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Heavy baryons as Skyrmions with $1/m_Q$ corrections

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We take into account the $1/m_Q$ corrections up to $1/N_c$ order in the heavy-meson-soliton bound-state approach for heavy baryons. With these corrections, the mass spectra of baryons with c quark as well as of those with b quark are well reproduced. For charmed baryons, however, the correction to the mass spectra amounts to about 300 MeV, which is not small compared to the leading order binding energy, $\sim$ 800 MeV.

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I. INTRODUCTION

The bound-state approach advocated by Callan and Klebanov (CK) [1] has been shown to work very well for static properties [2] of strange baryons. In the CK approach, strange baryons are described by properly quantized states of the K-meson(s)-soliton bound system. Rho *et al.* [3] extended the CK approach further to baryons containing a heavy flavor such as a charm or bottom quark. In particular, the mass spectra and magnetic moments [4] for charmed baryons are found to be strikingly close to the predictions of the quark model description. In these calculations, vector meson fields such as K^*, D^*, and B^* are eliminated in favor of a combination of a background and corresponding pseudoscalar fields, K, D, and B. This approximation is valid only when vector mesons are sufficiently heavier than corresponding pseudoscalar mesons as in the case of ρ and π ($m_\rho = 770$ MeV, $m_{\pi^0} = 135$ MeV: $m_{\pi^0}/m_\rho = 0.18$). For charmed mesons or bottom flavored mesons, however, vector mesons are only a few percent heavier than the corresponding pseudoscalar mesons: $m_{K^*} = 892$ MeV, $m_{K^0} = 498$ MeV ($m_{K^0}/m_{K^*} = 0.56$), $m_{D^*} = 2010$ MeV, $m_{D^0} = 1865$ MeV ($m_{D^0}/m_{D^*} = 0.93$), and $m_{B^*} = 5325$ MeV, $m_{B^0} = 5279$ MeV ($m_{B^0}/m_{B^*} = 0.99$). Thus we need to treat heavy vector mesons correctly on the same footing as heavy pseudoscalar mesons.

Heavy quark symmetry is a new spin and flavor symmetry of QCD in the limit of infinite heavy quark masses. As a heavy quark becomes infinitely heavy, the dynamics of a heavy quark in QCD depend only on its velocity and is independent of its mass and spin. This symmetry can be seen in weak semileptonic decays [5], mass splittings, and partial decay widths [6] of heavy mesons and heavy baryons, whose masses are much bigger than the QCD scale, Λ_{QCD}. Recently, effective heavy meson Lagrangians which have both chiral symmetry and heavy quark symmetry have been constructed by several authors [7–9]. Also, a lot of work on heavy baryons as Skyrmions in the manner of Callan and Klebanov has been reported [10–13]. In a series of papers [10], Jenkins *et al.* investigated the binding of a heavy meson with a soliton using such an effective Lagrangian. Nowak *et al.* [12] studied the heavy quark symmetry in heavy baryon mass spectra in connection with Berry's phase. Gupta *et al.* [13] discussed the roles of light vector-meson degrees of freedom such as ω and ρ. In these works, however, only the leading-order terms in the inverse of the heavy quark mass have been considered. Also, bound heavy mesons are assumed to sit at the center of the soliton with their wave functions taken as δ functions. As a result, heavy mesons appear to be too deeply bound and Σ_Q and Σ_Q^* are degenerate in mass.

In order to investigate more realistic cases with hyperfine splittings, one needs to include next-to-leading-order terms in $1/m_Q$. In Ref. [10], mass corrections are roughly estimated by including mass differences between heavy pseudoscalar mesons and heavy vector mesons, while keeping the δ-function-like wave functions. Although it may work well for bottom flavored baryons, we may have some doubts on the validity of such δ-function-like wave functions for charmed baryons: the finite mass corrections need to be included in the wave functions of heavy mesons, leading to different radial functions, though sharply peaked at the center of the soliton. In this paper, we attempt to establish a "smooth" connection between the CK approach for light baryons and the heavy-meson-soliton bound-state approach for heavy baryons

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by clarifying the above-mentioned problems. In our calculation, heavy pseudoscalar mesons and heavy vector mesons are treated on the same footing and the next-to-leading-order terms in $1/m_Q$ are incorporated properly.

In the following section, we introduce a simple Lagrangian which is relevant to our purpose. Then, a soliton-heavy-meson bound state is found in Sec. III by solving the equations of motion for the classical eigenmodes of heavy mesons moving in the soliton background. In Sec. IV, we discuss the mass formula for heavy baryons containing a heavy quark. We also discuss the heavy quark symmetry breaking by the Wess-Zumino term in the heavy baryon mass spectra. Section V contains a summary and conclusion.

II. MODEL LAGRANGIAN

In order to avoid any unnecessary complications, we work with a simple Lagrangian for the interaction of light Goldstone bosons with heavy mesons, which has the $SU(2)_L \times SU(2)_R$ chiral symmetry and the heavy quark symmetry in the heavy-mass limit. One may obtain such a Lagrangian from the Skyrme model Lagrangian by trimming away all the higher derivative terms or from the heavy quark effective Lagrangian by including the next-to-leading-order terms in $1/m_Q$.

Up to a single derivative on the Goldstone boson fields, the most general chirally invariant Lagrangian density[1] can be written in a form of [9]

$$\begin{aligned}\mathcal{L} = \mathcal{L}_M + (D_\mu\Phi)^\dagger D^\mu\Phi - M_\Phi^2\Phi^\dagger\Phi - \tfrac{1}{2}\Phi^{*\dagger}_{\mu\nu}\Phi^{*\mu\nu} + M_{\Phi^*}^2\Phi^{*\dagger}_\mu\Phi^{*\mu} \\ +f_Q\left(\Phi^\dagger A^\mu\Phi^*_\mu + \Phi^{*\dagger}_\mu A^\mu\Phi\right) + \tfrac{1}{2}g_Q\epsilon^{\mu\nu\lambda\rho}(\Phi^{*\dagger}_{\mu\nu}A_\lambda\Phi^*_\rho + \Phi^{*\dagger}_\rho A_\lambda\Phi^*_{\mu\nu}),\end{aligned} \tag{1}$$

where Φ and Φ^*_μ are the heavy pseudoscalar and the heavy vector-meson doublets[2] with masses M_Φ and M_{Φ^*}, respectively. For example, in the case of charmed mesons, we have

$$\Phi = \begin{pmatrix}\bar{D}^0 \\ D^-\end{pmatrix}, \quad \Phi^* = \begin{pmatrix}\bar{D}^{*0} \\ D^{*-}\end{pmatrix}.$$

The Lagrangian density for the Goldstone boson fields is

$$\mathcal{L}_M = \frac{f_\pi^2}{4}\mathrm{Tr}(\partial_\mu U^\dagger\partial^\mu U) + \frac{1}{32e^2}\mathrm{Tr}[U^\dagger\partial_\mu U, U^\dagger\partial_\nu U]^2, \tag{1a}$$

with

$$U \equiv \xi^2 = \exp\left(\frac{i}{f_\pi}\begin{pmatrix}\pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0\end{pmatrix}\right), \tag{1b}$$

and f_π being the pion-decay constant. The "Skyrme term" with a dimensionless parameter e is included to stabilize the soliton solution. Here f_Q and g_Q are the $\Phi\Phi^*\pi$ and $\Phi^*\Phi^*\pi$ coupling constants. The vector and axial-vector potentials V_μ and A_μ are defined in terms of ξ as

$$\begin{aligned}A_\mu &= \tfrac{i}{2}(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger), \\ V_\mu &= \tfrac{1}{2}(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger),\end{aligned} \tag{1c}$$

and the covariant derivative D_μ and the field strength $\Phi^*_{\mu\nu}$ are

$$D_\mu = \partial_\mu + V_\mu, \qquad \Phi^*_{\mu\nu} = D_\mu\Phi^*_\nu - D_\nu\Phi^*_\mu. \tag{1d}$$

Under $SU(2)_L \times SU(2)_R$ chiral transformations, the fields transform as

$$\begin{aligned}\xi \to \xi' = L\xi h^\dagger = h\xi R^\dagger \qquad (U \to U' = LUR^\dagger), \\ \Phi \to \Phi' = h\Phi, \qquad \Phi^*_\mu \to \Phi^{*\prime}_\mu = h\Phi^*_\mu,\end{aligned} \tag{2}$$

where L and R are global transformations in $SU(2)_L$ and $SU(2)_R$, respectively, and h is a special unitary matrix depending on L, R, and the Goldstone fields. Furthermore, the Lagrangian is invariant under the parity operation

$$\begin{aligned}U(\mathbf{r},t) &\to \mathcal{P}U\mathcal{P}^{-1} = U^\dagger(-\mathbf{r},t), \\ \Phi(\mathbf{r},t) &\to \mathcal{P}\Phi\mathcal{P}^{-1} = -\Phi(-\mathbf{r},t), \\ \Phi^*_\mu(\mathbf{r},t) &\to \mathcal{P}\Phi^*_\mu\mathcal{P}^{-1} = -\Phi^*_\mu(-\mathbf{r},t).\end{aligned} \tag{3}$$

Here, we have used the fact that pions and heavy mesons (both pseudoscalar mesons and vector mesons) carry negative intrinsic parity.

We have four parameters in the Lagrangian to be fixed: the pion-decay constant f_π, the Skyrme parameter e, and the coupling constants f_Q and g_Q. The pion-decay constant f_π and the Skyrme parameter e are fixed by fitting the nucleon and Δ masses in the SU(2) sector [14]. As for the heavy meson coupling constant f_Q and g_Q, little has been known except the upper bound [15]. Thus, we use the heavy quark symmetry as a guide line. We use the empirical masses for the heavy meson masses, M_Φ and M_{Φ^*}. In the heavy-mass limit, we have $M_\Phi \simeq M_{\Phi^*}$ with the mass difference being of order $1/m_Q$ at most and the two coupling constants f_Q and g_Q become related to each other by [9,16]

$$f_Q = 2M_{\Phi^*}g_Q, \tag{4}$$

due to the heavy-quark spin symmetry. Furthermore, g_Q approaches a universal constant g due to the heavy-quark flavor symmetry.

The nonrelativistic quark model estimate of g (-0.75)

[1] One may improve the model Lagrangian by including terms with more derivatives on the Goldstone boson fields and incorporating vector mesons such as ρ and ω [13].

[2] Here, we adopt a different convention for Φ and Φ^*_μ than that of Ref. [9]. Our $\Phi(\Phi^*_\mu)$ corresponds to their $\Phi^\dagger(\Phi^{*\dagger}_\mu)$.

[9] is consistent with the experimental value ($|g|^2 \lesssim 0.5$) measured via the D^* decay width [15] and the $D^{*+} \to D^+\pi^0$ and $D^{*+} \to D^0\pi^+$ branching ratios [17].

One may determine the two coupling constants f_Q and g_Q from a low-energy chiral theory. In Ref. [18], a Lagrangian for the interactions of K and K^* mesons with pions is derived on the basis of SU(3) chiral symmetry along the hidden gauge symmetry scheme. Comparing it with our Lagrangian, we get $f_Q/2M_{K^*} = -\frac{1}{\sqrt{2}} \sim -0.71$, which is very close to the nonrelativistic quark model prediction. Although the $\Phi^*\Phi^*\pi$ term proportional to g_Q is missing in Ref. [18], one can find such a term among the homogeneous solutions of the Wess-Zumino anomaly equation. (See Ref. [16] for further details.) Using the vector-meson dominance hypothesis and the empirical value on the $g_{K^*\pi\pi}$ coupling constant (~ 6), we obtain $g_Q \sim -0.7$ from the chiral Lagrangian of Ref. [16].

III. SOLITON-HEAVY-MESON BOUND STATE

The Lagrangian density $\mathcal{L}_M$ supports a stable SU(2) soliton solution of "hedgehog"-type:

$$U_0(\mathbf{r}) = \exp[i\boldsymbol{\tau}\cdot\hat{\mathbf{r}}F(r)], \tag{5}$$

with

$$F(0) = \pi \quad \text{and} \quad F(r) \overset{r\to\infty}{\longrightarrow} 0. \tag{5a}$$

The above solution carries a nontrivial winding number due to its nontrivial topological structure identified as the baryon number

$$B = -\frac{1}{24\pi^2}\int d^3r\, \varepsilon^{ijk}\mathrm{Tr}(U_0^\dagger\partial_i U_0 U_0^\dagger\partial_j U_0 U_0^\dagger\partial_k U_0)$$

$$= -\frac{2}{\pi}\int_0^\infty r^2 dr \frac{\sin^2 F}{r^2}F' = 1, \tag{5b}$$

and a finite mass

$$M_{\rm sol} = 4\pi\int_0^\infty r^2 dr \left\{\frac{f_\pi^2}{2}\left(F'^2 + 2\frac{\sin^2 F}{r^2}\right) + \frac{1}{2e^2}\frac{\sin^2 F}{r^2}\left(\frac{\sin^2 F}{r^2} + 2F'^2\right)\right\}, \tag{5c}$$

with $F' = \frac{dF}{dr}$.

Now, our problem is to find the eigenmodes of the heavy mesons moving in the static potentials provided by the $B = 1$ soliton configuration (5) sitting at the origin, viz.,

$$\begin{aligned} V^\mu &= (V^0, \mathbf{V}) = (0, iv(r)\hat{\mathbf{r}}\times\boldsymbol{\tau}), \\ A^\mu &= (A^0, \mathbf{A}) = (0, \tfrac{1}{2}[a_1(r)\boldsymbol{\tau} + a_2(r)\hat{\mathbf{r}}\boldsymbol{\tau}\cdot\hat{\mathbf{r}}]), \end{aligned} \tag{6}$$

with

$$\begin{aligned} v(r) &= \frac{\sin^2(F/2)}{r}, \\ a_1(r) &= \frac{\sin F}{r} \quad \text{and} \quad a_2(r) = F' - \frac{\sin F}{r}. \end{aligned} \tag{7}$$

The equations of motion can be read off from the Lagrangian (1):

$$(D_\mu D^\mu + M_\Phi^2)\Phi = f_Q A^\mu \Phi^*_\mu, \tag{8}$$

for the pseudoscalar-meson field Φ and

$$D_\mu \Phi^{*\mu\nu} + M_{\Phi^*}^2\Phi^{*\nu} = -f_Q A^\nu \Phi + g_Q \varepsilon^{\mu\nu\lambda\rho}A_\lambda \Phi^*_{\mu\rho} \tag{9}$$

for the vector-meson fields Φ^*_μ.

The conjugate momenta to the meson fields Φ and Φ^*_μ are

$$\begin{aligned} \Pi &= \frac{\partial\mathcal{L}}{\partial(\dot\Phi)} = (D_0\Phi)^\dagger, \\ \Pi^{*i} &= \frac{\partial\mathcal{L}}{\partial(\dot\Phi^*_i)} = (\Phi^{*i0})^\dagger - g_Q\epsilon^{ijk}\Phi^{*\dagger}_k A_j, \end{aligned} \tag{10}$$

respectively, and we get similar equations for $\Pi^\dagger$ and $\Pi^{*i\dagger}$. Since Π^*_0 vanishes identically, the Φ^*_0 cannot be an independent dynamical variable. We eliminate the complementary Φ^*_0 field by using Eq. (9)

$$\Phi^{*0} = -\frac{1}{M_{\Phi^*}^2}(D_i \Pi^{*i\dagger} + \tfrac{1}{2} g_Q \epsilon^{ijk} A_k \Phi_{ij}^*), \tag{11}$$

which results in a set of coupled equations

$$\begin{aligned}\dot{\mathbf{\Phi}}^* &= -\mathbf{\Pi}^{*\dagger} - g_Q \mathbf{A} \times \mathbf{\Phi}^* + \frac{1}{M_{\Phi^*}^2}\mathbf{D}(\mathbf{D}\cdot\mathbf{\Pi}^{*\dagger}) + \frac{g_Q}{M_{\Phi^*}^2}\mathbf{D}[\mathbf{A}\cdot(\mathbf{D}\times\mathbf{\Phi}^*)],\\ \dot{\mathbf{\Pi}}^{*\dagger} &= \mathbf{D}\times(\mathbf{D}\times\mathbf{\Phi}^*) + M_{\Phi^*}^2\mathbf{\Phi}^* + f_Q \mathbf{A}\Phi - g_Q \mathbf{A}\times\mathbf{\Pi}^{*\dagger} - g_Q^2 \mathbf{A}\times(\mathbf{A}\times\mathbf{\Phi}^*)\\ &\quad -\frac{2g_Q}{M_{\Phi^*}^2}\mathbf{A}\times\mathbf{D}\left\{\mathbf{D}\cdot\mathbf{\Pi}^{*\dagger} + g_Q \mathbf{A}\cdot(\mathbf{D}\times\mathbf{\Phi}^*)\right\},\end{aligned} \tag{12}$$

where $\mathbf{D} = \mathbf{\nabla} - \mathbf{V}$.

In order to express the equations of motion only in terms of Φ and $\mathbf{\Phi}^*$, we use the fact that the Φ_0^* field is of order $1/m_Q$ at most, viz.,

$$\Phi^{*0} \sim \frac{1}{M_{\Phi^*}^2} D_i \dot{\Phi}^{*i} = O(1/M_{\Phi^*}). \tag{13}$$

Keeping this leading-order term leads us to the equations of motion

$$\begin{aligned}\ddot{\mathbf{\Phi}}^* &= -2g_Q \mathbf{A}\times\dot{\mathbf{\Phi}}^* - \mathbf{D}\times(\mathbf{D}\times\mathbf{\Phi}^*) - M_{\Phi^*}^2\mathbf{\Phi}^*\\ &\quad - f_Q \mathbf{A}\Phi + \mathbf{D}(\mathbf{D}\cdot\mathbf{\Phi}^*).\end{aligned} \tag{14}$$

Because of the spin-isospin mixing in the hedgehog configuration of the classical background, the equations of motion (8) and (14) are invariant only under the rotation by the grand spin $\mathbf{K}$ defined by $\mathbf{K} = \mathbf{S} + \mathbf{I} + \mathbf{L}$ with $\mathbf{S}(\mathbf{I})$ being the spin (isospin) of the heavy mesons and $\mathbf{L}$ the orbital angular momentum. Thus, eigenmodes are classified by the quantum numbers k, m_k, and P [the parity P equals $(-1)^{\ell+1}$ with ℓ being the orbital angular momentum] as

$$\begin{aligned}\Phi(\mathbf{r},t) &= \sum_{k,m_k,P} \varphi_{k,m_k,P}(r,t)\mathcal{Y}_{k,P,m_k}(\hat{\mathbf{r}}),\\ \mathbf{\Phi}^*(\mathbf{r},t) &= \sum_{k,m_k,P,\kappa} \varphi^{*\kappa}_{k,m_k,P}(r,t)\boldsymbol{\mathcal{Y}}^{(\kappa)}_{k,P,m_k}(\hat{\mathbf{r}}),\end{aligned} \tag{15}$$

where $\mathcal{Y}_{k,P,m_k}$ and $\boldsymbol{\mathcal{Y}}_{k,P,m_k}$ are the generalized spherical spinor and vector harmonics, respectively, and κ is an index to label the possible vector spherical harmonics with the same k, m_k, and P. To avoid cumbersome notation, we will suppress the trivial indices k, m_k, and P of the radial functions as $\varphi(r,t)$ and $\varphi^{*\kappa}(r,t)$.

From now on, we will restrict our consideration to $k^P = \frac{1}{2}^+$ states, which are expected to have at least one bound state. Since pseudoscalar mesons do not carry spin, we have only one spherical spinor harmonic with $k^P = \frac{1}{2}^+$:

$$\mathcal{Y}_{\frac{1}{2},+,\pm\frac{1}{2}}(\hat{\mathbf{r}}) = \frac{1}{\sqrt{4\pi}}\boldsymbol{\tau}\cdot\hat{\mathbf{r}}\chi_\pm. \tag{16}$$

Here, $\chi_\pm$ is the isospin basis for the heavy meson doublets: i.e.,

$$\chi_+ = \begin{pmatrix}1\\0\end{pmatrix} \quad\text{and}\quad \chi_- = \begin{pmatrix}0\\1\end{pmatrix}. \tag{17}$$

For vector mesons with spin 1, we can construct two different $k^P = \frac{1}{2}^+$ vector spherical harmonics [19]: viz.,

$$\begin{aligned}\boldsymbol{\mathcal{Y}}^{(1)}_{\frac{1}{2},+,\pm\frac{1}{2}}(\hat{\mathbf{r}}) &= \frac{1}{\sqrt{4\pi}}\hat{\mathbf{r}}\chi_\pm,\\ \boldsymbol{\mathcal{Y}}^{(2)}_{\frac{1}{2},+,\pm\frac{1}{2}}(\hat{\mathbf{r}}) &= i\frac{1}{\sqrt{8\pi}}(\boldsymbol{\tau}\times\hat{\mathbf{r}})\chi_\pm.\end{aligned} \tag{18}$$

Putting

$$\Phi(\mathbf{r},t) = \varphi(r)e^{+i\omega t}\mathcal{Y}_{\frac{1}{2},+,\pm\frac{1}{2}}(\hat{\mathbf{r}}), \tag{19}$$

$$\begin{aligned}\mathbf{\Phi}^*(\mathbf{r},t) &= \varphi_1^*(r)e^{+i\omega t}\boldsymbol{\mathcal{Y}}^{(1)}_{\frac{1}{2},+,\pm\frac{1}{2}}(\hat{\mathbf{r}})\\ &\quad +\varphi_2^*(r)e^{+i\omega t}\boldsymbol{\mathcal{Y}}^{(2)}_{\frac{1}{2},+,\pm\frac{1}{2}}(\hat{\mathbf{r}}),\end{aligned} \tag{20}$$

into the equations of motion (8) and (14), we obtain three coupled differential equations for the radial functions:

$$\begin{aligned}\varphi'' + \frac{2}{r}\varphi' + \left(\omega^2 - M_\Phi^2 - \frac{2}{r^2}\right)\varphi &= 2v\left(v - \frac{2}{r}\right)\varphi + \frac{f_Q}{2}(a_1+a_2)\varphi_1^* - \frac{1}{\sqrt{2}}f_Q a_1\varphi_2^*,\\ \varphi_1^{*\prime\prime} + \frac{2}{r}\varphi_1^{*\prime} + \left(\omega^2 - M_{\Phi^*}^2 - \frac{2}{r^2}\right)\varphi_1^* &= \frac{f_Q}{2}(a_1+a_2)\varphi + 2v^2\varphi_1^*\\ &\quad +\sqrt{2}\left(g_Q a_1\omega - \frac{1}{r}v + v'\right)\varphi_2^*,\\ \varphi_2^{*\prime\prime} + \frac{2}{r}\varphi_2^{*\prime} + \left(\omega^2 - M_{\Phi^*}^2 - \frac{2}{r^2}\right)\varphi_2^* &= -\frac{f_Q}{\sqrt{2}}a_1\varphi + \sqrt{2}\left(\omega g_Q a_1 - \frac{1}{r}v + v'\right)\varphi_1^*\\ &\quad + \left(-\omega g_Q(a_1+a_2) - \frac{4}{r}v + 4v^2\right)\varphi_2^*.\end{aligned} \tag{21}$$

The wave functions are normalized such that each mode carries one corresponding heavy flavor number:

$$1 = \int_0^\infty r^2 dr \{2\omega\,[|\varphi|^2 + |\varphi_1^*|^2 + |\varphi_2^*|^2] + g_Q[(a_1 + a_2)|\varphi_2^*|^2 - \sqrt{2}a_1(\varphi_1^{*\dagger}\varphi_2^* + \varphi_2^{*\dagger}\varphi_1^*)]\}, \tag{22}$$

where we have kept terms up to the next-to-leading order in $1/m_Q$.

Near the origin, the equations of motion behave asymptotically as

$$\begin{aligned} \varphi'' + \frac{2}{r}\varphi' &= 0, \\ \varphi_1^{*\prime\prime} + \frac{2}{r}\varphi_1^{*\prime} - \frac{4}{r^2}\varphi_1^* &= -\frac{2\sqrt{2}}{r^2}\varphi_2^*, \\ \varphi_2^{*\prime\prime} + \frac{2}{r}\varphi_2^{*\prime} - \frac{2}{r^2}\varphi_2^* &= -\frac{2\sqrt{2}}{r^2}\varphi_1^*. \end{aligned} \tag{23}$$

They imply that we have three independent solution sets as

$$\begin{aligned} &\text{(a)}\ \ \varphi(r) = \varphi(0) + O(r^2), \\ &\qquad \varphi_i^*(r) = O(r^2), \\ &\text{(b)}\ \ \varphi(r) = O(r^2), \\ &\qquad \varphi_i^*(r) = \varphi_{bi}^*(0) + O(r^2), \\ &\text{(c)}\ \ \varphi(r) = O(r^4), \\ &\qquad \varphi_i^*(r) = \frac{1}{2}\varphi_{ci}^{*\prime\prime}(0)r^2 + O(r^4), \end{aligned} \tag{24}$$

with $\sqrt{2}\varphi_{b1}^*(0) = \varphi_{b2}^*(0)$ and $\varphi_{c1}^{*\prime\prime}(0) = -\sqrt{2}\varphi_{c2}^{*\prime\prime}(0)$. For sufficiently large $r(\gg 1/M_\Phi)$, the three equations decouple from each other: for example,

$$\varphi'' + \frac{2}{r}\varphi' + (\omega^2 - M_{\Phi^*}^2)\varphi = 0. \tag{25}$$

Thus the bound-state solutions ($\omega < M_\Phi$) are

$$\begin{aligned} \varphi(r) &= \alpha\frac{e^{-r\sqrt{M_\Phi^2-\omega^2}}}{r}, \\ \varphi_1^*(r) &= \alpha_1\frac{e^{-r\sqrt{M_{\Phi^*}^2-\omega^2}}}{r}, \\ \varphi_2^*(r) &= \alpha_2\frac{e^{-r\sqrt{M_{\Phi^*}^2-\omega^2}}}{r}, \end{aligned} \tag{26}$$

with three constants α, α_1, and α_2.

The lowest energy bound states are found numerically, and the results are shown in Table I and Fig. 1. In Table I, the input parameters are listed together with the numerical results on the lowest bound states. In Fig. 1, we give the radial functions $\varphi(r)$ and $\varphi_1^*(r)$ for the D and D^* mesons (solid curve) and the B and B^* mesons (dashed curves). By comparing the two cases, one can easily check that as the meson mass becomes larger, (1) the radial function becomes more sharply peaked at the origin and (2) the role of the vector mesons becomes important so that the radial function $\varphi_1^*(r)$ becomes comparable to $\varphi(r)$ [see also the ratio $\varphi_1^*(0)/\varphi(0)$]. The radial function $\varphi_2^*(r)$, though not shown in Fig. 1, is hardly distinguishable from $\sqrt{2}\varphi_1^*(r)$. This can be understood as follows: due to their heavy masses, heavy mesons are localized in the region $r \lesssim 1/M_\Phi$, where

$$\begin{aligned} [a_1(r) + a_2(r)] &\sim [-a_1(r)] \sim F'(0) + O(r^2), \\ v(r) &\sim \frac{1}{r} - \tfrac{1}{4}F'^2(0)r + \cdots, \end{aligned} \tag{27}$$

so that the equation of motion for $(\varphi_1^* - \frac{1}{\sqrt{2}}\varphi_2^*)$ is completely decoupled from those for φ and $(\varphi_1^* + \sqrt{2}\varphi_2^*)$.

It would be interesting to compare our radial functions with those of Ref. [3] and Ref. [10]. In Ref. [3], vector mesons are assumed to be sufficiently heavy and the following ansatz is made:

$$\Phi_\mu^* = \frac{\sqrt{2}}{M_{\Phi^*}}A_\mu\Phi, \tag{28}$$

which implies that

$$\begin{aligned} \varphi_1^*(r) &= \frac{1}{\sqrt{2}M_{\Phi^*}}[a_1(r) + a_2(r)]\varphi(r), \\ \varphi_2^*(r) &= -\frac{1}{M_{\Phi^*}}a_1(r)\varphi(r). \end{aligned} \tag{29}$$

As $\sqrt{2}\varphi_1^* \sim \varphi_2^*$ for heavy mesons due to Eq. (27), we have only to compare φ_1^* with φ in Eq. (29). In the heavy-mass limit, both should play equally important roles. But the ansatz strongly suppresses the role of vector mesons by a factor of $\sqrt{2}ef_\pi/M_{\Phi^*}$, since one obtains $F'(0) \sim -2ef_\pi$ in the Skyrme-term-stabilized soliton solution. For example, this factor amounts to 0.56, 0.25, and 0.09 for the cases of M_{K^*} (892 MeV), M_{D^*} (2010 MeV), and M_{B^*} (5325 MeV), respectively. Therefore, the ansatz of Eq. (28) is not valid unless the vector meson is

TABLE I. Summary on the input parameters and the numerical results on the bound state.

Q	f_π [a]	e [b]	M_Φ [a]	M_{Φ^*} [a]	f_Q [a]	g_Q [b]	ω_B [a]	$\sqrt{\langle r^2\rangle}$ [c]	c [b]	$\varphi_1^*(0)/\varphi(0)$
c	64.5	5.45	1872	2010	−3016	−0.75	1481	0.39	0.05	−0.828
b	64.5	5.45	5275	5325	−7988	−0.75	4722	0.29	0.02	−0.932

[a] In MeV unit.
[b] Dimensionless quantities.
[c] In fm unit.

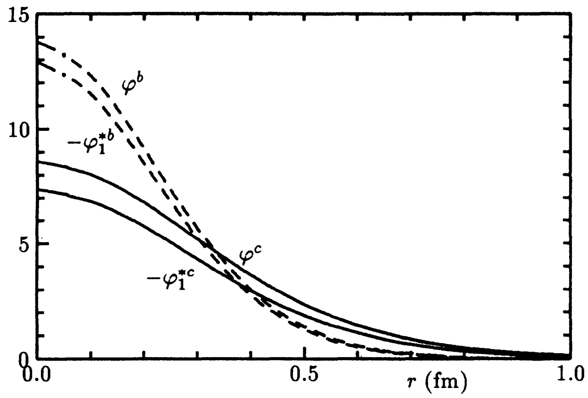

FIG. 1. $\varphi(r)$ and $\varphi_1^*(r)$ for $Q = c$ (solid) and b (dashed). $\varphi_2^*(r)$ is nearly equal to $\sqrt{2}\varphi_1^*(r)$ for both cases.

much heavier than the corresponding pseudoscalar meson.

The wave functions of Refs. [10,16] are obtained in the heavy-mass limit, $M_\Phi, M_{\Phi^*} \to \infty$ and can be written in our convention as

$$\begin{aligned} \Phi &\sim \frac{1}{2}\frac{1}{\sqrt{2M_{\Phi^*}}} f(r)\mathcal{Y}_{\frac{1}{2},+,\pm\frac{1}{2}}, \\ \Phi^* &\sim -\frac{1}{2}\frac{1}{\sqrt{2M_{\Phi^*}}} f(r)\left(\mathcal{Y}^{(1)}_{\frac{1}{2},+,\pm\frac{1}{2}} + \sqrt{2}\mathcal{Y}^{(2)}_{\frac{1}{2},+,\pm\frac{1}{2}}\right), \end{aligned} \tag{30}$$

where the radial function $f(r)$, normalized as $\int r^2 dr |f|^2 = 1$, is strongly peaked at the origin. It implies that

$$\varphi(r) = -\varphi_1^*(r) = -\frac{1}{\sqrt{2}}\varphi_2^*(r) \sim \tfrac{1}{2}\tfrac{1}{\sqrt{2M_{\Phi^*}}} f(r). \tag{30a}$$

These radial functions satisfy the normalization condition of Eq. (22) in the leading order in $1/m_Q$, viz.,

$$2\omega_B \int_0^\infty r^2 dr (|\varphi|^2 + |\varphi_1^*|^2 + |\varphi_2^*|^2) = 1. \tag{30b}$$

It is interesting to note that the pseudoscalar meson and three vector mesons contribute equally to the bound state.

Comparing our numerical results given in Table I with the binding energy $E_b = -\frac{3}{2}g_Q F'(0)$ of Refs. [10,16], which gives $\sim$ 800 MeV with the same input parameters, one can see that the $1/m_Q$ corrections amount to $\sim$ 200 MeV in the bottom sector and $\sim$ 300 MeV in the charm sector. This is one of the main results of this work.

In Ref. [10], the rms radii of the heavy flavor current in heavy baryons are essentially zero. Because of the $1/m_Q$ corrections, however, we have nonzero finite-size rms radii in our calculation, viz. $\sim$ 0.3 fm for bottom flavored baryons and $\sim$ 0.4 fm for charmed baryons. This implies that the rms radii of heavy flavored baryons become small as the masses become large. Because of this effect, the binding energy is smaller than the one obtained with δ-function-type solutions.

IV. HEAVY BARYONS AND HYPERFINE SPLITTINGS

So far we have considered soliton-heavy-meson bound states to the order N_c^0 with N_c being the number of color. The combined system of the soliton and a bound heavy meson carries a baryon number and a heavy flavor number, but does not have the spin and isospin of a heavy baryon. Up to order N_c^0, the soliton-heavy-meson bound state should be understood as a mixed state of three degenerate heavy baryons containing a heavy quark Q; Σ_Q, Λ_Q, and Σ_Q^*, whose mass is $M_{\rm sol} + \omega_B$. In order to give the spin and isospin quantum numbers and the hyperfine splittings, we have to go to the next order in $1/N_c$, i.e., $O(N_c^{-1})$. This is done by quantizing the zero modes associated with the simultaneous SU(2) rotation of the combined system. A standard collective coordinate quantization procedure leads us to the mass formula for a heavy baryon with spin J and isospin I:

$$\begin{aligned} M &= M_{\rm sol} + \omega_B \\ &\quad + \frac{1}{2\mathcal{I}}\left[cJ(J+1) + (1-c)I(I+1) + \frac{3}{4}c(c-1)\right] \\ &\quad + O(1/M^2). \end{aligned} \tag{31}$$

Here $\mathcal{I}$ is the moment of inertia of the soliton configuration against the SU(2) collective rotation:

$$\mathcal{I} = \frac{8\pi}{3}\int_0^\infty r^2 dr \sin^2 F\left[f_\pi^2 + \frac{1}{e^2}\left(F'^2 + \frac{\sin^2 F}{r^2}\right)\right], \tag{31a}$$

and c is the hyperfine splitting constant which can be obtained by directly applying the techniques developed in Ref. [18]:

$$\begin{aligned} c = \int_0^\infty r^2 dr \Bigg\{ & 2\omega_B\left[\left(|\varphi|^2 - \frac{1}{3}|\varphi_1^*|^2 - \frac{1}{3}|\varphi_2^*|^2\right) - \frac{4}{3}\cos^2(F/2)\left(|\varphi|^2 - |\varphi_1^*|^2\right)\right] \\ & + \frac{1}{3}g_Q\left[\left(F' - \frac{\sin 2F}{r}\right)|\varphi_2^*|^2 - \frac{1}{\sqrt{2}}\frac{\sin F}{r}(3\cos F + 1)(\varphi_1^{*\dagger}\varphi_2^* + \varphi_2^{*\dagger}\phi_1)\right]\Bigg\}. \end{aligned} \tag{31b}$$

We note that we have also kept terms of the next-to-leading order in $1/m_Q$. One can easily see that the hyperfine constant c is of order $1/m_Q$. The leading-order terms proportional to ω_B vanish identically when the radial functions of Eq. (30a) are used.

According to the formula, the masses of heavy baryons containing a single heavy quark have the following hyperfine splittings:

$$M_{\Sigma_Q^*} - M_{\Sigma_Q} = \frac{3}{2\mathcal{I}}\, c, \qquad M_{\Sigma_Q} - M_{\Lambda_Q} = \frac{1}{\mathcal{I}}\,(1-c). \tag{32}$$

By eliminating c from Eq. (32) we have a model-independent relation

$$\frac{1}{3}(2M_{\Sigma_Q^*} + M_{\Sigma_Q}) - M_{\Lambda_Q} = \frac{2}{3}(M_\Delta - M_N). \tag{33}$$

With the experimental values $M_{\Sigma_c}^{\rm expt}(= 2453$ MeV), $M_{\Lambda_c}^{\rm expt}(= 2285$ MeV), and $M_{\Lambda_b}^{\rm expt}(= 5641$ MeV), we predict the mass of Σ_c^* to be 2493 MeV and the averaged mass $\overline{M}_{\Sigma_b}$ $[\equiv \frac{1}{3}(2M_{\Sigma_b^*} + M_{\Sigma_b})]$ 5836 MeV. Since c is of order $1/m_Q$, the masses of Σ_Q and of Σ_Q^* are degenerate in the infinite mass limit as the heavy quark symmetry implies and Eq. (33) is reduced to $M_{\Sigma_Q} - M_{\Lambda_Q} = \frac{2}{3}(M_\Delta - M_N)$ as in Refs. [10–12].

Numerical results (Result I) on the heavy baryon masses are shown in Table II. They are in rough agreement with the experimental values. Result II is obtained by taking the two coupling constants as free parameters. To fit the experimental masses of Λ_c and Σ_c, one should have $f_Q/2M_{D^*} = -1.04$ and $g_Q = -0.40$, which implies that the heavy quark symmetric relation (4) is *strongly* broken in the charm sector. Note that as far as the two coupling constants are related by Eq. (4), the hyperfine constant is too small.

In order to improve the situation, one may consider higher-order terms in the $1/m_Q$ expansion or higher derivative terms of the pion fields. As a guide line, we may use the Skyrme Lagrangian [18] with the vector mesons included via the hidden gauge symmetry, since in the strangeness sector the heavy quark symmetry becomes no longer a good symmetry, but the SU(3) chiral symmetry becomes rather a good symmetry.

Among many possible terms which will be discussed below, the Wess-Zumino (WZ) term is known to play the most important role in CK approach [1]:

$$\mathcal{L}_{\rm WZ} = -\frac{iN_c}{4f_\Phi^2} B^\mu [\Phi^\dagger D_\mu \Phi - (D_\mu\Phi)^\dagger \Phi]. \tag{34}$$

Here, N_c is the number of color, f_Φ is the Φ-meson decay constant, and B_μ is the topological baryon number current. Although its role fades out in the heavy-mass limit, the WZ term should not be disregarded in the case of finite quark masses.

In addition, there are other contributions of order $1/m_Q$ to the binding potential. We introduce a typical $A\cdot A$ potential in the Lagrangian as in CK approach, which is next-to-leading order in the derivative of pion fields and turns out to have non-negligible effects in the strangeness sector:

$$\mathcal{L}_{(2)} = -\Phi^\dagger A_\mu A^\mu \Phi. \tag{35}$$

Now we discuss the effects of the above terms in detail. Let us write

$$\delta\mathcal{L} = \mathcal{L}_{\rm WZ} + \mathcal{L}_{(2)}. \tag{36}$$

This additional Lagrangian modifies the equations of motion for the pseudoscalar-meson field Φ as

$$(D_\mu D^\mu + M_\Phi^2)\Phi = f_Q A^\mu \Phi^*_\mu - \frac{2iN_c}{4f_\Phi^2} B_\mu D^\mu \Phi - A_\mu A^\mu \Phi, \tag{37}$$

while those for the vector-meson field Φ^* remain the same as Eq. (14). Consequently, the radial function of the $k^P = \frac{1}{2}^+$ eigenmodes is altered as

$$\varphi'' + \frac{2}{r}\varphi' + \left(\omega^2 - M_\Phi^2 - \frac{2}{r^2}\right)\varphi = \tfrac{1}{2} f_Q (a_1 + a_2)\phi_1^* - \tfrac{1}{\sqrt{2}} f_Q a_1 \phi_2^* - \left[2\omega\lambda - 2v\left(v - \frac{2}{r}\right) + \tfrac{1}{4}\left(3a_1^2 + 2a_1a_2 + a_2^2\right)\right]\varphi, \tag{38}$$

where

$$\lambda(r) = -\frac{N_c}{f_\Phi^2}\frac{1}{8\pi^2}\frac{\sin^2 F}{r^2} F'. \tag{39}$$

The WZ term contributes to the hyperfine splitting constant c as

$$\delta c = 2\int_0^\infty r^2 dr |\varphi|^2 \lambda, \tag{40}$$

and to the normalization condition of Eq. (22) by the same amount. Note that there is no direct contribution from $\mathcal{L}_{(2)}$ to this quantity.

TABLE II. Numerical results on the heavy baryon masses.

Q		$f_Q/2M_{\Phi^*}$	g_Q	ω_B[a]	c	M_{Λ_Q}[a]	M_{Σ_Q}[a]	$M_{\Sigma_Q^*}$[a]
	Expt[b]					2285	2453	—
c	I	−0.75	−0.75	1481	0.05	2348	2535	2548
	II	−1.04	−0.40	1419	0.14	2287	2454	2497
b	Expt[b]					5641	—	—
	I	−0.75	−0.75	4722	0.02	5589	5781	5786

[a]In MeV unit.

[b]Particle Data Group [20].

TABLE III. WZ term and strange baryon masses.

$\mathcal{L}_{WZ}$	$\mathcal{L}_{(2)}$	ω_B [a]	c	M_Λ [a]	M_Σ [a]	M_{Σ^*} [a]	$\sqrt{\langle r^2\rangle}$ [b]
off	off	389	0.098	1257	1433	1462	0.62
off	on	291	0.148	1160	1326	1369	0.56
on	off	191	0.717	1095	1151	1361	0.41
on	on	109	0.791	1022	1063	1295	0.39
Expt				1116	1192	1385	–

[a] In MeV unit.
[b] In fm unit.

We begin with the role of $\delta\mathcal{L}$ in the strangeness sector, where the above model Lagrangian does not work well as expected in this sector. In Table III, we show the numerical results for strange baryons obtained with the input parameters $f_\pi = 64.5$ MeV, $e = 5.45$, $M_K = 495$ MeV, $M_{K^*} = 892$ MeV, and $f_Q/2M_{K^*} = -\frac{1}{\sqrt{2}} = g_Q$. Note that the role of $\mathcal{L}_{(2)}$ in the binding energy is important (~ 80 MeV), while its effect on the hyperfine constant c is rather small. The Wess-Zumino term plays a crucial role in the hyperfine constant c, of which more than 80% comes from the WZ term. As shown in Fig. 2, the effect of the WZ term on the radial wave function is also remarkable; with the WZ term, the vector-meson contribution to the bound states is much suppressed compared with that of the pseudoscalar meson.

In the charm sector the role of the Wess-Zumino term in the heavy-mass limit is weakened as discussed in Ref. [11]. This results from the fact that the role of vector mesons balances that of pseudoscalar mesons in the heavy-mass limit. The decoupling of the WZ term in the heavy-mass limit is originally argued in Refs. [21,22]. In the chiral limit where pseudoscalar mesons predominate, the WZ term is entirely expressed in terms of these pseudoscalar mesons. To take into account the WZ term in the finite mass region, we consider the most characteristic expression of the WZ term [1,18] constructed in terms of pseudoscalar mesons and an adjustable parameter γ. The parameter γ contains the trace of cancellation between contributions of the vector and of the pseudoscalar mesons and should depend on $1/m_Q$. That is, we take

$$\delta\mathcal{L}' = \gamma\mathcal{L}_{WZ} + \epsilon\mathcal{L}_{(2)} \tag{41}$$

with the same $\mathcal{L}_{WZ}$ and $\mathcal{L}_{(2)}$ as given by Eqs. (34) and (35). The parameter ϵ has the role of turning on and off the effect of $\mathcal{L}_{(2)}$. Here, f_Φ is the D-meson decay constant f_D, which is known to be 1.8 times larger than the pion-decay constant f_π. Although the $\mathcal{L}_{(2)}$ plays a minor role for the heavy flavors such as charm, we keep it to compare its effects in the charm sector with those in the strangeness sector. In Fig. 3, we present ω_B and c as a function of the mitigating factor γ. The role of $\mathcal{L}_{(2)}$ is shown as narrow stripes, with ~ 30 MeV effect on the energy and ~ 0.04 on the hyperfine constant. However, as we can see in Fig. 3, the dependence of the mass spectrum on the parameter γ is not negligible. In order to fit the charmed baryon masses, we need to have $\omega_B = 1416$ MeV and $c = 0.16$. Then we have

$$\begin{aligned} M_{\Lambda_c} &= 2285 \text{ MeV}, \\ M_{\Sigma_c} &= 2449 \text{ MeV}, \\ M_{\Sigma_c^*} &= 2495 \text{ MeV}. \end{aligned} \tag{42}$$

Also from Fig. 3, one can estimate the mitigating factors as $\gamma \sim 0.25$ and $\epsilon = 1$, which reproduce the above mass spectra and then we have $\sqrt{\langle r^2\rangle_c} = 0.37$ fm. It implies that the increase in f_Φ alone is not enough to take fully into account the role of the Wess-Zumino term in the charm sector.

The dependence of γ on meson masses can be derived by showing how the Wess-Zumino term scales out as the mass increases. We are not in a position to illustrate

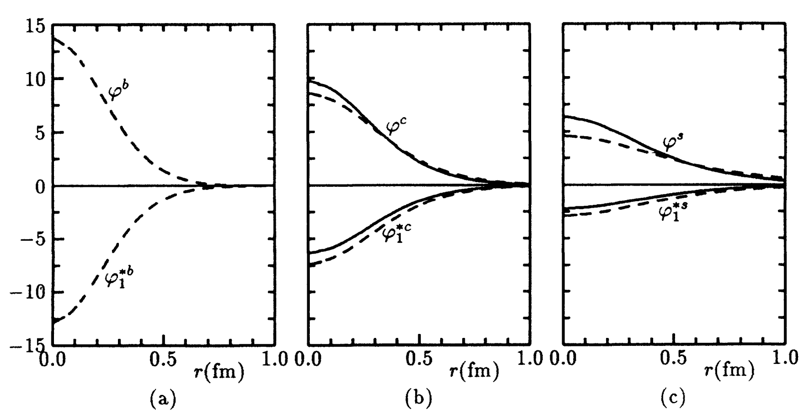

FIG. 2. $\varphi(r)$ and $\varphi_1^*(r)$ for (a) B and B^*, (b) D and D^*, (c) K and K^* with (solid) and without (dashed) the Wess-Zumino term. Each field is normalized as $\int dr r^2|\varphi|^2 = 1$.

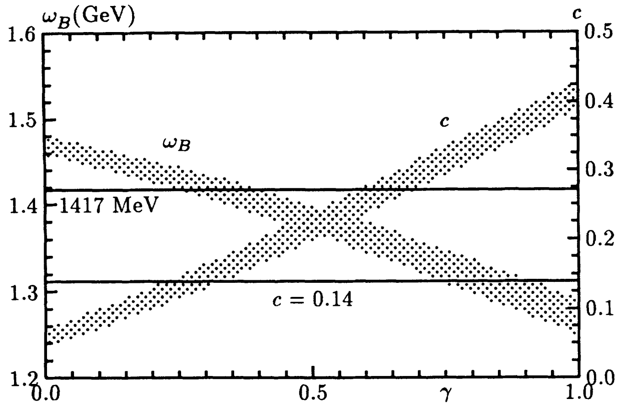

FIG. 3. ω_B and c vs γ obtained for charmed baryons.

this dependence yet. However, if we assume the dependence to be inversely proportional to the m_Φ, we find $\gamma \sim 0.5$ GeV$/m_\Phi$. Note the coincidence of the γ factor (~ 0.25) with the meson mass ratio m_K/m_D.

V. SUMMARY AND CONCLUSION

In this work, we have investigated the mass spectrum of heavy baryons containing a single heavy quark in the bound-state approach of the Skyrme model. To this end, we have worked with the heavy meson Lagrangian of Ref. [9] which includes the $1/m_Q$ order terms. The large binding energy obtained in the infinite mass limit is lowered by introducing $1/m_Q$ corrections. The binding energy is changed from $\sim$ 800 MeV to $\sim$ 500 MeV for $D(D^*)$ mesons and to $\sim$ 600 MeV for the $B(B^*)$. The effect may be crucial for the loosely bound exotic states such as "pentaquark" baryons [23,24]. However, due to the realization of the heavy quark symmetry, the hyperfine splitting constant comes out too small compared with the experimental one. For example, we get $c = 0.05$ for charmed baryons while it should be ~ 0.14 to reproduce the experimental masses. To resolve this problem, we introduce the WZ term in a mitigated form, which is known to have a crucial role in the strangeness sector. To reproduce the experimental masses for charmed baryons, its strength should be weakened by a factor of 4.

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[19] We combine first the spin basis and the orbital angular momentum basis to the total spin ($\mathbf{J} = \mathbf{S} + \mathbf{L}$) basis and then combine the isospin isospin. Then, $\mathcal{Y}^{(1)}_{\frac{1}{2},+,\pm\frac{1}{2}}(\hat{\mathbf{r}})$ and $\mathcal{Y}^{(2)}_{\frac{1}{2},+,\pm\frac{1}{2}}(\hat{\mathbf{r}})$ correspond to $J = 0$ and $J = 1$ states, respectively. One may obtain the vector spherical harmonics in other ways, for example, by combining first the isospin basis and the orbital angular momentum basis to $\mathbf{\Lambda}(= \mathbf{I} + \mathbf{L})$ basis and then the spin basis, which

leads us to

$$\mathcal{Y}'^{(1)}_{\frac{1}{2},+,\pm\frac{1}{2}}(\hat{\mathbf{r}}) = \frac{1}{\sqrt{12\pi}}\boldsymbol{\tau}\cdot\hat{\mathbf{r}}\boldsymbol{\tau}\chi_\pm = \sqrt{\frac{1}{3}}\mathcal{Y}^{(1)}_{\frac{1}{2},+,\pm\frac{1}{2}} + \sqrt{\frac{2}{3}}\mathcal{Y}^{(2)}_{\frac{1}{2},+,\pm\frac{1}{2}},$$

$$\mathcal{Y}'^{(2)}_{\frac{1}{2},+,\pm\frac{1}{2}}(\hat{\mathbf{r}}) = \frac{1}{\sqrt{24\pi}}(\boldsymbol{\tau}\cdot\hat{\mathbf{r}}\hat{\mathbf{r}} - 3\hat{\mathbf{r}})\chi_\pm = -\sqrt{\frac{2}{3}}\mathcal{Y}^{(1)}_{\frac{1}{2},+,\pm\frac{1}{2}} + \sqrt{\frac{1}{3}}\mathcal{Y}^{(2)}_{\frac{1}{2},+,\pm\frac{1}{2}}.$$

They are $\Lambda = \frac{1}{2}$ and $\Lambda = \frac{3}{2}$ states, respectively.
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