Generational seesaw mechanism in $[\mathrm{SU}(6)]^3 \times Z_3$

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For the gauge group $[\mathrm{SU}(6)]^3 \times Z_3$ which unifies nongravitational forces with flavors we analyze the generational seesaw mechanism. At the tree level we get $m_{\nu_e} = 0$ and $m_{\nu_\tau} \sim m_{\nu_\mu} \sim M_L^2/M_H$, where $M_L \sim 10^2$ GeV and $M_H \geq 10^5$ GeV are the weak and horizontal interactions mass scales, respectively. The right-handed neutrinos get Majorana masses M_R of the order of the scale where $\mathrm{SU}(2)_R$ is broken. A low energy exotic neutral lepton with a mass of the order of M_H^2/M_R is predicted. Radiative corrections can produce $m_{\nu_e} \neq 0$, four orders of magnitude smaller than the other neutrino masses.

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I. THE MODEL

In Refs. [1,2] we presented a model based on the gauge group $\mathrm{SU}(6)_L \otimes \mathrm{SU}(6)_C \otimes \mathrm{SU}(6)_R \times Z_3 \equiv G$ which unifies the known nongravitational interactions with flavors. Since G includes the so-called horizontal interactions, it leads to predictions for some masses and mixing angles of ordinary fermions. In G the three known families belong to a single irreducible representation, each family being defined by the dynamics of $\mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \otimes \mathrm{U}(1)_{Y_{(B-L)}}$, the left-right symmetric (LRS) extension of the standard model.

The seesaw mechanism [3] expresses the smallness of the neutrino masses in terms of the "large" masses of some other neutral fermions. This mechanism is very simple to implement for a single neutrino and, as far as we know, it has been implemented in a consistent way only for two families [4].

In this Brief Report we carry through in detail the diagonalization of the 18×18 electrically neutral mass matrix which appears in the context of our model. The detailed results obtained here confirm the qualitative results inferred in the second paper of Ref. [2]. Although our result does not provide a natural generational extension of the seesaw mechanism for three families, it provides a scenario where 3 of the 18 neutral particles in the model, mainly members of left-handed doublets, get tiny masses.

$\mathrm{SU}(6)_C$ in G is the color group which consists of three hadronic and three leptonic colors; it includes the $\mathrm{SU}(3)_C \otimes \mathrm{U}(1)_{Y_{(B-L)}}$ subgroup of the LRS. $\mathrm{SU}(6)_L \otimes \mathrm{SU}(6)_R$ is the flavor group which includes the $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R$ gauge group of the LRS.

The gauge bosons and Weyl fermions in G are clearly defined in [2]; let us specify here some of them. The known fermions are included in $\psi(108)_L = \psi(6,1,\bar{6})_L + \psi(1,\bar{6},6)_L + \psi(\bar{6},6,1)_L$, which has the particle content

$$\psi(\bar{6},6,1)_L = \begin{pmatrix} d_x^{-1/3} & d_y^{-1/3} & d_z^{-1/3} & E_1^- & L_1^0 & T_1^- \\ u_x^{2/3} & u_y^{2/3} & u_z^{2/3} & E_1^0 & L_1^+ & T_1^0 \\ s_x^{-1/3} & s_y^{-1/3} & s_z^{-1/3} & E_2^- & L_2^0 & T_2^- \\ c_x^{2/3} & c_y^{2/3} & c_z^{2/3} & E_2^0 & L_2^+ & T_2^0 \\ b_x^{-1/3} & b_y^{-1/3} & b_z^{-1/3} & E_3^- & L_3^0 & T_3^- \\ t_x^{2/3} & t_y^{2/3} & t_z^{2/3} & E_3^0 & L_3^+ & T_3^0 \end{pmatrix}_L \equiv \psi_a^\alpha, \tag{1}$$

where the rows (columns) represent color (flavor) degrees of freedom; $E_i^{-,0}$, $L_i^{+,0}$, and $T_i^{-,0}$, $i = 1,2,3$, stand for leptonic fields with electrical charges as indicated, and d, u, s, c, b, and t stand for the corresponding quark fields, eigenstates of G [x, y, and z stand for $\mathrm{SU}(3)_C$ color indices].

$\psi(1,\bar{6},6)_L \equiv \psi_\alpha^A$ stands for the 36 fields charge conjugated to the fields in $\psi(\bar{6},6,1)_L$, while $\psi(6,1,\bar{6})_L$ represents 36 exotic Weyl leptons, 9 with positive electric charges, 9 with negative (the charge conjugated to the positive ones), and 18 are neutrals. As is clear we are using $a,b,...,A,B,...,\alpha,\beta,... = 1,...,6$ as $\mathrm{SU}(6)_L$, $\mathrm{SU}(6)_R$, and $\mathrm{SU}(6)_C$ tensor indices respectively.

The most economical set of Higgs fields and vacuum expectation values (VEV's) which break the symmetry from G down to $\mathrm{SU}(3)_C \otimes \mathrm{U}(1)_{\mathrm{EM}}$ and at the same time give a tree level mass of order $M_L \sim 10^2$ GeV to the top quark (what we call the *modified horizontal survival hypothesis*) is [2]

$$\phi_1 = \phi(675) = \phi^{[A,B]}_{1,[a,b]} + \phi^{[\alpha,\beta]}_{1,[A,B]} + \phi^{[a,b]}_{1,[\alpha,\beta]}\ , \tag{2}$$

with VEV's $\langle\phi_1\rangle \equiv M$ in the directions $[a,b] = -[b,a] = [1,6] = -[2,5] = -[3,4]$, $[A,B]$ similar to $[a,b]$ and $[\alpha,\beta] = -[\beta,\alpha] = [5,6]$,

$$\phi_2 = \phi(1323) = \phi^{\{A,B\}}_{2,\{a,b\}} + \phi^{\{\alpha,\beta\}}_{2,\{A,B\}} + \phi^{\{a,b\}}_{2,\{\alpha,\beta\}}\ , \tag{3}$$

with VEV's $\langle\phi_2\rangle \equiv M'$ in the directions $\{a,b\} = \{b,a\} = \{1,4\} = -\{2,3\}, \{A,B\}$ similar to $\{a,b\}$ and $\{\alpha,\beta\} = \{\beta,\alpha\} = \{4,5\}$,

$$\phi_3 = \phi'(675) = \phi^{[A,B]}_{3,[a,b]} + \phi^{[\alpha,\beta]}_{3,[A,B]} + \phi^{[a,b]}_{3,[\alpha,\beta]}\ , \tag{4}$$

with VEV's $\langle\phi^{[A,B]}_{3,[a,b]}\rangle = \langle\phi^{[a,b]}_{3,[\alpha,\beta]}\rangle = 0$, and $\langle\phi^{[\alpha,\beta]=[4,6]}_{3,[A,B]=[4,6]}\rangle \equiv M_R$, and

$$\phi_4 = \phi(108) = \phi^A_{4,\alpha} + \phi^\alpha_{4,a} + \phi^a_{4,A}, \tag{5}$$

with VEV's such that $\langle\phi^A_\alpha\rangle = \langle\phi^\alpha_a\rangle = 0$ and $\langle\phi^a_A\rangle \equiv M_L \sim 10^2$ GeV, with values different from zero only in the directions $\langle\phi^2_2\rangle = \langle\phi^2_4\rangle = \langle\phi^2_6\rangle = \langle\phi^4_2\rangle = \langle\phi^4_4\rangle = \langle\phi^4_6\rangle = \langle\phi^6_2\rangle = \langle\phi^6_4\rangle = \langle\phi^6_6\rangle = M_L$.

According to the analysis presented in Ref. [2], $\langle\phi_1\rangle + \langle\phi_2\rangle$ breaks G down to the LRS group, and $\langle\phi_1\rangle + \langle\phi_2\rangle + \langle\phi_3\rangle$ breaks G down to $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. Also, since we are not interested in studying *CP* violation, we will assume throughout this paper that $\langle\phi_i\rangle$, $i = 1,2,3,4$, are real numbers.

II. GENERATIONAL SEESAW MECHANISM

The Higgs fields and VEV's presented in the previous section imply the Yukawa-type mass terms

$$\psi^\alpha_a\psi^\beta_b\langle\phi^{[a,b]}_{1,[\alpha,\beta]} + \phi^{\{a,b\}}_{2,\{\alpha,\beta\}}\rangle + \psi^A_\alpha\psi^B_\beta\langle\phi^{[\alpha,\beta]}_{1,[A,B]} + \phi^{\{\alpha,\beta\}}_{2,\{A,B\}} + \phi^{[\alpha,\beta]}_{3,[A,B]}\rangle + \psi^a_A\psi^b_B\langle\phi^{[A,B]}_{1,[a,b]} + \phi^{\{A,B\}}_{2,\{a,b\}}\rangle + \sum_{\alpha,a,A=1}^{6}\psi^\alpha_a\psi^A_\alpha\langle\phi^a_A\rangle + \text{H.c.} \tag{6}$$

The analysis of the tree level mass matrices for the quarks and the charged leptons produced by this expression was done already in Ref. [2]. In order to diagonalize the mass matrix for the neutral leptons, let us write it first in the basis defined by $\mathbf{N}_0 \equiv (E^0_1, E^0_2, E^0_3, T^0_1, T^0_2, T^0_3, L^{0c}_1, L^{0c}_2, L^{0c}_3, E^{0c}_1, E^{0c}_2, E^{0c}_3, T^{0c}_1, T^{0c}_2, T^{0c}_3, L^0_1, L^0_2, L^0_3)_L$, where the upper c symbol denotes the fields in $\psi(1,\bar{6},6)_L$. In this basis the mass matrix has the form

$$\mathcal{M}_{\text{tree}} = \begin{pmatrix}
0&0&0&0&0&0&0&0&0&M_L&M_L&M_L&0&0&0&0&-M'&0\\
0&0&0&0&0&0&0&0&0&M_L&M_L&M_L&0&0&0&M'&0&0\\
0&0&0&0&0&0&0&0&0&M_L&M_L&M_L&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&M_L&M_L&M_L&0&0&M\\
0&0&0&0&0&0&0&0&0&0&0&0&M_L&M_L&M_L&0&-M&0\\
0&0&0&0&0&0&0&0&0&0&0&0&M_L&M_L&M_L&M&0&0\\
0&0&0&0&0&0&0&0&0&0&M'&0&0&0&M&0&0&0\\
0&0&0&0&0&0&0&0&0&-M'&0&0&0&-M&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&M&0&0&0&0&0\\
M_L&M_L&M_L&0&0&0&0&-M'&0&0&0&0&0&0&0&0&0&0\\
M_L&M_L&M_L&0&0&0&M'&0&0&0&0&0&0&0&M_R&0&0&0\\
M_L&M_L&M_L&0&0&0&0&0&0&0&0&0&0&-M_R&0&0&0&0\\
0&0&0&M_L&M_L&M_L&0&0&M&0&0&0&0&0&0&0&0&0\\
0&0&0&M_L&M_L&M_L&0&-M&0&0&0&-M_R&0&0&0&0&0&0\\
0&0&0&M_L&M_L&M_L&M&0&0&0&M_R&0&0&0&0&0&0&0\\
0&M'&0&0&0&M&0&0&0&0&0&0&0&0&0&0&0&0\\
-M'&0&0&0&-M&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&M&0&0&0&0&0&0&0&0&0&0&0&0&0&0
\end{pmatrix}$$

Since the gauge bosons responsible for the horizontal transitions in the model get masses of order [2] M and M', and since the horizontal transitions include flavor-changing neutral currents, we must impose the experimental [5] constrain $M, M' > 100$ TeV. The other mass parameter in $\mathcal{M}_{\text{tree}}$, M_R, is the mass scale which characterizes the breaking of $SU(2)_R$ and produces mass terms to the right-handed neutrinos; since in most of the models it is responsible for the seesaw mechanism, $M_R \sim 10^{11,12}$ GeV. So it is natural to think that we can diagonalize $\mathcal{M}_{\text{tree}}$ under the assumption $M_R \gg M \sim M' \gg M_L$ by the use of a double perturbation theory.

According to the survival hypothesis [6] we identify the left- and right-handed neutrino states as the massless states which appear in the limit $M_L = M_R = 0$. In this limit $\mathcal{M}_{\text{tree}}$ is a rank-12 matrix with the zero eigenvalues associated with the following eigenvectors, (i) $[E^0_3;\ (ME^0_2 - M'T^0_3)/V;\ (ME^0_1 - M'T^0_2)/V]_L$, with $V = (M^2 + M'^2)^{1/2}$, which we define as $(\nu_1, \nu_2, \nu_3)_L$, due to the fact that they are a basis for the physical neutrinos ν_e, ν_μ, ν_τ [they are $SU(2)_L$ doublets, $SU(2)_R$ singlets]; (ii) $[E^{0c}_3;\ (ME^{0c}_2 - M'T^{0c}_3)/V;\ (ME^{0c}_1 - M'T^{0c}_2)/V]_L$, which we define as $(\nu^c_1, \nu^c_2, \nu^c_3)_L$, due to the fact that they are a basis for the right-handed neutrinos $\nu^c_e, \nu^c_\mu, \nu^c_\tau$ [they are $SU(2)_L$ singlets, $SU(2)_R$ doublets].

This suggests the use of a new basis defined by $\mathbf{N}_1 =$

(ν_1, ν_2, ν_3, N_1, N_2, N_3, L_1^c, L_2^c, L_3^c, ν_1^c, ν_2^c, ν_3^c, N_1^c, N_2^c, N_3^c, L_1, L_2, $L_3)_L$, where $N_1 = T_1^0$, $N_2 = (MT_3^0 + M'E_2^0)/V$, $N_3 = (MT_2^0 + M'E_1^0)/V$, $L_i = L_i^0$, $i = 1,2,3$, $N_1^c = T_1^{0c}$, etc.

A. First perturbation

From now on let $M = M' \equiv M_H$. In the first approximation given by $M_L = 0$ and in the basis $\mathbf{N}_1$, the squared mass matrix takes the form $\mathcal{M}_1^2 = \mathcal{M}_{1D}^2 + \mathcal{M}_{1N}^2$ where $\mathcal{M}_{1D}^2$ is an 18×18 diagonal matrix with entries given by $\mathcal{M}_{1D}^2 = \mathrm{diag}(0,0,0,M_H^2, 2M_H^2, 2M_H^2, 2M_H^2, 2M_H^2, M_H^2, M_R^2, M_R^2, M_R^2/\sqrt{2}, M_H^2, M_R^2{+}2M_H^2, M_R^2/2{+}2M_H^2, 2M_H^2, 2M_H^2, M_H^2)$ and $\mathcal{M}_{1N}^2$ is an 18×18 nondiagonal orthogonal matrix with the only entries different from zero given by $(\mathcal{M}_{1N}^2)_{14,7} = (\mathcal{M}_{1N}^2)_{7,14} = \sqrt{2}M_H M_R$ and $(\mathcal{M}_{1N}^2)_{15,12} = (\mathcal{M}_{1N}^2)_{12,15} = -M_R^2/2$.

$\mathcal{M}_1^2$ is a rank-15 matrix with the three zero eigenvalues related to the three left-handed neutrinos. We diagonalize it perturbatively under the assumption $M_R \gg M_H$. The list of eigenvalues and eigenvectors of $\mathcal{M}_1^2$ is, up to second order in perturbation theory, the following (where our expansion parameter is $\delta = M_H/M_R$): ν_1, ν_2, and ν_3 have eigenvalue zero; N_1, L_3^c, N_1^c, and L_3 have eigenvalue M_H^2; N_2, N_3, L_1, and L_2 have eigenvalue $V^2 = 2M_H^2$; ν_2^c has eigenvalue M_R^2; $\sqrt{2}(\delta - 3\delta^3)L_1^c + (1-\delta^2)N_2^c \equiv L_{1s}$ has eigenvalue $M_R^2(1+4\delta^2) \equiv \alpha_1^2$; $(1-\delta^2)L_1^c - \sqrt{2}(\delta - 3\delta^3)N_2^c \equiv N_{2s}$ has eigenvalue $2M_R^2\delta^4 \equiv \alpha_2^2$; $\frac{1}{\sqrt{2}}(1-\delta^2)\nu_3^c - \frac{1}{\sqrt{2}}(1+\delta^2)N_3^c \equiv \nu_{3s}$ has eigenvalue $M_R^2(1+\delta^2) \equiv \beta_1^2$; $\frac{1}{\sqrt{2}}(1+\delta^2)\nu_3^c + \frac{1}{\sqrt{2}}(1-\delta^2)N_3^c \equiv N_{3s}$ has eigenvalue $M_R^2\delta^2 \equiv \beta_2^2$; $(\delta+\delta^3/2)L_2^c + (1-\delta^2/2)\nu_1^c \equiv L_{2s}$ has eigenvalue $M_R^2(1+\delta^2) = \beta_1^2$; $(1-\delta^2/2)L_2^c - (\delta+\delta^3/2)\nu_1^c \equiv \nu_{1s}$ has eigenvalue $M_R^2\delta^2 = \beta_2^2$.

These eigenvectors define a new basis which we denote as $\mathbf{N}_2$. Notice in particular that the state N_{2s} has an eigenvalue of order M_H^4/M_R^2 which is a seesaw eigenvalue produced by the two mass scales M_H and M_R. This state is the only intermediate mass exotic predicted by this model.

B. Second perturbation

In the basis $\mathbf{N}_2$, $\mathcal{M}_1^2$ is diagonal up to second order in perturbation theory. To diagonalize $\mathcal{M}_1$ we apply an orthogonal transformation to $\mathbf{N}_2$ and obtain the new basis $\mathbf{N}_2'$ given by $\mathbf{N}_2' = (\nu_1$, ν_2, ν_3, N_1', N_2', N_3', $L_1'^c$, $L_2'^c$, $L_3'^c$, $\nu_1'^c$, $\nu_2'^c$, $\nu_3'^c$, $N_1'^c$, $N_2'^c$, $N_3'^c$, L_1', L_2', $L_3')_L$, where ν_i, $i = 1,2,3$, and ν_2^c are the same as in $\mathbf{N}_1$, but $N_1' = (N_1 + L_3)/\sqrt{2}$, $N_2' = (N_2 + L_1)/\sqrt{2}$, $N_3' = (N_3 + L_2)/\sqrt{2}$, $L_1' = (N_1 - L_3)/\sqrt{2}$, $L_2' = (N_3 - L_2)/\sqrt{2}$, $L_3' = (N_2 - L_1)/\sqrt{2}$, $L_1'^c = L_{1s}$, $L_2'^c = (L_{2s} + \nu_{3s})/\sqrt{2}$, $L_3'^c = (L_3^c + N_1^c)/\sqrt{2}$, $\nu_1'^c = (L_{2s} - \nu_{3s})/\sqrt{2}$, $\nu_3'^c = (N_{3s} + \nu_{1s})/\sqrt{2}$, $N_1'^c = (L_3^c - N_1^c)/\sqrt{2}$, $N_2'^c = N_{2s}$, and $N_3'^c = (N_{3s} - \nu_{1s})/\sqrt{2}$.

In this last basis the mass matrix $\mathcal{M}_{\mathrm{tree}}$ can be written as $\mathcal{M}_{\mathrm{tree}} = \mathcal{M}_D' + \mathcal{V}_m'$, where $\mathcal{M}_D'$ is an 18×18 diagonal mass matrix given by $\mathcal{M}_D' = \mathrm{diag}(0,0,0,M_H, \sqrt{2}M_H, -\sqrt{2}M_H, \alpha_1, \beta_1, M_H, -\beta_1, -M_R, -\beta_2, -M_H, -\alpha_2, \beta_2, -M_H, \sqrt{2}M_H, -\sqrt{2}M_H)$, and $\mathcal{V}_m'$ is a perturbation to $\mathcal{M}_D'$ proportional to M_L which can be written as

$$\mathcal{V}_m' = \begin{pmatrix} 0_{6\times 6} & A_{6\times 9} & 0_{6\times 3} \\ A_{9\times 6} & 0_{9\times 9} & B_{9\times 3} \\ 0_{3\times 6} & B_{3\times 9} & 0_{3\times 3} \end{pmatrix}, \quad A_{6\times 9} = \begin{pmatrix} A_{3\times 9} \\ B_{3\times 9} \end{pmatrix},$$
$$A_{9\times 6} = \begin{pmatrix} A_{9\times 3} & B_{9\times 3} \end{pmatrix}, \tag{7}$$

where $0_{n\times m}$ are zero matrices with n rows and m columns, and $A_{3\times 9} = A_{9\times 3}^T$ and $B_{3\times 9} = B_{9\times 3}^T$ are given by

$$A_{3\times 9} = \frac{M_L}{\sqrt{2}} \begin{pmatrix} \kappa_1 & \eta_1 & 0 & \eta_2 & 1 & \eta_3 & 0 & \kappa_2 & \eta_4 \\ 0 & \eta_5 & -1/\sqrt{2} & \eta_6 & \sqrt{2} & \eta_7 & 1/\sqrt{2} & 0 & \eta_8 \\ 0 & \eta_5 & -1/\sqrt{2} & \eta_6 & \sqrt{2} & \eta_7 & 1/\sqrt{2} & 0 & \eta_8 \end{pmatrix}, \tag{8}$$

$$B_{3\times 9} = \frac{M_L}{2} \begin{pmatrix} \kappa_1 & -1 & 1 & 1 & -1 & -\delta^2 & -1 & \kappa_2 & -\delta^2 \\ \sqrt{2}\kappa_1 & -\delta^2 & 1/\sqrt{2} & \sqrt{2} & 0 & \rho_1 & -1/\sqrt{2} & \sqrt{2}\kappa_2 & \rho_2 \\ \sqrt{2}\kappa_1 & -\delta^2 & 1/\sqrt{2} & \sqrt{2} & 0 & \rho_1 & -1/\sqrt{2} & \sqrt{2}\kappa_2 & \rho_2 \end{pmatrix}, \tag{9}$$

respectively, where $\kappa_1 = 1 - \delta^2$, $\kappa_2 = -\sqrt{2}\delta(1 - 3\delta^2)$, $\eta_1 = 1 - 3\delta^2/2$, $\eta_2 = 1 + \delta^2/2$, $\eta_3 = 1 - \delta - \delta^3/2$, $\eta_4 = -(1+\delta+\delta^3/2)$, $\eta_5 = \sqrt{2}(1 - 3\delta^2/4)$, $\eta_6 = \delta^2/2\sqrt{2}$, $\eta_7 = (1-\delta+\delta^2-\delta^3/2)/\sqrt{2}$, $\eta_8 = -(1+\delta+\delta^2+\delta^3/2)/\sqrt{2}$, $\rho_1 = (1-\delta-\delta^2-\delta^3/2)/\sqrt{2}$, and $\rho_2 = -(1+\delta-\delta^2+\delta^3/2)/\sqrt{2}$.

Now $\mathcal{V}_m'$ produces corrections to the eigenvalues and eigenvectors of $\mathcal{M}_D$ of the order of $M_L/M \equiv \xi, M_L/M_R \equiv \xi'$ and smaller (higher orders). These corrections are important only for the smaller eigenvalues, i.e., for the eigenvalues corresponding to ν_1, ν_2, ν_3, and $N_2'^{,c}$. For these states we use matrix perturbation theory [7]. The second order perturbative corrections to the 3×3 mass matrix for the states (ν_1, ν_2, ν_3) follow from the diagonalization of the mass matrix

$$C_{n,n'} = \sum_{m=1}^{9} \frac{(A_{3\times 9})_{n,m}(A_{9\times 3})_{m,n'}}{(\mathcal{M}_D)_{m+3,m+3}}. \tag{10}$$

Then, according to Eq. (8),

$$C = \begin{pmatrix} \theta_3 & \theta_1 & \theta_1 \\ \theta_1 & \theta_2 & \theta_2 \\ \theta_1 & \theta_2 & \theta_2 \end{pmatrix}, \tag{11}$$

where $\theta_1 = -M_L(\xi + 3\delta\xi')\sqrt{2}$, $\theta_2 = M_L\xi'$, and $\theta_3 = 6M_L\delta^2\xi' \sim 0$. C is a rank-2 matrix with the two eigenvalues different from zero given approximately by $M_L(\xi + 3\delta\xi' \pm \xi') \simeq M_L\xi = M_L^2/M_H$.

The state associated with $N_2^{0,c}$ is not degenerate, and so a straightforward second order perturbative calculation gives a mass correction $= M_L\xi'/\sqrt{2} = M_L^2/\sqrt{2}\, M_R$ which is smaller than its original value (M_H^2/M_R).

III. CONCLUSIONS

In the context of the model presented in Refs. [1,2] and for the mass hierarchy $M_R \gg M_H \geq 10^5$ GeV $\gg M_L \sim 10^2$ GeV we have diagonalized the 18×18 mass matrix for the neutral leptons. The original mass matrix includes only tree level mass terms and the diagonalization was done by using a double perturbation theory, with corrections up to second order in the parameters.

Our analysis gave four neutral leptons with small masses. Three of them are mainly members of left-handed doublets, one with zero mass and two with seesaw masses of order M_L^2/M_H. The fourth is mainly member of a right-handed doublet, left-handed singlet, with a seesaw mass of order M_H^2/M_R.

Under the assumption that the neutrinos do not oscillate we can identify the real neutrino states as the mass eigenstates (neutrino oscillations can be analyzed in the context of our model, but it is a tougher matter because it requires to identify simultaneously the known charged lepton states, which in turn requires a consistent treatment of the mass radiative corrections). We obtain at the tree level $m_{\nu_e} = 0$ and $m_{\nu_\mu} \simeq m_{\nu_\tau} \simeq M_L^2/M_H$.

These results and the direct experimental upper limit [8] $m_{\nu_\mu} < 0.27$ MeV imply $M_H > 10^7$ GeV, which is consistent with the experimental constraint [5] $M_H > 10^5$ GeV. If instead of using the direct experimental limit for m_{ν_μ} we use the more stringent cosmological constraint [9] $m_{\nu_\mu} \sim 10^2$ eV, we get $M_H \sim 10^{11}$ GeV, a value just allowed by the renormalization group equation analysis [2]. Also the experimental lower limit of 10 GeV for the mass of any exotic neutral lepton imposes, via the result for the $N_2'^c$ mass, the relation $M_R < M_H^2/10$ GeV$< 10^{13}$ GeV.

When the first order mass radiative corrections are included we expect modification in our results of the order of ϵ^2/M_H, where $\epsilon \sim 1$ GeV is the order of the radiative masses expected in our model (the radiative corrections must produce masses for all the known charged particles but the t quark). Then we should expect $m_{\nu_e} \sim \epsilon^2/M_H$, three or four orders of magnitude smaller than the other two neutrino masses.

Looking at our results we realize immediately that the prejudice of using $M_R \sim 10^{11,12}$ GeV in order to get the seesaw mechanism for the neutrinos is not well founded in the context of our model, because for the hierarchy $M_R \gg M_H \gg M_L$ it is the intermediate mass scale M_H that is responsible for the seesaw mechanism. If we repeat the calculations for the mass hierarchy $M_H \gg M_R \gg M_L$, then we get results very similar to the previous ones with the roles of M_H and M_R interchanged; that is, it is now the intermediate mass scale $M_R \sim 10^7$ GeV that is responsible for the seesaw mechanism. The fact is that with three mass scales, and due to the particular form of the matrix $\mathcal{M}_{\text{tree}}$, the two larger mass scales produce seesaw mechanisms, but obviously, the seesaw mechanism associated with the lower mass scale dominates.

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