Dynamical chiral symmetry breaking and confinement with an infrared-vanishing gluon propagator?

Frederick T. Hawes*
Department of Physics and SCRI, Florida State University, Tallahassee, Florida 32306-3016

Craig D. Roberts†
Physics Division, Argonne National Laboratory, Argonne, Illinois 60439-4843

Anthony G. Williams‡
Department of Physics and Mathematical Physics, University of Adelaide, Adelaide 5001, South Australia, Australia

We study a model Dyson-Schwinger equation for the quark propagator closed using an *Ansatz* for the gluon propagator of the form $D(q) \sim q^2/[(q^2)^2 + b^4]$ and two *Ansätze* for the quark-gluon vertex: the minimal Ball-Chiu form and the modified form suggested by Curtis and Pennington. Using the quark condensate as an order parameter, we find that there is a critical value of $b = b_c$ such that the model does not support dynamical chiral symmetry breaking for $b > b_c$. We discuss and apply a confinement test which suggests that, for all values of b, the quark propagator in the model *is not* confining. Together these results suggest that this *Ansatz* for the gluon propagator is inadequate as a model since it does not yield the expected behavior of QCD.

PACS number(s): 12.38.Aw, 11.30.Rd, 12.38.Lg

I. INTRODUCTION

Dynamical chiral symmetry breaking (DCSB) and confinement are two crucial features of quantum chromodynamics (QCD). Indeed, it might be argued that a realistic model of QCD should manifest both of these properties since they are responsible for the nature of the hadronic spectrum, DCSB ensuring the absence of low-mass scalar partners of the pion and confinement ensuring the absence of free quarks, for example.

A natural method for studying both DCSB and confinement in QCD, and models thereof, is the complex of Dyson-Schwinger equations (DSE's) [1]. The equations for the two-point functions of gluons and quarks have been used in many such studies. This manifestly relativistically covariant approach, recent reviews of which can be found in Refs. [2,3], has provided the foundation for a useful and successful understanding of the phenomena of low-energy QCD by facilitating the construction of realistic field-theoretic models [4].

Another goal in the DSE studies is to develop this nonperturbative approach to a point where it is as firmly founded as lattice QCD and calculationally competitive. Although much remains to be done in order to achieve this goal there has been a good deal of progress, especially in the study of Abelian gauge theories where direct and meaningful comparisons can be made, and agreement obtained, between the results of lattice and DSE studies [5].

In considering DSE studies it is important to note that they are hampered by the fact that there is an infinite tower of coupled equations: the equations for the two-point functions couple them to other two- and three-point functions; the equations for the three-point functions couple them to the four-point and higher n-point functions, etc. A commonly used resolution of this problem is to truncate the system at a finite number of coupled equations by making *Ansätze* for the higher n-point functions. For example, one may study the DSE for the quark propagator alone by choosing *Ansätze* for the gluon propagator and quark-gluon vertex thus closing the system. This is the nature of our study.

Herein we study the fermion DSE obtained with a model gluon propagator (two-point function) which vanishes at $q^2 = 0$:

$$D(q) \equiv \frac{1}{q^2[1 - \Pi(q^2)]} = \frac{q^2}{(q^2)^2 + b^4} , \qquad (1)$$

where $\Pi(q^2)$ is the gluon vacuum polarization function and b is a real parameter, in order to determine whether it can support DCSB and/or generate a confining quark propagator. Two properties we require of the propagator of a confined particle are (1) the absence of a Källen-Lehmann representation and (2) no singularity on the timelike q^2 axis [6]. The gluon propagator obtained from Eq. (1) satisfies both of these requirements.

Such a form for the gluon propagator, even though it may be argued to describe a confined gluon, is perhaps counterintuitive, since it would appear to provide a

---

*Electronic address: hawes@mailer.scri.fsu.edu
†Electronic address: cdroberts@anl.gov
‡On leave from Department of Physics and SCRI, Florida State University, Tallahassee, FL 32306-3016. Electronic address: williams@physics.adelaide.edu.au

weak interaction between quarks at small p^2, which corresponds to large distances. Indeed, some studies of the fermion DSE in QCD have employed quite a different *Ansatz*: one which behaves as $1/(q^2)^2$ for $q^2 \simeq 0$ [7,8]. This form for the infrared (IR) behavior of the gluon propagator is suggested by a number of studies of the DSE for the gluon propagator in both axial [9] and covariant gauges [10,11] using various approximation and/or truncation procedures; notably, they all effectively neglect the 4-gluon vertex. (We note that the results of the axial gauge studies [9] may be questioned on the basis that the gluon propagator therein is inconsistent with the known spectral representation in axial gauge [12].) In addition, this form of gluon propagator in the infrared is consistent with area law behavior of the Wilson loop [13], which has been observed in lattice gauge theory studies of QCD [14] and is often regarded as indicating confinement.

However, the form in Eq. (1) is suggested by a number of studies. It has been argued [15] that in order to completely eliminate Gribov copies [16], and hence to fix Landau or Coulomb gauge uniquely in lattice studies, one must introduce new ghost fields into QCD in addition to those associated with the Faddeev-Popov determinant in the continuum. Analyzing the lattice action thus obtained suggests that the gluon propagator vanishes as $(q^2)^\gamma$, with $\gamma > 0$ not determined. Subsequent analysis of a simplified model yields $\gamma = 1$ and, in fact, a gluon propagator of the form in Eq. (1) with b a finite constant in Landau gauge. Similar considerations in Ref. [16] yield the same result. A propagator of the form in Eq. (1) was also suggested in Refs. [17–19] as a result of an analysis of an approximate DSE for the gluon propagator. A recent lattice QCD simulation [20] also provides some support for this form. We also note that a gluon propagator of the type in Eq. (1) may arise in the field strength approach to QCD [21].

It is therefore important to study the phenomenological implications of Eq. (1); i.e, to determine whether it can support DCSB and confinement in QCD-based models of the type in Refs. [4]. It has been argued [15,17–19] that Eq. (1) represents confined gluons because there are no poles on the timelike real axis in the complex-q^2 plane and it allows the interpretation of the gluon as an unstable excitation which fragments into hadrons before observation (in a time of the order of $1/b$). It is also argued [19] that such a gluon propagator should lead to a quark propagator with similar structure in the complex plane, and hence a similar interpretation, but this result has not been proven. Learning enough about the quark propagator to make inferences about its analytic structure is therefore an important part of our study.

We study the implications of Eq. (1) for the structure of the quark propagator and DCSB using the fermion DSE. A similar study is undertaken in Ref. [22]. Our DSE is closed using two *Ansätze* for the quark-gluon vertex: the minimal Ball-Chiu vertex [23] and that of Curtis and Pennington [24]. These vertices are free of kinematic light-cone singularities. We find that in either case there are regions of DCSB and unbroken chiral symmetry characterized by a two-dimensional phase diagram in $(b^2, \ln \tau)$

space, where b appears in Eq. (1) above and τ will be introduced below in connection with an ultraviolet modification of Eq. (1). The phase transition is second order. We also employ a confinement test which suggests that, with either of the vertex *Ansätze* and for all values of b and τ, Eq. (1) leads to a quark propagator that *is not* confining. Given these results it appears that the gluon propagator of Eq. (1) is inadequate as a model, at least in our DSE framework, since it does not yield the behavior expected in QCD.

In Sec. II we present a model quark DSE which we solve numerically. We also discuss the gluon propagator and quark-gluon vertex *Ansätze* in some detail. In Sec. III we evaluate the quark condensate, which we use as an order parameter for DCSB, and determine the characteristic properties of the phase transition. We also demonstrate that the quark propagator is not confining. In Sec. IV we discuss and summarize our results and conclusions.

II. MODEL DYSON-SCHWINGER EQUATION

In Minkowski space, with metric $g_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$ and in a general covariant gauge, the inverse of the dressed quark propagator can be written as

$$S^{-1}(p) = \not{p} - m - \Sigma(p) \equiv Z^{-1}(p^2)\left[\not{p} - M(p^2)\right]$$
$$\equiv A(p^2)\,\not{p} - B(p^2), \tag{2}$$

with m the renormalized explicit chiral-symmetry-breaking mass (if present), $\Sigma(p)$ the self-energy, $M(p^2) = B(p^2)/A(p^2)$ the dynamical quark mass function, and $A(p^2) = Z^{-1}(p^2)$ the momentum-dependent renormalization of the quark wave function.

The unrenormalized DSE for the inverse propagator is

$$S^{-1}(p) = \not{p} - m^{\mathrm{bare}} - i\frac{4}{3}g^2 \int \frac{d^4k}{(2\pi)^4}\gamma^\mu S(k)\Gamma^\nu(k,p)$$
$$\times D_{\mu\nu}[(p-k)^2], \tag{3}$$

where Γ^ν is the proper quark-gluon vertex and $D_{\mu\nu}(q^2)$ is the dressed gluon propagator. Hereafter we set $m^{\mathrm{bare}} = 0$ so that we can concentrate on dynamical symmetry-breaking effects. (A study of explicit chiral symmetry breaking in such DSE models of QCD can be found in Refs. [25,26].) The renormalized, massless DSE is

$$S_R^{-1}(p) = Z_S\,\not{p} - iZ_\Gamma\frac{4}{3}g^2 \int^\Lambda \frac{d^4k}{(2\pi)^4}\gamma^\mu S_R(k)\Gamma_R^\nu(k,p)$$
$$\times D_{\mu\nu}^R[(p-k)^2], \tag{4}$$

where Z_S and Z_Γ are quark-propagator and quark-gluon-vertex renormalization constants, respectively, which depend on the renormalization scale μ and ultraviolet cutoff Λ. Neglecting ghost fields and explicit 3-gluon vertices, as we do in the following [see Eq. (7) below], one has $Z_S = 1 = Z_\Gamma$ at one loop in Landau gauge. Using this result here leads to our model renormalized DSE for the quark self-energy:

$$\Sigma(p) = i\frac{4}{3}g^2 \int^\Lambda \frac{d^4k}{(2\pi)^4} \gamma^\mu S(k)\Gamma^\nu(k,p)D_{\mu\nu}[(p-k)^2], \quad (5)$$

where here and hereafter we suppress the label R.

A. Model gluon propagator

In a general covariant gauge the dressed gluon propagator, which is diagonal in color space, can be written

$$D^{\mu\nu}(q^2) = \left[\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\frac{1}{1-\Pi(q^2)} - a\frac{q^\mu q^\nu}{q^2}\right]\frac{1}{q^2}, \quad (6)$$

where $\Pi(q^2)$ is the gluon vacuum polarization and a is the gauge parameter. It should be noted that in covariant gauges the longitudinal piece of this propagator is not modified by interactions. This follows from the Slavnov-Taylor identities in QCD. (See, for example, Ref. [27], pp. 42–45.)

The form of the propagator we consider herein has been argued to arise in Landau gauge ($a = 0$) and we shall use that gauge hereafter. [This has the benefit that the gluon propagator is purely transverse. In addition, Landau gauge is a fixed point of the renormalization group and therefore the gauge parameter does not "run." (See, for example, Ref. [27], pp. 135-136.)]

A commonly used approximation is to write

$$\frac{g^2}{4\pi}\frac{1}{1-\Pi(q^2)} = \alpha(-q^2), \quad (7)$$

where $\alpha(-q^2)$ is the running coupling in the gauge theory. In QCD this amounts to neglecting ghost contributions to the gluon vacuum polarization [11]. (In Abelian theories, of course, this is not an approximation but an exact result which follows from the fact that, in the absence of ghost fields and gauge-boson self-interactions, the renormalization constants for $\Pi(q^2)$ and g^2 are the same [28].) For $Q^2 = -q^2 \gg \Lambda_{\rm QCD}^2$, the one-loop contribution is dominant and the running coupling is

$$\alpha_{\rm S}(Q^2) = \frac{d\pi}{\ln(Q^2/\Lambda_{\rm QCD}^2)}, \quad d = \frac{12}{33 - 2N_f}. \quad (8)$$

(We use $N_f = 4$ herein. Our results are insensitive to changes in this value.) In QCD, the use of Eq. (7) in Eq. (6) is expected to be accurate at large Q^2 where it leads to the correct perturbative leading-log behavior [29].

In considering the small q^2 behavior we return to Eq. (1). We observe that in Refs. [17–19] a solution of the coupled DSE's for the gluon propagator and triple-gluon vertex was obtained, using rational polynomial *Ansätze*, (the coupling to the 4-gluon vertex was eliminated) and it was argued that Eq. (1) is a good approximation to the solution. A significant result in this study is that, in order to support a solution of the type in Eq. (1), the transverse part of the triple-gluon vertex, $\Gamma_{3T}(p_1,p_2,p_3)$, necessarily has kinematic light-cone singularities of the form $1/(p_i^2)$. These singularities, which cannot arise in perturbation

theory, are introduced in "solving" the Slavnov-Taylor identities for the triple-gluon vertex. In this connection we remark that Ref. [23] might be argued to suggest that the Slavnov-Taylor identities for both the triple-gluon vertex and quark-gluon vertex can be solved without introducing kinematic light-cone singularities. This latter approach would allow for nonperturbative corrections to the vertices and a simple connection with the perturbative vertices in the asymptotically free region. In this case the light-cone singularities in the triple-gluon vertex of Refs. [18,19] might be viewed as undesirable.

We note too that there has been an attempt to employ the gluon propagator described by Eq. (1) in a study of quarkonium spectra [30]. A Blankenbecler-Sugar reduction of a ladderlike approximation to the Bethe-Salpeter equation is used and it is argued that in the bound-state equation that results one can approximate the effect of Eq. (1) by a Coulomb potential for all r. The interesting structure in this equation arises because the bound fermions are described by propagators with complex conjugate poles just as the gluon propagator has [see Eq. (40) and associated discussion below]. The results in this study, however, are not competitive with detailed fits using potential models. It may be that the approximations and/or truncations made in Ref. [30] are partly responsible for this.

In connection with the small q^2 behavior of the quark-quark interaction the lattice results of Ref. [20] are also interesting. This study, on $16^3 \times 40$ and $24^3 \times 40$ lattices at $\beta = 6.0$, yielded a gluon propagator in a lattice Landau gauge which allowed a fit of the form in Eq. (1) at small q^2 but which could not rule out a fit using a standard massive particle propagator. Other lattice sizes and values of β were also studied. The results at $\beta = 6.3$ on a lattice of dimension 24^4 were not inconsistent with these results, but in this case the small physical size of the lattice was a problem. On a lattice of dimension $16^3 \times 24$ at $\beta = 5.7$ it was found that the gluon propagator was best fit with a standard massive vector boson propagator with mass ~ 600 MeV. (We note that the gauge fixing in this study did not include the modifications suggested by Ref. [15].) These studies represent an improvement in both technique and lattice sizes over earlier lattice studies of the gluon propagator [31], but the conclusions are not markedly different. The studies of Ref. [31], using $\beta = 5.6, 6.0$ on a $4^3 \times 8$ lattice and $\beta = 5.8$ on a $4^3 \times 10$ lattice, obtained results that were consistent with a free massive boson propagator with mass ~ 600 MeV. Clearly, further lattice studies would be of great interest.

At present, one can only say that Refs. [15–20] suggest that, in QCD,

$$\frac{1}{1-\Pi(q^2)} = \frac{(q^2)^2}{(q^2)^2 + b^4} \quad (9)$$

is not implausible, at least at small q^2. For this reason we study Eq. (1) in the DSE approach in order to determine whether it can lead to DCSB and a confining quark propagator. This will provide further insight into the validity of this form of gluon propagator.

Combining Eqs. (7) and (9) leads to the ultraviolet-

improved model gluon propagator that we consider herein; i.e, we study the phenomenological implications of a model gluon propagator obtained with

$$\frac{g^2}{4\pi} \frac{1}{1 - \Pi(q^2)} = \alpha(\tau; -q^2) \frac{(q^2)^2}{(q^2)^2 + b^4} , \qquad (10)$$

where $(Q^2 \equiv -q^2)$

$$\alpha(\tau; Q^2) = \frac{d\pi}{\ln\left[\tau + \dfrac{Q^2}{\Lambda_{QCD}^2}\right]} , \qquad (11)$$

in Eq. (6). Here $\tau > 1$ is an IR regularization parameter introduced so that the logarithmic singularity is shifted to $Q^2 = -\tau\Lambda_{QCD}^2$, which ensures that the piece derived from Eq. (9) dominates in the spacelike IR region [32,33].

B. Model quark-gluon vertex

In choosing an *Ansatz* for the vertex we note that in QCD the Slavnov-Taylor identity [34]

$$q_\mu \Gamma^\mu(k,p) \left[1 + b(q^2)\right] = \left[1 - B(q,p)\right] S^{-1}(k)$$

$$-S^{-1}(p)\left[1 - B(q,p)\right] , \qquad (12)$$

where $q = (p - k)$, $b(q^2)$ is the ghost self-energy, and $B(p,q)$ is the ghost-quark scattering kernel, constrains the longitudinal part of the vertex. Clearly, the often used "rainbow" approximation, in which $\Gamma_\mu(k,p)$ is simply taken to be γ_μ, cannot satisfy Eq. (12) in an interacting theory.

Neglecting ghosts, as we have done so far, this relation takes the form of the Ward-Takahashi identity in QED:

$$q_\mu \Gamma^\mu(k,p) = S^{-1}(k) - S^{-1}(p) . \qquad (13)$$

The constraints that this relation places on the vertex can now be inferred from the QED studies [23,24,35]. Taking these into account one is led to a vertex of the form

$$\Gamma^\mu(k,p) = \Gamma_{BC}^\mu(k,p) + \Gamma_{CP}^\mu(k,p), \qquad (14)$$

where

$$\Gamma_{BC}^\mu(k,p) = \frac{A(p^2) + A(k^2)}{2}\gamma^\mu + \frac{(p+k)^\mu}{p^2 - k^2}\left\{\left[A(p^2) - A(k^2)\right]\frac{[\slashed{p} + \slashed{k}]}{2} - \left[B(p^2) - B(k^2)\right]\right\} , \qquad (15)$$

$$\Gamma_{CP}^\mu(k,p) = \frac{\gamma^\nu(k^2 - p^2) - (k+p)^\nu(\slashed{k} - \slashed{p})}{2d(k,p)}\left[A(k^2) - A(p^2)\right] , \qquad (16)$$

with

$$d(k,p) = \frac{1}{(k^2 + p^2)}\left\{(k^2 - p^2)^2 + \left[\frac{B^2(k^2)}{A^2(k^2)} + \frac{B^2(p^2)}{A^2(p^2)}\right]^2\right\} . \qquad (17)$$

In these equations, Γ_{BC} is the Ball-Chiu vertex of Ref. [23] and Γ_{CP} is the additional piece suggested and studied by Curtis and Pennington in Ref. [24].

Equation (14) specifies our vertex *Ansatz* which has the properties that it satisfies the Ward-Takahashi identity, is free of kinematic singularities, reduces to the bare vertex in the absence of interactions, transforms correctly under charge conjugation and Lorentz transformations, and preserves multiplicative renormalizability in the quark DSE. (Of these properties the minimal Ball-Chiu vertex satisfies all but the last.)

For the most part in the following we neglected $\Gamma_{CP}^\mu(k,p)$; i.e, we used a minimal Ball-Chiu *Ansatz*. As we show below, this has the virtue of simplifying the integral equations. We did include this term for a single value of τ and a number of values of b in Eq. (10) and found, as shown below, that it generates a small quantitative change in some of the characteristic quantities calculated in the model, but does not alter its qualitative features.

C. Wick rotation and Euclidean space

The model is now almost completely specified. Up to this point, however, we have not considered the possible complications that may arise in employing a Wick rotation to obtain the DSE in Euclidean space where a numerical solution is most easily obtained. In all model fermion DSE's that have been studied so far the Wick rotation is not allowed in the sense that the rotation of the k_0 contour always encounters at least a pole and very often a branch cut [22,36]. Indeed, in some models it is not possible to rotate the contour at all [37]. This point is discussed in Refs. [3,6]. We shall simply complete the definition of our model DSE by *specifying* it in Euclidean space, with metric $\delta_{\mu\nu} = \mathrm{diag}(1,1,1,1)$ and with γ_μ Hermitian:

$$\Sigma(p) = \frac{4}{3}g^2 \int^\Lambda \frac{d^4k}{(2\pi)^4}\gamma^\mu S(k)\Gamma^\nu(k,p)D_{\mu\nu}[(p-k)^2], \qquad (18)$$

where

$$S^{-1}(p) = i\gamma \cdot p + \Sigma(p) = i\gamma \cdot p A(p^2) + B(p^2), \qquad (19)$$

and all of the other elements in this equation are taken to be specified by the expressions given above, evaluated at Euclidean (spacelike) values of their arguments.

Using Eqs. (6), (10), (14), (15), and (18) we obtain the following pair of coupled integral equations for the scalar functions that specify the model quark propagator:

$$A(p^2) = 1 + \frac{16\pi}{3} \int^\Lambda \frac{d^4k}{(2\pi)^4} \frac{\alpha(\tau; (p-k)^2)}{(p-k)^2} \frac{1}{A^2(k^2)k^2 + B^2(k^2)}$$
$$\times \left\{ A(k^2) \frac{A(k^2) + A(p^2)}{2} \frac{1}{p^2} [3p \cdot k - h(p,k)] - A(k^2)\Delta A(k^2, p^2) \left[k^2 - \frac{(k \cdot p)^2}{p^2} + \frac{k \cdot p}{p^2} h(p,k) \right] \right.$$
$$\left. - B(k^2)\Delta B(k^2, p^2) \frac{h(p,k)}{p^2} \right\}, \qquad (20)$$

$$B(p^2) = \frac{16\pi}{3} \int^\Lambda \frac{d^4k}{(2\pi)^4} \frac{\alpha(\tau; (p-k)^2)}{(p-k)^2} \frac{1}{A^2(k^2)k^2 + B^2(k^2)}$$
$$\times \left\{ 3B(k^2) \frac{A(k^2) + A(p^2)}{2} + \left[B(k^2)\Delta A(k^2, p^2) - A(k^2)\Delta B(k^2, p^2) \right] h(p,k) \right\}, \qquad (21)$$

where $h(p,k) = 2 \left[k^2 p^2 - (k \cdot p)^2 \right] / q^2$ and $\Delta F(k,p) = [F(k^2) - F(p^2)]/[k^2 - p^2]$.
Including the additional Curtis-Pennington term in the vertex, Eq. (16), these equations are modified as follows:

$$A(p^2) = \text{RHS of } (20) + \frac{16\pi}{3} \int^\Lambda \frac{d^4k}{(2\pi)^4} \frac{\alpha(\tau; (p-k)^2)}{(p-k)^2} \frac{A(k^2)\Delta A(k^2, p^2)}{A^2(k^2)k^2 + B^2(k^2)} \frac{(k^2 - p^2)}{2d(k,p)} \frac{3(k^2 - p^2)k \cdot p}{p^2}, \qquad (22)$$

$$B(p^2) = \text{RHS of } (21) + \frac{16\pi}{3} \int^\Lambda \frac{d^4k}{(2\pi)^4} \frac{\alpha(\tau; (p-k)^2)}{(p-k)^2} \frac{B(k^2)\Delta A(k^2, p^2)}{A^2(k^2)k^2 + B^2(k^2)} \frac{(k^2 - p^2)}{2d(k,p)} 3(k^2 - p^2) . \qquad (23)$$

These equations were solved numerically by iteration on a logarithmic grid of $x = p^2/\Lambda^2_{QCD}$ and $y = k^2/\Lambda^2_{QCD}$ points. In doing this we ensured that our results were independent of the seed solution and grid choice. Our results were independent of the UV cutoff, which was $\Lambda^2 = 5 \times 10^8 \Lambda^2_{QCD}$, and this value was also sufficient to ensure that the leading-log behavior of the mass function, Eq. (25) below, had become evident.

III. ANALYSIS OF ORDER PARAMETERS
A. Quark condensate

We are interested in determining whether the model gluon propagator specified by Eqs. (6) and (10) can support DCSB—a crucial feature of QCD. The quark condensate, which is gauge invariant, is an order parameter for DCSB and it is easily related to the trace of the quark propagator, which is the focus of our DSE study:

$$\langle \bar{q}q \rangle_\mu = -\frac{3}{4\pi^2} \ln \left(\frac{\mu^2}{\Lambda^2_{QCD}} \right)^d \lim_{\Lambda^2 \to \infty} \left[\ln \left(\frac{\Lambda^2}{\Lambda^2_{QCD}} \right)^{-d} \int_0^{\Lambda^2} ds\, s \frac{B(s)}{sA(s)^2 + B(s)^2} \right], \qquad (24)$$

where $d = 12/(33 - 2n_f)$ and μ is the renormalization point for the condensate, which is usually fixed at 1 GeV^2. This is the parameter that is used to study DCSB in lattice QCD and in many of the model studies in the continuum. [It is clear from Eq. (24) that an equivalent order parameter is $B(s = 0)$ since if this is zero then so is the condensate.]

As we remarked above, Eqs. (10) and (11) in the quark DSE ensure that the leading-log behavior of QCD is retained in the model so that [25,29,33]

$$B(p^2)\big|_{p^2 \to \infty} \to -\frac{4\pi^2 d}{3} \frac{\left(\ln \left[\mu^2/\Lambda^2_{QCD} \right] \right)^{-d} \langle \bar{q}q \rangle_\mu}{p^2 \left(\ln \left[p^2/\Lambda^2_{QCD} \right] \right)^{1-d}} . \qquad (25)$$

This provides another means of extracting the condensate and hence a check on its evaluation.

In cases for which our iterative solution procedure for the DSE converged quickly, with relative errors of less than 1×10^{-6}, the condensate could be obtained easily. However, for values of b^2 near a phase transition the con-
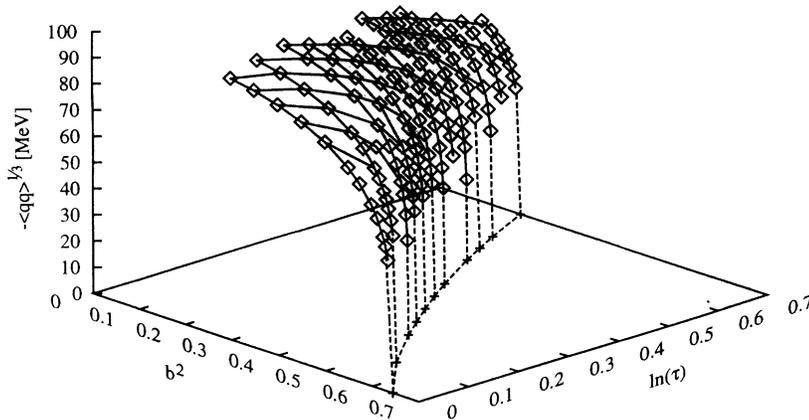
FIG. 1. Criticality plot for $-\langle \bar{q}q \rangle_{\mu}^{\frac{1}{3}}$ as a function of $\ln \tau$ and b^2. The condensate, $-\langle \bar{q}q \rangle_{\mu}^{\frac{1}{3}}$, is in units of MeV, scaled to $\mu^2 = 1$ GeV, and b^2 is in units Λ_{QCD}^2; the gluon regulator τ is dimensionless.

vergence could be extremely slow. In those cases, the numerical solution was examined at constant intervals through the run (say, every 50th cycle), and the condensate evaluated in each case. Aitken extrapolation [38] was then used to find the "infinite-cycles" limit. In several cases the program was subsequently run until the solutions had converged to within 1×10^{-6} and the extrapolated result always matched the actual result to within a few parts in 10^{-6}.

B. Critical behavior of the condensate

We solved Eqs. (20) and (21) for values of $\ln \tau$ in the domain $[0.0, 0.7]$ and b^2 in $[0.1, 1.0]$ using the minimal Ball-Chiu vertex and we plot the condensate obtained from our solutions in Fig. 1. This figure shows regions of unbroken and dynamically broken chiral symmetry.

Our numerical results suggest that the condensate rises continuously from the transition boundary and hence that the transition is second order. As a consequence we assumed that the order parameter, $\langle \bar{q}q \rangle_{\mu}$, behaves as

$$\langle \bar{q}q \rangle_{\mu}(z) \approx C \left(1 - \frac{z}{z_c} \right)^{\beta} \qquad (26)$$

for $z \to z_c^-$ (for z equal to either $\ln \tau$ or b^2) and extracted the critical points, z_c, and critical exponents, β, using ratio-of-logs methods adapted from Gaunt and Guttmann [39]. We list these quantities in Table I and, in Fig. 2, plot the critical curve in the $(b^2, \ln \tau)$ plane.

We also solved Eqs. (20) and (21) with the Curtis-Pennington additions, Eqs. (22) and (23), using $\ln \tau = 0.6$. The critical curve (in b^2) in this case is shown in Fig. 3 along with the minimal Ball-Chiu results for the same value of $\ln \tau$. The effect of the Curtis-Pennington addition is to lower the critical value of b^2 but, as we show below, the critical exponent is unchanged. This curve illustrates the point that the qualitative features of the model are not affected by this modification of the model quark-gluon vertex.

1. Critical parameters for the chiral phase transition

From Table I we find

$$\beta_{\text{BC}} = 0.575 \ , \quad \sigma_{\beta} = 0.024 \ . \qquad (27)$$

We note that the critical exponent obtained with $\ln \tau = 0$ is quite different from the others. This is a special case

TABLE I. The critical points and exponents extracted for various values of $\ln \tau$; the cumulative result is $\beta_{\text{BC}} = 0.575$, with $\sigma_{\beta} = 0.024$; excluding the point with $\ln \tau = 0.0$, $\beta_{\text{BC}} = 0.572$ with $\sigma_{\beta} = 0.020$.

$\ln \tau$	b_C^2 critical b^2 value	β critical exponent	σ_{β} standard deviation in β
0.00	0.6439	0.609	0.03
0.10	0.5448	0.579	0.021
0.20	0.4642	0.570	0.021
0.25	0.4278	0.579	0.021
0.30	0.3932	0.573	0.021
0.35	0.3601	0.570	0.0195
0.40	0.3289	0.567	0.021
0.50	0.2706	0.567	0.021
0.55	0.2437	0.570	0.021
0.60	0.2180	0.561	0.021
0.70	0.1710	0.579	0.021
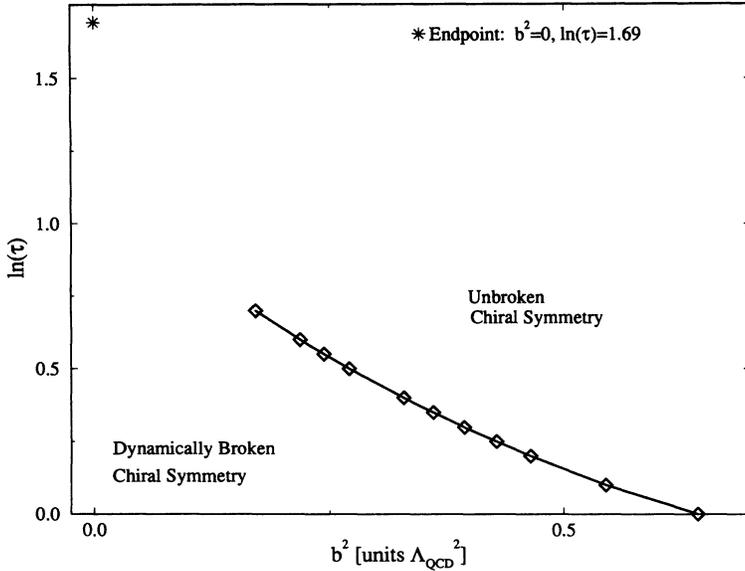
FIG. 2. Critical curve for the phase transition in the $(\ln \tau, b^2)$ plane. The asterisk is the result extracted from Ref. [33].

since for this value the propagator does not vanish in the infrared:

$$\frac{g^2}{4\pi} D(q^2) = d\pi \frac{\Lambda^2_{QCD}}{b^4} . \tag{28}$$

If we neglect this point in our analysis then we find

$$\beta_{BC} = 0.572 \ , \quad \sigma_\beta = 0.020 . \tag{29}$$

The results in Eqs. (27) and (29) are in agreement with those of Ref. [33] where it is argued that $\beta = 0.589 \pm 0.031$. That study used $b^2 = 0$ and found a critical value of $\ln \tau = 1.69$ which complements the results reported herein, as will be seen in Fig. 2.

We also calculated the critical exponent using our numerical DSE solutions obtained with the Curtis-Pennington addition to the vertex at $\ln \tau = 0.6$:

$$\beta_{CP} = 0.579 \ , \quad \sigma_\beta = 0.015 . \tag{30}$$

This suggests that the vertex modification does not alter the critical exponent of the transition; a conclusion that is also supported by the observation that the vertex used in Ref. [33] was not of either of the above forms but was, effectively, a simple modified rainbow approximation:

$$\Gamma_\mu(k, p) = A(k^2) \gamma_\mu . \tag{31}$$

C. Confinement test

It is important to determine whether the model gluon propagator specified by Eqs. (6) and (10) leads to quark confinement, i.e., the absence of free quarks in the QCD spectrum. This form of gluon propagator has been con-
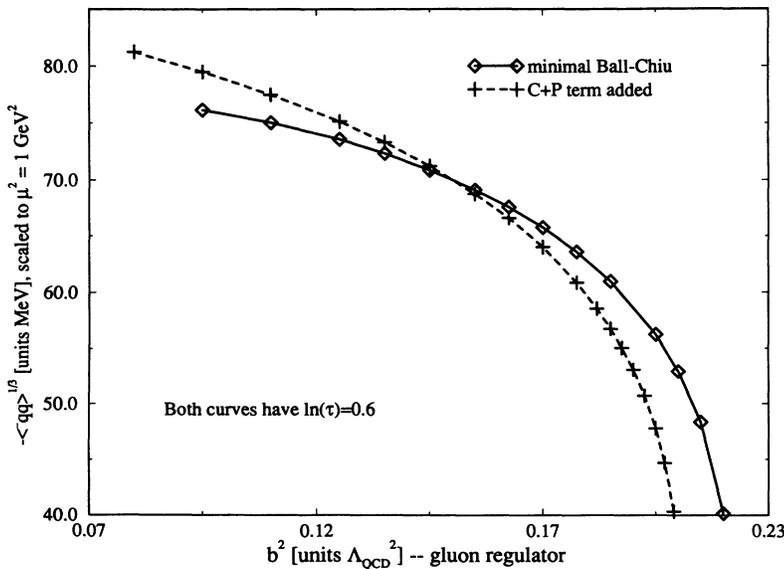
FIG. 3. Comparison of the $-\langle \bar{q}q \rangle^{\frac{1}{3}}_\mu$ condensate curves for the minimal Ball-Chiu and Curtis-Pennington *Ansätze* for the proper vertex. Both curves have $\ln \tau = 0.6$. Diamonds, $\diamond$, connected with solid lines are the results from the BC vertex; plus signs, $+$, connected with dashed lines are results from the CP vertex.

structed so that the dominant IR behavior ensures that it vanishes at $q^2 = 0$. In order to determine whether the quark propagator obtained as a solution to our DSE can represent a confined particle, we follow Ref. [40] and adapt a method commonly used in lattice QCD to estimate bound-state masses.

We write

$$\sigma_S(p^2) = \frac{B(p^2)}{p^2 A(p^2)^2 + B(p^2)^2}, \qquad (32)$$

and define

$$\Delta_S(T, \mathbf{x}) = \int \frac{d^4 p}{(2\pi)^4} e^{i(p_4 T + \mathbf{p} \cdot \mathbf{x})} \sigma_S(p^2) . \qquad (33)$$

This is the scalar part of the Schwinger function of the model quark propagator. If we now define

$$\Delta_S(T) = \int d^3 x \, \Delta_S(T, \mathbf{x}), \qquad (34)$$

and, for notational convenience,

$$E(T) = -\ln\left[\Delta_S(T)\right], \qquad (35)$$

then it follows that if there is a stable *asymptotic state* with the quantum numbers of this Schwinger function

$$\lim_{T \to \infty} \frac{dE(T)}{dT} = m , \qquad (36)$$

where $m \geq 0$ is the mass of this excitation; i.e, this limit yields the dynamically generated quark mass. A finite value of m indicates that the quarks are not confined since it ensures that the cluster decomposition property is satisfied by this Schwinger function [6].

We argue, therefore, that if the limit in Eq. (36) exists for a given propagator then the associated excitation *is not* confined: this is our definition of a nonconfining propagator.

In order to illustrate this point we note that the calculation of the "constituent quark mass" in the Nambu–Jona-Lasinio model [41] can be understood in just this fashion: in this model m, as defined above, is finite and quarks are not confined.

In contrast, one can consider the model of Ref. [37]. Applied to this model one finds

$$\frac{dE(T)}{dT} \overset{T \to \infty}{\sim} \kappa T, \qquad (37)$$

where κ is a constant and hence the limit in Eq. (36) does not exist. In this case the Schwinger function does not satisfy the cluster decomposition property and hence the quarks are confined [6]. Alternatively one may say that through self-interaction the quark acquires an infinite dynamical mass. This provides another way of understanding the claim that the model of Ref. [37] is confining.

Another application of this method, which is of direct interest here, is the IR vanishing gluon propagator of Eq. (1). In this case one has a boson propagator and finds for the analogue of $\Delta_S(T)$

$$\Delta(T) \propto \frac{1}{b\sqrt{2}} \exp\left(-\frac{bT}{\sqrt{2}}\right) \left[\cos\left(\frac{bT}{\sqrt{2}}\right) - \sin\left(\frac{bT}{\sqrt{2}}\right)\right] . \qquad (38)$$

We remark that the Schwinger function in this case is not positive definite, which is an easily identifiable signal in $\Delta(T)$ that is due to the pair of complex conjugate poles, and this violates the axiom of reflection positivity. It follows from this that Eq. (38) describes a field with a complex mass spectrum and/or residues that are not positive. This is appropriate for particles that decay and forms the basis of the argument [15,17–19] that a propagator of the type in Eq. (1) allows colored states to exist only for a finite time (of the order of $1/b$) before hadronizing, i.e., that the propagator describes confined gluons.

1. Confinement and dressed-quark masses

In applying this method here it is obvious that numerical evaluation of the Fourier transforms required in using Eq. (36) will be hindered by numerical noise as T is increased. In order to minimize the effect of this noise on the derivative, we fitted $\Delta_S(T)$ to a form

$$C \exp\left(-mT\right), \qquad (39)$$

and extracted the derivative from this fit. [Importantly, we found no indication of the structure suggested by Eq. (38) in our results.] This was particularly useful with the propagators obtained using small values of b which had large dynamical masses (as one would expect since the condensate is large in this case) and hence a rapid decline with T.

TABLE II. Asymptotic dressed-quark-mass values for the family of propagators with $\ln \tau = 0.1$, the propagator which showed maximal DCSB (at $\ln \tau = 0$ and $b^2 = 0.35$), and for $\ln \tau = 0.6$, $b^2 = 0.1$ with both the Ball-Chiu and Curtis-Pennington vertices.

$\ln \tau$	b^2	m_{free}	C	Comments
0.1	0.25	0.410	0.664	BC vertex
0.1	0.3	0.354	0.650	"
0.1	0.35	0.296	0.633	"
0.1	0.4	0.237	0.624	"
0.1	0.45	0.176	0.619	"
0.1	0.475	0.143	0.619	"
0.1	0.49	0.122	0.619	"
0.1	0.5	0.107	0.621	"
0.1	0.51	0.0913	0.619	"
0.1	0.52	0.0739	0.619	"
0.1	0.525	0.0644	0.619	"
0.1	0.53	0.0539	0.619	"
0.1	0.535	0.0421	0.619	"
0.1	0.5375	0.0353	0.619	"
0.1	0.539	0.0308	0.619	"
0.1	0.54	0.0275	0.619	"
0.0	0.35	0.406	0.667	
0.6	0.1	0.210	0.648	BC vertex
0.6	0.1	0.210	0.507	CP vertex
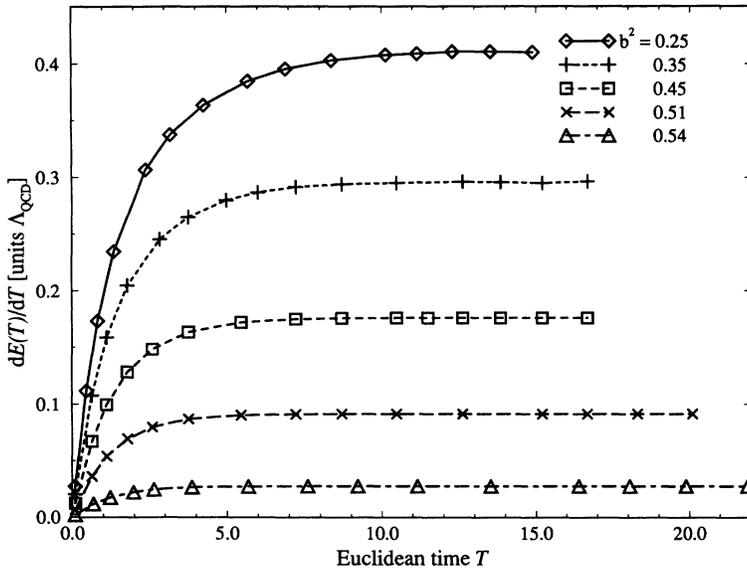
FIG. 4. Dressed-quark-mass curves for the family of propagators with the minimal Ball-Chiu vertex and $\ln \tau = 0.1$. The masses are in units of Λ_{QCD}.

We applied the confinement test in the following cases: (1) The propagators obtained with $\ln \tau = 0.1$ and b^2 in the range $[0.1, 1.0]$; (2) the propagator obtained with $\ln \tau = 0$ and $b^2 = 0.35$ which yields the largest value of $-\langle \bar{q}q \rangle_\mu$ on the $(b^2, \ln \tau)$ domain considered; (3) two propagators obtained with $(b^2, \ln \tau) = (0.1, 0.6)$—one using the Ball-Chiu vertex and another using the Curtis-Pennington addition. The results obtained by fitting the form Eq. (39) to our numerical output are presented in Table II. In Fig. 4 we present plots of $E'(T)$ for the family of propagators obtained with $\ln \tau = 0.1$ and this clearly illustrates that an unambiguous determination of the dressed quark mass is possible in this model. It will be observed that the mass decreases with increasing b^2. This is easily understood in terms of the chiral phase transition: as b increases beyond b_c there is no DCSB and massless current quarks remain massless. Since the behavior of all the other solutions we obtained was qualitatively the same as that described by the results presented in Table II and Fig. 4 we infer that the model considered here does not yield a confining quark propagator. We also note that the rainbow approximation studies of the fermion DSE with Eq. (1) in Ref. [22], which address the question of confinement by a direct continuation to Minkowski momentum space, found a quark propagator with a pole at timelike p^2, i.e., a nonconfining propagator.

We remark that, within numerical noise, the Curtis-Pennington addition made no difference to the dressed-quark-mass value extracted in the cases considered and only slightly reduced the normalization constant C. Clearly, the Curtis-Pennington addition leads only to a minor quantitative effect in this part of our study, too.

IV. DISCUSSION AND CONCLUSION

It has been suggested [19] that the model gluon propagator in Eq. (1) would lead to a confining quark propagator of the form

$$\frac{i\gamma \cdot p + c_0}{(i\gamma \cdot p + c_1)(i\gamma \cdot p + c_1^*)}, \qquad (40)$$

where $c_0 \in \mathbb{R}$ and $c_1 \in \mathbb{C}$ are constants, i.e., to a fermion propagator with complex conjugate poles just as the gluon propagator has, confinement being realized through the absence of poles on the real timelike axis. As Eq. (38) shows, such a propagator has a characteristic signature in $\Delta_S(T)$ which we do not see in Fig. 4. In our model then it is clear that a fermion propagator of the type in Eq. (40) does not arise. This does not eliminate the possibility that it can arise in the approach of Refs. [17–19], however, since the rational-polynomial *Ansätze* employed for the vertex functions therein may lead to a completely different quark-gluon vertex to that used here. We simply remark that our results suggest that a fermion propagator of the type in Eq. (40) cannot arise if the quark-gluon vertex is free of kinematic singularities.

In conclusion, we have studied a model DSE (Dyson-Schwinger equation) for the quark propagator using a model gluon propagator that vanishes as $q^2 \to 0$, Eq. (1), and a light-cone-regular model quark-gluon vertex, Eqs. (14)–(17). This is the first study to analyze the phenomenological implications of Eq. (1) in the framework of the fermion DSE. Our results suggest that this model can only support DCSB for values of $\ln \tau$ and b^2 less than certain critical values (see Fig. 2) and does not confine quarks. Qualitatively similar results are obtained in Ref. [22]. As a consequence we believe that this model gluon propagator is unlikely to be a useful foundation for a chiral-dynamical model of QCD of the general coupled Dyson-Schwinger–Bethe-Salpeter equation type considered in Ref. [4]. Indeed, taken in a broader context, our results do not support the contention that Eq. (1) is the correct form of the quark-quark interaction at small q^2 in QCD.

[1] F. J. Dyson, Phys. Rev. **75**, 1736 (1949); J. S. Schwinger, Proc. Natl. Acad. Sci. U.S.A. **37**, 452, 455 (1950).

[2] A. Hädicke, Int. J. Mod. Phys. A **6**, 3321 (1991).

[3] C. D. Roberts, in *Proceedings of The Workshop on QCD Vacuum Structure*, American University of Paris, 1992, edited by H. M. Fried and B. Müller (World Scientific, New York, 1993).

[4] See, for example, D. W. McKay and H. J. Munczek, Phys. Rev. D **32**, 266 (1985); R. T. Cahill and C. D. Roberts, *ibid.* **32**, 2419 (1985); J. Praschifka, C. D. Roberts, and R. T. Cahill, *ibid.* **36**, 209 (1987); D. W. McKay, H. J. Munczek, and B.-L. Young, *ibid.* **37**, 195 (1988); C. D. Roberts, R. T. Cahill, and J. Praschifka, Ann. Phys. (N.Y.) **188**, 20 (1988); Y. Dai, C. Huang, and D. Liu, Phys. Rev. D **43**, 1717 (1991); K.-I. Aoki, T. Kugo, and M. G. Mitchard, Phys. Lett. B **266**, 467 (1991); R. T. Cahill, Nucl. Phys. **A543**, 63 (1992); P. Jain and H. Munczek, Phys. Rev. D **46**, 438 (1992); M. Buballa and S. Krewald, Phys. Lett. B **294**, 19 (1992); C. D. Roberts, R. T. Cahill, M. E. Sevior, and N. Iannella, Phys. Rev. D **49**, 125 (1994).

[5] C. J. Burden, in *Proceedings of The Workshop on QCD Vacuum Structure*, in [3]; C. J. Burden and C. D. Roberts, Phys. Rev. D **44**, 540 (1991).

[6] C. D. Roberts, A. G. Williams, and G. Krein, Int. J. Mod. Phys. A **7**, 5607 (1992).

[7] N. Brown and C. Pennington, Phys. Rev. D **38**, 2266 (1988).

[8] L. v. Smekal, P. Amunsden, and R. Alkofer, Nucl. Phys. **A529**, 663 (1991).

[9] M. Baker, J. S. Ball, and F. Zachariasen, Nucl. Phys. **B186**, 531 (1980); **B186**, 560 (1980); **B226**, 455 (1983).

[10] S. Mandelstam, Phys. Rev. D **20**, 3223 (1979); N. Brown and M. R. Pennington, Phys. Lett. B **202**, 257 (1988); N. Brown and M. R. Pennington, Phys. Rev. D **39**, 2723 (1989).

[11] U. Bar-Gadda, Nucl. Phys. **B163**, 312 (1980).

[12] G. B. West, Phys. Rev. D **27**, 1878 (1983).

[13] G. B. West, Phys. Lett. **115B**, 468 (1982).

[14] K. G. Wilson, Phys. Rev. D **10**, 2445 (1974); S. W. Otto and J. D. Stack, Phys. Rev. Lett. **52**, 2328 (1984); *ibid.* **53**, 1028(E) (1984).

[15] D. Zwanziger, Nucl. Phys. **B364**, 127 (1991).

[16] V. N. Gribov, Nucl. Phys. **B139**, 1 (1979).

[17] M. Stingl, Phys. Rev. D **34**, 3863 (1986); **36**, 651(E) (1987).

[18] U. Häbel, R. Könning, H.-G. Reusch, M. Stingl, and S. Wigard, Z. Phys. A **336**, 423 (1990).

[19] U. Häbel, R. Könning, H.-G. Reusch, M. Stingl, and S. Wigard, Z. Phys. A **336**, 435 (1990).

[20] C. Bernard, C. Parrinello, and A. Soni, Phys. Rev. D **49**, 1585 (1994).

[21] M. Schaden, *et al.*, Nucl. Phys. **B339**, 595 (1990); L. v. Smekal (private communication).

[22] R. Alkofer and A. Bender, work in progress and private communication.

[23] J. S. Ball and T.-W. Chiu, Phys. Rev. D **22**, 2542 (1980).

[24] D. C. Curtis and M. R. Pennington, Phys. Rev. D **42**, 4165 (1990); **44**, 536 (1991); **46**, 2663 (1992).

[25] A. G. Williams, G. Krein, and C. D. Roberts, Ann. Phys. (N.Y.) **210**, 464 (1991).

[26] F. T. Hawes and A. G. Williams, Phys. Lett. B **268**, 271 (1991).

[27] P. Pascual and R. Tarrach, *QCD: Renormalization for the Practitioner*, Vol. 194 of *Lecture Notes in Physics* (Springer-Verlag, New York, 1984).

[28] See, for example, C. Itzykson and J. -B. Zuber, *Quantum Field Theory* (McGraw-Hill, New York, 1980), Chap. 13.

[29] See, for example, K. Higashijima, Phys. Lett. **124B**, 257 (1983); P. I. Fomin, V. P. Gusynin, V. A. Miransky, and Yu. Sitenko, Riv. Nuov. Cim. **6**, 1 (1983); K. Higashijima, Phys. Rev. D **29**, 1228 (1984); V. P. Gusynin and Yu. A. Sitenko, Z. Phys. C **29**, 547 (1985); P. Castorini and S.-Y. Pi, Phys. Rev. D **31**, 411 (1985); V. A. Miransky and P. I. Fomin, Fiz. Elem. Chatstis At Yadra **16**, 469 (1985) [Sov. J. Part. Nucl. Phys. **16**, 203 (1985)].

[30] M. Becker, G. Löffler, A. Pesch, M. Stingl, and R. Rosenfelder, Phys. Lett. B **267**, 261 (1991).

[31] J. Mandula and M. Ogilvie, Phys. Lett. B **185**, 127 (1987).

[32] D. Atkinson and P. W. Johnson, Phys. Rev. D **37**, 2296 (1988).

[33] C. D. Roberts and B. H. J. McKellar, Phys. Rev. D **41**, 672 (1990).

[34] W. Marciano and H. Pagels, Phys. Rep. **36**, 137 (1978).

[35] C. J. Burden and C. D. Roberts, Phys. Rev. D **47**, 5581 (1993).

[36] D. Atkinson and D. W. E. Blatt, Nucl. Phys. **B151**, 342 (1979); S. J. Stainsby and R. T. Cahill, Phys. Lett. A **146**, 467 (1990); P. Maris and H. Holties, Int. J. Mod. Phys. A **7**, 5369 (1992); S. J. Stainsby and R. T. Cahill, Int. J. Mod. Phys. A **7**, 7541 (1992).

[37] C. J. Burden, C. D. Roberts, and A. G. Williams, Phys. Lett. B **285**, 347 (1992).

[38] A. C. Aitken, Proc. R. Soc. Edinburgh sect. A **46**, 289 (1926); J. Wimp, *Sequence Transformations and their*

Applications (Academic, New York, 1981).

[39] D. S. Gaunt and A. J. Guttmann, in *Phase Transitions and Critical Phenomena*, edited by C. Domb and M. S. Green (Academic Press, London, 1974), Vol. 3.

[40] L. C. L. Hollenberg, C. D. Roberts, and B. H. J. McKellar, Phys. Rev. C **46**, 2057 (1992).

[41] See, for example, S. P. Klevansky, Rev. Mod. Phys. **64**, 649 (1992).