# Is the Gepner three generation model phenomenologically viable?

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The low-energy spectrum of the Gepner three generation model constructed from the discrete series of the $N=2$ superconformal theory below the intermediate scale is studied. It is found that, aside from the usual standard model leptons and quarks, there appear four Higgs doublets, six exotic light neutral particles, and a pair of light color Higgs triplets. Although the proton is stable, the presence of these new particles and the difficulty for the up and down quarks and the first two lepton generations to grow nonzero masses may rule out this model phenomenologically. Despite this discouraging result we do find that the nonrenormalizable interactions can provide a viable mechanism to solve the lepton-quark mass hierarchy problem, due to the large VEV growth of the standard model singlet fields at the intermediate symmetry-breaking scale.

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## I. INTRODUCTION

The last few years have witnessed a remarkable intertwining between two classes of viable phenomenological models of the heterotic string theory [1]: those based on the Calabi-Yau compactifications [2] on the one hand and those based on the $N=2$ superconformal field theories [3] on the other. It is generally believed now that a superconformal construction actually corresponds to a mirror pair of the Calabi-Yau models with the Hodge numbers $h^{1,1}$ and $h^{2,1}$ of the internal manifold interchanged, and vice versa [4]. According to this correspondence, the family in one model is in fact the mirror family in the other. In particular, there exists a three generation model, the Gepner-Schimmrigk model [5,6], that enjoys constructions from both approaches. (We call the three generation model constructed from the Calabi-Yau manifold based on $CP^2\times CP^3$ [5] the Schimmrigk model, and that constructed from the tensorial product of one level 1 and three level 16 models from the discrete series of the $N=2$ superconformal theory [6] the Gepner model, which in turn corresponds to the symmetric Schimmrigk model.) Given the fact that all the Yukawa couplings are rigorously known [7–9] and the possible nonvanishing nonrenormalizable interactions have been studied [10] for the Gepner model [6], it is thus surprising that there exist in the literature very few works [11–13] devoted to the low-energy phenomenological aspects of the model. It is our purpose to study the low-energy phenomenology of this model in this paper.

The Gepner model has a gauge group $E_6$ which is broken into $[SU(3)]^3$ via flux breaking at the compactification scale $M_C$. Of two possible flux-breaking patterns [12], we are interested in the one which produces nine lepton, six mirror-lepton, three quark, three antiquark and no mirror-quark or mirror antiquark generations [7,11,13]. These massless fields can be represented by their $SU(3)_C\times SU(3)_L\times SU(3)_R$ quantum numbers as $9L(1,3,\bar{3})\oplus 6\bar{L}(1,\bar{3},3)\oplus 3Q(3,\bar{3},1)\oplus 3Q^c(\bar{3},1,3)$ where $L\oplus Q\oplus Q^c$ furnish the 27's, and $\bar{L}\oplus\bar{Q}\oplus\bar{Q}^c$, the $\overline{27}$'s, of $E_6$. In addition to the chiral generations, there are 61 gauge singlets, $\phi_i$, $i=1,\ldots,61$. In terms of the standard model quantum numbers, these massless multiplets are $L=[l=(\nu,e);e^c;H;H';\nu^c,N]$, $Q=[q=(u,d);H_3\equiv D]$, and $Q^c=[q^c=(u^c,d^c);H_3'\equiv D^c]$, where $l$, $H$, $H'$, and $q^a$ are $SU(2)_L$ doublets, $e^c$, $u^c$, and $d^c$ are the conjugate singlets, $D$ and $D^c$ are the color Higgs triplets of the SU(5) 5 and $\bar{5}$ representations, $\nu^c$ is an SU(5) singlet, and $N$ an O(10) singlet.

Recently, we have studied the intermediate scale of breaking $[SU(3)]^3$ further into the standard model $SU(3)_C\times SU(2)_L\times U(1)_Y$ signaled by the scalar (mass)$^2$ turning negative, via the renormalization group equations [14]. It is found there that the model does admit a very large intermediate breaking scale $M_I\gtrsim 10^{16}$. One lesson we have learned in [14] is that the mixing in the generation space can produce, through loops, some new Yukawa couplings which are zero valued at the tree level. These new Yukawa couplings are generally very small compared to the ones with nonvanishing tree level values and hence provide a possible new mechanism to solve the quark-lepton mass hierarchy problem. The $E_6$ gauge singlets were found to play a very important role in determining the intermediate symmetry-breaking scale as well as the direction of this breaking. Actually, had they been absent, the color $SU(3)_C$ subgroup could have been broken at a very high scale $\gtrsim 10^{17}$ GeV. The introduction of the gauge singlet couplings into the renormalization group equation analysis shifts the symmetry-breaking direction from the quark and antiquark sector to the lepton and mirror-lepton sector, for the gauge singlet and gauge nonsinglet scalar mass ratio $\gtrsim 10$. If this ratio is small $\sim 1$, the gauge singlet (mass)$^2$ turns negative first, due to its large interactions with the gauge nonsinglets. This opens an avenue for the gauge singlets to grow nonvanishing vacuum expectation values (VEV's) first, so that the lepton and the mirror-lepton families pair up to form superheavy states and hence decouple. This process cannot go on forever, because the index theorem guarantees three light generations. This motivated us to suggest a scenario in [14] in which the gauge singlets grow VEV's in the following order: (i) $\phi_{58}$ grows a VEV, then $L_7$ and $\bar{L}_4$ pair up and decouple, (ii) $\phi_{57}$ grows a VEV, then $L_6$ and $\bar{L}_1$ pair up and decouple, (iii) $\phi_{60}$ grows a VEV, then $L_4$ and $\bar{L}_6$ pair up and decouple, (iv) $\phi_{61}$ grows a VEV, then $L_5$ and $\bar{L}_5$ pair up and decouple, where the notation for the gauge singlets follows that of Ref. [9]. These are expected to occur at a very large scale $M_{\rm singlet} \approx 5.0 \times 10^{17}$ GeV. After the decoupling of four generations of lepton–mirror-lepton pairs, we are left with five lepton ($L_i$, $i=1,2,3,8,9$), two mirror-lepton ($\bar{L}_{\bar{i}}$, $\bar{i}=2,3$), and three intact quark and conjugate-quark generations. Of all 52 independent Yukawa couplings listed in Table I of [14], only 15 survive. We may run the renormalization group (RG) equations with these nonets and Yukawa couplings and it can be expected that the gauge symmetry breaking will take place at a scale, say, greater than $1.0 \times 10^{16}$ GeV.

In the next section, we will study the intermediate scale breaking, the low-energy spectrum, and the proton stability for this scenario. We summarize our results here before going into detailed analyses. In addition to the usual standard model particles, we have found quite a few more light states: there are four Higgs doublets, six exotic $SU(3)_C \times SU(2)_L \times U(1)_Y$ singlet fields, and a pair of color Higgs triplets. Though the proton is stabilized against fast decay, the presence of extra electroweak Higgs doublets may trigger large flavor-changing neutral currents. This requires further investigation. Furthermore, the recent data [15] from the CERN $e^+e^-$ collider LEP favor two Higgs doublets. The new massless neutral particles can give rise to the lepton-family-violating processes $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ [16] and $\tau \rightarrow \mu\mu\mu$ and $\tau \rightarrow \mu e\bar{e}$ [17], which may be accessible to experimental detection. The massless color Higgs triplets have not yet been observed, and the structure of the Yukawa couplings reveals that they are unlikely to become heavier than the top quark. We also note that the model has a problem in generating masses for the up and down quarks and in turn predicts a zero value for the Cabibbo angle. Moreover, the first two lepton generations are massless. Therefore, the Gepner three generation model has serious difficulties to overcome if it is to be phenomenologically viable.

## II. INTERMEDIATE BREAKING, LIGHT GENERATIONS, AND PROTON STABILITY

### A. Matter parity

A four dimensional effective field theory arising from the underlying heterotic string theory has a very large number of massless modes. Many couplings among these massless modes can mediate fast proton decay. In order to suppress dangerous dimension 4 interactions, one generally needs a new symmetry, such as *matter parity,* to guarantee the longevity of proton [18]. For the Gepner three generation model the matter parity operator is [11] $M_2 = U_z \times C_2$, where

$$U_z = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}_C \otimes \begin{bmatrix} -1 & & \\ & -1 & \\ & & 1 \end{bmatrix}_L \otimes \begin{bmatrix} -1 & & \\ & -1 & \\ & & 1 \end{bmatrix}_R \qquad (2.1)$$

acts on the gauge group, and $C_2$ is a discrete operator which changes the sign of $L_8$, $L_9$, $\bar{L}_1$, and $\bar{L}_3$ and leaves the rest of the massless modes invariant. Hence it is useful to classify the massless modes according to their transformation properties under matter parity (Table I).

TABLE I. $M_2$ classification of the standard model states.

| $M_2$ even states | $M_2$ odd states |
|---|---|
| $H_i, H_i', N_i$; $i=1,2,\ldots,7$; | $l_i, e_i^c, \nu_i^c$; $i=1,2,\ldots,7$; |
| $l_8, e_8^c, \nu_8^c; l_9, e_9^c, \nu_9^c$; | $H_8, H_8', N_8; H_9, H_9', N_9$; |
| $D_j, D_j^c$; $j=1,2,3$; | $q_j, q_j^c$; $j=1,2,3$; |
| $\bar{H}_k, \bar{H}'_k, \bar{N}_k$; $k=2,4,5,6$; | $\bar{l}_k, \bar{e}_k^c, \bar{\nu}_k^c$; $k=2,4,5,6$; |
| $\bar{l}_1, \bar{e}_1^c, \bar{\nu}_1^c; \bar{l}_3, \bar{e}_3^c, \bar{\nu}_3^c$. | $\bar{H}_1, \bar{H}'_1, \bar{N}_1; \bar{H}_3, \bar{H}'_3, \bar{N}_3$. |

Study shows that the matter parity so defined is no longer invariant once one takes into account the $[1(\overline{27}27)]$ interactions—there is no consistent way to assign matter parity quantum numbers to the $E_6$ gauge singlets which preserves the matter parity. The many nonrenormalizable interactions which are not forbidden by selection rules, if all nonzero, also violate the matter parity. We will discuss the effect of this on proton stability below.

### B. Intermediate scale symmetry breaking

From the discussion in the last section, we know that the gauge group $SU(3)_C \times SU(3)_L \times SU(3)_R$ must be broken at very large scale $M_I \geq 10^{16}$ GeV. In this section, we investigate the breaking of gauge symmetry in the scenario suggested in the last section.

The symmetry breaking at $M_I$ is governed by the following.

(1) A soft supersymmetry-breaking scalar mass term $V_m$.

(2) An $F$ part of the potential that arises from contributions from both the renormalizable and nonrenormalizable interactions. In the scenario we suggested, the renormalizable interactions read

$$\begin{aligned} \mathbf{W}_3 = & \sum_{i=1}^{3} \lambda^3{}_{i89} \det(L_i L_8 L_9) + \sum_{i=1}^{3} \{\lambda^4{}_{2i2} \mathrm{Tr}(Q_2 L_i Q^c{}_2) + \lambda^4{}_{3i3} \mathrm{Tr}(Q_3 L_i Q^c{}_3)\} + \sum_{i=1}^{3} \{\lambda^4{}_{2i3} \mathrm{Tr}(Q_2 L_i Q^c{}_3 + Q_3 L_i Q^c{}_2)\} \\ & + \lambda^1{}_{122}[\det(Q_1 Q_2 Q_2) + \det(Q^c{}_1 Q^c{}_2 Q^c{}_2)] + \lambda^1{}_{123}[\det(Q_1 Q_2 Q_3) + \det(Q^c{}_1 Q^c{}_2 Q^c{}_3)] \\ & + \lambda^1{}_{133}[\det(Q_1 Q_3 Q_3) + \det(Q^c{}_1 Q^c{}_3 Q^c{}_3)] \ . \end{aligned} \qquad (2$$

In Table II, we recreate part of the Table I of Ref. [14] to show the sizes of the Yukawa couplings present in Eq. (2.2) at the intermediate scale $M_I$. The relevant nonrenormalizable terms are

$$\mathbf{W}_{\rm nr}=\frac{\alpha_1}{M_c}[\mathrm{Tr}(L_3\bar{L}_2)]^2+\frac{\alpha_2}{(M_c)^3}\mathrm{Tr}(L_3\bar{L}_2)\mathrm{Tr}(L_8\bar{L}_3)\mathrm{Tr}(L_9\bar{L}_3)+(27\overline{27})^4+\text{higher order terms}\ , \tag{2.3}$$

where $M_c$ is the compactification scale. The first and second terms in Eq. (2.3) are the only terms not forbidden by selection rules [9], up to the order of $(27\overline{27})^3$, which survive (i)–(iv) of the last section and hence can contribute to VEV growth. Notice that these two terms and the Yukawa couplings listed in Eq. (2.2) preserve the matter parity. Other terms of order $(27\overline{27})^4$ or higher await future calculations [10].

(3) A $D$ part of the potential which is governed by the gauge transformations under $SU(3)_L\times SU(3)_R$ and is given by

$$D=\tfrac{1}{8}\sum[D_L^aD_L^{a\dagger}+D_R^aD_R^{a\dagger}]\ , \tag{2.4}$$

where

$$D_L^a=\tfrac{1}{2}g_L\sum[(L_i)_r^{l\dagger}(L_i)_r^{l'}-(\bar{L}_{\bar{i}})_l^{r\dagger}(\bar{L}_{\bar{i}})_{l'}^{r}](t^a)_{ll'}\ , \tag{2.5}$$

and

$$D_R^a=\tfrac{1}{2}g_R\sum[(L_i)_r^{l\dagger}(L_i)_{r'}^{l}-(\bar{L}_{\bar{i}})_l^{r\dagger}(\bar{L}_{\bar{i}})_l^{r'}](t^a)_{rr'}\ . \tag{2.6}$$

Here $t^a$ are the Gell-Mann matrices and $g_{L,R}$ are the $SU(3)_{L,R}$ gauge coupling constants.

It was proven in [19] that the lowest-lying vacuum solutions of the potential are those that preserve $SU(2)_L\times U(1)_Y$ invariance and the matter parity, and VEV growth must be along $C$-even $N$ ($\bar{N}$) and $C$-odd $\nu^c$ ($\bar{\nu}^c$) directions. It was also shown [19] that these solutions are almost $D$ flat. Hence, in the following analysis, we will impose these two conditions. The first of these tells us that the possible fields that can grow VEV's are $N_1$, $N_2$, $N_3$, $\bar{N}_2$, $\nu_8^c$, $\nu_9^c$, and $\bar{\nu}_3^c$ (Table I). The $D$ flatness condition requires that all these possible fields must grow VEV's simultaneously. Actually, one sees that the VEV's must satisfy

$$\langle\bar{N}_2\rangle^2\cong\langle N_1\rangle^2+\langle N_2\rangle^2+\langle N_3\rangle^2\ ,\qquad \langle\bar{\nu}_3^c\rangle^2\cong\langle\nu_8^c\rangle^2+\langle\nu_9^c\rangle^2\ . \tag{2.7}$$

The potential reads, after discarding the $D$ terms,

$$\mathbf{V}=V_m+\sum_i\left|\frac{\partial\mathbf{W}}{\partial L_i}\right|^2+\sum_i\left|\frac{\partial\mathbf{W}}{\partial\bar{L}_i}\right|^2\ , \tag{2.8}$$

where $\mathbf{W}=\mathbf{W}_3+\mathbf{W}_{\rm nr}$ is the superpotential, with $\mathbf{W}_3$ containing only the first term in Eq. (2.2). Minimizing the potential $\mathbf{V}$ with respect to the VEV's, $\langle N_1\rangle$, $\langle N_2\rangle$, $\langle N_3\rangle$, $\langle\bar{N}_2\rangle$, as well as $\langle\nu_8^c\rangle$, $\langle\nu_9^c\rangle$, and $\langle\bar{\nu}_3^c\rangle$, we found that the VEV's are all of order

$$\mathrm{VEV}\sim1.0\times10^{15}-1.0\times10^{16}\ (\mathrm{GeV}) \tag{2.9}$$

or higher, for soft breaking scalar masses $\sim1$ TeV. In deriving (2.9), we have assumed some nonrenormalizable interactions of the form $(27\overline{27})^4$ that contain $L_1$ and $L_2$ nonets to ensure that $N_1$ and $N_2$ grow nonzero VEV's.

### C. Light generations

Because of the large VEV growth of scalar particles discussed above, many massless modes will grow superheavy masses, and this mass growth is governed by the Yukawa couplings listed in Table II and the nonrenormalizable interactions of Eq. (2.3). We consider only those terms up to order $(27\overline{27})^3$, partly because we do not have any information about the nonrenormalizable interactions beyond this order.

In the quark (antiquark) sector, only $D$ quarks grow superheavy mass, via the Yukawa couplings of form $\mathrm{Tr}QLQ^c$, and the mass matrix reads

$$\begin{array}{c|ccc} & D_1 & D_2 & D_3\\ \hline D^c{}_1 & 0 & 0 & 0\\ D^c{}_2 & 0 & a & c\\ D^c{}_3 & 0 & c & b\end{array}\ , \tag{2.10}$$

where

TABLE II. Yukawa couplings.

| Coupling | Tree value | One-loop value | Coupling | Tree value | One-loop value |
|---|---|---|---|---|---|
| $\lambda^1{}_{122}$ | | $-2.470\times10^{-3}$ | $\lambda^2{}_{122}$ | | $-2.470\times10^{-3}$ |
| $\lambda^1{}_{123}$ | 0.654 | 0.629 | $\lambda^2{}_{123}$ | 0.654 | 0.629 |
| $\lambda^1{}_{133}$ | 0.537 | 0.511 | $\lambda^2{}_{133}$ | 0.537 | 0.511 |
| $\lambda^3{}_{189}$ | | $-1.868\times10^{-3}$ | $\lambda^3{}_{289}$ | 0.635 | 0.630 |
| $\lambda^3{}_{389}$ | | $-1.038\times10^{-3}$ | | | |
| $\lambda^4{}_{212}$ | | $-6.040\times10^{-3}$ | $\lambda^4{}_{313}$ | | $-5.408\times10^{-3}$ |
| $\lambda^4{}_{213}$ | 0.577 | 0.560 | $\lambda^4{}_{312}$ | 0.577 | 0.560 |
| $\lambda^4{}_{222}$ | 0.577 | 0.545 | $\lambda^4{}_{323}$ | 0.390 | 0.358 |
| $\lambda^4{}_{223}$ | | $-5.189\times10^{-3}$ | $\lambda^4{}_{322}$ | | $-5.189\times10^{-3}$ |
| $\lambda^4{}_{232}$ | | $-9.077\times10^{-4}$ | $\lambda^4{}_{333}$ | 1.054 | 1.005 |
| $\lambda^4{}_{233}$ | | $-3.925\times10^{-3}$ | $\lambda^4{}_{332}$ | | $-3.925\times10^{-3}$ |

$$a=\sum_{i=1}^{3}\lambda^{4}{}_{2i2}\langle N_i\rangle\ ;$$
$$b=\sum_{i=1}^{3}\lambda^{4}{}_{3i3}\langle N_i\rangle\ ; \qquad (2.11)$$
$$c=\sum_{i=1}^{3}\lambda^{4}{}_{2i3}\langle N_i\rangle\ .$$

Thus only two of three color Higgs triplets $D_{2,3}$ become superheavy, with mass $\gtrsim 10^{15}$ GeV. The nonrenormalizable couplings of any order cannot make $D_1$ heavy, because there are no mirror- (anti-) quark generations which can pair up with $D_1$. For example, there appears a coupling of the form [10]

$$(Q_1\bar{L}_2)(Q^c{}_1\bar{L}_2)=\epsilon^{ll'l''}\epsilon_{rr'r''}Q_1{}^{a}_{l}(\bar{L}_2)^{r}_{l'}(Q^c_2)^{r'}_{a}(\bar{L}_2)^{r''}_{l''} \qquad (2.12)$$

at the $(27\overline{27})^2$ level, but this does not contribute to the mass growth for $Q_1$ or $Q^c{}_1$. This is because the VEV growth can only take place along the $\bar{N}_2$ direction and the $\epsilon^{ll'l''}$ and $\epsilon_{rr'r''}$ symbols prevent this term from giving rise to masses to any fields involved in the interaction. Therefore, the light particles in the quark sector are $D_1$ and $D^c_1$ [which are $SU(2)_L\times U(1)_Y$ singlets], in addition to three $SU(2)_L$ doublets $q_i$ and their conjugate fields of the standard model.

In the lepton sector, the terms relevant for mass growth are

$$\begin{aligned}V^L_{\rm mass}=&\sum_{i=1}^{3}H_i(A_il_8+B_il_9)+\sum_{i=1}^{3}l_i(A_iH_8+B_iH_9)\\&+C(H_8H'_9+H_9H'_8)+D(H_3\bar{H}_2+H'_3\bar{H}'_2+e^c_3\bar{e}^c_2+\nu^c_3\bar{\nu}^c_2+N_3\bar{N}_2+l_3\bar{l}_2)\\&+(E_1H_8+E_2H_9)\bar{H}_3+(E_1H'_8+E_2H'_9)\bar{H}'_3+(E_1l_8+E_2l_9)\bar{l}_3+(E_1e^c_8+E_2e^c_9)\bar{e}^c_3\\&+(E_1\nu^c_8+E_2\nu^c_9)\bar{\nu}^c_3+(E_1N_8+E_2N_9)\bar{N}_3+(27\overline{27})^4+\text{higher order terms}\ .\end{aligned} \qquad (2.13)$$

Here we define

$$A_i=\lambda^{3}{}_{i89}\langle\nu^c_9\rangle,\quad B_i=\lambda^{3}{}_{i89}\langle\nu^c_8\rangle,\quad i=1,2,3\ ,$$
$$C=\sum_{i=1}^{3}\lambda^{3}{}_{i89}\langle N_i\rangle\ ,$$
$$D=\frac{2\alpha_1}{M_c}\langle N_3\rangle\langle\bar{N}_2\rangle+\frac{\alpha_2}{(M_c)^3}\langle\nu^c_8\rangle\langle\nu^c_9\rangle\langle\bar{\nu}^c_3\rangle^2\ , \qquad (2.14)$$
$$E_1=\frac{\alpha_2}{(M_c)^3}\langle N_3\rangle\langle\bar{N}_2\rangle\langle\nu^c_9\rangle\langle\bar{\nu}^c_3\rangle\ ,$$
$$E_2=\frac{\alpha_2}{(M_c)^3}\langle N_3\rangle\langle\bar{N}_2\rangle\langle\nu^c_8\rangle\langle\bar{\nu}^c_3\rangle\ .$$

The sizes of these parameters, from Eq. (2.9), are $A_i\sim B_i\sim C\sim 10^{15}-10^{16}$ GeV, $D\sim 10^{12}-10^{14}$ GeV, and $E_1\sim E_2\sim 10^{6}-10^{10}$ GeV, respectively. Therefore, the mass matrix for the lepton sector decomposes to two parts: a matter-parity-even part

$$\begin{array}{c|ccc|}
 & l_8 & l_9 & \bar{H}_2\\
\hline
H_1 & A_1 & B_1 & 0\\
H_2 & A_2 & B_2 & 0\\
H_3 & A_3 & B_3 & D\\
\bar{l}_3 & E_1 & E_2 & 0
\end{array}\ , \qquad (2.15)$$

and a matter-parity-odd part

$$\begin{array}{c|cccccc|}
 & l_1 & l_2 & l_3 & H'_8 & H'_9 & \bar{H}_3\\
\hline
H_8 & A_1 & A_2 & A_3 & 0 & C & E_1\\
H_9 & B_1 & B_2 & B_3 & C & 0 & E_2\\
\bar{l}_2 & 0 & 0 & D & 0 & 0 & 0\\
\bar{H}'_3 & 0 & 0 & 0 & E_1 & E_2 & 0
\end{array}\ . \qquad (2.16)$$

It is easy to read off the light particles from Eqs. (2.10), (2.13), (2.15), and (2.16). The light particles which will survive down to the electroweak scale are as follows.

*Leptons.* The three lepton doublets are $l_\pm$ and $\tilde{l}$, where

$$l_\pm=a_\pm l_1+b_\pm l_2+c_\pm\bar{H}_3 \qquad (2.17)$$

are the two states orthogonal to $l=a_0l_1+b_0l_2+c_0\bar{H}_3$, where

$$a_0=\frac{\lambda^{3}{}_{189}}{\{(\lambda^{3}{}_{189})^2+(\lambda^{3}{}_{289})^2+(\rho)^2\}^{1/2}}\ ,$$
$$b_0=\frac{\lambda^{3}{}_{289}}{\{(\lambda^{3}{}_{189})^2+(\lambda^{3}{}_{289})^2+(\rho)^2\}^{1/2}}\ ,$$
$$c_0=\frac{\rho}{\{(\lambda^{3}{}_{189})^2+(\lambda^{3}{}_{289})^2+(\rho)^2\}^{1/2}}\ , \qquad (2.18)$$
$$\rho=\frac{\alpha_2}{(M_c)^3}\langle N_3\rangle\langle\bar{N}_2\rangle\langle\bar{\nu}^c_3\rangle\ ,$$

and $\tilde{l}$ is given by

$$\tilde{l}=\alpha l_8-\beta l_9\ , \qquad (2.19)$$

where

$$\alpha=\frac{\langle\nu^c_8\rangle}{\{\langle\nu^c_8\rangle^2+\langle\nu^c_9\rangle^2\}^{1/2}},\quad \beta=\frac{\langle\nu^c_9\rangle}{\{\langle\nu^c_8\rangle^2+\langle\nu^c_9\rangle^2\}^{1/2}}\ . \qquad (2.20)$$

Note that $\rho\approx 10^{-6}$, the heavy lepton $l$ lies almost entirely along $a_0l_1+b_0l_2$, thus the two light leptons, $l_\pm$ lie almost totally along $b_0l_1-a_0l_2$ and $\bar{H}_3$ directions, respectively.

Three right-handed electrons are $e^c_{1,2}$ and

$$\tilde{e}^{\,c}=\alpha e^c_8-\beta e^c_9\ . \qquad (2.21)$$

Light Higgs bosons. There are four light Higgs bosons $H_\pm$ and $H'_{1,2}$, where

$$H_\pm = a_\pm H_1 + b_\pm H_2 + c_\pm \bar{l}_3 \approx \begin{bmatrix} b_0 H_1 - a_0 H_2 \\ \bar{l}_3 \end{bmatrix}, \quad (2.22)$$

are the two states orthogonal to $H = a_0 H_1 + b_0 H_2 + c_0 \bar{l}_3 \sim a_0 H_1 + b_0 H_2$.

Quarks. There are three quark doublets $q_i$, $i=1,2,3$, and their conjugate fields $q_i^c$, $i=1,2,3$.

Other exotic particles. There are 6 exotic particles coming from lepton nonets, $\nu^c_{1,2}$, $N_{1,2}$, and

$$\tilde{\nu}^c = \alpha \nu^c_8 - \beta \nu^c_9, \quad \tilde{N} = \alpha N_8 - \beta N_9, \quad (2.23)$$

and the color Higgs triplet $D_1$ and its conjugate field $D^c_1$.

### Couplings among light generations

The couplings among the light fields can be very easily read off from Eq. (2.2) to be

$$\lambda^3{}_{i89}[H_i(\nu^c{}_8 l_9 + \nu^c{}_9 l_8) + H'_i(e^c{}_8 l_9 + e^c{}_9 l_8)] + \lambda^4{}_{2i2}[q_2 H_i u^c{}_2 + q_2 H'_i d^c{}_2] + \lambda^4{}_{3i3}[q_3 H_i u^c{}_3 + q_3 H'_i d^c{}_3]$$
$$+ \lambda^4{}_{2i3}[q_2 H_i u^c{}_3 + q_2 H'_i d^c{}_3 + q_3 H_i u^c{}_2 + q_3 H'_i d^c{}_2], \quad (2.24)$$

where the sum over $i=1,2$ is implied. Note that Eqs. (2.24) and (2.35) below do not contain any couplings among $D_1$ ($D^c_1$) and leptons. This will have a very important effect on proton stability which we will study in the next subsection.

The couplings among leptons can be rewritten, in terms of the light fields, as

$$-2\alpha\beta[(\lambda^3{}_{189}H_1 + \lambda^3{}_{289}H_2)\tilde{\nu}^c\bar{l} + (\lambda^3{}_{189}H'_1 + \lambda^3{}_{289}H'_2)\tilde{e}^c\bar{l}]; \quad (2.25)$$

one can easily convince oneself that the first almost decouples because the heavy Higgs boson lies along $\lambda^3{}_{189}H_1 + \lambda^3{}_{289}H_2$. Then Eq. (2.25) reduces to

$$-2\alpha\beta[\delta H_- \tilde{\nu}^c\bar{l} + \gamma H' \tilde{e}^c \bar{l}], \quad (2.26)$$

where $\delta \approx 10^{-6}$, $\gamma \approx 1$, and $H' \sim \lambda^3{}_{189}H'_1 + \lambda^3{}_{289}H'_2$. Thus the mass ratio for the "electron" $\tilde{e}$ and the "neutrino" $\tilde{\nu}$ is

$$\frac{m_{\tilde{\nu}}}{m_{\tilde{e}}} \approx 10^{-6} \times \frac{\langle H_- \rangle}{\langle H' \rangle} \sim 3.0 \times 10^{-5}, \quad (2.27)$$

for $\langle H_- \rangle \sim 30 \langle H' \rangle$. We may identify these particles as the third generation of leptons. Thus this ratio is reasonable to explain the lepton mass hierarchy. Notice that this ratio comes from the second nonrenormalizable interaction of Eq. (2.3), and hence implies that the nonrenormalizable interactions are a viable source to explain the lepton-quark hierarchy. The first two lepton generations remain massless, due to the lack of their couplings to the light Higgs bosons.

The mass matrices for the quarks are

$$\begin{array}{c|cc} & u_2 & u_3 \\ \hline u^c_2 & v_{22} & v_{23} \\ u^c_3 & v_{23} & v_{33} \end{array}, \quad \begin{array}{c|cc} & d_2 & d_3 \\ \hline d^c_2 & v'_{22} & v'_{23} \\ d^c_3 & v'_{23} & v'_{33} \end{array}, \quad (2.28)$$

where

$$\left.\begin{aligned} v_{ij} &= \lambda^4{}_{i1j}\langle H_1 \rangle + \lambda^4{}_{i2j}\langle H_2 \rangle \\ v'_{ij} &= \lambda^4{}_{i1j}\langle H'_1 \rangle + \lambda^4{}_{i2j}\langle H'_2 \rangle \end{aligned}\right\} \; i,j=2,3. \quad (2.29)$$

These two matrices are easily diagonalized as

$$\begin{bmatrix} u_+ \\ u_- \end{bmatrix} = \begin{bmatrix} \sin\theta_u & \cos\theta_u \\ -\cos\theta_u & \sin\theta_u \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix}, \quad \begin{bmatrix} d_+ \\ d_- \end{bmatrix} = \begin{bmatrix} \sin\theta_d & \cos\theta_d \\ -\cos\theta_d & \sin\theta_d \end{bmatrix} \begin{bmatrix} d_2 \\ d_3 \end{bmatrix}, \quad (2.30)$$

where

$$\tan 2\theta_u = \frac{2v_{23}}{v_{33} - v_{22}}, \quad \tan 2\theta_d = \frac{2v'_{23}}{v'_{33} - v'_{22}}, \quad (2.31)$$

and the masses are given by

$$m_u^\pm = \frac{v_{22} + v_{33} \pm [(v_{22} - v_{33})^2 + 4(v_{23})^2]^{1/2}}{2}, \quad m_d^\pm = \frac{v'_{22} + v'_{33} \pm [(v'_{22} - v'_{33})^2 + 4(v'_{23})^2]^{1/2}}{2}, \quad (2.32)$$

respectively. Then, the Kobayashi-Maskawa (KM) matrix reads

$$\begin{bmatrix} d_1 \\ d'_+ \\ d'_- \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_u - \theta_d) & -\sin(\theta_u - \theta_d) \\ 0 & \sin(\theta_u - \theta_d) & \cos(\theta_u - \theta_d) \end{bmatrix} \begin{bmatrix} d_1 \\ d_+ \\ d_- \end{bmatrix}. \quad (2.33)$$

Here,

$$\begin{bmatrix} d'_+ \\ d'_- \end{bmatrix} = \begin{bmatrix} \sin\theta_u & \cos\theta_u \\ -\cos\theta_u & \sin\theta_u \end{bmatrix} \begin{bmatrix} d_2 \\ d_3 \end{bmatrix}. \quad (2.34)$$

Notice that $d_1$ and $u_1$ are massless; it is natural to identify them as the first quark generation. Then Eq. (2.33) predicts a zero value for the Cabibbo angle.

### D. Proton stability

The proton stability is a crucial test for any grand unified theory. As shown above, there appears a dangerous color Higgs triplet $D_1$ in the low-energy spectrum of the model, which might mediate rapid proton decay. From the structure of the Yukawa couplings, Eq. (2.2), we see that the first generation of quarks does not couple to any leptons. Also, $D_1$ and its conjugate $D^c{}_1$ do not couple to $q_1$:

$$\epsilon_{aa'a''}[\lambda^1{}_{122}d_2^a u_2^{a'} + \lambda^1{}_{133}d_3^a u_3^{a'} + \lambda^1{}_{123}(d_2^a u_3^{a'} + d_3^a u_2^{a'})]D_1^{a''} + \epsilon^{aa'a''}[\lambda^1{}_{122}d_{2a}^c u_{2a'}^c + \lambda^1{}_{133}d_{3a}^c u_{3a'}^c + \lambda^1{}_{123}(d_{2a}^c u_{3a'}^c + d_{3a}^c u_{2a'}^c)]D_{1a''}^c \ . \quad (2.35)$$

Hence if we identify $q_1$ as the usual standard model up and down quark doublet, we can assign a baryon number to these light color Higgs triplets without having them trigger the fast proton decay. Then proton decay is mediated only by $D_{2,3}$ and their conjugate fields, $D^c{}_{2,3}$, which are superheavy with mass $\approx 10^{15}-10^{16}$ GeV. Another remarkable factor that prevents fast proton decay is that, as we mentioned in Sec. II, there is no mirror generation of quarks and their conjugate fields. Hence, of the many channels leading to fast proton decay discussed in [20], only the first (via heavy color Higgs triplets) appears, and because of the superheavy mass of these fields, decay will be sufficiently inhibited.

## III. CONCLUSIONS

We have already summarized the results in the Introduction. The Gepner three generation model has great problems to overcome for it to be phenomenologically viable. The above analysis shows that the nonrenormalizable interactions of the form $(27\overline{27})^n$, where $n \geq 3$, provide a viable mechanism to solve the lepton-quark mass hierarchy problem. For example, although the first two generations of the leptons remain massless, the mass ratio of the third generation of leptons, Eq. (2.27), does come out correctly, due to the smallness of the parameter $\rho$, which comes from the nonrenormalizable interactions, as shown in Eqs. (2.3) and (2.18). Recall that the Gepner model corresponds only to the symmetric Schimmrigk model where all the moduli are zero. Therefore, one may have to go beyond the symmetric point (i.e., the Schimmrigk model with nonzero moduli) to find a phenomenologically more interesting model, as shown in Ref. [13] for a simple case with only one nonvanishing modulus, where an interesting model has been found which demonstrates an automatic *CP* violation. It is thus of interest to study the model represented in Ref. [13] in more detail.

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