple to S_{ij}. We drop the terms such as ${}^t\phi_1\phi_1$ by hand, as in the previous models [6,7].

$$W = W_0 + g\bar{\psi}_H\sigma_{ij}\psi_H A_{ij} + g_\chi \left(\bar{\psi}_H\psi_H - \mu^2\right)\chi + M\, {}^t\phi_2\phi_2. \tag{5}$$

The vacuum states now satisfy the equations

$$-2m_A A_{ij} - \lambda_A \left(A\cdot S + S\cdot A\right)_{ij} + g\bar{\psi}_H\sigma_{ij}\psi_H = 0,$$

$$2m_S S_{ij} + \lambda_A \left(\left(A^2\right)_{ij} - \frac{\delta_{ij}}{10}\mathrm{Tr}A^2\right) + 3\lambda_S \left(\left(S^2\right)_{ij} - \frac{\delta_{ij}}{10}\mathrm{Tr}S^2\right) = 0,$$

$$\left(g\sigma_{ij}A_{ij} + g_\chi\chi\right)\psi_H = 0,$$

$$g_\chi \left(\bar{\psi}_H\psi_H - \mu^2\right) = 0. \tag{6}$$

The original solution Eq. (2) does not satisfy the above condition Eq. (6) for the vacuum due to the presence of the $g\bar{\psi}_H\sigma_{ij}\psi_H A_{ij}$ term. We show this more explicitly, making an ansatz

$$A_{ij} = \begin{pmatrix} ai\sigma_2 & & & & \\ & ai\sigma_2 & & & \\ & & ai\sigma_2 & & \\ & & & bi\sigma_2 & \\ & & & & bi\sigma_2 \end{pmatrix},$$

$$S_{ij} = \begin{pmatrix} 2\times 1\!\!1 & & & & \\ & 2\times 1\!\!1 & & & \\ & & 2\times 1\!\!1 & & \\ & & & -3\times 1\!\!1 & \\ & & & & -3\times 1\!\!1 \end{pmatrix} s,$$

$$\psi_H^t = \left(\uparrow\otimes\uparrow\otimes\uparrow\otimes\uparrow\otimes\uparrow\right)v. \tag{7}$$

See the Appendix for the notation of the spinor in SO(10). Then, Eq. (6) leads to

$$\begin{aligned} -2m_A a - 4\lambda_A a s + g v^2 &= 0,\\ -2m_A b + 6\lambda_A b s + g v^2 &= 0,\\ 10 m_S s - \lambda_A\left(a^2 - b^2\right) - 15\lambda_S s^2 &= 0,\\ \left(g\left(6a + 4b\right) + g_\chi\chi\right)v &= 0,\\ v^2 - \mu^2 &= 0. \end{aligned} \tag{8}$$

The second equation clearly shows that $b = 0$ is not a solution unless $g = 0$.

We now find a solution to Eq. (8) perturbatively assuming $g \ll 1$ (we take later[2] $g \sim 10^{-6}$), and all mass parameters except for of SUSY-breaking ones are $O(M_{\rm GUT}) \sim 10^{16}$ GeV. Up to $O(g)$ we find

$$\begin{aligned} a &= a_0 + g\frac{5\mu^2}{4\lambda_A^2 a_0^2}\left(m_S - 3\lambda_S s_0\right),\\ b &= g\,\mu^2/5m_A,\\ s &= s_0 + g\,\mu^2/4\lambda_A a_0,\\ v &= \mu,\\ \chi &= -g\,6a_0/g_\chi, \end{aligned} \tag{9}$$

where a_0, s_0 are given in Eq. (4). Note that b vanishes in the limit $g \to 0$.

In this vacuum we find a mass matrix for two pairs of Higgs doublet chiral multiplets:

$$\left(\bar{H}_1\bar{H}_2\right)\begin{pmatrix} 0 & g_\phi b\\ g_\phi b & M\end{pmatrix}\begin{pmatrix} H_1\\ H_2\end{pmatrix}. \tag{10}$$

Note that $\phi_i(10)$ $(i = 1, 2)$ consist of $SU(2)_L$ doublets H_i and $\bar{H}_i$ and $SU(3)_C$ triplets H_i^C and $\bar{H}_i^C$. When $g_\phi b \ll M$, one finds the seesaw hierarchy [10] in two eigenvalues as

$$\begin{aligned} m_1 &\sim (g_\phi b)^2/M,\\ m_2 &\sim M. \end{aligned} \tag{11}$$

To obtain the correct electroweak breaking scale, we impose $m_1 \sim m_W$: namely,

$$g_\phi b \sim \sqrt{M m_W}\,. \tag{12}$$

On the other hand, the colored Higgs triplet masses are given by

$$\left(\bar{H}_1^C\bar{H}_2^C\right)\begin{pmatrix} 0 & g_\phi a\\ g_\phi a & M\end{pmatrix}\begin{pmatrix} H_1^C\\ H_2^C\end{pmatrix}. \tag{13}$$

To avoid too-fast proton decay via dimension five operators [11] one must take

$$(g_\phi a)^2/M \geq M_{\rm GUT} \sim 10^{16}\ \text{GeV}. \tag{14}$$

Combining Eq. (12) and Eq. (14), we obtain the consistent hierarchy between Higgs doublet and triplet masses if

$$b/a \sim \sqrt{m_W/M_{\rm GUT}} \simeq 10^{-7}. \tag{15}$$

This desired ratio is realized by taking $g \sim 10^{-6}$, as seen from Eq. (9).

Since there is only one pair of Higgs doublets at the weak scale, we have the well-known problem in quark and lepton masses in SO(10) [12]. Namely, there is too much mass degeneracy leading to the vanishing quark mixing. To avoid this problem we furthermore introduce three families of chiral matter multiplets $\varphi_\alpha(10)$ $(\alpha = 1\text{–}3)$. The superpotential responsible for the quark and lepton masses is

$$W_{\rm Yukawa} = h_\alpha\psi_\alpha\psi_\alpha\phi_1 + f_{\alpha\beta}\psi_\alpha\varphi_\beta\psi_H + \tfrac{1}{2}M'_\alpha\varphi_\alpha\varphi_\alpha, \tag{16}$$

where $\psi_\alpha(16)$ $(\alpha = 1\text{–}3)$ are three families of usual quark and lepton multiplets and we have used the basis where the Yukawa coupling ϕ_1 to ψ_α and the mass term of φ_α are diagonal. Since the only vacuum expectation value in this superpotential is $\langle\psi_H(16)\rangle = \mu$, there is an SU(5) invariance, and we use SU(5) language hereafter. The vacuum expectation value $\langle\psi_H\rangle \neq 0$ induces mixing be-

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[2]We do not think that this coupling $g \sim 10^{-6}$ is unnaturally small, since the Yukawa coupling constant of electron is also of the same order of this coupling. We hope that such small coupling constants may be explained by some underlying physics.

tween $\mathbf{5^*}$ of ψ_α and of φ_β through the second term in Eq. (16), and massless $\mathbf{5^*}$ of three families are mixture of these $\mathbf{5^*}$'s. This generates the desired disparity between Yukawa couplings of[3] $\phi_1[5^*]$ and $\phi_1[5]^3$ to $\mathbf{10}\times\mathbf{10}$ and $\mathbf{5^*}\times\mathbf{10}$ and yields nonvanishing quark mixing angles.

To see this explicitly, we take M'_α universally as M'. After the vacuum expectation value of ψ_H is inserted to the superpotential in Eq. (16), the relevant parts are written using SU(5) decomposition as

$$\begin{aligned} W^{(1)}_{\rm Yukawa} = h_\alpha(&\psi_\alpha[10]\psi_\alpha[10]\phi_1[5] + 2\psi_\alpha[10]\psi_\alpha[5^*]\phi_1[5^*] \\ &+2\psi_\alpha[1]\psi_\alpha[5^*]\phi_1[5]) \\ &+ (f_{\alpha\beta}\mu\psi_\alpha[5^*] + M'\varphi_\beta[5^*])\,\varphi_\beta[5]. \end{aligned} \tag{17}$$

When the matrix $f_{\alpha\beta}\mu$ is diagonalized by biunitary transformation as

$$f_{\alpha\beta}\mu = \left(U^\dagger M_D V\right)_{\beta\alpha}, \tag{18}$$

where $M_D \equiv \mathrm{diag}(M_{D1}, M_{D2}, M_{D3})$, the last term in Eq. (17) becomes

$$\left(\varphi[5]U^\dagger\right)_\alpha \left\{M_{D\alpha}\left(V\psi[5^*]\right)_\alpha + M'\left(U\varphi[5^*]\right)_\alpha\right\} . \tag{19}$$

This shows that three massless matter fields ($\psi^{\rm light}[5^*_\alpha]$) and three heavy ones ($\psi^{\rm heavy}[5^*_\alpha]$) are given by

$$\begin{aligned} \psi^{\rm light}_\alpha[5^*] &= \frac{1}{\sqrt{M'^2+M^2_{D\alpha}}}\{M'(V\psi[5^*])_\alpha \\ &\qquad -M_{D\alpha}(U\varphi[5^*])_\alpha\} , \\ \psi^{\rm heavy}_\alpha[5^*] &= \frac{1}{\sqrt{M'^2+M^2_{D\alpha}}}[M_{D\alpha}(V\psi[5^*])_\alpha \\ &\qquad +M'(U\varphi[5^*])_\alpha] . \end{aligned} \tag{20}$$

Therefore, the Yukawa coupling of the Higgs fields, $\phi_1[5]$ and $\phi_1[5^*]$, to the light matter fields are given by

$$\begin{aligned} W^{(2)}_{\rm Yukawa} &= h_\alpha\psi_\alpha[10]\psi_\alpha[10]\phi_1[5] \\ &\quad +2h_\alpha V^\dagger_{\alpha\beta}\frac{M'}{\sqrt{M'^2+M^2_{D\beta}}}\psi_\alpha[10]\psi^{\rm light}_\beta[5^*]\phi_1[5^*] \\ &\quad +2h_\alpha V^\dagger_{\alpha\beta}\frac{M'}{\sqrt{M'^2+M^2_{D\beta}}}\psi_\alpha[1]\psi^{\rm light}_\beta[5^*]\phi_1[5]. \end{aligned} \tag{21}$$

Unless $V_{\alpha\beta}$ is proportional to an unit matrix, we have non-vanishing quark and lepton mixing angles.[4]

It should be stressed that the SO(10) gauge coupling constant remains in the perturbative regime below Planck mass scale although it is not asymptotic free. This is the crucial difference from the previous attempts [6,7], where the gauge coupling constant blows up around 10^{17} GeV.

Two comments are in order:

(1) Because of the small coupling $g\sim 10^{-6}$, both $\psi_H[5^*]$, $\bar\psi_H[5]$ and $\psi_H[10]$, $\bar\psi_H[10^*]$ remain at 10^{10} GeV. This is very dangerous, since the color-triplet component in $\psi_H[5^*]$ causes a fast proton decay via dimension six operators.[5] We need a mechanism to give $\psi_H[5^*]$ and $\bar\psi_H[5]$ a larger mass (at least 10^{11} GeV). One example is to assume a nonrenormalizable interaction such as

$$W = \frac{1}{M_{\rm Pl}}\left(\psi_H\psi_H\right)\left(\bar\psi_H\bar\psi_H\right), \tag{22}$$

which produces the mass of $\psi_H[5^*]$ and $\bar\psi_H[5]$ of order of $\langle\psi_H\rangle^2/M_{\rm Pl}\sim 10^{13}$ GeV. This operator may be induced by some underlying physics at the Planck scale. Since all Higgs multiplets put in the intermediate scale are complete multiplets of SU(5), the success of the gauge coupling constant unification is not affected.

(2) Since right-handed neutrinos in $\psi_\alpha(16)$ are massless, we have Dirac neutrinos with masses of the order of lepton masses. Thus, one needs some mechanism to generate right-handed neutrino masses. This problem is also easily solved by invoking nonrenormalizable new interactions such as

$$W = \frac{\kappa_{\alpha\beta}}{M_{\rm Pl}}\left(\psi_\alpha\psi_\beta\right)_{126}\left(\psi_H\psi_H\right)_{126}, \tag{23}$$

which produces large Majorana masses of right-handed neutrinos of order of $\frac{\kappa_{\alpha\beta}}{M_{\rm Pl}}\langle\psi_H\psi_H\rangle\sim\kappa\times 10^{13}$GeV. This is consistent with the Mikheyev-Smirnov-Wolfenstein (MSW) explanation of the solar neutrino deficit [13]. Furthermore, we can see that the Dirac mass matrix of neutrinos is proportional to the lepton mass matrix and then the simultaneous diagonalization of both mass matrices is possible [see Eq. (21)]. This means that the neutrino mixing angle comes only from the off-diagonal elements of the Majorana mass matrix for the right-handed neutrinos and hence the mixing angles are expected to be small.[6] The MSW solution indeed suggests a very small mixing angle [13].

In summary, we have extended the SO(10) SUSY-GUT proposed by Srednicki to a realistic one. The hierarchical doublet-triplet splitting has been achieved without fine-tuning among the parameters. This model requires a single small coupling constant $g\sim 10^{-6}$, and the weak-scale

[3]The numbers in square brackets denote dimensions of the SU(5) representations.

[4]One can further induce disparity between lepton and down-quark mass matrices by adding terms $k_{\alpha\beta}{}^t\varphi_\alpha A\varphi_\beta$.

[5]These $\psi_H[5^*]$, $\bar\psi_H[5]$ do not cause the proton decay via dimension five operators, since $\bar\psi_H[5]$ has no Yukawa coupling to the matter fields.

[6]For example, the mixing angle $\theta_{e\mu}$ between ν_e and ν_μ is given by $\theta_{e\mu}\simeq(m_e/m_\mu)(C/A)$, where A and C are defined by the Majorana mass matrix of right-handed neutrinos $M=\begin{pmatrix} A & -C \\ -C & B\end{pmatrix}$. Therefore, the mixing angle is expected to be small unless there is a hierarchy in the Majorana mass, $C/A\sim(m_\mu/m_e)$.

mass of Higgs doublets is given by the seesaw mechanism in the Higgs sector. Moreover, the gauge coupling constant remains small up to the Planck scale, and hence this model can be regarded as a realistic low-energy effective theory of the supergravity.

APPENDIX: NOTATION OF SPINORS IN SO(10)

In this appendix, we summarize our notation of spinors in SO(10) and Γ matrices. The gamma matrices Γ^i, ($i = 1, \ldots, 10$) are 32×32 matrices, which can be written as a tensor product of five Pauli-matrices. We take

$$\begin{aligned}
\Gamma_1 &= \sigma_1 \otimes 1 \otimes 1 \otimes 1 \otimes 1,\\
\Gamma_2 &= \sigma_2 \otimes 1 \otimes 1 \otimes 1 \otimes 1,\\
\Gamma_3 &= \sigma_3 \otimes \sigma_1 \otimes 1 \otimes 1 \otimes 1,\\
\Gamma_4 &= \sigma_3 \otimes \sigma_2 \otimes 1 \otimes 1 \otimes 1,\\
\Gamma_5 &= \sigma_3 \otimes \sigma_3 \otimes \sigma_1 \otimes 1 \otimes 1,\\
\Gamma_6 &= \sigma_3 \otimes \sigma_3 \otimes \sigma_2 \otimes 1 \otimes 1,\\
\Gamma_7 &= \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_1 \otimes 1,\\
\Gamma_8 &= \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_2 \otimes 1,\\
\Gamma_9 &= \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_1,\\
\Gamma_{10} &= \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_2,
\end{aligned}$$

and, $\Gamma_{11} = -i\Gamma_1\Gamma_2\Gamma_3\Gamma_4\Gamma_5\Gamma_6\Gamma_7\Gamma_8\Gamma_9\Gamma_{10} = \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_3$.

The spinors can be also represented as a tensor product of five two-component spinors, $\uparrow$ or $\downarrow$. The **16** representation is defined as a spinor ψ satisfying $\frac{1+\Gamma_{11}}{2}\psi = \psi$, and **16*** as $\bar{\psi}$ satisfying $\frac{1-\Gamma_{11}}{2}\bar{\psi} = \bar{\psi}$. Therefore, a spinor belongs to **16** when it has even number of $\downarrow$.

After the breaking of SO(10), the matter $\psi_\alpha(16)$ has the following decomposition. It is convenient to use parentheses to separate the first three and the last two two-component spinors, since $SU(3)_C$ operates only on the first three spinors, and $SU(2)_L$ on the last two spinors:

$$\begin{aligned}
\nu_R^c &: (\uparrow \otimes \uparrow \otimes \uparrow) \otimes (\uparrow \otimes \uparrow),\\
e_R^c &: (\uparrow \otimes \uparrow \otimes \uparrow) \otimes (\downarrow \otimes \downarrow),\\
u_R^c &: (\downarrow \otimes \downarrow \otimes \uparrow) \otimes (\uparrow \otimes \uparrow), (\downarrow \otimes \uparrow \otimes \downarrow) \otimes (\uparrow \otimes \uparrow), (\uparrow \otimes \downarrow \otimes \downarrow) \otimes (\uparrow \otimes \uparrow),\\
d_L &: (\uparrow \otimes \uparrow \otimes \downarrow) \otimes (\uparrow \otimes \downarrow), (\uparrow \otimes \downarrow \otimes \uparrow) \otimes (\uparrow \otimes \downarrow), (\downarrow \otimes \uparrow \otimes \uparrow) \otimes (\uparrow \otimes \downarrow),\\
u_L &: (\uparrow \otimes \uparrow \otimes \downarrow) \otimes (\downarrow \otimes \uparrow), (\uparrow \otimes \downarrow \otimes \uparrow) \otimes (\downarrow \otimes \uparrow), (\downarrow \otimes \uparrow \otimes \uparrow) \otimes (\downarrow \otimes \uparrow),\\
d_R^c &: (\downarrow \otimes \downarrow \otimes \uparrow) \otimes (\downarrow \otimes \downarrow), (\downarrow \otimes \uparrow \otimes \downarrow) \otimes (\downarrow \otimes \downarrow), (\uparrow \otimes \downarrow \otimes \downarrow) \otimes (\downarrow \otimes \downarrow),\\
\nu_L &: (\downarrow \otimes \downarrow \otimes \downarrow) \otimes (\downarrow \otimes \uparrow),\\
e_L &: (\downarrow \otimes \downarrow \otimes \downarrow) \otimes (\uparrow \otimes \downarrow).
\end{aligned}$$

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