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QCD sum rules: Δ-*N* and Σ^0-Λ mass splittings

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We use the method of QCD sum rules to investigate both the Δ-*N* and Σ^0-Λ mass splittings. In the case of the Δ-*N* mass splitting, our numerical results indicate that the mass splitting is dominated primarily by the quark-gluon condensate $\langle 0|g_c\bar{q}\sigma_{\mu\nu}(\lambda^a/2)G_a^{\mu\nu}q|0\rangle$ and the observed mass splitting may be understood. In the case of Σ^0-Λ mass splitting, we obtain a value of about 66 MeV, which is slightly smaller than the observed value of 77 MeV.

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I. INTRODUCTION

The method of QCD sum rules [1] has now been recognized as a powerful tool in studying hadron physics on the basis of QCD. Among efforts by many different authors, we have recently considered the isovector and isoscalar axial coupling constants [2] and the isospin symmetry-breaking effect such as the neutron-proton mass difference [3]. In spite of the fact that QCD sum rules have been generally successful in studying baryon masses, discrepancies remain in certain problems. For instance, consider the Δ-*N* mass splitting. Some authors obtained the Δ mass sum rules taking into account only the chiral-symmetry-breaking (quark condensates) effect [4,5], contrary to the well-known result that, in a quark model, the Δ-*N* mass splitting is often attributed to the one-gluon exchange potential [6–8]. Belyaev and Ioffe [9] derived the sum rules up to dimension nine, but failed to understand the observed mass splitting. On the other hand, Dosch, Jamin, and Narison [10] have shown that if the quark-gluon condensate increases from 0.8 to 0.9 GeV^2, the Δ mass is 2% larger. Nevertheless, they obtained a mass difference of wrong sign in the case of $M_{\Sigma^0}-M_\Lambda$ [10].

Therefore, we have in the present paper adopted a systematic method, which we developed earlier [3], to simultaneously study the Δ-*N* and the Σ^0-Λ mass splittings. Our results are rather encouraging—not only could we understand the observed Δ-*N* mass splitting as coming primarily from the quark-gluon condensate, but we also reproduce the major portion of the observed Σ-Λ mass difference. We thus believe that the inconsistency in the existing literature is caused primarily by the fact that the relevant sum rules have not been considered consistently up to a certain dimension, and partly by the fact that these sum rules should be analyzed in a consistent manner.

The rest of this paper is organized as follows. In Sec. II we discuss the case of the Δ-*N* mass splitting while in Sec. III we treat the Σ^0-Λ mass difference. A brief summary is given in Sec. IV.

II. Δ-*N* MASS SPLITTING

To study the Δ-*N* mass splitting, we consider the two-point Green's function

$$\Pi^\Delta_{\mu\nu}(p)=i\int d^4x\, e^{ipx}\langle 0|T[\eta^\Delta_\mu(x)\bar{\eta}^\Delta_\nu(0)]|0\rangle \ , \tag{1}$$

where we may introduce [4]

$$\eta^\Delta_\mu=\epsilon^{abc}(u^{aT}C\gamma_\mu u^b)u^c \ . \tag{2}$$

Here the field η^Δ_μ may couple to particles of either $J=\frac{3}{2}$ or $\frac{1}{2}$:

$$\begin{aligned}\langle 0|\eta^\Delta_\mu(0)|\tfrac{3}{2}^+\rangle&=\lambda^+ v_\mu(p) \ ,\\ \langle 0|\eta^\Delta_\mu(0)|\tfrac{3}{2}^-\rangle&=\lambda^-\gamma_5 v_\mu(p) \ ,\end{aligned} \tag{3}$$

and

$$\begin{aligned}\langle 0|\eta^\Delta_\mu(0)|\tfrac{1}{2}^+\rangle&=(\alpha^+p_\mu+\beta^+\gamma_\mu)\gamma_5 v(p) \ ,\\ \langle 0|\eta^\Delta_\mu(0)|\tfrac{1}{2}^-\rangle&=(\alpha^-p_\mu+\beta^-\gamma_\mu)v(p) \ .\end{aligned} \tag{4}$$

Here v_μ is a vectorial spinor and satisfies $(\hat{p}-M_X)v_\mu=0$, $\bar{v}_\mu v^\mu=-2M_X$, and $\gamma_\mu v^\mu=p_\mu v^\mu=0$ (in the Rarita-Schwinger formalism). Moreover, because of $\gamma_\mu\eta^\mu_\Delta=0$ and $(\hat{p}-m)v=0$, we have $(\alpha^+\hat{p}+4\beta^+)\gamma_5 v=0$, i.e., $\alpha^+=4\beta^+/m^+$. Similarly we have $\alpha^-=-4\beta^-/m^-$.

At the hadronic level [the right-hand side (RHS)], we rewrite the Green's function via dispersion relation as

$$\Pi^\Delta_{\mu\nu}(p)=\frac{1}{\pi}\int_0^\infty \frac{\text{Im}\Pi^\Delta_{\mu\nu}(p')}{p'^2-p^2-i\varepsilon}dp'^2 \ , \tag{5}$$

where

$$\text{Im}\Pi^\Delta_{\mu\nu}(p')=\pi\sum_X \delta(p'^2-M_X^2)\langle 0|\eta^\Delta_\mu(0)|X\rangle\langle X|\bar{\eta}^\Delta_\nu(0)|0\rangle \ . \tag{6}$$

Using Eqs. (3) and (4), we obtain

$$\langle 0|\eta_\mu^\Delta(0)|\tfrac{3}{2}^\pm\rangle\langle\tfrac{3}{2}^\pm|\bar\eta_\nu^\Delta(0)|0\rangle$$
$$=-\lambda_\Delta^2\left[g_{\mu\nu}-\frac{1}{3}\gamma_\mu\gamma_\nu\mp\frac{p_\mu\gamma_\nu-p_\nu\gamma_\mu}{3m}-\frac{2p_\mu p_\nu}{3m^2}\right](\hat p\pm m)$$
$$=-\lambda_\Delta^2\left[g_{\mu\nu}\hat p-\frac{1}{3}\gamma_\mu\gamma_\nu\hat p+\frac{p_\nu\gamma_\mu-p_\mu\gamma_\nu}{3}-\frac{2p_\mu p_\nu\hat p}{3m^2}\pm m\left[g_{\mu\nu}-\frac{1}{3}\gamma_\mu\gamma_\nu+\frac{(p_\nu\gamma_\mu-p_\mu\gamma_\nu)\hat p}{3m^2}-\frac{2p_\mu p_\nu}{3m^2}\right]\right], \quad (7)$$

and

$$\langle 0|\eta_\mu^\Delta(0)|\tfrac{1}{2}^\pm\rangle\langle\tfrac{1}{2}^\pm|\bar\eta_\nu^\Delta(0)|0\rangle=-\beta^2\left[\gamma_\mu\gamma_\nu\hat p-(p_\nu\gamma_\mu-p_\mu\gamma_\nu)+3(p_\nu\gamma_\mu+p_\mu\gamma_\nu)-\frac{16p_\mu p_\nu\hat p}{m^2}\right]$$
$$\mp m\beta^2\left[\gamma_\mu\gamma_\nu-\frac{4(p_\nu\gamma_\mu-p_\mu\gamma_\nu)\hat p}{m^2}+\frac{8p_\mu p_\nu}{m^2}\right]. \quad (8)$$

At the quark level [the left-hand side (LHS)], we have, from Eqs. (1) and (2),

$$\langle 0|T[\eta_\mu^\Delta(x)\bar\eta_\nu^\Delta(0)]|0\rangle=-2i\epsilon^{abc}\epsilon^{a'b'c'}(\mathrm{Tr}\{S_u^{aa'}(x)\gamma_\nu C[S_u^{bb'}(x)]^TC\gamma_\mu\}S_u^{cc'}(x)+2S_u^{aa'}(x)\gamma_\nu C[S_u^{bb'}(x)]^TC\gamma_\mu S_u^{cc'}(x)). \quad (9)$$

Here we have used the definition,

$$iS^{ab}(x)\equiv\langle 0|T[q^a(x)\bar q^b(0)]|0\rangle,$$

as in Ref. [3]. In the present paper, we carry out the calculation at the quark level up to dimension eight, which turns out to be a necessity, as will be discussed later. The enumeration of the diagrams consistently to a certain order is already given in one of our earlier papers [3] and will not be repeated here. After performing the Borel transformation [1,3] and comparing the structures of both sides (both the quark level and hadron level), we obtain four sum rules for the Δ mass, according to the Lorentz structure (which is indicated in parentheses in front of each equation):

$$(g_{\mu\nu}\hat p):\quad \frac{M^6}{5}L^{4/27}E_2^{\{1\}}-\frac{5}{72}bM^2L^{4/27}E_0^{\{1\}}+\frac{4}{3}a_u^2L^{28/27}-\frac{7a_u^2m_0^2}{9M^2}L^{14/27}=\tilde\lambda_\Delta^2e^{(-M_\Delta^2/M^2)}, \quad (10)$$

$$(g_{\mu\nu}):\quad \frac{4}{3}a_uM^4L^{16/27}E_1^{\{2\}}-\frac{2}{3}a_um_0^2M^2L^{2/27}E_0^{\{2\}}-\frac{a_ub}{18}L^{16/27}=\tilde\lambda_\Delta^2M_\Delta e^{-(M_\Delta^2/M^2)}, \quad (11)$$

$$(p_\mu p_\nu\hat p):\quad \frac{3M^4}{20}L^{4/27}E_1^{\{3\}}+\frac{b}{48}L^{4/27}=\frac{\tilde\lambda_\Delta^2}{M_\Delta^2}e^{-(M_\Delta^2/M^2)}, \quad (12)$$

$$(p_\mu p_\nu):\quad a_uM^2L^{16/27}E_0^{\{4\}}-a_um_0^2L^{2/27}+\frac{a_ub}{6M^2}L^{16/27}=\frac{\tilde\lambda_\Delta^2}{M_\Delta}e^{-(M_\Delta^2/M^2)}, \quad (13)$$

where we have adopted the definition [3]

$$a_q\equiv-(2\pi)^2\langle\bar qq\rangle,\quad q=u\text{ or }d,$$
$$b\equiv\langle g_c^2G^2\rangle,$$
$$a_qm_0^2\equiv(2\pi)^2\langle g_c\bar q\sigma\cdot Gq\rangle, \quad (14)$$
$$\tilde\lambda_\Delta^2\equiv(2\pi)^4\lambda_\Delta^2,$$
$$E_n^{(m)}=1-e^{-x_{(m)}}\left[1+x_{(m)}+\cdots+\frac{1}{n!}x_{(m)}^n\right],$$
$$\text{with } x_{(m)}=W_{(m)}^2/M^2.$$

In what follows, we neglect the light (*up* and *down*) quark masses and do not distinguish a_u from a_d. Or we use $a_q=0.545$ GeV^3, $b=0.474$ GeV^4, and $m_0=0.8$ GeV^2 [3,2]. These values correspond to the choice of the QCD scale parameter $\Lambda=0.1$ GeV and the normalization point $\mu=0.5$ GeV. Note that there are four mass sum rules about the baryon of $I(J^P)=\frac{3}{2}(\frac{3}{2}^+)$. The first two are obtained from the structure $g_{\mu\nu}\hat p$ and $g_{\mu\nu}$ and are completely contributed by particles with $J=\frac{3}{2}$. The other two sum rules, ($p_\mu p_\nu\hat p$ and $p_\mu p_\nu$), are determined by particles with both $J=\frac{3}{2}$ and $\frac{1}{2}$.

We would like to note that we may also obtain two additional sum rules for baryons with $I(J^P)=\frac{3}{2}(\frac{1}{2}^\pm)$:

$$(\gamma_\mu p_\nu+\gamma_\nu p_\mu):\quad \frac{M^6}{240}L^{4/27}E_2+\frac{5}{1728}bM^2L^{4/27}E_0-\frac{1}{18}a_u^2L^{28/27}-\frac{7a_u^2m_0^2}{216M^2}L^{14/27}=\tilde\beta_\Delta^2e^{-(M_\Delta^2/M^2)}, \quad (15)$$

$$(\gamma_\mu\gamma_\nu+\tfrac{1}{3}g_{\mu\nu}):\quad \frac{1}{36}a_uM^4L^{16/27}E_1-\frac{1}{18}a_um_0^2M^2L^{2/27}E_0+\frac{a_ub}{108}L^{16/27}=\tilde\beta_{\Delta^+}^2M_{\Delta^+}e^{-(M_{\Delta^+}^2/M^2)}-\tilde\beta_{\Delta^-}^2M_{\Delta^0}e^{-(M_{\Delta^-}^2/M^2)}, \quad (16)$$

where $\tilde{\beta}_\Delta^2=(2\pi)^4\beta^2$. These sum rules will not be investigated in this paper. Note that the anomalous dimensions which we have already included in Eqs. (10)–(13) and (15)–(16) are described in our earlier paper [3], except that the anomalous dimension [11] of η_Δ is $-\frac{2}{27}$.

To study the Δ-N mass difference, we recall [3] the two well-known mass sum rules for the nucleon:

$$(\hat{p}):\quad \frac{M^6}{8}L^{-4/9}E_2+\frac{bM^2}{32}L^{-4/9}E_0+\frac{a_u^2}{6}L^{4/9}-\frac{a_u^2m_0^2}{24M^2}L^{-2/27}=\beta_p^2e^{-(M_p^2/M^2)}, \quad (17)$$

$$(1):\quad \frac{a_dM^4}{4}E_1-\frac{a_db}{72}=\beta_p^2M_pe^{-(M_p^2/M^2)}, \quad (18)$$

which follow from a choice of the current [3,12]:

$$\eta_N=\epsilon^{abc}(u^{aT}C\gamma_\mu u^b)\gamma_5\gamma^\mu d^c . \quad (19)$$

In our numerical analysis, we obtain a reliable estimate of the threshold $W_{(m)}$ for the continuum making use of the method developed by Belyaev and Ioffe [9] (which was also used in Ref. [3]) and choose to perform the optimization in a Borel mass range, in which the contribution from the continuum is less than 50% while at the quark level the highest dimension correction is no more than 15%.

In Fig. 1, we plot the contributions, in percentage of the total, from the continuum and highest dimension as a function of the square of the Borel mass M^2. These corrections are defined through Eqs. (10) and (17). In the case of Eq. (10) (for Δ), the solid curve is the contribution from the continuum while the dashed curve is the contribution of the highest dimension at the quark level. In the case of Eq. (17) (for the nucleon), the curve represented by triangles is the contribution from the continuum while the dotted curve represents the contribution of the highest dimension.

Along the same line, we plot in Fig. 2 the contributions from the continuum and highest dimension as a function of the square of the Borel mass M^2, where all the corrections are defined through Eqs. (11) and (18).

In this way, we may analyze the Δ mass sum rules with the structures $g_{\mu\nu}\hat{p}$ and $g_{\mu\nu}$, and obtain $\tilde{\lambda}_\Delta^2=2.58$ GeV^6, $M_\Delta=1.38$ GeV, $W_{\{1\}\Delta}^2=3.55$ GeV^2, and $W_{\{2\}\Delta}^2=3.40$ GeV^2. The relevant Borel mass range is 1.44 GeV$^2<M^2<1.69$ GeV^2. Our results differ slightly from those obtained by Belyaev and Ioffe [9], especially on $W_{\{1,2\}}^2$; this may be caused by some errors in the anomalous dimensions quoted in their formulas.

Note that, in our studies, the choice of the Borel mass range depends on how many terms which we have kept at the quark level. Such choice is not arbitrary. For instance, it is necessary to keep the dimension up to eight at the quark level in the case of Δ; otherwise, the highest-dimension term will be so large in the series that a suitable Borel range cannot be found.

Because of similar structures between Eqs. (10) and (17), we choose to use the two equations to analyze properties of the mass splitting. In the analysis, we take the logarithm of both sides of Eqs. (10) and (17), and then apply the differential operator $M^4\partial/\partial M^2$ to both sides [3]. In Fig. 3, the $M_\Delta-M_N$ is plotted as a function of the square of the Borel mass M^2. The solid curve is the result obtained by using the common parameters indicated earlier. The long-dashed curve is obtained by assuming $m_0^2=0$, i.e., $\langle g_c\bar{q}\sigma Gq\rangle=0$, while the short-dashed curve is obtained by assuming $\langle g_c^2G^2\rangle=0$.

Comparing the difference between the solid curve and the dashed curve, we find that the size of the effect due to the quark-gluon condensate on the Δ-N mass splitting is 215±65 MeV, which is much larger than that obtained by Ioffe [4] (≈20 MeV). Analogously, we may compare the solid curve with the short-dashed curve and conclude

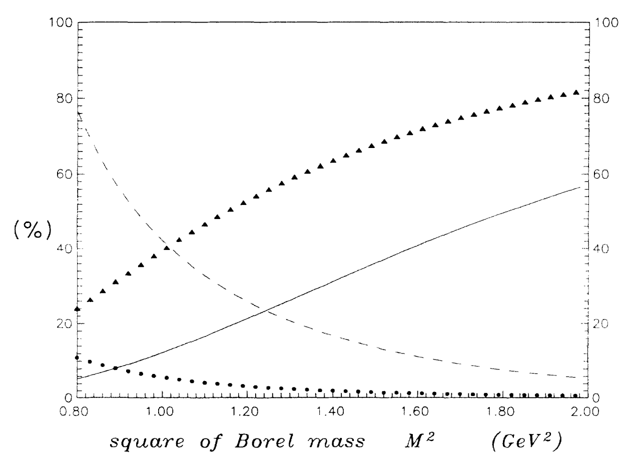

FIG. 1. The contributions from the continuum and highest dimension are plotted as a function of the square of the Borel mass M^2. These corrections are defined through Eqs. (10) and (17). In the case of Eq. (10) (for Δ), the solid curve is the contribution from the continuum while the dashed curve is the contribution of the highest dimension at the quark level. In the case of Eq. (17) (for the nucleon), the curve represented by triangles is the contribution from the continuum while the dotted curve represents the contribution of the highest dimension.

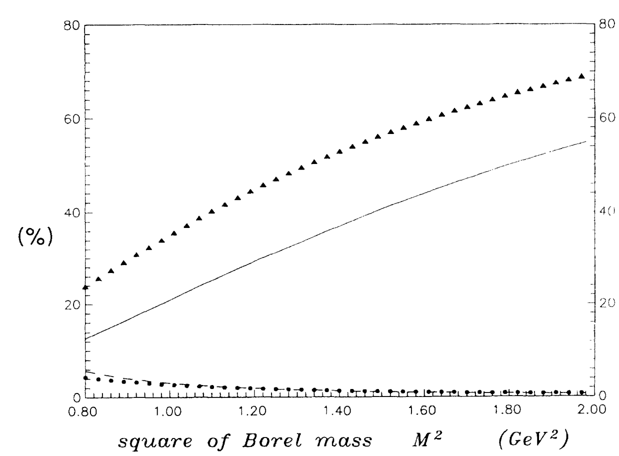

FIG. 2. The contributions from the continuum and highest dimension are plotted as a function of the square of the Borel mass M^2, while all the corrections are defined through Eqs. (11) and (18). See Fig. 1, for captions.

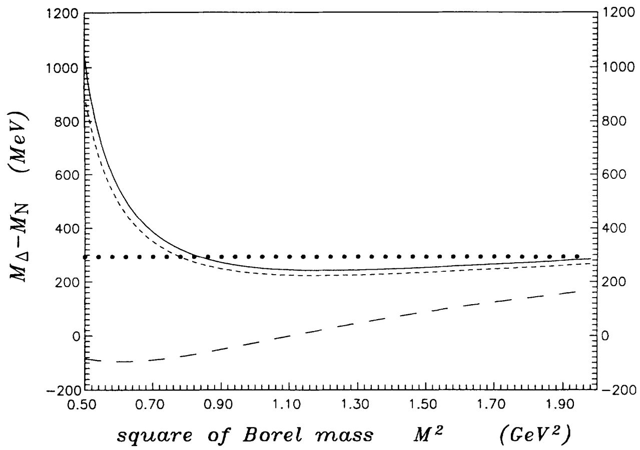

FIG. 3. The M_Δ-M_N mass splitting is plotted as a function of the square of the Borel mass M^2, obtained on the basis of Eqs. (10) and (17). The solid curve is the result obtained by using the common parameters indicated in the text. The long-dashed curve is obtained by assuming $m_0^2=0$, i.e., $\langle g_c\bar{q}\sigma Gq\rangle=0$, while the short-dashed curve is obtained by assuming $\langle g_c^2G^2\rangle=0$. In solid dots is the experimental value of the mass splitting.

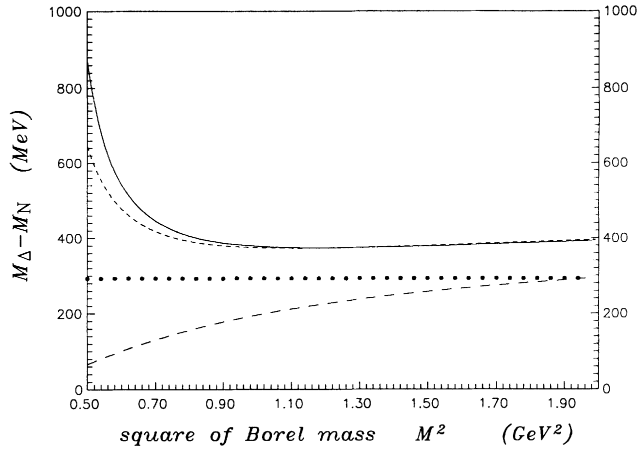

FIG. 4. The M_Δ-M_N mass splitting is plotted as a function of the square of the Borel mass M^2, obtained on the basis of Eqs. (11) and (18). See Fig. 3.

that the size of the effect due to the gluon condensate is small, only about 20 MeV.

Along the same line, we study properties of the mass splitting making use of Eqs. (11) and (18). In Fig. 4, we plot the results as in Fig. 3. Here we find that the contribution due to the quark-gluon condensate $\langle g_c\bar{q}\sigma Gq\rangle$ on the Δ-N mass splitting is 145±45 MeV while the contribution due to the gluon condensate vanishes. These results are consistent with those given in Fig. 3, taking into consideration that terms of higher dimensions are not included in Eqs. (11) and (18).

III. Σ^0-Λ MASS SPLITTING

The field operators relevant for the derivation of the Σ^0 and Λ mass sum rules are given by [13]

$$\begin{aligned}\eta_{\Sigma^0}&=\epsilon^{abc}\frac{1}{\sqrt{2}}[(u^{aT}C\gamma_\mu d^b)\gamma_5\gamma^\mu s^c\\&\quad+(d^{aT}C\gamma_\mu u^b)\gamma_5\gamma^\mu s^c]\ ,\\ \eta_\Lambda&=\epsilon^{abc}(\tfrac{2}{3})^{1/2}[(u^{aT}C\gamma_\mu s^b)\gamma_5\gamma^\mu d^c\\&\quad-(d^{aT}C\gamma_\mu s^b)\gamma_5\gamma^\mu u^c]\ .\end{aligned}\tag{20}$$

The derivation of the relevant QCD sum rules is similar to our earlier paper [3] and it is sufficient to record only the final results. For the Σ^0, we have the QCD mass sum rules

$$\begin{aligned}&\frac{M^6}{8}L^{-4/9}E_2+\frac{bM^2}{32}L^{-4/9}E_0+\frac{a_ua_d}{6}L^{4/9}-\frac{a_ua_dm_0^2}{24M^2}L^{-2/27}-\frac{[m_sa_s-(a_u-a_d)(m_d-m_u)]M^2}{4}L^{-4/9}E_0\\&\qquad-\frac{m_sa_sm_0'^2}{24}L^{-26/27}-\frac{[a_d(3m_u-m_d)+a_u(3m_d-m_u)]m_0^2}{48}L^{-26/27}=\beta_{\Sigma^0}^2e^{-(M_{\Sigma^0}^2/M^2)}\ ,\end{aligned}\tag{21}$$

and

$$\begin{aligned}&\frac{a_sM^4}{4}E_1-\frac{a_sb}{72}+\frac{\alpha_s}{\pi}\frac{[-(a_u^2+a_d^2)+36a_ua_d]a_s}{81M^2}L^{-1/9}+\frac{m_sM^6}{4}L^{-8/9}E_2-\frac{m_sbM^2}{32}L^{-8/9}E_0+\frac{m_sa_ua_d}{3}\\&\qquad+\frac{a_s(4m_ua_d+4m_da_u-m_ua_u-m_da_d)}{12}=\beta_{\Sigma^0}^2M_{\Sigma^0}e^{-(M_{\Sigma^0}^2/M^2)}\ .\end{aligned}\tag{22}$$

For the Λ, we have the QCD mass sum rules

$$\begin{aligned}&\frac{M^6}{8}L^{-4/9}E_2+\frac{bM^2}{32}L^{-4/9}E_0+\frac{2a_s(a_u+a_d)-a_ua_d}{18}L^{4/9}-\frac{(a_u+a_d)a_s(m_0'^2+m_0^2)-a_ua_dm_0^2}{72M^2}L^{-2/27}\\&\quad-\frac{M^2}{12}L^{-4/9}E_0m_s[3a_s-2(a_u+a_d)]-\frac{M^2}{12}L^{-4/9}E_0[3(m_ua_u+m_da_d)+m_ua_d+m_da_u-2(m_u+m_d)a_s]\\&\qquad-\frac{L^{-26/27}}{24}(m_u+m_d-m_s)a_sm_0'^2-\frac{L^{-26/27}}{48}[2m_s(a_u+a_d)m_0^2-(a_u-a_d)(m_d-m_u)m_0^2]\\&\qquad=\beta_{\Lambda^0}^2e^{-(M_{\Lambda^0}^2/M^2)}\ ,\end{aligned}\tag{23}$$

and

$$\frac{(2a_u+2a_d-a_s)M^4}{12}E_1-\frac{(2a_u+2a_d-a_s)b}{216}+\frac{\alpha_s}{\pi}\frac{L^{-1/9}}{243M^2}[108a_ua_da_s+a_s(a_u^2+a_d^2)-2(a_ua_d+a_s^2)(a_u+a_d)]$$
$$+\frac{M^6}{12}L^{-8/9}E_2(2m_u+2m_d-m_s)-\frac{bM^2}{96}L^{-8/9}E_0(2m_u+2m_d-m_s)+\frac{1}{36}[12m_sa_ua_d-2m_sa_s(a_u+a_d)]$$
$$+\frac{1}{36}[12a_s(m_ua_d+m_da_u)+a_s(m_ua_u+m_da_d)-2(m_u+m_d)a_ua_d]=\beta^2_{\Lambda^0}M_{\Lambda^0}e^{-(M^2_{\Lambda_0}/M^2)} . \quad (24)$$

Note that we obtain these sum rules by keeping terms of up to dimension nine, as in Ref. [3]. Also note that the electromagnetic correction is trivial in the case of the Σ^0-Λ mass difference and for the purpose of the present paper it can be neglected.

In the following numerical analysis, we keep all light current quark masses, $m_u=5.1$ MeV, $m_d=8.9$ MeV, and $m_s=175$ MeV [15,3], and distinguish $\langle\bar{d}d\rangle$ from $\langle\bar{u}u\rangle$ [3],

$$\gamma=(\langle\bar{d}d\rangle/\langle\bar{u}u\rangle)-1=-0.006\,57 .$$

We also introduce

$$a_s=-(2\pi)^2\langle\bar{s}s\rangle ,$$
$$a_sm_0'^2\equiv(2\pi)^2\langle g_c\bar{s}\sigma Gs\rangle , \quad (25)$$

and take the value [5,13,16]

$$\langle\bar{s}s\rangle=0.8(\langle\bar{u}u\rangle+\langle\bar{d}d\rangle)/2 .$$

We also use the relation

$$\langle g_c\bar{s}\sigma Gs\rangle/\langle g_c\bar{u}\sigma Gu\rangle\simeq\langle\bar{s}s\rangle/\langle\bar{u}u\rangle ,$$

i.e., $m_0'^2\simeq m_0^2$, which has been examined in some detail in octet baryons [16]. In Ref. [16], the absolute value of the quark-gluon condensate $\langle g_c\bar{\Psi}\sigma G\Psi\rangle$ has been determined by QCD sum rules, and it decreases as the quark mass increases (at least in the region $m_q\le m_s$). Here we accept this result as a reasonable approximation.

In Figs. 5 and 6, we plot, again in percentage of the total, the contributions of the continuum (at the hadron level) and highest dimension (at the quark-gluon level) for Σ^0 and Λ. From Figs. 5 and 6, we obtain the suitable range of Borel mass to be 0.9 GeV$^2<M^2<1.3$ GeV^2 for both of the particles, Σ^0 and Λ. Following the method of the last section, we may obtain the properties of Σ^0 and Λ as follows:

$$\beta^2_\Lambda=0.41\text{ GeV}^6,\quad \beta^2_{\Sigma^0}=0.47\text{ GeV}^6 ,$$
$$M_\Lambda=1.118\text{ GeV},\quad M_{\Sigma_0}=1.184\text{ GeV} , \quad (26)$$
$$W^2_\Lambda=3.06\text{ GeV}^2,\quad W^2_{\Sigma^0}=3.47\text{ GeV}^2 .$$

We may proceed to analyze the Σ^0-Λ mass splitting using Eqs. (21) and (23) as the basis. Again, we take the logarithm of both sides of Eqs. (21) and (23), then apply the differential operator $M^4\partial/\partial M^2$ to both sides [3], just as in the last section. The results are depicted in Fig. 7. The standard result is shown as a solid curve, which yields $M_{\Sigma^0}-M_\Lambda=65\pm15$ MeV in the suitable Borel range of 0.9 GeV$^2<M^2<1.3$ GeV^2 (as from Fig. 5). Thus, the observed value of 77 MeV can be reasonably understood.

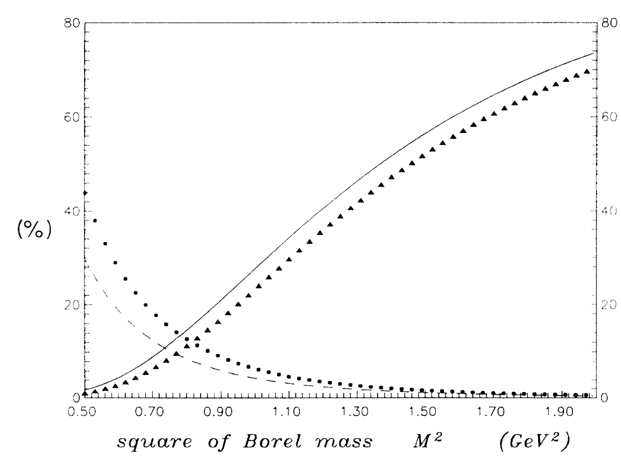

FIG. 5. The contributions from the continuum and highest dimension are plotted as a function of the square of the Borel mass M^2. These corrections are defined through Eq. (21) and Eq. (23). In the case of Eq. (21) (for Σ^0), the solid curve is the contribution from the continuum while the dashed curve is the contribution of the highest dimension at the quark level. In the case of Eq. (23) (for Λ), the curve represented by triangles is the contribution from the continuum while the dotted curve represents the contribution of the highest dimension.

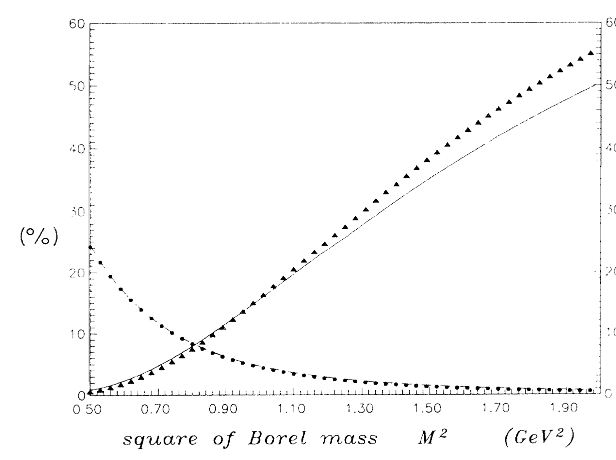

FIG. 6. The contributions from the continuum and highest dimension are plotted as a function of the square of the Borel mass M^2, where all the corrections are defined through Eq. (22) and Eq. (24). See Fig. 5.

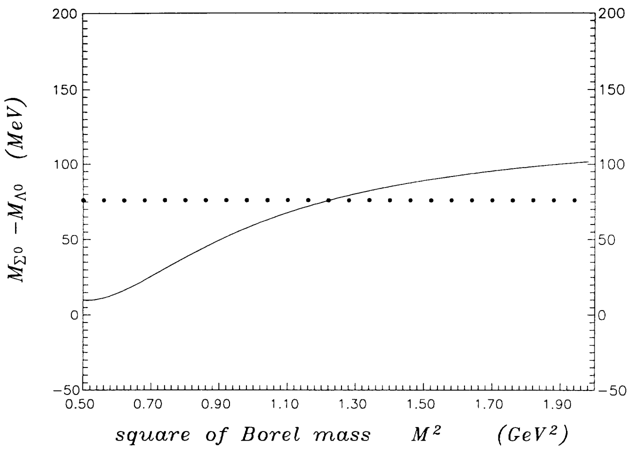

FIG. 7. The M_Σ-M_Λ mass splitting is plotted as a function of the square of the Borel mass M^2, obtained on the basis of Eqs. (21) and (23). The solid curve is the result obtained by using the parameters indicated in the text.

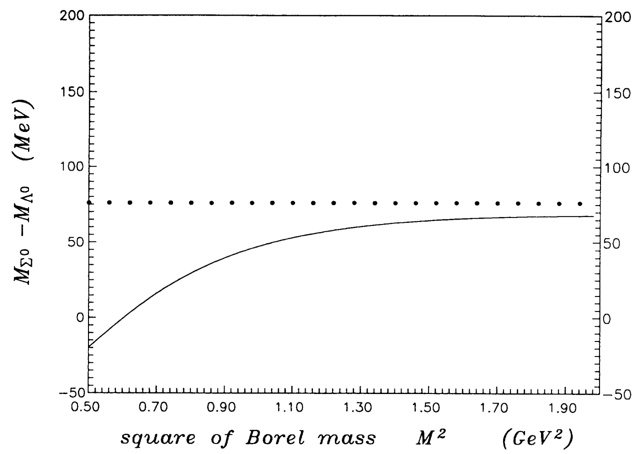

FIG. 8. The M_Σ-M_Λ mass splitting is plotted as a function of the square of the Borel mass M^2, obtained on the basis of Eqs. (22) and (24). See Fig. 7.

The same consideration may be applied to Eqs. (22) and (24). The results are shown in Fig. 8, which yields 54±14 MeV on the Σ^0-Λ mass difference in a fairly wide Borel range of 0.9–1.9 GeV^2. Note that Figs. 7 and 8 are consistent among themselves [3], yielding a theoretic estimate of about 66 MeV on the M_{Σ^0}-M_Λ mass splitting [which is slightly smaller than the experimental value 77 MeV; the reason might be that we underestimate (and/or overestimate) the value(s) of m_s (and/or $|\langle \bar{s}s \rangle|$)].

IV. SUMMARY

In this paper, we have used the method of QCD sum rules to investigate both the Δ-*N* and Σ^0-Λ mass splittings. In the case of the Δ-*N* mass splitting, we find that the mass splitting is dominated primarily by the quark-gluon condensate

$$\left\langle 0 \left| g_c \bar{q} \sigma_{\mu\nu} \frac{\lambda^a}{2} G_a^{\mu\nu} q \right| 0 \right\rangle ,$$

which gives rise to a contribution of 215±65 MeV [Eqs. (10) and (17)] or 145±45 MeV [Eqs. (11) and (18)], and the overall observed mass splitting may be reasonably understood. In the case of Σ^0-Λ mass splitting, we have obtained a value of about 66 MeV, which is slightly smaller than the observed value of 77 MeV.

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