

Light hadron spectrum and decay constants in quenched lattice QCD

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We present results for light hadrons composed of both degenerate and nondegenerate quarks in quenched lattice QCD. We calculate masses and decay constants using 60 gauge configurations with an $O(a)$ -improved fermion action at $\beta=6.2$. Using the ρ mass to set the scale we find hadron masses within two to three standard deviations of the experimental values (given in parentheses): $m_{K^*} = 868_{-8}^{+9}$ MeV (892 MeV), $m_\phi = 970_{-10}^{+20}$ MeV (1020 MeV), $m_N = 820_{-60}^{+90}$ MeV (938 MeV), $m_\Delta = 1300_{-100}^{+100}$ MeV (1232 MeV), and $m_\Omega = 1650_{-50}^{+70}$ MeV (1672 MeV). Direct comparison with experiment for decay constants is obscured by uncertainty in current renormalizations. However, for ratios of decay constants we obtain $f_K/f_\pi = 1.20_{-2}^{+3}$ (1.22) and $f_\phi/f_\rho = 1.13_{-3}^{+2}$ (1.22).

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I. INTRODUCTION

Within the quenched approximation, it is currently possible to study lattice QCD numerically in a box of linear size around 2 fm, with a lattice spacing of less than 0.1 fm, corresponding to a cutoff above 2 GeV. Although extrapolation to the chiral limit is still a necessary ingredient when u and d quarks are involved, such lattices allow the direct simulation of hadrons containing s quarks, at the cost of fixing one additional mass parameter. A significantly wider range of physical quantities thereby becomes calculable, with which to probe the reliability of lattice QCD.

In this paper, we extend an earlier study [1,2] of light hadron masses and decay constants to include the effects of SU(3)-flavor-symmetry breaking. The earlier study was based on 18 configurations, and the correlation functions were evaluated for hadrons composed of degenerate quarks, for five different values of the quark mass, using both the Wilson and clover fermion actions. The results presented in this paper were obtained from our complete data set of 60 configurations, using the clover fermion action, for three of the five previously used quark masses. In order to study flavor-symmetry-breaking effects, we construct mesons using all possible quark-mass combinations.

In the Wilson formulation, the bare quark mass m is

given in terms of the hopping parameter κ by

$$m = \frac{1}{2} \left(\frac{1}{\kappa} - \frac{1}{\kappa_{\text{crit}}} \right). \quad (1)$$

κ_{crit} is the value of the hopping parameter at zero quark mass, which is taken to be the point at which the mass of the pseudoscalar meson, m_P , vanishes and the quark and antiquark are degenerate. With each quark flavor, we need to associate a value of κ corresponding to its experimentally observed mass. It is a good approximation to take the physical light quark (u and d) masses to be zero, i.e., to set $\kappa_u = \kappa_d = \kappa_{\text{crit}}$. Here we are interested in computing the effect of SU(3)-flavor-symmetry breaking on the spectrum and decay constants, and so we need to associate a nonzero mass with the strange quark. The corresponding κ_s can be determined, for example, by first extrapolating the vector meson mass $m_V(\kappa_1, \kappa_2)$ to $\kappa_1 = \kappa_2 = \kappa_{\text{crit}}$ and then fitting the data for the ratio $m_P^2(\kappa_1, \kappa_2)/m_V^2(\kappa_{\text{crit}}, \kappa_{\text{crit}})$ to some function of the two quark masses, extrapolating in κ_1 to κ_{crit} and using κ_2 to fix the ratio to the experimental value of m_K^2/m_ρ^2 . Alternatively, κ_s may be determined from the degenerate-quark data alone, avoiding the second chiral extrapolation, by using the ratio m_Ω/m_ρ .

This procedure requires some assumption about how the hadron masses depend on the quark masses. In most previous calculations (see, for example, Refs. [2–6]), it has been assumed that the pseudoscalar meson mass obeys the PCAC (partial conservation of axial-vector current) relation

$$m_P^2(\kappa_1, \kappa_2) = b_P \left[\frac{1}{2\kappa_1} + \frac{1}{2\kappa_2} - \frac{1}{\kappa_{\text{crit}}} \right], \quad (2)$$

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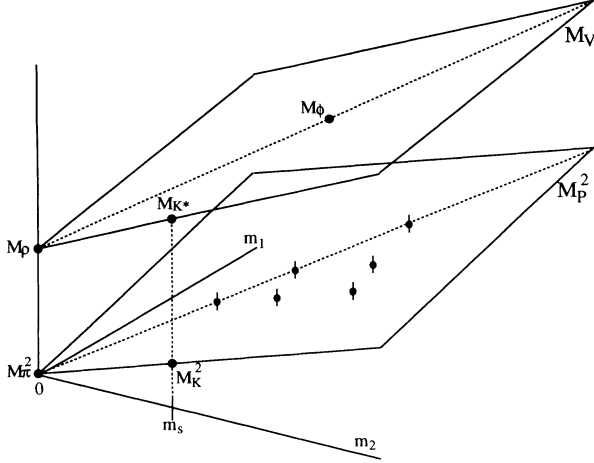


FIG. 1. Schematic of the fit planes defined by Eq. (4).

and that the vector meson mass obeys

$$m_V(\kappa_1, \kappa_2) = a_V + b_V \left[\frac{1}{2\kappa_1} + \frac{1}{2\kappa_2} - \frac{1}{\kappa_{\text{crit}}} \right]. \quad (3)$$

The original work of Martinelli *et al.* [7] supported this assumption, albeit on the basis of rather limited statistics, by combining quark propagators computed for different values of κ . More recent studies of strange hadrons [8,9] have similarly utilized nondegenerate quarks, and this is the procedure that we adopt here.

We fit our data for m_P^2 , m_V , f_P , etc., to the following function of the two quark masses m_1 and m_2 :

$$a_1 + \frac{a_2}{2}(m_2 + m_1) + \frac{a_3}{2}|m_2 - m_1| \quad (4)$$

$$S_F^W = \sum_x \left[\bar{q}(x)q(x) - \kappa \sum_\mu [\bar{q}(x)(1 - \gamma_\mu)U_\mu(x)q(x + \hat{\mu}) + \bar{q}(x + \hat{\mu})(1 + \gamma_\mu)U_\mu^\dagger(x)q(x)] \right], \quad (6)$$

and $F_{\mu\nu}$ is a lattice definition of the field strength tensor. We have computed propagators at three values of κ , 0.14144, 0.14226, and 0.14262, using an over-relaxed minimal residual algorithm with red-black preconditioning and point sources and sinks. Although there are advantages to using smeared sources and/or sinks to extract ground-state properties from two-point functions [12], constraints imposed by other parts of the UKQCD program did not allow us this option.

We construct correlators for mesons composed of quarks of flavors 1 and 2 using the local interpolating fields

$$P = \bar{q}_1 \gamma_5 q_2, \quad (7)$$

$$A_4 = \bar{q}_1 \gamma_4 \gamma_5 q_2, \quad (8)$$

$$V_i = \bar{q}_1 \gamma_i q_2, \quad (9)$$

to test the assumption that $a_3 = 0$ in Eqs. (2) and (3). The expression (4), or its equivalent with $a_3 = 0$, defines a plane through the data, as shown in Fig. 1. By means of a combination of extrapolation and interpolation using such fits, we are able to calculate the masses and decay constants of the K , K^* , and ϕ mesons and the mass of the Ω baryon, in addition to those of the usual light hadrons.

In Sec. II we summarize our statistics and present details of our fitting criteria and procedures. Our main results are given in Sec. III where we describe the chiral extrapolations, the interpolation to the strange quark mass, and the masses and decay constants in physical units. Section IV contains our conclusions.

II. COMPUTATIONAL DETAILS

A. Statistics

We have analyzed 60 gauge field configurations at an inverse coupling $\beta = 6.2$ on a lattice of size $24^3 \times 48$. The gauge configurations and quark propagators were produced on the 64-node i860 Meiko Computing Surface at the University of Edinburgh. The SU(3) gauge fields were generated using the hybrid overrelaxed algorithm, defined in Ref. [2]. The gauge configurations are separated by 2400 sweeps, beginning at configuration 16 800. The quark propagators were calculated using an $O(a)$ -improved clover action [10,11]

$$S_F^C = S_F^W - i \frac{\kappa}{2} \sum_{x,\mu,\nu} \bar{q}(x) F_{\mu\nu}(x) \sigma_{\mu\nu} q(x). \quad (5)$$

S_F^W is the standard Wilson lattice action,

and correlators for baryons composed of degenerate quarks using

$$N = \epsilon_{abc} (u^a C \gamma_5 d^b) u^c, \quad (10)$$

$$\Delta_\mu = \epsilon_{abc} (u^a C \gamma_\mu u^b) u^c. \quad (11)$$

For the vector meson we average our correlators over the three polarization states, for the nucleon we average the 11 and 22 spinor indices of the correlator, and for the Δ we project out the spin- $\frac{3}{2}$ component and average over the four spin projections. Our quark propagators incorporate the rotations required to ensure that using these interpolating fields yields $O(a)$ -improved correlators. This and our computational procedure are described in detail in Ref. [2].

Except where explicitly stated otherwise, the errors quoted in this paper are purely statistical and are calcu-

lated according to the prescription: create 1000 bootstrap samples from the original dataset of 60 configurations by randomly choosing, with replacement, 60 configurations per sample; for each bootstrap sample, perform all the mass fits and extrapolations as for the original data; obtain the errors on a given quantity from the 68% confidence limits of the corresponding bootstrap distribution.

B. Fitting procedure

We construct two-point meson correlation functions from quark propagators with all combinations of the three κ values. We perform least- χ^2 fits to the zero-momentum time-symmetrized time-slice correlators to single cosh functions. For the pseudoscalar channel, we fit over the time range $t=14-22$ for all κ combinations. For the vector channel, we use the fitting range $t=13-23$ for all except the heaviest degenerate- κ case, where we use $t=15-23$. For the baryons, we construct two-point correlation functions only for the degenerate cases. We fit the appropriate average of the forwards and backwards, zero-momentum time-slice correlators to single exponential functions, choosing the time ranges $t=16-22$ for the nucleon and $t=16-21$ for the Δ . We take account of time correlations in the least- χ^2 fits.

We carried out an extensive investigation of the most appropriate fitting ranges, before arriving at the above choices. With reference to the effective mass plots in Figs. 2 and 3, we fixed t_{\max} to be as large as possible and reduced t_{\min} until the χ^2/N_{DF} showed a significant increase. In this way, we attempted to fit as many time slices as possible. We followed the reasoning of Ref. [13] in fitting our data as far out as possible to avoid contamination from excited states at earlier times.

In Fig. 4 we show the variation of the degenerate-quark vector meson mass estimates and of the corresponding χ^2/N_{DF} with the position of the first time slice, t_{\min} , in a variable window, fixing $t_{\max}=23$. We observe a slow monotonic decrease from $t_{\min}=14$ onwards, at all κ values, although there is no significant variation in χ^2/N_{DF} from $t_{\min}=13$ and the mass estimates agree within statistical errors for the two lightest κ values. The case of the heaviest κ value is more problematic given the small statistical errors. We adopt the compromise choice of $t_{\min}=15$ for this, which is consistent with a wider range of other choices of t_{\min} . We attempt to quantify in Table I the systematic error arising from the choice of fit range, by taking the lowest and highest mass estimates from fits with values of t_{\min} acceptable according to the above criteria. We note that the monotonic decrease in

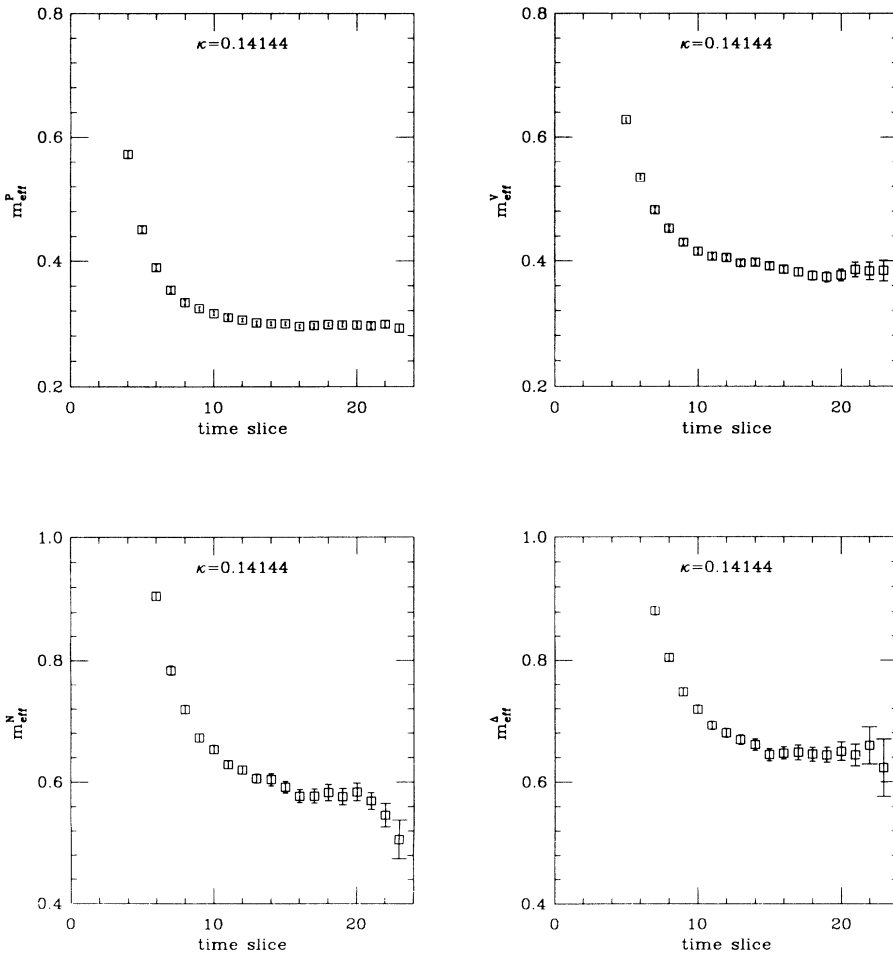


FIG. 2. Effective mass plots for the pseudoscalar, vector, nucleon, and Δ at $\kappa=0.14144$.

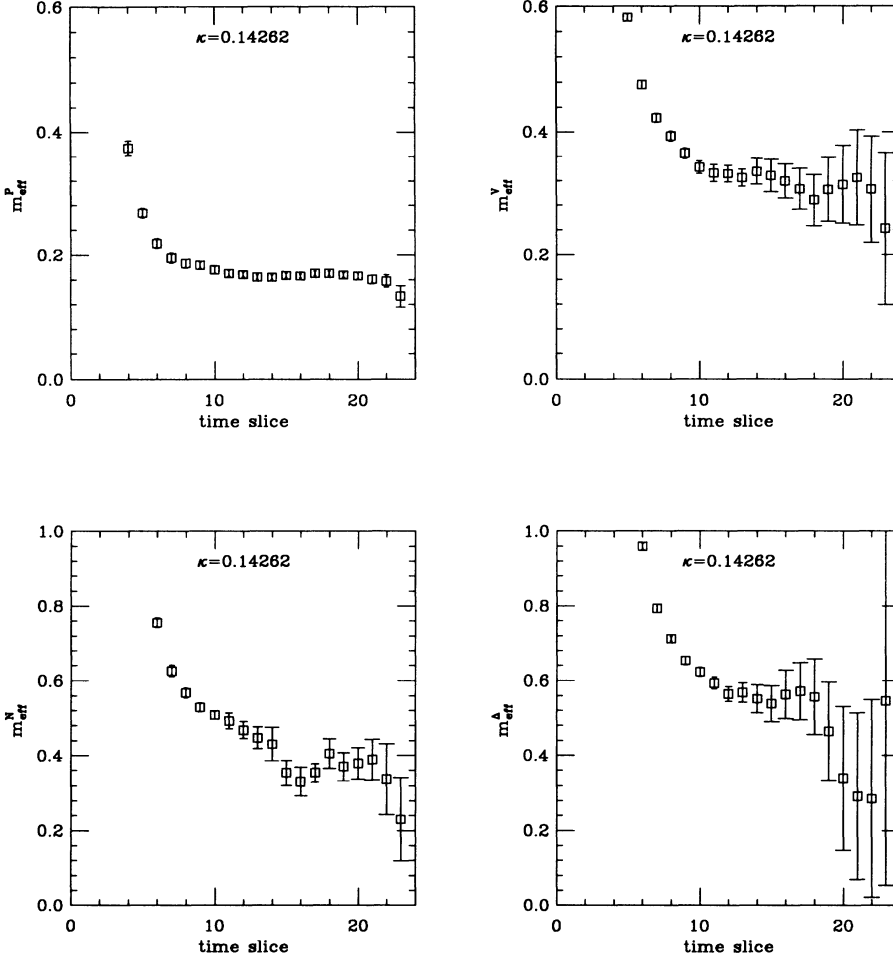


FIG. 3. Effective mass plots for the pseudoscalar, vector, nucleon, and Δ at $\kappa=0.14262$.

our mass estimates with increasing t_{\min} is reflected in the asymmetry of the systematic error estimates towards lower mass values. However, beyond this, we feel unable to quote quantitative estimates of this error in our results.

In Fig. 5 and Table II we show the corresponding data for the nucleon. These provide convincing evidence of plateaus at all three κ values, beginning at $t_{\min}=16$ for the heaviest and at $t_{\min}=14$ for the two lightest masses. We note that the large upper error bars in the latter data at $t_{\min}=14$ and 15 may indicate the sensitivity of the bootstrap sampling to the tail of the excited-state contributions. For this reason, we take $t_{\min}=16$ for all three κ values. The systematic error estimates in Table II are well within the statistical errors, giving us confidence in our choice of fit range.

We calculate the pseudoscalar decay constant from the ratio

$$\frac{\sum_x \langle A_4(x,t)P^\dagger(0) \rangle}{\sum_x \langle P(x,t)P^\dagger(0) \rangle} \sim \frac{f_P m_P}{Z_A \langle 0|P|P \rangle} \tanh m_P(L_t/2-t), \quad (12)$$

fitting to the same time slice range as in the fit to the pseudoscalar correlator. We use the parameter from this latter fit to fix m_P and the matrix element $\langle 0|P|P \rangle$. We find that this particular ratio of correlators gives the cleanest signal from which to extract f_P [12]. We determine f_V by fitting to

$$\sum_{j=1}^3 \sum_x \langle V_j(x,t)V_j^\dagger(0) \rangle \sim \frac{3m_V^3}{2Z_V^2 f_V^2} e^{-m_V L_t/2} \cosh m_V(L_t/2-t). \quad (13)$$

TABLE I. Highest and lowest fit masses for the vector meson with degenerate quarks, based on an analysis of the fit regions 13–23 to 18–23. We take the difference between the highest (lowest) mass and the best-fit mass as a measure of the systematic error.

κ	m_V^{high}	m_V^{low}	m_V	Error	
				Statistical	Systematic
0.141 44	0.395	0.376	0.389	+0.007–0.006	+0.006–0.013
0.142 26	0.351	0.312	0.343	+0.009–0.007	+0.008–0.031
0.142 62	0.335	0.284	0.319	+0.014–0.013	+0.016–0.035

TABLE II. Highest and lowest fit masses for the nucleon, based on an analysis of the fit regions 16–22 to 19–22 for the heaviest and 14–22 to 19–22 for the others. We take the difference between the highest (lowest) mass and the best-fit mass as a measure of the systematic error.

κ	m_N^{high}	m_N^{low}	m_N	Error	
				Statistical	Systematic
0.141 44	0.573	0.568	0.573	+0.015–0.007	+0.000–0.005
0.142 26	0.475	0.452	0.462	+0.020–0.014	+0.013–0.010
0.142 62	0.392	0.372	0.372	+0.024–0.016	+0.020–0.000

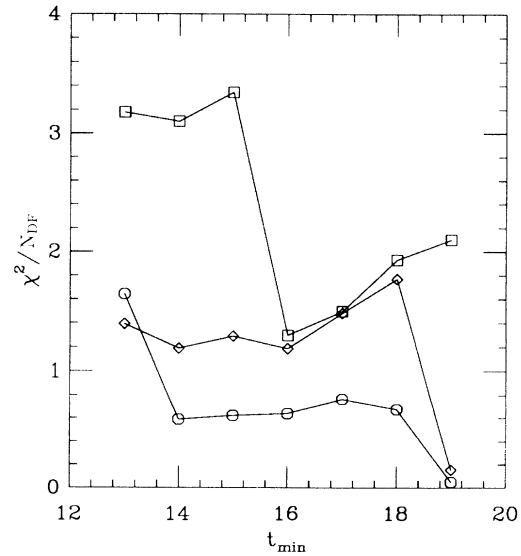
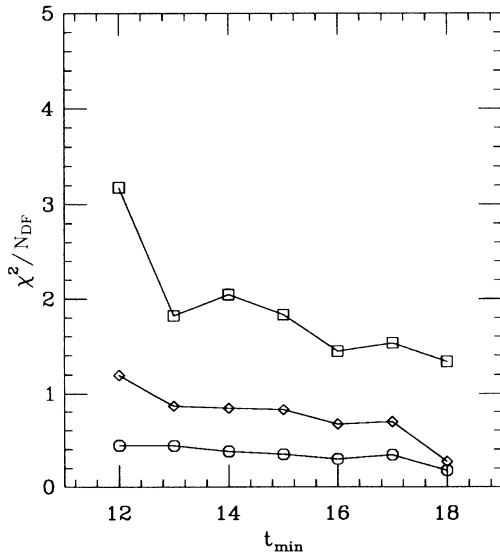
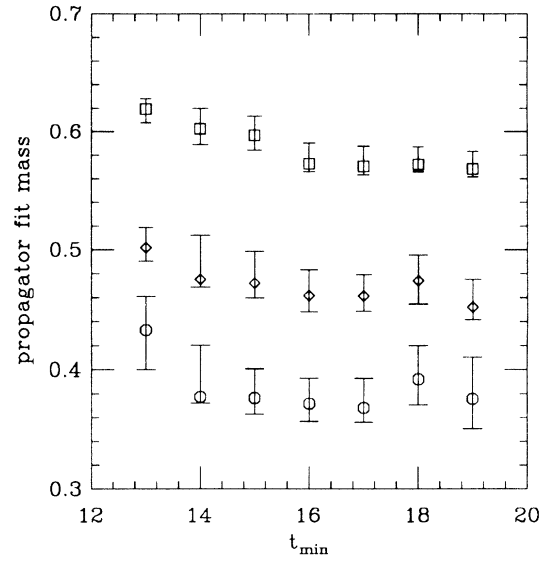
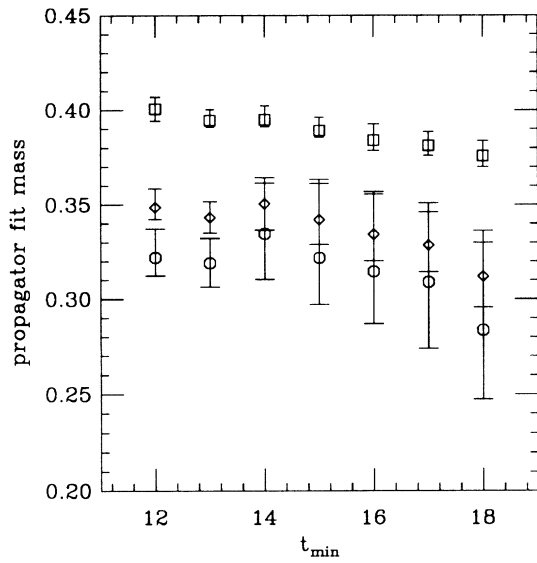


FIG. 4. Fit stability plots for the vector meson. The top graph shows the variation of fit mass with t_{\min} , fixing $t_{\max} = 23$. The bottom graph shows the corresponding χ^2/N_{DF} . The different symbols refer to different κ values: squares = 0.141 44, diamonds = 0.142 26, and circles = 0.142 62.

FIG. 5. Fit stability plots for the nucleon. The top graph shows the variation of fit mass with t_{\min} , fixing $t_{\max} = 22$. The bottom graph shows the corresponding χ^2/N_{DF} . The different symbols refer to different κ values: squares = 0.141 44, diamonds = 0.142 26, and circles = 0.142 62.

Here, Z_A and Z_V are the factors required to ensure that the lattice currents obey the correct current algebra in the continuum limit [14,15].

III. RESULTS

A. Masses and decay constants in lattice units

In Table III we present the masses and decay constants of the vector and pseudoscalar mesons calculated from both degenerate-quark and nondegenerate-quark correlators. The χ^2/N_{DF} are all satisfactory, generally being between 0.5 and 2. The degenerate-quark data may be compared with our results from the first 18 configurations, presented in Ref. [2], where our fit range was 12–16 for all hadrons. Our new estimates are within 1σ of the 18 configuration estimates. The errors are reduced by a factor of approximately 2.

As noted previously [1,2], experimental data suggests that the hyperfine splitting $m_V^2 - m_P^2$ should be only weakly dependent on the quark masses for light hadrons, and this we observed within large errors. Our higher-statistics results quoted in Table III, are shown in Fig. 6. The new data for both degenerate and nondegenerate quarks is entirely consistent with our earlier results, but with significantly smaller errors.

In Table IV we present the masses of the nucleon and Δ computed using degenerate quarks only. Again, the χ^2/N_{DF} of the fits is acceptable. Our estimates for the nucleon mass are about 2σ below our estimates based on the first 18 configurations [2], with only slightly smaller errors. It is clear from Fig. 5 that $t_{\text{min}} = 12$ is not in the plateau region when using 60 configurations. We have used the increase in sample size primarily to reduce the systematic error in the mass due to contamination by excited states. However, this may not be the only effect. We observe increasing fluctuations in the nucleon mass with decreasing quark mass, and fluctuations which decrease the nucleon mass tend to dominate the statistical average. The influence of such fluctuations in the present analysis may be more pronounced because we fit the nucleon correlator further from the source.

Apart from at the highest quark mass, where our new estimate is 3σ lower, our estimates for m_Δ agree with our

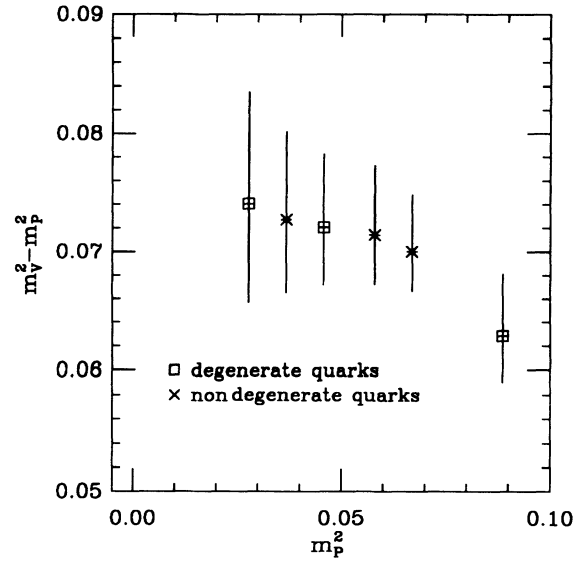


FIG. 6. Vector-pseudoscalar meson mass splitting for both degenerate- and nondegenerate-quark data.

previous results. The errors have not decreased, probably because of the extended fitting range.

We have looked for evidence of correlations between successive configurations, by varying the bin size in a jackknife error analysis. This did not reveal any significant effects in the hadron time slice correlators.

In Fig. 7 we show the Edinburgh plot for our degenerate-quark data. The corresponding mass ratios are given in Table V. The apparently alarming fall of the data points may not indicate any discrepancy with experiment, as we shall see that the chirally extrapolated value of m_N/m_V is 1.07^{+11}_-8 , within 2σ of the experimental value, as may be deduced from Table X. Indeed, our higher-statistics results are only $1-2\sigma$ below our earlier result using local sources and sinks [1]. We attribute this difference to our new lower estimates for the nucleon mass. We remark that were the systematic errors in the vector meson mass, discussed above, to be included somehow, the tendency would be for the upper and right-hand error bars to increase.

TABLE III. Masses and decay constants in lattice units of mesons composed of degenerate and nondegenerate quarks.

	Degenerate			Nondegenerate		
κ_1	0.141 44	0.142 26	0.142 62	0.141 44	0.142 62	0.142 26
κ_2	0.141 44	0.142 26	0.142 26	0.142 26	0.141 44	0.142 62
m_P	0.298^{+2}_-2	0.214^{+3}_-3	0.167^{+3}_-4	0.259^{+2}_-2	0.241^{+3}_-3	0.192^{+3}_-3
χ^2/N_{DF}	8.7/7	6.9/7	7.2/7	6.3/7	5.3/7	6.9/7
m_V	0.389^{+7}_-6	0.343^{+9}_-7	0.319^{+14}_-13	0.370^{+6}_-5	0.360^{+8}_-6	0.331^{+11}_-10
χ^2/N_{DF}	13/7	7.8/9	4.0/9	12/9	9.1/9	5.4/9
$m_V^2 - m_P^2$	0.063^{+5}_-4	0.072^{+6}_-5	0.074^{+9}_-8	0.070^{+5}_-3	0.071^{+6}_-4	0.073^{+7}_-6
f_P/Z_A	0.0624^{+7}_-13	0.0512^{+6}_-15	0.0452^{+8}_-21	0.0567^{+7}_-13	0.0539^{+6}_-15	0.0482^{+7}_-17
χ^2/N_{DF}	12/8	9.2/8	8.3/8	11/8	9.2/8	8.1/8
$1/(f_V Z_V)$	0.314^{+8}_-7	0.345^{+8}_-9	0.356^{+9}_-17	0.332^{+6}_-7	0.336^{+8}_-9	0.350^{+9}_-12
χ^2/N_{DF}	13/7	7.8/9	4.0/9	12/9	9.1/9	5.4/9

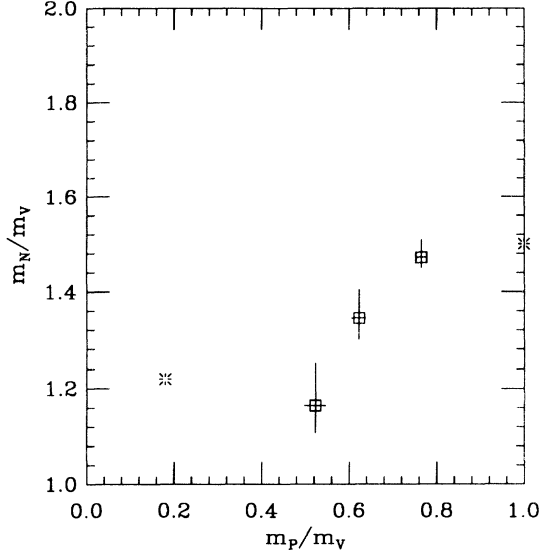


FIG. 7. Edinburgh plot for degenerate-quark data.

B. Quark-mass dependences

We fit our lattice estimates for the meson masses, decay constants, and their ratios, for all six quark-mass combinations, to the expression (4) with a_3 , the coefficient of $|m_2 - m_1|$, (A) unconstrained and (B) constrained to be zero. In Table VI we present values for the fit coefficients. For all the quantities, the values obtained for a_3 in fit (A) are consistent with zero to within one or two standard deviations. The χ^2/N_{DF} for all the fits are satisfactory, although mostly a little larger for fit (B), and in each case the two fits give completely consistent values for a_1 and a_2 . This is the numerical evidence that these physical quantities depend only on the sum of the quark masses. As there is no theoretical justification for $a_3 \neq 0$, nor any support for this from our data, from here on we use only the results of fit (B).

C. Chiral extrapolations

First, we fit our estimates of the pseudoscalar meson mass for all six quark-mass combinations to the form in Eq. (2), and obtain κ_{crit} from extrapolating the fit in both κ 's to $m_P^2(\kappa_{\text{crit}}, \kappa_{\text{crit}}) = 0$. This gives

$$\kappa_{\text{crit}} = 0.14315_{-2}^{+2}, \quad (14)$$

in good agreement with, although significantly more ac-

TABLE IV. Masses in lattice units of baryons composed of degenerate quarks.

κ	0.141 44	Degenerate 0.142 26	0.142 62
m_N	0.573_{-6}^{+15}	0.462_{-14}^{+20}	0.372_{-24}^{+24}
χ^2/N_{DF}	6.5/5	5.9/5	3.2/5
m_Δ	0.646_{-10}^{+12}	0.577_{-21}^{+31}	0.556_{-46}^{+66}
χ^2/N_{DF}	0.7/4	1.1/4	1.7/4

TABLE V. Mass ratios from degenerate-quark data, used in the Edinburgh plot.

κ	m_P/m_V	m_N/m_V
0.141 44	0.77_{-1}^{+1}	1.47_{-2}^{+4}
0.142 26	0.62_{-2}^{+1}	1.35_{-4}^{+6}
0.142 62	0.52_{-2}^{+2}	1.17_{-6}^{+9}

curate than, our estimate from 18 configurations [2]. In Fig. 8, we present the plot of m_P^2 vs $1/2\kappa_{\text{eff}}$ from which we derive κ_{crit} , defining an effective κ as

$$\frac{1}{\kappa_{\text{eff}}} = \left[\frac{1}{2\kappa_1} + \frac{1}{2\kappa_2} \right]. \quad (15)$$

The fact that both the degenerate- and nondegenerate-quark data agree well with the fit is graphical evidence of our claim in the previous section that the pseudoscalar meson mass depends only on the sum of the quark masses, and confirms the observation of Ref. [7]. The chiral extrapolation of m_V to κ_{crit} using Eq. (3) is also

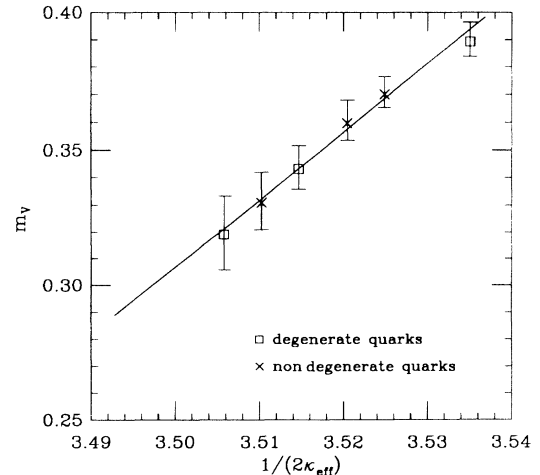
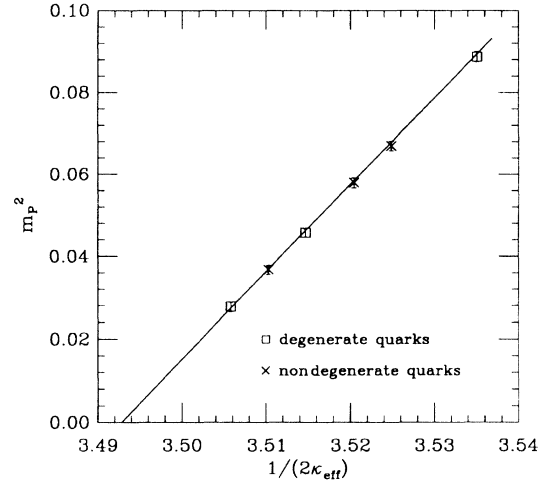


FIG. 8. Chiral extrapolations of the pseudoscalar and vector meson masses.

TABLE VI. Fit parameters for masses, decay constants, and ratios, using the fit form described in Eq. (4), (A) with a_3 unconstrained, and (B) with $a_3 = 0$.

	Fit (A) parameters			χ^2/N_{DF}
	a_1	a_2	a_3	
m_P^2	0.0	2.12^{+4}_{-3}	-0.01^{+1}_{-1}	4.3/3
m_V	0.29^{+2}_{-1}	2.3^{+4}_{-3}	0.1^{+1}_{-1}	1.9/3
f_P/Z_A	0.041^{+1}_{-2}	0.53^{+3}_{-2}	-0.01^{+1}_{-1}	4.8/3
$1/(f_V Z_V)$	0.38^{+1}_{-2}	-1.4^{+4}_{-2}	0.0^{+1}_{-1}	1.2/3
$f_P/(m_V Z_A)$	0.142^{+6}_{-9}	0.5^{+2}_{-2}	-0.10^{+6}_{-7}	3.1/3
	Fit (B) parameters			χ^2/N_{DF}
	a_1	a_2	a_3	
m_P^2	0.0	2.12^{+4}_{-3}	0.0	9.5/4
m_V	0.29^{+1}_{-1}	2.5^{+3}_{-3}	0.0	2.5/4
f_P/Z_A	0.040^{+1}_{-2}	0.53^{+3}_{-2}	0.0	7.1/4
$1/(f_V Z_V)$	0.38^{+1}_{-2}	-1.4^{+3}_{-2}	0.0	1.3/4
$f_P/(m_V Z_A)$	0.142^{+6}_{-9}	0.4^{+2}_{-2}	0.0	5.6/4

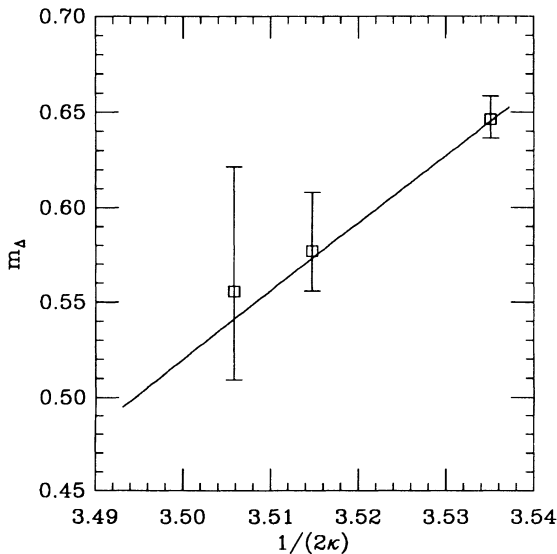
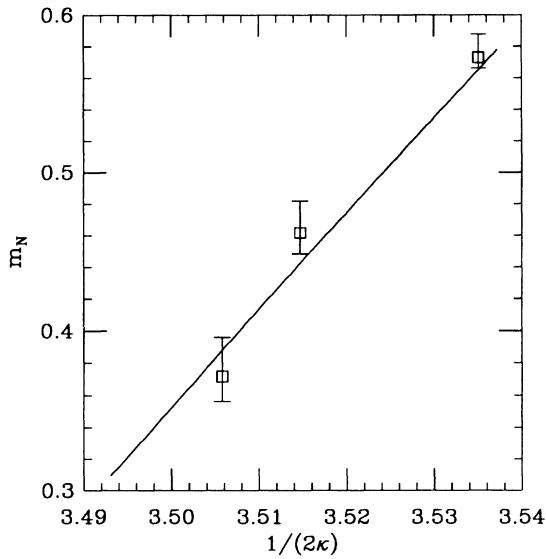


FIG. 9. Chiral extrapolations of m_N and m_Δ for degenerate-quark data only.

shown in Fig. 8; again, we conclude from the good agreement between the data and the fit that m_V depends only on the sum of the quark masses.

We present in Fig. 9 the chiral extrapolations of the nucleon and Δ . It is evident that for the nucleon the quality of the linear fit is rather poor. This is supported by the fact that the χ^2/N_{DF} for this fit is 5.1, compared to the value 2.4 obtained for the pseudoscalar meson fit and 0.63 obtained for the vector meson fit. Negative curvature of the nucleon has been observed before (see, for example, [9,16]). A linear fit to our data for m_N^2 gives a smaller χ^2/N_{DF} of 2.7, as well as a substantially smaller estimate for the nucleon mass in the chiral limit. However, because there is no theoretical justification for this choice of extrapolation, and because we cannot reliably compare different choices with only three data points, we quote results only from the linear extrapolation of m_N , noting that there is significant uncertainty in this procedure. We see no such problem with the chiral extrapolation of m_Δ , although the errors are larger.

The inverse lattice spacing a^{-1} in physical units, obtained from each of the ρ , nucleon and Δ masses, is given in Table VII. Throughout the remainder of this paper we will use m_ρ to set the scale. In so far as there is good agreement between the scales from the string tension and m_ρ , our results in physical units are not especially dependent on the chiral extrapolation. The corresponding values for the nucleon and Δ masses in physical units are given in Table X.

TABLE VII. Chirally extrapolated lattice masses and corresponding scales compared with the scale from the string tension [2].

Physical quantity	Lattice value	a^{-1} (GeV)
m_ρ	0.29^{+1}_{-1}	2.7^{+1}_{-1}
m_N	0.31^{+3}_{-2}	3.0^{+2}_{-3}
m_Δ	0.50^{+4}_{-3}	2.5^{+2}_{-2}
\sqrt{K}	0.161(3)	2.73(5)

D. Determination of the strange quark mass

We calculate the value of κ_s from the fit to the ratio $m_P^2(\kappa_1, \kappa_2)/m_\rho^2$ for all six quark-mass combinations, extrapolating κ_1 to κ_{crit} , and using κ_2 to match the ratio to its experimental value, $m_K^2/m_\rho^2=0.413$, giving

$$\kappa_2 = \kappa_s = 0.1419_{-1}^{+1}. \quad (16)$$

This value agrees well with that obtained from degenerate-quark data with smeared sinks in Ref. [2]. We note that m_s lies between two of our quark masses, so that our strange-quark results are obtained by interpolation, an intrinsically more robust procedure than extrapolation.

Our value of κ_s corresponds, in the lattice regularization, to a strange quark mass in physical units of

$$m_s(a) = \frac{1}{2a} \left[\frac{1}{\kappa_s} - \frac{1}{\kappa_{\text{crit}}} \right] = 82_{-8}^{+8} \text{ MeV}, \quad (17)$$

where we have used m_ρ to determine the lattice spacing in physical units. From this we can determine the renormalized strange quark mass in the modified minimal subtraction ($\overline{\text{MS}}$) scheme at a reference renormalization scale of $\mu=2 \text{ GeV}$:

$$m_s^{\overline{\text{MS}}}(2 \text{ GeV}) = 109_{-11}^{+11}, \quad (18)$$

where we have used the perturbative value of the renormalization constant relating $m_s(a)$ and $m_s^{\overline{\text{MS}}}(\mu)$ [17], together with the effective coupling defined in Ref. [18]. This result agrees with the conclusion from previous lattice evaluations of the strange quark mass (see Ref. [19] for a simulation with Wilson fermions and Ref. [20] for one with the clover action at $\beta=6.0$), that $m_s^{\overline{\text{MS}}}(2 \text{ GeV})$ is at the lower end of expectations [21,22]. The value given in Eq. (18) can be compared to $100 \pm 6 \text{ MeV}$ quoted in Ref. [19] and $89 \pm 9 \text{ MeV}$ quoted in Ref. [20].

E. Spectrum and decay constants in physical units

We present the results for meson masses and decay constants, extrapolated and/or interpolated to the physical κ values, in Table VIII, using the parameters of fit (B). As indicated in Table VIII, the ratios of the decay constants need to be multiplied by renormalization constants before they can be compared with experimental numbers. The perturbative estimates for these renormalization constants [15], using the effective coupling, are

$$Z_A \simeq 0.97, \quad Z_V \simeq 0.83. \quad (19)$$

Incorporating these values for the renormalization constants and using the lattice scale from m_ρ in Table VII, we obtain the meson masses and decay constants in physical units presented in Table IX.

The results for m_{K^*} , $m_{K^*}^2 - m_K^2$, and m_ϕ , although $2-3\sigma$ below experiment, provide support for our determination of κ_s . However, they are open to the interpretation that the vector meson masses may be slightly underestimated relative to the pseudoscalar meson masses, as a result of the suppression of spin splittings in the

TABLE VIII. Extrapolated and/or interpolated values of meson masses and decay constants in lattice units, calculated using the fit (B) parameters in Table VI.

m_{η_s}	0.251_{-11}^{+13}
m_{K^*}	0.326_{-12}^{+13}
m_ϕ	0.364_{-11}^{+12}
$m_{K^*}^2 - m_K^2$	0.075_{-5}^{+6}
f_π/Z_A	0.040_{-2}^{+1}
f_K/Z_A	0.048_{-2}^{+1}
f_K/f_π	1.20_{-2}^{+3}
$1/(f_\rho Z_V)$	0.380_{-16}^{+9}
$1/(f_K^* Z_V)$	0.359_{-11}^{+6}
$1/(f_\phi Z_V)$	0.337_{-7}^{+4}
f_ϕ/f_ρ	1.13_{-3}^{+2}
f_{K^*}/f_ρ	1.06_{-2}^{+1}
$f_\pi/(m_\rho Z_A)$	0.142_{-9}^{+6}
$f_K/(m_\rho Z_A)$	0.165_{-8}^{+7}
$f_K/(m_{K^*} Z_A)$	0.148_{-6}^{+4}

quenched approximation. Our result for m_{K^*} is significantly more precise than that obtained with the standard Wilson action at $\beta=6.0$ by Lipps *et al.* [4] who quote 930 (40) MeV, and by Loft and DeGrand [9] who obtain 761 (122) MeV. It is comparable with the value 896 (17) MeV obtained using a renormalization-group-improved action by Iwasaki [8]. Lipps *et al.* and Iwasaki both use m_ϕ to determine κ_s , based on the assumption that the ϕ is pure $s\bar{s}$, and so we are only able to compare our result for m_ϕ with that of Loft and DeGrand, who give 868 (114) MeV. Loft and DeGrand determine κ_s from the η_s , which is the hypothetical pure $s\bar{s}$ pseudoscalar meson, whose theoretically expected mass of 686 MeV [4] agrees well with our calculation.

Maiani and Martinelli [5] noted that the ratio $(m_{K^*} - m_\rho)/(m_K^2 - m_\pi^2)$ typically gives a value for the inverse lattice spacing which is lower than that obtained using other physical quantities. The ratio is estimated by assuming that m_V and m_P^2 are linear in the sum of the quark masses, Eqs. (2) and (3), so that

$$m_V(\kappa_1, \kappa_2) = a_V + \frac{b_V}{b_P} m_P^2(\kappa_1, \kappa_2) \quad (20)$$

and

$$\frac{m_{K^*} - m_\rho}{m_K^2 - m_\pi^2} = \frac{m_V(\kappa_s, \kappa_{\text{crit}}) - m_V(\kappa_{\text{crit}}, \kappa_{\text{crit}})}{m_P^2(\kappa_s, \kappa_{\text{crit}}) - m_P^2(\kappa_{\text{crit}}, \kappa_{\text{crit}})} = \frac{b_V}{b_P}. \quad (21)$$

We obtain $2.3_{-3}^{+3} \text{ GeV}$, in good agreement with our low-statistics results [2]. This is consistent with the scales we obtain from other physical quantities, although it remains on the low side.

Our results for the pseudoscalar decay constants are $5-7\sigma$ below their experimental values. This has been noted recently in simulations using the standard Wilson action [23]. These authors argue that a smaller value of

TABLE IX. Values of meson masses and decay constants in physical units, using the scale from m_ρ .

	Lattice estimates	Experiment
m_{η_s}	670_{-10}^{+10} MeV	"686 MeV"
m_{K^*}	868_{-8}^{+9} MeV	892 MeV
m_ϕ	970_{-10}^{+20} MeV	1020 MeV
$m_{K^*}^2 - m_K^2$	0.53_{-1}^{+1} (GeV) ²	0.55 (GeV) ²
f_π	102_{-7}^{+6} MeV	132 MeV
f_K	123_{-6}^{+5} MeV	160 MeV
$1/f_\rho$	0.316_{-13}^{+7}	0.28
$1/f_{K^*}$	0.298_{-9}^{+5}	
$1/f_\phi$	0.280_{-6}^{+3}	0.23
f_π/m_ρ	0.138_{-9}^{+6}	0.172
f_K/m_ρ	0.160_{-8}^{+7}	0.208
f_K/m_{K^*}	0.144_{-6}^{+4}	0.179

the decay constant is to be expected in the quenched approximation than in the full theory, as a consequence of the smaller wave function at the origin. This discrepancy may also be partly due to our use of the perturbative value for Z_A . Recent nonperturbative calculations at $\beta=6.0$, and at a single value of the quark mass, put the value of $Z_A \sim 1.09(3)$ [24], somewhat higher than the perturbative value. Thus, it may be hoped that a full nonperturbative evaluation of Z_A at $\beta=6.2$ will raise our estimates of the decay constants, bringing them closer to the experimental values. The uncertainty in the renormalization constant is removed in the ratio $f_K/f_\pi = 1.20_{-2}^{+3}$, which agrees well with the experimental value of 1.22. This suggests that, although we are working in the quenched approximation, we obtain correctly the dependence on the strange quark mass. We cannot attribute the problem with the overall normalization to quenching until we have a precise nonperturbative determination of the axial-vector current renormalization.

Experimentally, the ratio of the pseudoscalar decay constant to the vector meson mass is fairly insensitive to the SU(3)-flavor-symmetry breaking. In Fig. 10 we plot this ratio versus m_p^2 for both the degenerate- and nondegenerate-quark data. The slope agrees well with that of the experimental data and we note that an increase in Z_A of order 25% would give excellent agreement between the two.

Our estimates for the vector meson decay constants, $1/f_\rho$ and $1/f_\phi$, given in Table IX, lie above the experimental values. This is shown for all our data in Fig. 11. The discrepancy is small for the ρ , but it becomes significant for the ϕ , although the slope of the data is consistent with experiment. The sign of the discrepancy is opposite to that expected from the suppression of the wave function at the origin by the quenched approximation, suggesting that other effects are important. The difference is less likely than in the case of the pseudoscalar decay constant to be due to our use of the perturba-

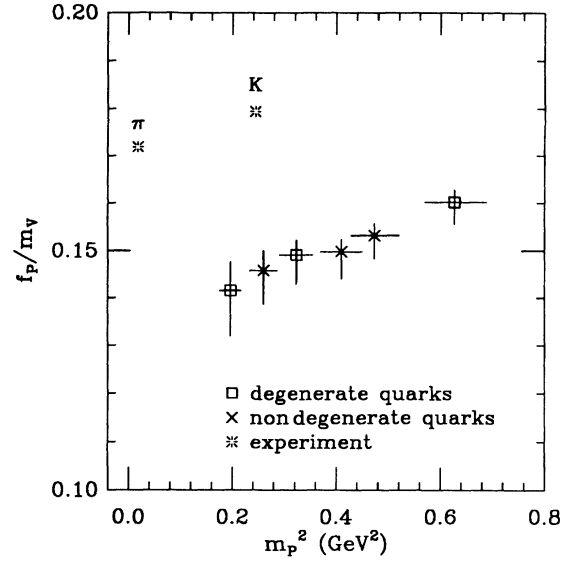


FIG. 10. f_p/m_V against m_p^2 in physical units using the scale from m_ρ . Experimental points for the pion and kaon are also shown.

tive value of the renormalization constant, because at least at $\beta=6.0$ Z_V is known to be close to its nonperturbative value [24]. Thus, we suspect that significant discretization and/or finite-volume errors are present.

For the baryon data in Table IV, we perform extrapolations and interpolations in the single κ value to obtain our predictions for the physical masses. The Δ and Ω baryons both have $J^P = \frac{3}{2}^+$, so by interpolating our Δ fits to κ_s we can obtain m_Ω , as shown in Table X. We are encouraged by the good correspondence between our computed values and the experimental masses presented in

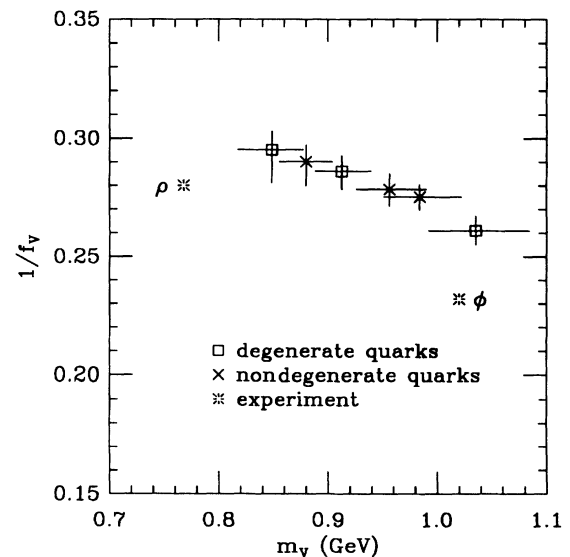


FIG. 11. $1/f_V$ against m_V in physical units using the scale from m_ρ . Experimental points for the ρ and ϕ mesons are also shown.

TABLE X. Physical values of the baryon masses using m_ρ to set the scale, including the ratio m_Ω/m_N .

	Extrapolated values		Experiment
	Lattice units	MeV	
m_N	0.31^{+3}_{-2}	820^{+90}_{-60}	938 MeV
m_Δ	0.50^{+4}_{-3}	1300^{+100}_{-100}	1232 MeV
m_Ω	0.62^{+2}_{-2}	1650^{+70}_{-50}	1672 MeV
m_Ω/m_N	2.0^{+2}_{-2}		1.78

the table. However, the low confidence we have in the linear chiral extrapolation of the nucleon means that we cannot attach much significance to the comparison between the computed and experimental values for m_N . The extrapolation of m_Δ does not have this problem, and the interpolation to κ_s gives a value of m_Ω very close to the experimental number. This indicates that our calculation of κ_s can be applied sensibly to the baryon sector. Our value for m_Ω is to be compared with that of Lipps *et al.* [4] who quote 1650 (150) MeV, Loft and DeGrand [9] who quote 1512 (144) MeV, and Iwasaki [8] who quotes 1737 (77) MeV.

Our determination of the spin- $\frac{3}{2}$ baryon mass provides us with an alternative means of determining κ_s , by interpolating our lattice data to the physical value of m_Ω . Using m_ρ to set the scale, we obtain in this way $\kappa_s = 0.1417^{+4}_{-3}$, $m_{\eta_s} = 740^{+50}_{-80}$ MeV and $m_\phi = 1010^{+20}_{-40}$ MeV. Although there remains an implicit dependence of these estimates on the chiral extrapolation, through our use of m_ρ , this can be avoided by, for example, taking the scale from the string tension. It is evident from Table VII that this would give similar values. Thus, this method of determining κ_s depends only on the mild assumption that the baryon mass varies smoothly with quark mass close to the strange quark mass. The agreement, within the somewhat larger statistical errors, with the value of κ_s obtained from m_K , in Eq. (16), and with the corresponding mass estimates in Table IX, provides a further check on our determination of κ_s . Finally, we note that assuming that the ϕ is pure $s\bar{s}$ does not, in practice, permit the determination of κ_s from hadrons composed solely of s quarks, because the numerical data for the ratio $m_\Omega/m_V(\kappa, \kappa)$ is only weakly dependent on κ .

IV. CONCLUSIONS

Despite working in the quenched approximation, our simulations using the clover action at $\beta=6.2$ demonstrate good agreement for meson masses and decay constants with the dependence on light quark masses, including SU(3)-flavor-symmetry breaking, expected from chiral

perturbation theory. In particular, we find evidence from simulations with nondegenerate quarks that the dependences on the bare quark masses,

$$m_P^2(\kappa_1, \kappa_2) = b_P \left[\frac{1}{2\kappa_1} + \frac{1}{2\kappa_2} - \frac{1}{\kappa_{\text{crit}}} \right], \quad (22)$$

$$m_V(\kappa_1, \kappa_2) = a_V + b_V \left[\frac{1}{2\kappa_1} + \frac{1}{2\kappa_2} - \frac{1}{\kappa_{\text{crit}}} \right], \quad (23)$$

hold at least for quark masses up to that of the strange quark. Our results for the strange-particle spectrum are encouragingly close to the experimental values, and the dependence of both the pseudoscalar and vector decay constants on m_s is consistent with experiment. The only serious problem in the meson sector shows up in the actual values of the decay constants. For the pseudoscalar case, the sign of the discrepancy is consistent with the expectation that the wave function at the origin is reduced in the quenched approximation, but whether the magnitude is entirely attributable to this effect or is due in part to our use of the perturbative value for the axial current renormalization is not known. Our results for the vector decay constants are more difficult to interpret, as the discrepancy is in the opposite direction, which may signal the presence of other effects. It is clearly important to calculate the current renormalizations nonperturbatively at $\beta=6.2$ for at least two values of the quark mass.

We have not explored SU(3)-flavor-symmetry breaking in the baryon sector, but only present results for degenerate quarks. Compared with the chiral extrapolations for the other hadrons, a linear chiral extrapolation for the nucleon is not well supported by our data, and leads to a nucleon mass which is lower than is typical of quenched simulations to date, although in better agreement with experiment. On the other hand, our results for the Δ and Ω are less problematic. They are also in good agreement with experiment, and the result for the Ω , along with our results for mesons, encourages us to believe that we have good control of the strange-quark physics that can be obtained from two-point functions.

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