PHYSICAL REVIEW D VOLUME 49, NUMBER 9 1 MAY 1994

Electroweak theory of SU(3)×U(1)

Daniel Ng
TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia, Canada V6T 2A3
(Received 28 December 1992; revised manuscript received 31 August 1993)

An electroweak model of the SU(3)×U(1) gauge group is studied. By matching the gauge coupling constant we obtain the mass of the new neutral gauge boson to be less than 3.1 TeV. Including the constraint from muon decay, the allowed ranges of the new gauge boson masses are 1.3 TeV $\leq M_{Z_2} \leq 3.1$ TeV and 270 GeV $\leq M_Y \leq 550$ GeV. Within these mass ranges, the decay $Z_2 \to Y^{++}Y^{--}$ with $Y^{\pm\pm} \to 2l^{\pm} (l=e,\mu,\tau)$ is allowed, providing a spectacular signature at future colliders. The low energy experiments further constrain the Z-Z' mixing angle to be $-5\times 10^{-3} \leq \theta \leq 7\times 10^{-4}$.

PACS number(s): 12.60.Cn, 12.15.Mm, 13.10.+q, 14.70.Pw

I. INTRODUCTION

A model of $SU(3)_L \times U(1)_X$ was introduced by Pisano and co-workers [1] and Frampton [2] recently. The former authors argues that Y^{--} is necessary in order to avoid unitarity violation for $e^-e^- \to W^-Y^-$ at high energies; the latter looked for a simple solution, which included dileptons $Y^{\pm\pm}$. $Y^{\pm\pm}$ and $Y^{\pm}$ are called dileptons because they couple to two leptons; thus, they have two units of lepton number. Many other electroweak models [3] of SU(3)×U(1) were suggested some years ago with different choices of particle content. Here, this model has minimal particle content yielding some interesting new physics, such as stringent constraints on the new gauge boson masses.

The anomaly in this model is not canceled within each generation. However, one generation of quark representations is chosen in such a way that the anomaly is canceled among three generations. Thus the number of generations is a multiple of 3. Note that the first- and the third-generation quark multiplets were chosen arbitrarily by the authors in Refs. [1] and [2], respectively. These two representations are *identical*, namely, we can map from one representation to another by a unitary transformation. By matching the gauge coupling constants at SU(3)×U(1) symmetry breaking, we find that the new neutral gauge boson Z' cannot be heavier than 3.1 TeV. Unlike most extended models such as E_6 and left-right (LR) symmetric models, the masses of the new neutral gauge boson in this model are bounded from above. Hence, this model provides an interesting phenomenology, which will be discovered or ruled out at future colliders [4].

Limits of doubly charged gauge bosons $(Y^{\pm\pm})$ have been studied [5] in e^+e^- collision, yielding the mass lower bound $M_{Y^{++}} > 210$ GeV (95% C.L.). The mass limit of $(Y^{\pm})$ obtained from muon decay [6] yields the bound $M_{Y^+} > 270$ GeV (90% C.L.). The process $e^-e^- \to \mu^-\mu^-$ would be the best experiment testing the existence of a double charge gauged boson. However, the only machine relevant to this process is operated at the center-of-mass energy 1.112 GeV [7].

In this paper, we consider this model in detail. We first review this model in Sec. II; in Sec. III, the limits on gauge boson masses are discussed; Sec. IV investigates flavor-changing neutral current processes; low-energy experiments are studied in Sec. V; finally, the conclusions are presented in Sec. VI.

II. REVIEW OF THE MODEL

The simplest anomaly-free solution [1,2], which includes the standard model (SM), of a gauge symmetry $SU(3)_C \times SU(3)_L \times U(1)_X$ is given as

$$\psi_{1,2,3} = \begin{pmatrix} e_1 \\ \nu_{e_1} \\ e_1^c \end{pmatrix}, \begin{pmatrix} e_2 \\ \nu_{e_2} \\ e_2^c \end{pmatrix}, \begin{pmatrix} e_3 \\ \nu_{e_3} \\ e_3^c \end{pmatrix} : \ (1,3^*,0)\,, \tag{2.1a}$$

$$Q_{1,2} = \begin{pmatrix} u_1 \\ d_1 \\ D_1 \end{pmatrix}, \begin{pmatrix} u_2 \\ d_2 \\ D_2 \end{pmatrix} : \ (3,3,-\tfrac{1}{3})\,, \tag{2.1b}$$

$$Q_3 = \begin{pmatrix} u_3 \\ d_3 \\ T \end{pmatrix} : \ (3,3^*,\tfrac{2}{3})\,, \tag{2.1c}$$

$$d_1^c, d_2^c, d_3^c: \ (3^*,1,\tfrac{1}{3})\,, \tag{2.1d}$$

$$u_1^c, u_2^c, u_3^c: \ (3^*,1,-\tfrac{2}{3})\,, \tag{2.1e}$$

$$D_1^c, D_2^c: \ (3^*,1,\tfrac{4}{3})\,, \tag{2.1f}$$

$$T^c: \ (3^*,1,-\tfrac{5}{3})\,, \tag{2.1g}$$

where the subscripts denote the generations, and $D_{1,2}$ and T are new quarks. For a minimal particle content, the anomaly is not canceled within each generation, but canceled among three generations by choosing the third quark generation [2] as an $SU(3)_L$ antitriplet. The first quark generation is chosen in Ref. [1]. However, two different choices of quark representations are physically *identical*.

$SU(3)_L \times U(1)_X$ will first be broken down to the standard model $SU(2)_L \times U(1)_Y$ by a nonzero vacuum expectation value (VEV) of a triplet scalar $\langle \Phi \rangle^T = (0,0,u/\sqrt{2})$, yielding a massive neutral gauge boson (Z') and two charged gauge bosons (Y^+, Y^{++}) as well as new quarks ($D_{1,2}, T$). The breaking of $SU(2)_L \times U(1)_Y$ to $U(1)_Q$ can be achieved by $\langle \Delta \rangle^T = (0, v/\sqrt{2}, 0)$ and $\langle \Delta' \rangle^T = (v'/\sqrt{2}, 0, 0)$. In order to obtain acceptable masses for charged leptons, a sextet η is necessary. Hence, the minimally required scalar multiplets are summarized as

$$\Phi = \begin{bmatrix} \phi^{++} \\ \phi^{+} \\ \phi^{0} \end{bmatrix} : (1,3,1) , \tag{2.2a}$$

$$\Delta = \begin{bmatrix} \Delta_1^{+} \\ \Delta^{0} \\ \Delta_2^{-} \end{bmatrix} : (1,3,0) , \tag{2.2b}$$

$$\Delta' = \begin{bmatrix} \Delta'^{0} \\ \Delta'^{-} \\ \Delta'^{--} \end{bmatrix} : (1,3,-1) , \tag{2.2c}$$

and

$$\eta = \begin{bmatrix} \eta_1^{++} & \eta_1^{+}/\sqrt{2} & \eta^{0}/\sqrt{2} \\ \eta_1^{+}/\sqrt{2} & \eta'^{0} & \eta_2^{-}/\sqrt{2} \\ \eta^{0}/\sqrt{2} & \eta_2^{-}/\sqrt{2} & \eta_2^{--} \end{bmatrix} : (1,6,0) . \tag{2.2d}$$

For completeness, the Yukawa interactions corresponding to the scalar multiplets Φ, Δ, Δ', and η are given as

$$-\mathcal{L}(\Phi) = h_{D_{1,2}}^{1,2} Q_{1,2}(D_1^c, D_2^c)\Phi^* + h_T^3 Q_3 T^c \Phi + \text{H.c.} , \tag{2.3a}$$

$$-\mathcal{L}(\Delta) = h_{u_i}^3 Q_3 (u_i^c)\Delta + h_{d_i}^{1,2} Q_{1,2}(d_i^c)\Delta^* + h_e^{ij}\psi_i\psi_j\Delta^* + \text{H.c.} , \tag{2.3b}$$

$$-\mathcal{L}(\Delta') = h_{d_i}^3 Q_3 (d_i^c)\Delta' + h_{u_i}^{1,2} Q_{1,2}(u_i^c)\Delta'^* + \text{H.c.} , \tag{2.3c}$$

and

$$-\mathcal{L}(\eta) = y_c^{ij}\psi_i\psi_j\eta + \text{H.c.} , \tag{2.3d}$$

where the summation of generation indices are implicit. h_e^{ij} and y_e^{ij} are antisymmetric [8] and symmetric matrices in flavor space, respectively.

III. GAUGE BOSON MASSES

To obtain the gauge interactions, let us first define the covariant derivative for triplets

$$D_\mu = \partial_\mu - ig\frac{\lambda^a}{2}W_\mu^a - ig_X X \frac{\lambda^9}{2} V_\mu , \tag{3.1}$$

where $\lambda^a (a = 1, \ldots, 8)$ are the $SU(3)_L$ generators, and $\lambda^9 = \sqrt{2/3}$ diagonal(1,1,1) are defined such that $\text{Tr}(\lambda^a\lambda^b) = 2\delta^{ab}$ and $\text{Tr}(\lambda^9\lambda^9) = 2$. g and g_X are the gauge coupling constants for $SU(3)_L$ and $U(1)_X$ with their gauge bosons W^a and V, respectively. The covariant derivative for the sextet is

$$D_\mu \eta^{\alpha\beta} = \partial_\mu \eta^{\alpha\beta} - i\frac{g}{2}W_\mu^a[\lambda_{\beta'}^{a\beta}\eta^{\alpha\beta'} + \lambda_{\alpha'}^{a\alpha}\eta^{\alpha'\beta}] . \tag{3.2}$$

As the triplet scalar Φ acquires a VEV, the symmetry $SU(3)_L \times U(1)_X$ breaks down to $SU(2)_L \times U(1)_Y$, where $Y \equiv \sqrt{3}(\lambda^8 + \sqrt{2}X\lambda^9)$ is the hypercharge. By matching the gauge coupling constants at the $SU(3)_L \times U(1)_X$ breaking, the coupling constant of $U(1)_Y, g'$, is given by

$$\frac{1}{g'^2} = 3\left[\frac{1}{g^2} + \frac{2}{g_X^2}\right] . \tag{3.3}$$

Therefore we obtain

$$\frac{g_X^2}{g^2} = \frac{6\sin^2\theta_W(M_{Z'})}{1 - 4\sin^2\theta_W(M_{Z'})} , \tag{3.4}$$

where $g'/g = \tan\theta_W$. Therefore, $\sin^2\theta_W(M_{Z'})$ has to be smaller than 1/4. At the energy scale below $M_{Z'}$, there are three Higgs doublets (Δ_1^+, Δ^0), (Δ'^0, Δ'^-), and (η^0, η_2^-) and one triplet $(\eta_1^{++}, \eta_1^+, \eta'^0)$. There are three singlets η_2^{--}, Δ_2^- and Δ'^{--}, but their masses are on the order of $M_{Z'}$. Let us first consider the one-loop running of $\sin^2\theta_W$, which includes the contributions from the Higgs doublets and triplets as well as the usual fermions. Therefore, the upper bound of the $M_{Z'}$ can be computed from the constraint $\sin^2\theta_W(M_{Z'}) \leq 1/4$. Since the result is very sensitive to the value of $\sin^2\theta_W$ given at M_Z, we plot in Fig. 1 the upper bound of $M_{Z'}$ as a function of $\sin^2\theta_W(M_Z)$ for $\alpha_{\text{em}}^{-1} = 127.9$ [9] in the modified minimal subtraction scheme ($\overline{\text{MS}}$) scheme. In particular, for $\sin^2\theta_W(M_Z) = 0.2333$ [9], we obtain that $M_{Z'}$ is less than 3.1 TeV.

The breaking of the SM to $U(1)_Q$ can be achieved by $\langle \Delta^0 \rangle = v/\sqrt{2}$, $\langle \Delta'^0 \rangle = v'/\sqrt{2}$ and $\langle \eta^0 \rangle = w/\sqrt{2}$, where $\langle \eta'^0 \rangle = 0$ is assumed for lepton number conservation. The charged gauge bosons

$$W^+ = (W^1 - iW^2)/\sqrt{2} , \tag{3.5a}$$

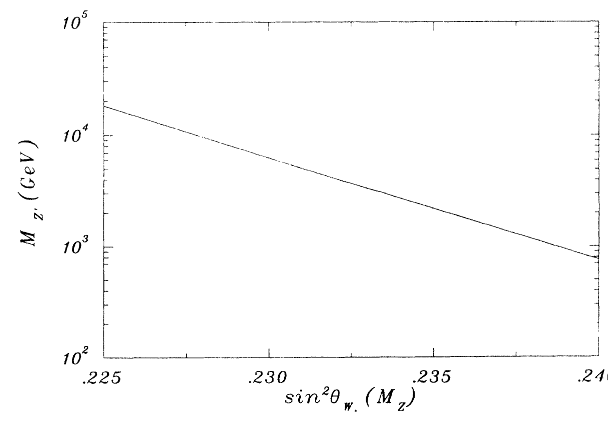

FIG. 1. The upper bound of $M_{Z'}$, obtained from $\sin^2\theta_W(M_{Z'}) \leq 1/4$, is plotted as a function of $\sin^2\theta_W(M_Z)$.

$$Y^{+}=(W^{6}-iW^{7})/\sqrt{2}\ , \tag{3.5b}$$

and

$$Y^{++}=(W^{4}-iW^{5})/\sqrt{2}\ , \tag{3.5c}$$

acquire masses

$$M_W^2=\frac{1}{4}g^2(v^2+v'^2+w^2)\ , \tag{3.6a}$$

$$M_{Y^+}^2=\frac{1}{4}g^2(u^2+v^2+w^2)\ , \tag{3.6b}$$

and

$$M_{Y^{++}}^2=\frac{1}{4}g^2(u^2+v'^2+4w^2)\ , \tag{3.6c}$$

respectively. Hence, the breaking of $SU(2)_L\times U(1)_Y \rightarrow U(1)_Q$ give rise to the mass splitting of $Y^{\pm}$ and $Y^{\pm\pm}$.

The mass-squared matrix for the neutral gauge bosons $\{W^3, W^8, V\}$ is given by

$$\begin{bmatrix} \frac{1}{4}g^2(v^2+v'^2+w^2) & -\frac{1}{4\sqrt{3}}g^2(v^2-v'^2+w^2) & -\frac{1}{2\sqrt{6}}gg_Xv'^2 \\ -\frac{1}{4\sqrt{3}}g^2(v^2-v'^2+w^2) & \frac{1}{12}g^2(4u^2+v^2+v'^2+w^2) & -\frac{1}{6\sqrt{2}}gg_X(2u^2+v'^2) \\ -\frac{1}{2\sqrt{6}}gg_Xv'^2 & -\frac{1}{6\sqrt{2}}gg_X(2u^2+v'^2) & \frac{1}{6}g_X^2(u^2+v'^2) \end{bmatrix}\ . \tag{3.7}$$

We can easily identify the photon field γ as well as the massive bosons Z and Z':

$$\gamma=+\sin\theta_W W^3+\cos\theta_W(\sqrt{3}\tan\theta_W W^8+\sqrt{1-3\tan^2\theta_W}V)\ , \tag{3.8a}$$

$$Z=+\cos\theta_W W^3-\sin\theta_W(\sqrt{3}\tan\theta_W W^8+\sqrt{1-3\tan^2\theta_W}V)\ , \tag{3.8b}$$

and

$$Z'=-\sqrt{1-3\tan^2\theta_W}W^8+\sqrt{3}\tan\theta_W V\ , \tag{3.8c}$$

where the mass-squared matrix for $\{Z,Z'\}$ is given by

$$\mathcal{M}^2=\begin{bmatrix} M_Z^2 & M_{ZZ'}^2 \\ M_{ZZ'}^2 & M_{Z'}^2 \end{bmatrix} \tag{3.9}$$

with

$$M_Z^2=\frac{1}{4}\frac{g^2}{\cos^2\theta_W}(v^2+v'^2+w^2)\ , \tag{3.10a}$$

$$M_{Z'}^2=\frac{1}{3}g^2\left[\frac{\cos^2\theta_W}{1-4\sin^2\theta_W}u^2+\frac{1-4\sin^2\theta_W}{4\cos^2\theta_W}(v^2+v'^2+w^2)+\frac{3\sin^2\theta_W}{1-4\sin^2\theta_W}v'^2\right]\ , \tag{3.10b}$$

$$M_{ZZ'}^2=\frac{1}{4\sqrt{3}}g^2\frac{\sqrt{1-4\sin^2\theta_W}}{\cos^2\theta_W}\left[(v^2+w^2)-\left[\frac{1+2\sin^2\theta_W}{1-4\sin^2\theta_W}\right]v'^2\right]\ . \tag{3.10c}$$

The mass eigenstate are

$$Z_1=\cos\theta Z-\sin\theta Z' \tag{3.11a}$$

and

$$Z_2=\sin\theta Z+\cos\theta Z'\ , \tag{3.11b}$$

where the mixing angle is given by

$$\tan^2\theta=\frac{M_Z^2-M_{Z_1}^2}{M_{Z_2}^2-M_Z^2}\ , \tag{3.12}$$

with $M_{Z_1}^2$ and $M_{Z_2}^2$ being the masses for Z_1 and Z_2. Here, Z_1 corresponds to the standard model neutral gauge boson and Z_2 corresponds to the additional neutral gauge boson. From Eq. (3.10c), the limit for $M_{ZZ'}$ is

$$-\frac{1+2\sin^2\theta_W}{\sqrt{3}\sqrt{1-4\sin^2\theta_W}}M_Z^2\le M_{ZZ'}^2\le\frac{\sqrt{1-4\sin^2\theta_W}}{\sqrt{3}}M_Z^2\ . \tag{3.13}$$

Combining Eqs. (3.12) and (3.13), we plot (dashed curves) the bounds of the mixing angle as a function of M_{Z_2} in Fig. 2, where $M_{Z_1}=91.175$ GeV is used. The area within the dashed lines is the allowed region obtained from this minimal Higgs structure. The result would be modified for extended Higgs structures.

From the symmetry-breaking hierarchy $u>v,v',w$, we obtain the lower mass bound of Z_2:

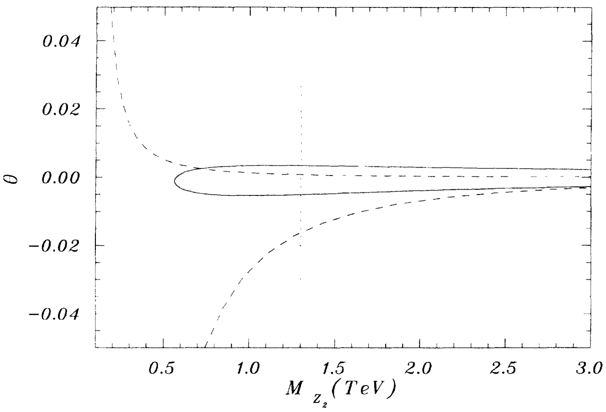

FIG. 2. The bounds on the mixing angle θ and M_{Z_2} from the Higgs structure (dashed lines), from muon decay (dotted line), and the 90% C.L. contour (solid line) from low-energy experiments.

$$M_{Z_2} > \left[\frac{4}{3}\right]^{1/2} \frac{\cos^2\theta_W(M_{Z_2})}{\sqrt{1-4\sin^2\theta_W(M_{Z_2})}} M_{Z_1}$$
$$> 280 \text{ GeV} . \quad (3.14)$$

On the other hand, the masses of the new charged gauge boson $Y^{\pm}$ and $Y^{\pm\pm}$ are constrained from the lepton collider experiments [5] and muon decay [6]. Thus, $M_{Y^{++}}$ and $M_{Y^{+}}$ are greater than 210 GeV (95% C.L.) and 270 GeV (90% C.L.), respectively.

$M_{Y^{++}}$ and $M_{Y^{+}}$ are approximately related to M_{Z_2} as $M_{Y^{+}} \simeq M_{Y^{++}}$

$$= M_Y \simeq \left[\frac{3}{4}\right]^{1/2} \frac{\sqrt{1-4\sin^2\theta_W(M_Y)}}{\cos\theta_W(M_Y)} M_{Z_2} . \quad (3.15)$$

Hence, we can further constrain M_{Z_2} by using the lower bound $M_Y \geq 270$ GeV; it yields $M_{Z_2} \geq 1.3$ TeV. With the same token, M_Y is also bounded from above. Therefore, the allowed ranges for the new gauge bosons are

$$1.3 \text{ TeV} \leq M_{Z_2} \leq 3.1 \text{ TeV} , \quad (3.16a)$$

$$270 \text{ GeV} \leq M_Y \leq 550 \text{ GeV} . \quad (3.16b)$$

The mass lower bound $M_{Z_2} \geq 1.3$ TeV is plotted (dotted line) in Fig. 2. The intersection points of the dotted line and dashed curves in Fig. 2 yield the limits on the mixing angle:

$$-1.6\times 10^{-2} \leq \theta \leq 7\times 10^{-4} . \quad (3.17)$$

Further constraint on θ and M_{Z_2} will be considered in Sec. V.

Within the allowed range of M_{Z_2}, we find that M_Y is always less than 0.5 M_{Z_2}. Therefore, the decay $Z_2 \to Y^{++}Y^{--}$ and $Y^{\pm\pm} \to 2l^{\pm}$ ($l = e,\mu,\tau$) is allowed, leading to a spectacular signature in future colliders. In many extensions of the SM, the masses of the additional neutral gauge bosons are not bounded from above. They can be as heavy as the unification scale. Here, this model is very restrictive. Hence, it would be either ruled out or realized in the near future.

In Ref. [1], the authors obtained the mass lower bound for Z_2 to be 40 TeV from the K^0-$\bar{K}^0$ mixing. In the next section, we will explain this controversial disagreement.

IV. FLAVOR-CHANGING NEUTRAL CURRENT PROCESSES

Since $M_{Z'}$ is expected to be smaller than 3.1 TeV, flavor-changing neutral current processes induced by Z' would be important constraints for this model. Let us first write out the gauge interactions for Z and Z' explicitly.

Gauge interaction of Z boson. The gauge interaction is given by

$$\mathcal{L}(Z) = \frac{g}{\cos\theta_W} Z^{\mu} J_{\mu} , \quad (4.1)$$

where

$$J_{\mu} = \bar{f}\gamma_{\mu}\left[g_L(f)\frac{1-\gamma_5}{2} + g_R(f)\frac{1+\gamma_5}{2}\right] f \quad (4.2a)$$

$$= \bar{f}\gamma_{\mu}[g_V(f) + g_A(f)\gamma_5] f . \quad (4.2b)$$

The neutral current coupling coefficients are given as

$$g_{L,R}(f) = T^3(f_{L,R}) - Q(f)\sin^2\theta_W , \quad (4.3a)$$

$$g_V(f) = \frac{g_R(f)+g_L(f)}{2}, \quad g_A(f) = \frac{g_R(f)-g_L(f)}{2} . \quad (4.3b)$$

T^3 is the isospin generation. $T^3(Q)=0$ for $Q = D_{1,2}$ and T, leading to $g_L(Q)=g_R(Q)$. Hence, the gauge couplings of Q to Z are vectorlike. As in the standard model, the gauge couplings of the usual fermions to Z are the same for all three generations, hence there is no flavor changing neutral current (FCNC) at the tree level.

Gauge interaction of Z′ boson. The gauge interaction is given by

$$\mathcal{L}(Z') = \frac{g}{\cos\theta_W} Z'^{\mu} J'_{\mu} , \quad (4.4)$$

where

$$J'_{\mu} = \bar{f}\gamma_{\mu}\left[g'_L(f)\frac{1-\gamma_5}{2} + g'_R(f)\frac{1+\gamma_5}{2}\right] f \quad (4.5a)$$

$$= \bar{f}\gamma_{\mu}[a^f + b^f\gamma_5] f , \quad (4.5b)$$

where the new neutral current coupling coefficients are given as

$$g'_{L,R}(f) = -\frac{\sqrt{1-4\sin^2\theta_W}}{2\sqrt{3}} Y(f_{L,R}) + \frac{1-\sin^2\theta_W}{\sqrt{3}\sqrt{1-4\sin^2\theta_W}} X(f_{L,R}) , \quad (4.6a)$$

$$a^f=\frac{g_R'(f)+g_L'(f)}{2}, \quad b^f=\frac{g_R'(f)-g_L'(f)}{2} . \tag{4.6b}$$

In the left-handed sector, the third generation have a different X charge from the other two generations; their gauge couplings to Z' are different, leading to the FCNC. In particular, the FCNC in the down sector is given by

$$\mathcal{L}_{\rm FCNC}=\frac{g}{\cos\theta_W}[-\sin\theta Z_1^\mu+\cos\theta Z_2^\mu]\delta_L \bar{d}_3\gamma_\mu\frac{1-\gamma_5}{2}d_3 \tag{4.7a}$$

$$=\frac{g}{\cos\theta_W}[-\sin\theta Z_1^\mu+\cos\theta Z_2^\mu] \times\delta_L U_{3\alpha}^* U_{3\beta}\bar{\alpha}\gamma_\mu\frac{1-\gamma_5}{2}\beta \tag{4.7b}$$

and

$$\delta_L=g_L(d_3)-g_L(d_1)=\frac{1-\sin^2\theta_W}{\sqrt{3}\sqrt{1-4\sin^2\theta_W}} , \tag{4.7c}$$

where U, the unitary matrix relating the gauge states to mass eigenstates, is defined as $d_i=U_{i\alpha}\alpha$ with $i=1,2,3$ and $\alpha=d,s,b$. There is no FCNC for the right-handed currents as the right-handed fermions transform identically.

From Eqs. (4.7), K^0-$\bar{K}^0$ and B^0-$\bar{B}^0$ mixing are calculated to be

$$\Delta M_K\simeq\frac{4\pi\alpha}{3\sin^2\theta_W\cos^2\theta_W}|U_{3d}^*U_{3s}|^2 \times\delta_L^2\left[\frac{\cos^2\theta}{M_{Z_2}^2}+\frac{\sin^2\theta}{M_{Z_1}^2}\right]B_K f_K^2 M_K , \tag{4.8}$$

$$\Delta M_{B_d}\simeq\frac{4\pi\alpha}{3\sin^2\theta_W\cos^2\theta_W}|U_{3d}^*U_{3b}|^2 \times\delta_L^2\left[\frac{\cos^2\theta}{M_{Z_2}^2}+\frac{\sin^2\theta}{M_{Z_1}^2}\right]B_B f_B^2 M_B , \tag{4.9}$$

where $f_{K,B}$ and $B_{K,B}$ are the decay constants and bag factors of a K and B mesons. From the allowed mass range for Z_2, the contributions from first term in Eqs. (4.8) and (4.9) are at least 10 times bigger than that of the second term. Taking $\sin^2\theta_W=0.23$, $\sqrt{B_B}f_B=\sqrt{B_K}f_K=160$ MeV, and assume the new contributions to the mixing to be $\Delta M_K\le 1\times10^{-15}$ GeV and $\Delta M_B\le 1\times10^{-13}$ GeV, we obtain

$$|U_{3d}^*U_{3s}|\le 5\times10^{-3} , \tag{4.10}$$

$$|U_{3d}^*U_{3b}|\le 2\times10^{-2} . \tag{4.11}$$

The ansatz adopted in Ref. [1] is that the up-type quarks are chosen to be the mass eigenstates; thus, the mixing of the down quark sector is exactly the Cabibbo-Kobayaski-Maskawa (CKM) matrix. Hence, the authors obtained the lower Z_2 to be 40 TeV as $U_{3d}^*U_{3s}\to\sin\theta_C\sim0.22$. In the standard model, the gauge interactions are universal among generations. Thus, we are allowed to redefine quarks be the mass eigenstates in the gauge interactions of the Z. Here, in this model, the gauge interactions of Z' for the first two generations are different from those for the third generation. Therefore, the ansatz adopted in Ref. [1] was not appropriate.

V. LOW-ENERGY EXPERIMENTS

Models with additional neutral gauge bosons, such as E_6 and LR models, have been intensively investigated [10]. In particular, the mass lower bounds at 90% C.L. for Z_2 from neutrino-hadron scattering for E_6 (χ model) and LR models are 555 GeV and 795 GeV [11], respectively. The corresponding mixing angles are bound to be less than 6×10^{-3} and 5×10^{-3}. Here, the coupling strength of Z' to quarks in this model is much stronger than that of leptons because of the enhancement factor of $1/\sqrt{1-4\sin^2\theta_W}$. Therefore, low-energy experiments such as neutrino-nucleus scattering and atomic parity violation measurement would be important processes to further constrain this model.

In the low-energy limit, the four-fermion neutral-current Hamiltonian, in the limit of small mixing, is given by

$$\frac{4G_F}{\sqrt{2}}\left[1+\frac{M_{Z_2}^2}{M_{Z_1}^2}\theta^2\right]\left[J_\mu J^\mu-2\theta J_\mu' J^\mu+\frac{M_{Z_1}^2}{M_{Z_2}^2}J_\mu' J'^\mu\right] . \tag{5.1}$$

It would be important to include the radiative corrections to the measurables, denoted as starred functions [12,13]. If we ignore the effects due to the combination of mixing and radiative corrections, we can express the $Z-Z'$ mixing effects as shifts of the starred functions.

Atomic parity violation. The weak charge is given by

$$Q_W=Q_{W_*}+2[\delta C_1(u)(2Z+N)+\delta C_1(d)(Z+2N)] , \tag{5.2a}$$

where

$$\delta C_1(u)=\left[\frac{1}{2}-\frac{4}{3}s_0^2\right]\frac{M_{Z_2}^2}{M_{Z_1}^2}\theta^2 -2\theta\left[a^u+\left[1-\frac{8}{3}s_0^2\right]b^e\right]+8\frac{M_{Z_1}^2}{M_{Z_2}^2}b^e a^u , \tag{5.2b}$$

$$\delta C_1(d)=\left[-\frac{1}{2}+\frac{2}{3}s_0^2\right]\frac{M_{Z_2}^2}{M_{Z_1}^2}\theta^2 -2\theta\left[a^d+\left[-1+\frac{4}{3}s_0^2\right]b^e\right]+8\frac{M_{Z_1}^2}{M_{Z_2}^2}b^e a^d . \tag{5.2c}$$

Neutrino nucleus scattering. The left- and right-handed coupling coefficients are given as

$$\epsilon_\lambda(q)=g_\lambda^0(q)_*\left[1+\frac{M_{Z_2}^2}{M_{Z_1}^2}\theta^2-4\theta a^v\right]-\left[\theta-4a^v\frac{M_{Z_1}^2}{M_{Z_2}^2}\right]g'_\lambda(q)\ , \tag{5.3}$$

for $\lambda=R$ and L. Hence we obtain the quantities, which can be extracted from the neutrino-nucleus experiments:

$$g_L^2=\epsilon_L(u)^2+\epsilon_L(d)^2$$

$$=g_{L_*}^2+\left[\frac{1}{2}-s_0^2+\frac{5}{9}s_0^4\right]\left[2\frac{M_{Z_2}^2}{M_{Z_1}^2}\theta^2-8\theta a^v\right]-2\left[\left[\frac{1}{2}-\frac{2}{3}s_0^2\right]g'_L(u)+\left[-\frac{1}{2}+\frac{1}{3}s_0^2\right]g'_L(d)\right]\left[\theta-4a^v\frac{M_{Z_1}^2}{M_{Z_2}^2}\right] \tag{5.4a}$$

and

$$g_R^2=\epsilon_R(u)^2+\epsilon_R(d)^2$$

$$=g_{R_*}^2+\frac{5}{9}s_0^4\left[2\frac{M_{Z_2}^2}{M_{Z_1}^2}\theta^2-8\theta a^v\right]-2\left[-\frac{2}{3}s_0^2\,g'_R(u)+\frac{1}{3}s_0^2\,g'_R(d)\right]\left[\theta-4a^v\frac{M_{Z_1}^2}{M_{Z_2}^2}\right]. \tag{5.4b}$$

$s_0^2=\sin^2\theta_W(q^2=0)$. a's and b's are also evaluated at $q^2=0$.

The starred functions Q_{W_*}, $g_{L_*}^2$, and $g_{R_*}^2$ are modified by the radiative corrections including oblique corrections. The explicit expressions are given in Ref. [13]. The oblique corrections depend on the masses of the top quark and Higgs scalars. Unless the mass splittings of Higgs scalars are large [14], the main contribution comes from the top quark. A detailed calculation of the oblique corrections from the new particles as well as the global constraint for θ and M_{Z_2} are considered in Ref. [15]. Here, we take the top-quark mass and the standard model Higgs boson mass, $m_t=150$ GeV and $m_H=1000$ GeV, and use experimental values given in Ref. [16], we obtain a 90% C.L. contour plotted in Fig. 2. We find that the Z-Z' mixing angle is further constrained, which becomes

$$-5\times10^{-3}\le\theta\le7\times10^{-4}. \tag{5.5}$$

VI. CONCLUSION

In conclusion, we have studied an electroweak theory of SU(3)×U(1) in detail. Two different representations are adopted in [1,2], but they are *identical* as we can map from one representation to another by a unitary transformation. By matching the coupling constants at the SU(3)×U(1) symmetry breaking, we find that $\sin^2\theta_W(M_{Z'})$ should be less than 1/4, leading to $M_{Z'}\le3.1$ TeV if we take $\sin^2\theta_W(M_Z)=0.2333$. From the muon decay experiments, M_Y is found to be at least 270 GeV at 90% confidence level. Hence, we obtain narrow windows for M_Y and M_{Z_2}, 270 GeV $\le M_Y\le550$ GeV and 1.3 TeV $\le M_{Z_2}\le3.1$ TeV. Furthermore, the limits of the Z-Z' mixing, further constrained from the low-energy experiments, is $-5\times10^{-3}\le\theta\le7\times10^{-4}$. Within the allowed range of M_{Z_2}, the masses of the new charged gauge bosons $Y^\pm$ and $Y^{\pm\pm}$ are less than a half of M_{Z_2}. Therefore the allowed decay $Z_2\to Y^{++}Y^{--}$ with $Y^{\pm\pm}\to2l^\pm$ provides an unique signature in future colliders. Thus the model can either be realized or ruled out in the near future.

ACKNOWLEDGMENTS

The author would like to thank James T. Liu for useful discussions. The work was supported in part by the Natural Science and Engineering Research Council of Canada.

[1] F. Pisano and V. Pleitez, Phys. Rev. D **46**, 410 (1992); R. Foot, O. Hernández, F. Pisano, and V. Pleitez, *ibid.* **47**, 4158 (1993).
[2] P. H. Frampton, Phys. Rev. Lett. **69**, 2889 (1992).
[3] M. Singer, J. W. F. Valle, and J. Schechter, Phys. Rev. D **22**, 738 (1980), and references therein.
[4] P. H. Frampton *et al.*, UNC Report No. IFP-466-UNC, 1993.
[5] P. H. Frampton and Daniel Ng, Phys. Rev. D **45**, 4240 (1991), ($M_{Y^{++}}>210$ GeV is obtained for $g_{3l}=g$).
[6] I. Beltrami *et al.*, Phys. Lett. B **194**, 326 (1987).
[7] W. C. Barber *et al.*, Phys. Rev. D **3**, 2796 (1971).
[8] An antisymmetric matrix alone will give an unacceptable mass relation $m_\mu=m_\tau$.
[9] Paul Langacker and Ming-xin Luo, Phys. Rev. D **44**, 817 (1991).
[10] For example, Paul Langacker and Ming-xing Luo, Phys. Rev. D **45**, 278 (1992); J. Rosner, *ibid.* **42**, 3107 (1990).
[11] F. del Aquilo, J. M. Moreno, and M. Quiros, Phys. Lett. B **254**, 497 (1991).

[12] D. Kennedy and B. W. Lynn, Nucl. Phys. **B322**, 1 (1989).
[13] M. E. Peskin and T. Takeuchi, Phys. Rev. Lett. **65**, 964 (1990); Phys. Rev. D **46**, 381 (1992).
[14] Because of the Veltman's screening effect, Higgs boson mass dependent in oblique corrections is logarithmically suppressed unless the scalar mass splittings are large, see for example, A. Denner *et al.*, Phys. Lett. B **240**, 438 (1990).
[15] J. T. Liu and D. Ng, Z. Phys. C (to be published).
[16] Particle Data Group, K. Hikasa *et al.*, Phys. Rev. D **45**, S1 (1992).