



TABLE I.  $\Delta\beta$  as predicted by Eq. (6) as the function of the quark mass.

$m$	0.025	0.05	0.1	0.2	0.3	0.4	0.5	0.75	1.0
$\Delta\beta$	0.203	0.168	0.133	0.096	0.074	0.059	0.048	0.029	0.020

The leading term of the effective action is the usual continuum gauge action  $(1/g_0^2)F_{\mu\nu}F_{\mu\nu}$  where the coefficient  $1/g_0^2$  can be calculated by evaluating the two-legged graphs in Eq. (5). The quantity  $1/g_0^2$  can also be calculated starting with Eq. (4) and using the method presented in Ref. [5] for adjoint scalars. It happens that this technique is actually incorrect for adjoint fields but correct for fundamental ones.

The result is given by a four-dimensional lattice integral

$$\frac{1}{g_0^2} = \frac{\tilde{N}_f}{4} \int \frac{d^4p}{(2\pi)^4} \text{Tr} \left\{ Q(p_\mu) S(p) Q(p_\mu) \frac{\partial^2}{\partial p_\nu^2} S(p) \right\}, \quad (6)$$

where  $S(p)$  is the lattice fermion propagator

$$S^{-1}(p) = \frac{1}{2\kappa} - r \sum_{\mu} \cos(p_\mu) - i \sum_{\mu} \gamma_{\mu} \sin(p_\mu) \quad (7)$$

and  $Q(p)$  is given by

$$Q(p_\mu) = ir \sin(p_\mu) + \gamma_{\mu} \cos(p_\mu). \quad (8)$$

The integral reduces to the hopping parameter expansion result in the  $\kappa \rightarrow 0$  limit

$$\begin{aligned} \frac{1}{g_0^2} &= 2\tilde{N}_f \kappa^4, \quad r = 1, \\ \frac{1}{g_0^2} &= 1\tilde{N}_f \kappa^4, \quad r = 0. \end{aligned} \quad (9)$$

For small  $ma$  ( $\kappa \rightarrow 0.125$ ) it has a logarithmic singularity

$$\frac{1}{g_0^2} = 16 \frac{\tilde{N}_f}{24\pi^2} \ln \frac{\pi^2}{m^2 a^2}, \quad r = 0. \quad (10)$$

The effective action has additional terms containing more derivatives and/or external gluon legs. These graphs are multiplied by negative powers of  $m$  and are suppressed for heavy fermions [3]. In the limit where the higher order terms can be neglected the effective action is indeed a pure gauge action with bare coupling constant given by Eq. (6). In terms of the plaquette action lattice model it corresponds to an effective plaquette term with coefficient  $\Delta\beta = 6/g_0^2$ . Table I shows  $\Delta\beta$  for several  $m$  values for  $r = 0$  and  $N_f = 16\tilde{N}_f = 1$  fermion flavor.

### III. VALIDITY OF THE EFFECTIVE ACTION

In this section we investigate under what conditions a single plaquette effective action can describe the fermionic theory at low energies.

The question is twofold: (1) Can the nonlocal effective action Eq. (4) indeed be replaced by a single plaquette

term and (2) how well does Eq. (6) predict the coefficient of this term? It is possible to have a pure gauge effective action in a region where Eq. (6) is no longer valid. We will consider the second point in the next chapter and now investigate the first question.

The low-energy effective theory can be considered pure gauge if the gluonic spectrum characterized by the  $\Lambda$  parameter is much lower than the fermionic mass scale that is characterized by the fermion mass  $m$ .

The one-loop definition of the lattice  $\Lambda$  parameter is

$$\Lambda_{\text{latt}} = \frac{1}{a} \exp \left\{ -\frac{\beta_{\text{eff}}}{12\beta_0} \right\}, \quad (11)$$

where  $\beta_0 = 11N_c/48\pi^2$  is the first (universal) coefficient of the  $\beta$  function and  $\beta_{\text{eff}} = \beta + \Delta\beta$ . The condition  $\Lambda_{\text{latt}} \ll m$  can be expressed as

$$\frac{\beta + \Delta\beta}{12\beta_0} + \ln(am) \gg 0. \quad (12)$$

As  $\Delta\beta$  is proportional to  $N_f$ , this condition can be translated into a lower limit on the fermion flavors. For example, for  $\beta = 5.7$ ,  $ma = 0.1$ , assuming the validity of Eq. (6), if  $N_f \gg -28$  (i.e., for any physical  $N_f$ ) the heavy fermions and the gluonic sector decouple.

It is interesting to consider the  $\beta = 0$  strong gauge coupling limit. For small  $m$   $\Delta\beta$  is logarithmically divergent leading to the condition for decoupling

$$N_f > \frac{33}{2} \quad (13)$$

in the  $a \rightarrow 0$  limit (recall  $N_f = 16\tilde{N}_f$ ). The minimum number of flavors coincides with the value where the  $\beta$  function of the gauge-fermion system changes sign and becomes nonasymptotically free. Since the presence of a gauge term ( $\beta \neq 0$ ) relaxes the limit on  $N_f$ , if Eq. (6) holds in an SU(3) gauge theory with 17 or more fermions, then the fermions always decouple from the low lying gluonic spectrum. An identical condition was found in Ref. [3] using dimensional regularization. An interesting consequence is that the  $m = 0$  theory is always deconfined even in the strong gauge coupling limit for  $N_f \geq 17$  flavors, assuming  $\Delta\beta$  diverges as in Eq. (10). One should keep in mind, however, that this derivation is valid only if the higher order terms in the effective action can be neglected.

### IV. EXAMPLES

Now we want to address the second question. When a gauge-fermion theory with massive fermions can be replaced by a gauge theory with a single plaquette action, is the shift in  $\beta$  induced by the fermions given by Eq. (6)? The easiest way to explore the shift in  $\beta$  due to fermions is by tracking the confinement-deconfinement transition as a function of quark mass and number of flavors.

The quenched phase transition at  $N_T = 4$  is at  $\beta_c^Q = 5.69(1)$  [6]. Introducing  $N_f$  flavors of fermions with mass  $m$  will shift the transition to  $\beta_c^{N_f} = \beta_c^Q - \Delta\beta$ . If  $m$  is such that the fermionic action can be considered pure gluonic at low energies then  $\Delta\beta(N_f, m) = N_f \Delta\beta_1(m)$ . If,

in addition,  $m$  is large enough that the perturbative formula is valid,  $\Delta\beta_1(m)$  is given by Eq. (6). Thus we expect the following behavior for the shift  $\Delta\beta(N_f, m)$ : For  $m \gg \Lambda$  where the fermion and gluon mass scales are well separated we expect to see universal behavior  $\Delta\beta/N_f = f(m)$  where  $f(m)$  is given by Eq. (6). For smaller  $m$  we expect Eq. (6) to fail quantitatively. However, it might happen that  $\Delta\beta/N_f$  is still some universal function of the quark mass. Finally, when the fermion scale is the same order as the gauge scale one can no longer replace the fermions by an effective gauge action. The shift  $\Delta\beta/N_f$  would then be different for different  $N_f$ ,  $N_T$ . Measuring the finite temperature transition for different  $N_f$  and  $m$  values makes it possible to distinguish the different scenarios.

The finite temperature transition is first order for the pure gauge theory, and is stable under the inclusion of heavy fermions. With decreasing quark mass the location of the transition shifts downward in  $\beta$ . At some point the deconfinement transition line terminates (at sufficiently light quark mass). We might still be able to track the crossover point as a function of  $N_f$  and  $m$ . As long as the fermionic spectrum remains heavy compared to the low energy gluon spectrum, the system could still be described by an effective gauge action and Eq. (6) could be valid.

At very small or zero quark mass (depending on the number of light flavors) there is a second transition whose behavior is thought to be primarily chirally restoring. At this transition the role of the fermions is fundamental and one would not expect the decoupling of the gluonic and fermionic spectrum.

In a model with  $N_l$  light flavors of mass  $m_l$  and  $N_h$  heavy flavors of mass  $m_h$ , we also expect that the transition should be shifted by the heavy flavors:  $\beta_c(N_l, m_l, N_h, m_h) = \beta_c(N_l, m_l) + \Delta\beta(N_h, m_h)$  where  $\Delta\beta$  is given by Eq. (6). It is a phenomenologically interesting question to ask, “How light is still heavy?” For example, several groups have recently performed simulations with  $N_l=2$  and  $N_h=1$  in an attempt to model the deconfinement transition in the real world of two light ( $u, d$ ) quarks and one strange quark. To the extent that Eq. (6) predicts the shift in lattice critical coupling, the heavy flavor is merely renormalizing the gauge coupling and contributing no new physics.

In the above consideration we had to assume the relation  $\beta = 6/g_0^2$ —the induced gauge coupling is expressed through the bare continuum coupling  $g_0^2$  while in a lattice simulation one uses the coefficient of the plaquette term  $\beta$ .  $\beta = 6/g_0^2$  should hold in the continuum, large  $\beta$  limit; one expects to encounter deviations when the finite temperature transition happens in the strong coupling (small  $\beta$ ) region.

Now we consider a number of cases. We have chosen to focus on staggered fermions since it is easier to make a connection to Eq. (6) with them than with Wilson fermions. In Sec. V we discuss a possible connection to the confinement-deconfinement transition for Wilson fermions.

In the following we will translate numerical data to ex-

press the shift in the gauge coupling caused by one of the fermions only,  $\Delta\beta_1 = (\beta_c^Q - \beta_c^{N_f})/N_f$ . Here  $\beta_c^Q$  is the Monte Carlo quenched critical coupling and  $\beta_c^{N_f}$  is the Monte Carlo  $N_f$  flavor critical coupling. This way we can compare simulations with different  $N_f$  and  $N_T$  values.

#### A. $N_f = 24$

Table II shows the result of a 24 flavor staggered fermion simulation on  $6^3 \times 4$  lattices for several mass values. These simulations were done by us and use a version of a code written by the MILC collaboration [7]. We employ the hybrid molecular dynamics algorithm described in Ref. [8]. We have defined the dynamical fermion fields on all sites of the lattice, so that the “natural” number of flavors in the simulation is a multiple of 8. Simulations with large  $N_f$  require a very small time step compared to ones with small  $N_f$  since the fermion force in the microcanonical evolution equation scales linearly in  $N_f$ . For example for  $m=0.5$  step size  $\Delta t = 0.020$  is needed. The  $N_f=24$  transition is very sharp. Figure 1 shows the expectation value of the Polyakov loop at  $m=0.5$ . At  $\beta=4.62$  the time evolution shows tunneling between two states (Fig. 2), the transition is probably first order. The data points agree with the analytical prediction for  $m > 0.25$ . At  $m=0.25$  simulations with  $N_f \leq 8$  agree with the analytic prediction. The deviation here should be attributed to the fact that  $\beta_c = 3.90(5)$  is a very strong coupling where  $\beta = 6/g_0^2$  does not hold anymore.

#### B. $N_f = 17$

These data (shown in Table III) are from runs using the Langevin updating algorithm on  $N_T=4$  lattices [9]. The analytic formula consistently overestimates the shift in  $\beta_c$ . This is hard to understand given that the  $N_f=24$  and  $N_f=8$  simulations (see below) are well represented by the formula. However, the Langevin time step  $\Delta t_L$  is related to the time step of microcanonical simulations  $\Delta t_M$  by [10]  $\Delta t_L = (\Delta t_M)^2/2$ . These simulations are performed at  $\Delta t_L = 0.01$  corresponding to  $\Delta t_M = 0.14$ , which is known to be large enough to induce sizable integration time step errors.

TABLE II.  $N_f=24$  simulations on  $6^3 \times 4$  lattices performed by us. All the phase transitions, except the  $m=1.00$  one are very sharp, probably first order. At  $m=1.00$  there is only a broad crossover around  $\beta=5.24$ .

$m$	$\beta_c$	$\Delta\beta^{\text{MC}}/N_f$	$\Delta\beta^{\text{anal}}/N_f$
1.00	5.24(4)	0.0188(16)	0.0175
0.75	5.00(2)	0.0287(8)	0.0286
0.60	4.76(2)	0.0387(8)	0.0388
0.50	4.62(2)	0.0446(8)	0.0478
0.25	3.90(5)	0.0746(20)	0.0840

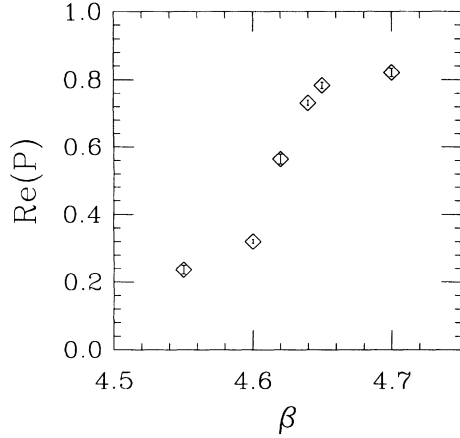


FIG. 1. The real part of the Polyakov loop for  $m=0.5, N_f=24$ .

### C. $N_f=8$

These data, shown in Table IV, are also from runs using the Langevin updating algorithm on  $N_T=4$  and 6 lattices. For smaller mass values the results are very sensitive to the step size used in the simulations. For too large  $\Delta t$  the transition is generally overestimated, so the shift  $\Delta\beta$  is underestimated. The analytic formula accurately predicts the location of the transition or crossover point for the larger values of the quark mass studied. For smaller quark mass values the agreement is still reasonable though  $\Delta\beta^{\text{MC}}$  is consistently smaller than the analytic prediction. At very small  $m$  and  $N_T > 8$  a number of authors [11–13] have seen a transition which may be a

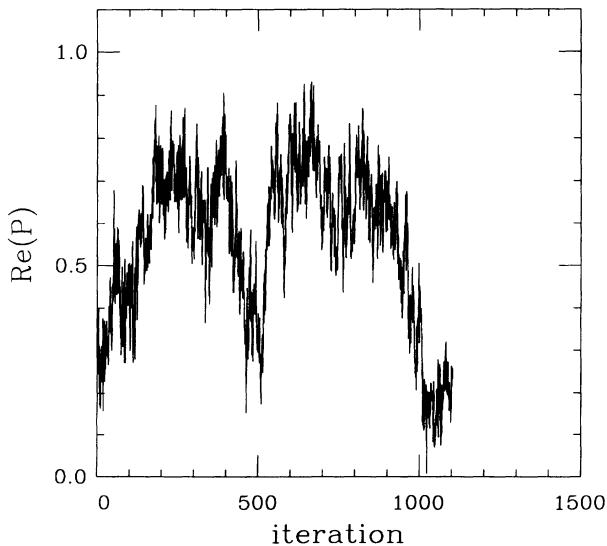


FIG. 2. Time evolution of the real part of the Polyakov loop for  $m=0.5, N_f=24$ .

TABLE III.  $N_f=17$  simulations, from Ref. [9], at  $N_T=4$ .

$m$	$\beta_c$	$\Delta\beta^{\text{MC}}/N_f$	$\Delta\beta^{\text{anal}}/N_f$
0.50	5.025	0.039	0.0478
0.25	4.6(1)	0.064	0.084
0.10	4.3(1)	0.082	0.133

bulk transition. Our analytic formula does not predict this transition. On the other hand at such small mass values Eq. (6) is not expected to be valid anymore.

### D. $N_f=4$

These simulations, shown in Table V, do not show a phase transition at moderate values of the quark mass. At small  $m$  they show a first-order transition which is believed to be associated with chiral restoration. The location of the transition and/or crossover is well predicted by Eq. (6) down to  $m=0.05$ . For  $N_T=4$  at  $m=0.073$  the first-order chiral transition switches on [6]. It is surprising that Eq. (6) is still valid. One can see deviation from the analytic formula for  $m \leq 0.025$ .

### E. $N_f=2$

Most of the  $N_f=2$  simulations were performed at very light values of the quark mass. They do not show a phase transition; instead, they show a smooth crossover from a chirally broken phase to a chirally restored one. Nevertheless, the location of the crossover point is very well tracked by the analytic formula, even at very light value of the quark mass. The results are collected in Table VI.

### F. Summary

Figure 3 contains our  $N_f=24$  data and  $N_f=17, 8, 4$ , and 2 data for  $N_T=4-8$ . Some of these data points correspond to real first-order transitions, others describe just a crossover. For larger masses they correspond to the  $Z_3$  transition, for smaller masses they describe the chiral transition. The agreement with the analytic prediction, especially with smaller  $N_f$ , is remarkable even for masses as small as  $m=0.05$  or below. The fact that the data ap-

TABLE IV.  $N_f=8$  simulations, from Ref. [11].

$m$	$\beta_c$	$\Delta\beta^{\text{MC}}/N_f$	$\Delta\beta^{\text{anal}}/N_f$	$N_T$
1.0	5.54	0.0187	0.0175	4
0.50	5.31	0.0475	0.0478	4
0.25	5.025	0.083	0.084	4
0.10	4.80(1)	0.111(1)	0.133	4
0.10	4.95	0.12(1)	0.133	6
0.05	4.75	0.14(1)	0.168	6

TABLE V.  $N_f=4$  simulations performed by several groups.

$m$	$\beta_c$	$\Delta\beta^{\text{MC}}/N_f$	$\Delta\beta^{\text{anal}}/N_f$	$N_T$	Ref.
0.5	5.50	0.04	0.048	4	[13]
0.5	5.45	0.055	0.048	4	[15]
0.4	5.42	0.063	0.059	4	[15]
0.3	5.35	0.08	0.074	4	[13]
0.2	5.255(5)	0.104(2)	0.0958	4	[16]
0.1	5.130(5)	0.136(2)	0.1334	4	[16]
0.05	5.04	0.163	0.168	4	[17]
0.0375	4.99	0.173	0.183	4	[18]
0.0375	5.02	0.168	0.183	4	[17]
0.025	4.98(2)	0.175	0.203	4	[16]
0.0125	4.919	0.190	0.240	4	[18]
0.01	4.95	0.185	0.25	4	[17]
0.25	5.509	0.090	0.084	6	[19]
0.10	5.322	0.137	0.133	6	[19]
0.075	5.25	0.155	0.149	6	[19]
0.065	5.22	0.162	0.155	6	[19]
0.05	5.175	0.174	0.168	6	[19]
0.025	5.130	0.187	0.203	6	[17]
0.01	5.08	0.199	0.25	6	[17]
0.025	5.25	0.188	0.203	8	[20]
0.01	5.15(5)	0.213	0.25	8	[21]

pear to lie on a universal curve is a signal that the fermions induce an effective  $\beta$  whose strength is linear in  $N_f$  at fixed quark mass, down to very small mass.

One can conclude that the effect of dynamical fermions for finite temperature transition is no more than an induced effective gauge coupling even for fairly small ( $m \geq 0.05$ ) fermion masses.

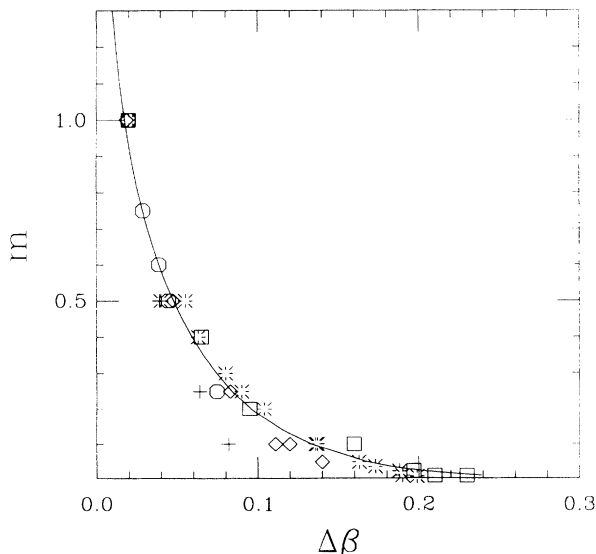


FIG. 3. The induced gauge coupling divided by the number of flavors,  $\Delta\beta/N_f$ , from the simulations described in this paper, compared with the curve from Eq. (6), as a function of quark mass. Data are labeled with octagons for  $N_f=24$ , pluses for  $N_f=17$ , diamonds for  $N_f=8$ , bursts for  $N_f=4$ , and squares for  $N_f=2$ .

## V. LIGHT AND HEAVY FLAVORS TOGETHER

### A. $N_f=2+1$

Several groups studied QCD with two light and one heavy flavors. In these simulations the heavy fermion mass was between 0.1 and 0.25 and 4 to 20 times heavier than the light species. According to the previous chapter the effect of fermions with  $m=0.1-0.25$  on the finite temperature transition is well described by an effective action with induced gauge coupling given by Eq. (6). Unless there is a strong interaction between the light and heavy flavors one would expect the same here. This hypothesis can be tested by comparing the shifts caused by a light species in the  $N_f=2$  and  $N_f=2+1$  simulations assuming the effect of the heavy flavor can be described by Eq. (6). Table VII shows this comparison for recent simulations. Here we use the notation

$$\begin{aligned} \Delta\beta_l^{2+1} &= [\beta_c^Q - (\beta_c^{2+1} + \Delta\beta_h^{\text{anal}})]/2, \\ \Delta\beta_l^2 &= (\beta_c^Q - \beta_c^2)/2, \end{aligned} \quad (14)$$

where  $\beta_c^Q$  is the Monte Carlo quenched critical coupling,  $\beta_c^{2+1}$  is the Monte Carlo  $N_f=2+1$  critical coupling, and  $\beta_c^2$  is the Monte Carlo  $N_f=2$  critical coupling.  $\Delta\beta_h^{\text{anal}}$  is the analytically predicted induced gauge coupling due to the heavy fermions. If  $\Delta\beta_l^{2+1} = \Delta\beta_l^2$ , the shift in coupling due to the light quarks is independent of the presence of the heavy quark. If Eq. (6) is valid for the light species, too, we expect  $\Delta\beta_l^{2+1} = \Delta\beta_l^2 = \Delta\beta_l^{\text{anal}}$ . The shifts from the  $N_f=2+1$  and  $N_f=2$  simulations agree within errors though  $\Delta\beta_l^{2+1}$  is consistently smaller than  $\Delta\beta_l^2$  which agrees with the analytic prediction  $\Delta\beta_l^{\text{anal}}$ .

TABLE VI.  $N_f=2$  simulations performed by several groups.

$m$	$\beta_c$	$\Delta\beta^{\text{MC}}/N_f$	$\Delta\beta^{\text{anal}}/N_f$	$N_T$	Ref.
1.0	5.63	0.02	0.018	4	[13]
0.4	5.54	0.065	0.059	4	[13]
0.2	5.48	0.095	0.096	4	[13]
0.1	5.38	0.15	0.133	4	[15]
0.05	5.34	0.165	0.168	4	[13]
0.025	5.2875	0.197	0.203	4	[15]
0.0125	5.271	0.21	0.24	4	[22]
0.01	5.265(10)	0.212	0.25	4	[2]
0.025	5.445	0.212	0.203	6	[23]
0.0125	5.42(1)	0.225(10)	0.239	6	[23]
0.0125	5.5375	0.23	0.239	8	[24]

### B. $N_f=2+8$

As another test of the interplay of light and heavy flavors, we performed simulations with two light flavors and eight heavy flavors on  $8^3 \times 4$  lattices.

We were motivated to perform these studies by consideration of the deconfinement transition with Wilson fermions. Wilson fermions have 15 doublers for each light species. What is the role of the doublers? Do they only generate an effective gauge coupling or can they influence the low energy spectrum in a nontrivial way? Perturbatively, the 4 doublers sitting at the nearest edges of the Brillouin zone with one component of momentum equal to  $\pi$  and three components of momentum equal to zero are much lighter than the others. It is plausible to assume that they give the most important contribution to the effective action, i.e., 2 flavors of Wilson fermions can be modeled as 2+8 flavors of staggered fermions. (Wilson fermions and 2+8 flavors of staggered fermions are of course not identical, since they have different flavor and chiral symmetry properties. This approach just models the effect of doublers.)

For the heavy flavors we chose mass values  $m_h=0.88, 0.77, 0.665$ , and  $0.4$ , corresponding roughly to the bare Wilson doubler masses at  $\kappa=0.17-0.21$ . The light masses were chosen as listed in Table VIII.

These runs were performed on the Intel iPSC/860 hypercube at the San Diego Supercomputer Center. The iPSC/860 and the code are described briefly elsewhere [7]. We used a truly hybrid algorithm for these simulations: the eight heavy flavors were simulated using the  $\Phi$  algorithm of Ref. [8], with a random noise term for the fermions which was refreshed at the start of each micro-canonical trajectory. (The fermion fields were defined on all sites of the lattice to produce eight flavors.) The two light flavors were simulated using the  $R$  algorithm of Ref.

[8]; the noise estimator for their determinant was updated throughout the simulation. We also performed a two flavor simulation at  $m=0.04$  for comparison. We used integration time steps of  $\Delta t=0.1$  for the  $m_l=0.2$  and  $0.1$  simulations. The smaller quark mass simulations were more sensitive to  $\Delta t$  systematics. We used  $\Delta t=0.05$  away from the transition for all the  $m=0.04$  simulations and switched to  $\Delta t=0.02$  near the transitions.

We display plots of the Polyakov loop and  $\bar{\psi}\psi$  for the light quark from our simulations with light quark mass  $0.04$  in Fig. 4. The heavy quark masses are  $0.4, 0.665$ , and  $\infty$ . The smaller step size points are shown as squares in the figure. The transition for the system with two light and eight heavy flavors appears to be much sharper than the transition for the system containing only two light flavors. It might be first order. Note that at this value of the light quark mass the  $N_f=2$  transition is a smooth crossover and the  $N_f=8$  system does not have a first-order transition for  $m_h \geq 0.25$  either.

In the previous section we concluded that the gauge coupling induced by the heavy flavors is well described by the analytic formula. Using Eq. (6) we compute the shift caused by one of the light flavors as in Sec. V A and compare it to the shift observed in the  $N_f=2$  simulations. We present these results in Table VIII. Our results for  $m_l=0.1$  and  $0.2$  reproduce the  $N_f=2$  simulation results and the analytical prediction, as we would expect following the successes recorded in the last section. Neither of our  $m_l=0.04$  results agree with the analytical formula. That could be explained simply as a breakdown of the analytic formula at light quark mass. However, with  $N_f=2$  or  $4$ , simulations at  $m_q=0.05$  still agree with the analytic formula, as can be seen by comparing Tables V, VI, and the last entry of Table VII. What is even more surprising, the  $m_h=0.665$  and  $0.4$  data show a different shift in  $\beta$  from the same light quarks  $m_l=0.04$ . These facts, cou-

TABLE VII.  $N_f=2+1$  simulations from Refs. [2] and [1].

$m_l$	$m_h$	$\beta_c^{2+1}$	$\Delta\beta_h^{\text{anal}}$	$\Delta\beta_l^{2+1}$	$\Delta\beta_l^2$	$\Delta\beta_l^{\text{anal}}$	$N_T$
0.025	0.025	5.132(2)	0.20	0.18	0.20	0.20	4
0.025	0.10	5.171	0.13	0.20	0.20	0.20	4
0.0125	0.25	5.199(2)	0.084	0.20	0.23	0.24	4
0.00833	0.1667	5.325(25)	0.11	0.22		0.26	6

TABLE VIII.  $N_f=2+8$  simulations performed by us.

$m_l$	$m_h$	$\beta_c^{8+2}$	$\Delta\beta_h^{\text{anal}}$	$\Delta\beta_l^{2+8}$	$\Delta\beta_l^2$	$\Delta\beta_l^{\text{anal}}$
0.04	0.665	5.065(5)	0.271	0.170(3)	0.183(5)	0.18
0.04	0.40	4.89(1)	0.474	0.155(5)	0.183(5)	0.18
0.10	0.77	5.18(1)	0.22	0.138(5)	0.15	0.133
0.20	0.88	5.275(25)	0.18	0.11(2)	0.095	0.096
0.04	$\infty$	5.324(25)	0		0.183(5)	0.18

pled with the qualitative sharpening of the transition at smaller  $m_h$ , lead us to conclude that the eight heavy flavors have an observable influence on the light flavors in addition to an induced gauge coupling. The assumption that the heavy flavors are unimportant at low energies does not seem to hold.

One might expect that this result would be even stronger if the light fermions were lighter.

One might also expect similar behavior for Wilson fermions. In fact, one might expect an even stronger effect, since Wilson fermions include explicit interactions between the light quarks and the doublers which are not present in this  $2+8$  flavor system.

## VI. $\beta=0$ LIMIT

128 flavors of fermions with mass  $m \approx 0.4$  induce a gauge coupling  $6/g^2 = \beta_{\text{ind}} \approx 7.6$ . That is large enough to deconfine an  $N_T=4$  system even when the plaquette gauge coupling is zero. With large number of flavors one should see a confining-deconfining pure gauge phase transition in the  $\beta=0$  limit as the function of the fermion mass.

The naive analytical prediction in Sec. II predicted that for  $N_f > 16$  flavors the fermions always decouple

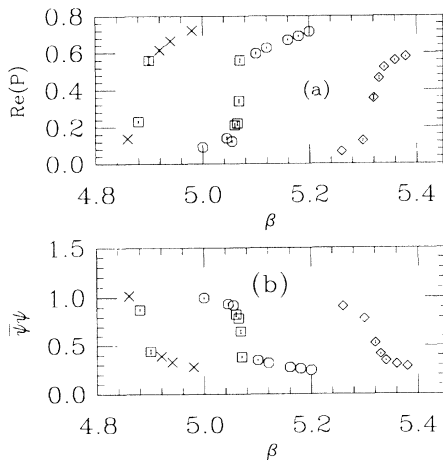


FIG. 4. Plots of (a) the Polyakov loop and (b)  $\bar{\psi}\psi$  for simulations with two flavors of light quarks ( $m_l=0.04$ ) and either nothing else (diamonds) or eight flavors of heavy quarks, of mass  $m_h=0.665$  (octagons and squares) or  $0.4$  (crosses and squares). The squares show data points from simulations with  $\Delta t=0.02$ ; all other data points used  $\Delta t=0.05$ .

from the low-lying gauge spectrum even in the  $\beta=0$  strong coupling limit. Does that mean that for  $N_f > 16$  at  $\beta=0$  one will always find a deconfining phase transition for some value of the quark mass?

We obviously cannot check this scenario numerically but we can study the  $N_T=4$  finite temperature phase transitions in  $m$  at  $\beta=0$  for different  $N_f$  values. Figure 5(a) shows  $m_{\text{crit}}$  and Fig. 5(b) shows  $\beta_{\text{ind}}$  at the phase transition calculated from Eq. (6) as the function of  $N_f$ . Since we observed strong metastability in all cases, we conclude that the phase transition with so many fermions is first order.

The induced  $\beta$  lies in the range  $7.5-8.5$  for  $N_f \geq 80$ . The constancy of this result over a wide range of  $N_f$  indicates that the fermions do induce an effective gauge coupling which scales with  $N_f$ . This  $\beta_{\text{ind}}$  is not consistent with the quenched critical coupling  $\beta_c^Q=5.69$  indicating that  $6/g^2$ , the coefficient of  $F_{\mu\nu}F_{\mu\nu}$ , does not equal  $\beta$  for small  $\beta$  values.

## VII. CONCLUSION

We demonstrated that the effects of fermions on the finite temperature phase transition can be described by an induced effective plaquette term for masses as low as

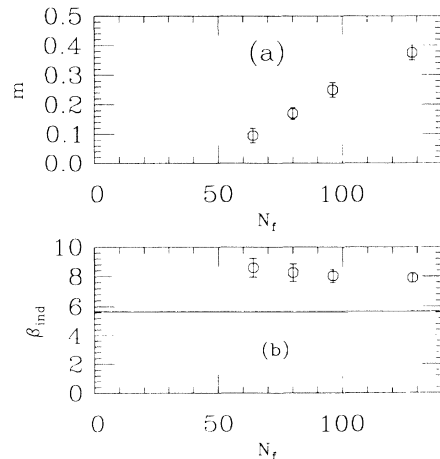


FIG. 5. (a) Plot of the location of the confinement-deconfinement transition at  $\beta=0$  as a function of quark mass for several values of  $N_f$ . (b) The same data, but now interpreted as a function of the induced coupling inferred from the analytic expression. The line shows the location of the quenched deconfinement transition.

$m \simeq 0.05$ . The induced coupling is proportional to the flavor number and is independent of  $N_T$ . The proportionality constant is given by a simple one-loop formula. It is amazing that the simple formula for a fermion-induced shift in  $\beta$  works so well down to such small quark mass, for degenerate mass fermions. From the point of view of lattice simulations of QCD, our results show that some dynamical quarks must be very light to cause interesting effects. A finite temperature simulation at some quark mass ought to show an induced  $\beta$  which is not given by the one-loop formula, before one could claim that a  $T=0$  simulation at the same mass would be sensitive to the effects of dynamical quarks. This is just barely the case in contemporary dynamical fermion simulations. For example, the spectroscopy of the High Energy Monte Carlo Grand Challenge (HEMCGC) simulations at  $\beta=5.6$  with  $N_f=2$  and  $m=0.025$  and  $0.01$  has been mapped onto quenched simulations at  $\beta=5.935$  and  $5.95$ , respectively [14]. These comparisons correspond to shifts per flavor of  $\Delta\beta=0.1675$  and  $0.175$ , respectively, to be contrasted with  $\Delta\beta^{\text{anal}}=0.20$  and  $0.25$  and finite temperature Monte Carlo shifts of about  $0.20$  and  $0.21$ . Thus they are in a regime where the sea quarks might be important for long distance dynamics.

We simulated systems with 2 light and 8 heavy flavors

to study the interaction of heavy and light quarks. For light masses  $m_l \geq 0.1$  we found no observable effect. For  $m_l=0.04$  interaction with the heavy fermions as heavy as  $m=0.665$  can be observed in the finite temperature phase transition. We found the  $N_f=8+2$  transition is very sharp and its location cannot be predicted from the  $N_f=2$  transition assuming that the effect of the heavy flavors is described by an induced gauge coupling. These results may have applications to technicolor models, where one has to deal with the low energy effects of large numbers of heavy fermions as well as a small number of light fermions. Consequences of these results for Wilson fermions remain an open problem.

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