Radiative corrections to $H^i \to t\bar{t}$ in the minimal supersymmetric model

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We compute the $O(\alpha m_t^2/m_W^2)$ corrections to $H^i \to t\bar{t}$ in the minimal supersymmetric standard model. The analytic expressions of such corrections to the decay rates are given. Numerical examples are presented, which show that the corrections to the rates of $H, A \to t\bar{t}$ typically imply a few percent reduction in the widths.

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I. INTRODUCTION

The standard model (SM) has been remarkably successful. Nevertheless, there are a number of unsolved theoretical puzzles which suggest that new physics beyond the standard model must exist at an energy scale of a few TeV or below. By far the most intensively studied class of theories as a possible candidate for new physics beyond the SM is supersymmetry (SUSY), especially the minimal supersymmetric extension of the standard model (MSSM) [1] in which two Higgs doublets are necessary, giving masses separately to up- and down-type fermions and assuring cancellation of anomalies. Three neutral and charged Higgs bosons $H, h, A, H^{\pm}$, of which H and h are CP even and A is CP odd are introduced by this extension of the Higgs sector. However, the four masses m_h, m_H, m_A, and $m_{H^\pm}$ are not independent. At the tree level, because of the restrictions imposed by SUSY, several relations between the Higgs-boson masses and couplings exist [2] and the Higgs sector contains only two free input parameters, conveniently chosen to be m_A and $\tan\beta = v_1/v_2$, so that the other three masses are calculable in terms of them. These relations impose a strong hierarchical structure on the mass spectrum [$m_h < m_Z$, $m_A < m_H$, and $m_W < m_{H^\pm}$]. However, the large radiative corrections to the Higgs-boson masses due to a heavy top quark [3] may alter this situation significantly; these corrections grow as the fourth power of the top quark mass and can shift the upper limit of m_h from m_Z to ~ 140 GeV. Therefore, the search for neutral and charged Higgs bosons has a very high priority in the experimental program of present and future colliders. In order to facilitate such experimental searches, it is necessary to study not only the production mechanisms, but also all possible decay modes of the charged and neutral Higgs bosons.

In the SM, as analyzed by Zerwas [4], up to masses of 140 GeV, the Higgs particle is very narrow: $\Gamma(H) < 10$ MeV. After opening the gauge boson channels, the state becomes rapidly wider, reaching ~ 1 GeV at the ZZ threshold. The width cannot be measured directly in the intermediate mass range. Only above $M_H > 200$ GeV does it become wide enough to be resolved experimentally. Moreover, it is pointed out in Ref. [5] that if the SM Higgs-boson mass M_H exceeds the $t\bar{t}$ threshold, then the $Ht\bar{t}$ coupling may be experimentally accessible at the CERN Large Hadron Collider (LHC) and the Superconducting Super Collider (SSC) if $B(H \to t\bar{t}) > 10^{-2}$, where $B(H \to t\bar{t}) \simeq \Gamma(H \to t\bar{t})/\Gamma(H \to t\bar{t}, ZZ, W^+W^-)$. The analysis of Ref. [5] has shown that this branching ratio can be as large as 30% at $m_t \sim M_H/3$, clearly warranting further study of this mode.

In the MSSM, the widths of Higgs bosons H and A can both be up to a few GeV for large Higgs-boson masses [6]. If the Higgs bosons are heavy enough, such as $m_H, m_A > 2m_t$, the decay channels $H, A \to t\bar{t}$ open up. Since the $Ht\bar{t}$ and $At\bar{t}$ couplings are proportional to $1/\sin\beta$ and $1/\tan\beta$, respectively, such decay modes are suppressed for large $\tan\beta$ and enhanced for small $\tan\beta$. For large Higgs-boson and top masses and small $\tan\beta$, $H, A \to t\bar{t}$ are the dominant decay modes. For example, with $\tan\beta = 1.5$, $m_t = 140$ GeV, and $M_{H,A} > 300$ GeV, the branching ratios for $H, A \to t\bar{t}$ are almost $\sim 100\%$ [6]. Thus, like the SM case analyzed in Ref. [5], these couplings may be experimentally accessible at the LHC and SSC. If such a decay mode is detected, we have to distinguish between the SM and the MSSM. Therefore, precise calculations are needed for the $Ht\bar{t}$ couplings both in the SM and in the MSSM, and the radiative corrections to

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such couplings are important to theoretical predictions for the width and the branching fraction of Higgs bosons, although it may be difficult to detect these corrections at future colliders, depending on the measuring accuracy. The radiative corrections to $H\to ZZ, W^+W^-$ in the MSSM have been calculated [7,8], and the one-loop SUSY QCD corrections to $H^i(H,A,h)\to t\bar{t}$ were given in Ref. [9]. Also, Mendez and Pomarol [7] calculated the $O(\alpha m_t^2/m_W^2)$ corrections to the couplings $H^it\bar{t}$, where they worked in the two Higgs doublet model (2HDM) and finally extended their work to the MSSM. But they did not take into account the virtual effects of genuine SUSY particles as well as the vertex corrections. Because of the importance of the decay channels $H, A\to t\bar{t}$ in the case of large Higgs-boson masses and small $\tan\beta$, in this paper we present the complete calculation of $O(\alpha m_t^2/m_W^2)$ corrections to them in the MSSM to complement the previous work. Such corrections arise from the virtual effects of the third family (top and bottom) quarks and squarks, charginos, and neutralinos, charged and neutral Higgs bosons, as well as the Goldstone bosons.

The structure of this paper is as follows. In Sec. II we give the analytic results in terms of the well-known standard notation of one-loop integrals. In Sec. III we present some numerical examples with a brief discussion. The lengthy expression for the vertex corrections is given in Appendix A, and the factors we use in the analytic results are summarized in Appendix B.

II. CALCULATIONS

We perform the calculation in the 't Hooft–Feynman gauge and use dimensional regularization to regulate all the ultraviolet divergences in the virtual loop corrections utilizing the on-mass-shell renormalization scheme [10], in which the fine-structure constant α and the physical masses are chosen to be the renormalized parameters, and the finite parts of the counterterms are fixed by the renormalization conditions. As far as the parameters β and α, for the MSSM we are considering, they have to be renormalized, too. In the MSSM they are not independent. Nevertheless, we follow the approach of Mendez and Pomarol [7] in which we consider them as independent renormalized parameters and fix the corresponding renormalization constants by a renormalization condition that the on-mass shell $H^+\bar{l}\nu_l$ and h^0ll couplings keep the forms of Eq. (3) of Ref. [7] to all orders of perturbation theory. For simplicity we consider the case of unmixed squarks, i.e., the mixing angle between left- and right-handed squarks $\theta=0$, and we assume the different mass eigenstates of squarks have the same mass value.

The relevant Feynman diagrams are shown in Fig. 1. The renormalized amplitudes can be expressed as

$$M_{\rm ren}(H\to t\bar{t})=M_0(H\to t\bar{t})(1+\Lambda_{\rm ct}^H+\delta\Gamma^H) \qquad (1)$$

and

$$M_{\rm ren}(A\to t\bar{t})=M_0(A\to t\bar{t})(1+\Lambda_{\rm ct}^A+\delta\Gamma^A)\;, \qquad (2)$$

respectively. Here, $\delta\Gamma^{H,A}$ are the vertex corrections and $\Lambda_{\rm ct}^{H,A}$ are the counterterms whose explicit expressions are given by

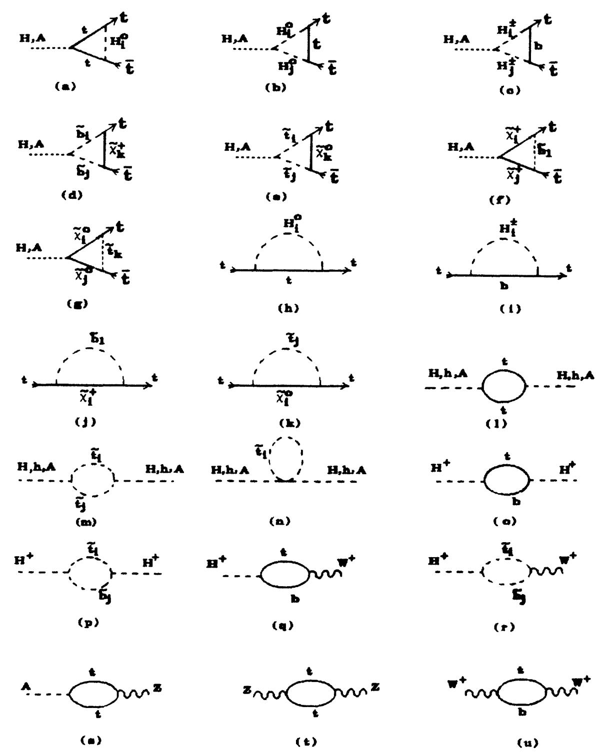

FIG. 1. Feynman diagrams contributing to $O(\alpha m_t^2/m_W^2)$ corrections in the MSSM: (a)–(g) vertex diagrams; (h)–(u) self-energy diagrams. The H_i^0 represent H, h, A, and G^0. The $H_i^\pm$ represent $H^\pm$ and $G^\pm$.

$$\begin{aligned}\Lambda_{\rm ct}^H=&\frac{\delta e}{e}+\frac{\delta m_Z^2}{2m_Z^2}-\frac{\delta m_W^2}{2m_W^2}+\frac{\delta m_W^2-\delta m_Z^2}{2(m_Z^2-m_W^2)}\\&+\tfrac{1}{2}(\delta Z_{H^+}+\delta Z_H-\delta Z_h)+m_W\cot\beta Z_{HW}^{1/2}\\&+\cot\alpha(Z_{Hh}^{1/2}+Z_{hH}^{1/2})+\frac{\delta m_t}{m_t}+\tfrac{1}{2}\delta Z_t^L+\tfrac{1}{2}\delta Z_t^R\;,\end{aligned} \qquad (3)$$

$$\begin{aligned}\Lambda_{\rm ct}^A=&\frac{\delta e}{e}+\frac{\delta m_Z^2}{m_Z^2}-\frac{\delta m_W^2}{m_W^2}+\frac{\delta m_W^2-\delta m_Z^2}{m_Z^2-m_W^2}\\&+\tfrac{1}{2}(\delta Z_{H^+}+\delta Z_A)+m_W\cot\beta Z_{HW}^{1/2}-m_Z\tan\beta Z_{ZA}^{1/2}\\&+\frac{\delta m_t}{m_t}+\tfrac{1}{2}\delta Z_t^L+\tfrac{1}{2}\delta Z_t^R\;,\end{aligned} \qquad (4)$$

where the renormalization constants are defined as

$$\begin{aligned}&e_0=e+\delta e\;,\quad m_{W0}^2=m_W^2+\delta m_W^2\;,\\&m_{Z0}^2=m_Z^2+\delta m_Z^2\;,\quad m_t^0=m_t+\delta m_t\;,\\&\tan\beta_0=(1+\delta Z_\beta)\tan\beta\;,\quad \sin\alpha_0=(1+\delta Z_\alpha)\sin\alpha\;,\\&H_0=(1+\delta Z_H)^{1/2}H+Z_{Hh}^{1/2}h\;,\\&h_0=(1+\delta Z_h)^{1/2}h+Z_{hH}^{1/2}H\;,\\&A_0=(1+\delta Z_A)^{1/2}\;,\quad H_0^\pm=(1+\delta Z_{H^+})^{1/2}H^\pm\;,\\&W_0^{\pm\mu}=Z_W^{1/2}W^{\pm\mu}+iZ_{HW}^{1/2}\partial^\mu H^\pm\;,\\&Z_0^\mu=Z_Z^{1/2}Z^\mu+Z_{ZA}^{1/2}\partial^\mu A\;,\\&\psi_{t0}=(1+\delta Z_t^RP_R+\delta Z_t^LP_L)^{1/2}\psi_t\;.\end{aligned} \qquad (5)$$

The renormalized decay rates are then obtained from

$$\Gamma_{\rm ren}^{H,A}(\alpha)=\Gamma_0^{H,A}(\alpha)(1+2\,{\rm Re}\Lambda_{\rm ct}^{H,A}+2\,{\rm Re}\delta\Gamma^{H,A}) , \tag{6a}$$

in the so-called α scheme, and

$$\Gamma_{\rm ren}^{H,A}(G_F)=\Gamma_0^{H,A}(G_F)(1+2\,{\rm Re}\Lambda_{\rm ct}^{H,A}+2\,{\rm Re}\delta\Gamma^{H,A}-\Delta r) , \tag{6b}$$

in the so-called G_F scheme, respectively. Here Δr depends on all the parameters of the model, especially on the mass of the top quark. Since the additional contributions to Δr in the MSSM contain no $O(\alpha m_t^2/m_W^2)$ terms, Δr is given by [11]

$$\Delta r \sim -\frac{\alpha N_C c_W^2 m_t^2}{16\pi^2 s_W^4 m_W^2} , \tag{7}$$

for heavy top quark. The vertex corrections are presented in Appendix A. The renormalization constants are given by

$$\delta m_W^2=-\frac{\sqrt{2}G_F N_c}{4\pi^2} m_W^2 m_t^2 \left[\frac{\Delta}{2}+F_1^{(Wtb)}-F_0^{(Wtb)} \right] , \tag{8}$$

$$\delta m_Z^2=-\frac{\sqrt{2}G_F N_c}{4\pi^2} m_Z^2 m_t^2 \left[\frac{\Delta}{2}-\frac{F_0^{(Ztt)}}{2} \right] , \tag{9}$$

$$\delta Z_H=\frac{N_c}{16\pi^2}\left\{4(V_t^H)^2\left[-\frac{\Delta}{2}+F_1^{(Htt)}-2m_t^2G_0^{(Htt)}+m_H^2G_1^{(Htt)}\right]-\sum_{i,j}(\tilde{V}_{tij}^H)^2G_0^{(H\tilde{t}_i\tilde{t}_j)}\right\} , \tag{10}$$

$$\delta Z_{H^+}=\frac{N_c}{16\pi^2}\left\{8g_t^2\left[-\frac{\Delta}{2}+F_1^{(H^+tb)}-m_t^2G_0^{(H^+tb)}+m_{H^+}^2G_1^{(H^+tb)}\right]+\sum_{i,j}(\tilde{V}_{ij}^{H^+})^2G_0^{(H^+\tilde{t}_i\tilde{b}_j)}\right\} , \tag{11}$$

$$\begin{aligned} Z_{hH}^{1/2}=-\frac{N_c}{16\pi^2}\frac{1}{m_h^2-m_H^2}&\left\{4V_t^HV_t^h\left[3m_t^2\Delta-\frac{m_H^2}{2}\Delta+\overline{A}_0(m_t)-2m_t^2F_0^{(Htt)}+m_H^2F_1^{(Htt)}\right]\right.\\ &\left.-\sum_{i,j}\tilde{V}_{tij}^H\tilde{V}_{tij}^h\left[\Delta-F_0^{(H\tilde{t}_i\tilde{t}_j)}\right]+\sum_i\tilde{U}_{tii}^{Hh}\left[\tilde{m}_{ti}^2\Delta+\overline{A}_0(\tilde{m}_{ti})\right]\right\} , \end{aligned} \tag{12}$$

$$\delta Z_h=\delta Z_H|_{H\to h}, \quad Z_{Hh}^{1/2}=Z_{hH}^{1/2}|_{(H,h)\to(h,H)} , \tag{13}$$

$$Z_{HW}^{1/2}=\frac{gN_c}{16\pi^2\sqrt{2}m_W^2}\left\{4g_tm_t\left[\frac{\Delta}{2}+F_1^{(H^+tb)}-F_0^{(H^+tb)}\right]+\tilde{V}_{11}^{H^+}\left[2F_1^{(H^+\tilde{t}_1\tilde{b}_1)}-F_0^{(H^+\tilde{t}_1\tilde{b}_1)}\right]\right\} \tag{14}$$

$$\delta Z_A=\frac{N_c}{16\pi^2}\left\{4(V_t^A)^2\left[-\frac{\Delta}{2}+F_1^{(Att)}+m_A^2G_1^{(Att)}\right]-\sum_{i,j}(\tilde{V}_{tij}^A)^2G_0^{(A\tilde{t}_i\tilde{t}_j)}\right\} , \tag{15}$$

$$Z_{ZA}^{1/2}=-\frac{gN_cm_t}{16\pi^2m_Wm_Z}V_t^A[\Delta-F_0^{(Att)}] , \tag{16}$$

$$\begin{aligned} \delta Z_t^R=\frac{\sqrt{2}G_Fm_t^2}{16\pi^2}&\left\{\sum_{i=H^+,W^+}\eta_i^2[-\Delta+2F_1^{(tbi)}+2m_t^2G_1^{(tbi)}]+\sum_{i=A,Z}\eta_i^2\left[-\frac{\Delta}{2}+F_1^{(tti)}+2m_t^2G_1^{(tti)}-2m_t^2G_0^{(tti)}\right]\right.\\ &\left.+\sum_{i=H,h}\eta_i^2\left[-\frac{\Delta}{2}+F_1^{(tti)}+2m_t^2G_1^{(tti)}+2m_t^2G_0^{(tti)}\right]\right\}\\ &+\frac{\sqrt{2}G_Fm_W^2}{4\pi^2}\lambda_t^2\left\{V_{j2}^2\left[-\frac{\Delta}{2}+F_1^{(t\tilde{\chi}_j^+\tilde{b})}+m_t^2G_1^{(t\tilde{\chi}_j^+\tilde{b})}\right]+N_{j4}^2\left[-\frac{\Delta}{2}+F_1^{(t\tilde{\chi}_j^0\tilde{t})}+2m_t^2G_1^{(t\tilde{\chi}_j^0\tilde{t})}\right]\right\} , \end{aligned} \tag{17}$$

$$\begin{aligned} \delta Z_t^L=\frac{\sqrt{2}G_Fm_t^2}{16\pi^2}&\left\{\sum_{i=H^+,W^+}\eta_i^22m_t^2G_1^{(tbi)}+\sum_{i=A,Z}\eta_i^2\left[-\frac{\Delta}{2}+F_1^{(tti)}+2m_t^2G_1^{(tti)}-2m_t^2G_0^{(tti)}\right]\right.\\ &\left.+\sum_{i=H,h}\eta_i^2\left[-\frac{\Delta}{2}+F_1^{(tti)}+2m_t^2G_1^{(tti)}+2m_t^2G_0^{(tti)}\right]\right\}\\ &+\frac{\sqrt{2}G_Fm_W^2}{4\pi^2}\lambda_t^2\left\{V_{j2}^2m_t^2G_1^{(t\tilde{\chi}_j^+\tilde{b})}+N_{j4}^2\left[-\frac{\Delta}{2}+F_1^{(t\tilde{\chi}_j^0\tilde{t})}+2m_t^2G_1^{(t\tilde{\chi}_j^0\tilde{t})}\right]\right\} , \end{aligned} \tag{18}$$

$$\begin{aligned} \frac{\delta m_t}{m_t}=-\frac{\sqrt{2}G_Fm_t^2}{16\pi^2}&\left\{\sum_{i=H^+,W^+}\eta_i^2\left[-\frac{\Delta}{2}+F_1^{(tbi)}\right]+\sum_{i=A,Z}\eta_i^2\left[\frac{\Delta}{2}-F_0^{(tti)}+F_1^{(tti)}\right]+\sum_{i=H,h}\eta_i^2\left[-\frac{3\Delta}{2}+F_1^{(tti)}+F_0^{(tti)}\right]\right\}\\ &-\frac{\sqrt{2}G_Fm_W^2}{4\pi^2}\lambda_t^2\left\{\tfrac{1}{2}V_{j2}^2\left[-\frac{\Delta}{2}+F_1^{(t\tilde{\chi}_j^+\tilde{b}}\right]+N_{j4}^2\left[-\frac{\Delta}{2}+F_1^{(t\tilde{\chi}_j^0\tilde{t})}\right]\right\} . \end{aligned} \tag{19}$$

Here, $\Delta = 1/\epsilon - \gamma_E + \ln 4\pi$. The factors appearing in the above are presented in Appendix B. A_t, μ, α, and β are SUSY parameters. The functions $\overline{A}_0$, $F_{0,1}$, and $G_{0,1}$ are defined as

$$\overline{A}_0(m) = m^2\left[1 - \ln\frac{m^2}{\mu^2}\right],$$
$$F_n^{(i,j,k)} = F_n(m_i, m_j, m_k) = \int_0^1 dx\, x^n \ln\frac{m_i^2 x^2 - (m_i^2 + m_j^2 - m_k^2)x + m_i^2}{\mu^2},$$
$$G_n^{(i,j,k)} = \left.\frac{\partial F_n(p, m_j, m_k)}{\partial p^2}\right|_{p^2 = m_i^2}. \tag{20}$$

III. NUMERICAL EXAMPLES AND CONCLUSION

Here we present some numerical examples. Note that all these numerical results are given in the α scheme and the G_F scheme. The difference between the α scheme and the G_F scheme is shown in Eqs. (6a) and (6b) from which it is simple to transfer the results in the α scheme to the G_F scheme. In the numerical calculation, α, G_F, M_Z, and the masses of physical Higgs bosons and top quarks are used as input parameters whose values can be found in Ref. [12] and m_W is determined through [13]

$$m_W^2\left[1 - \frac{m_W^2}{M_Z^2}\right] = \frac{\pi\alpha}{\sqrt{2}G_F}\frac{1}{1 - \Delta r}, \tag{21}$$

where Δr is given in Eq. (7). As far as the parameters $\tan\beta$ and $\sin\alpha$ are concerned, as pointed out in Ref. [7], their experimental values used as input for numerical calculation are different from the renormalized parameters $\tan\beta$ and $\sin\alpha$ appearing in the above. The relations between $\tan\beta$ and $\tan\overline{\beta}$, $\sin\alpha$ and $\sin\overline{\alpha}$ are given by [7]

$$\tan\beta = \tan\overline{\beta}\left[1 - \frac{N_C G_F m_t^2}{16\sqrt{2}\pi^2}\frac{m_W^2}{M_Z^2 - m_W^2}\right] \tag{22}$$

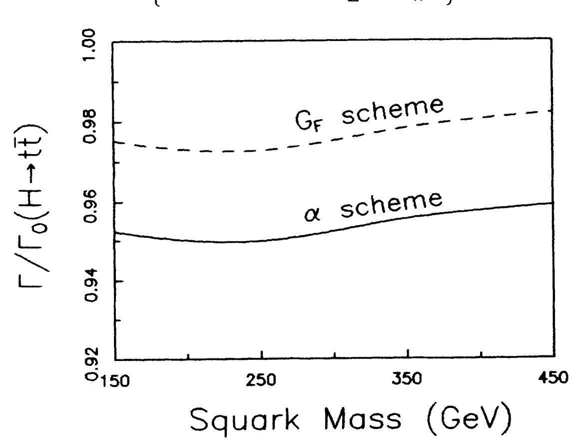

FIG. 2. $\Gamma/\Gamma_0(H\to t\bar{t})$ versus squark mass with $m_t = 150$ GeV, $m_A = 350$ GeV, and $\tan\beta = 2$ in the α scheme and the G_F scheme.

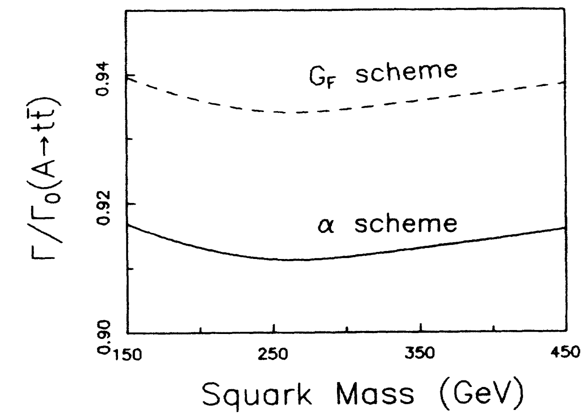

FIG. 3. As Fig. 2, but for $A\to t\bar{t}$.

and

$$\sin\alpha = \sin\overline{\alpha}\left[1 - \cos^2\beta\frac{N_C G_F m_t^2}{16\sqrt{2}\pi^2}\frac{m_W^2}{M_Z^2 - m_W^2}\right], \tag{23}$$

respectively. Moreover, we use the relations [14] between the Higgs-boson masses $m_{H,h,A,H^\pm}$ and parameters α, β at one loop, and choose m_A and $\tan\beta$ as two independent input parameters. As explained in Ref. [7], there is a small inconsistency in doing so since the parameters α and β of Ref. [7] are not the ones defined by the conditions given by Eq. (3) of Ref. [7]. Nevertheless, Mendez and Pomarol have shown [7] that this difference would only induce a higher order change. Other SUSY parameters M, μ, and A_t are fixed to be $M = 50$, $\mu = -30$, and $A_t = 0.1$.

We present some numerical results in Figs. 2–6. Figures 2 and 3 are the plots of Γ/Γ_0 versus squark mass $\tilde{m}_q$ for $m_t = 150$ GeV, $m_A = 350$ GeV, and $\tan\beta = 2$. For $\tilde{m}_q > 250$ GeV, the corrections decrease slightly as $\tilde{m}_q$ increases, showing the decoupling effect of squarks. But the corrections do not vanish for large squark mass since such corrections arise not only from the virtual squarks but also from virtual quarks, Higgs bosons, and Goldstone bosons. In Figs. 4 and 5 we present Γ/Γ_0 versus m_A with $m_t = 140$ GeV, $\tilde{m}_q = 200$ GeV, and $\tan\beta = 2$. As

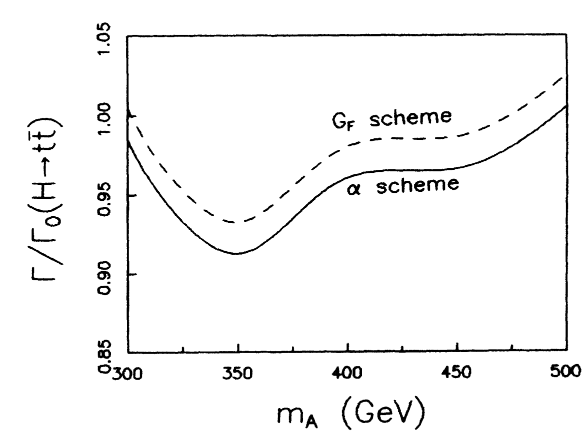

FIG. 4. $\Gamma/\Gamma_0(H\to t\bar{t})$ versus m_A with $m_t = 140$ GeV, $\tilde{m}_q = 200$ GeV, and $\tan\beta = 2$ in the α scheme and the G_F scheme.

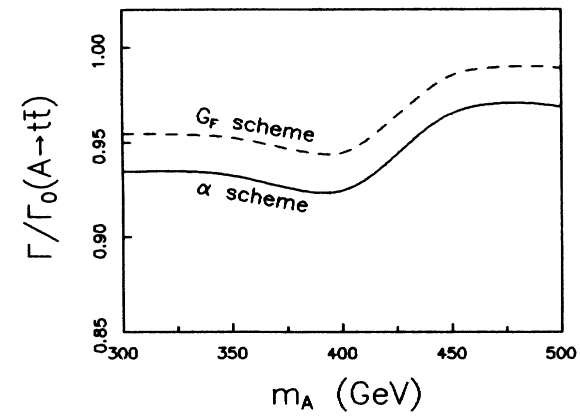

FIG. 5. As Fig. 4, but for $A \to t\bar{t}$.

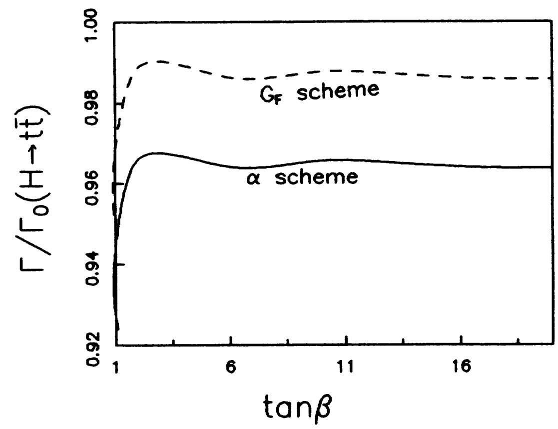

FIG. 6. $\Gamma/\Gamma_0(H \to t\bar{t})$ versus $\tan\beta$ with $m_t = 150$ GeV, $m_A = 350$ GeV, and $\tilde{m}_q = 200$ GeV in the α scheme and the G_F scheme.

shown, the corrections depend strongly on the value of m_A. Figure 6 shows the dependence of the corrections on the value of $\tan\beta$. The corrections are not sensitive to the value of $\tan\beta$ for $\tan\beta > 2$, but as $\tan\beta$ gets small, in the range of $\tan\beta < 2$, they increase rapidly and can be quite large. Note that the results of Ref. [7] show that the corrections are typically a few percent and in some special cases, for instance, if $m_t = 200$ GeV and $\tan\beta < 1$, they can reach values up to $\sim 20\%$. From Figs. 2–6, we find that the $O(\alpha m_t^2/m_W^2)$ corrections to decay rates of $H, A \to t\bar{t}$ in the MSSM are also typically a few percent and, as shown in Fig. 6, if $\tan\beta < 1$ they can reach quite large values which are comparable to the maximum value given in Ref. [7]. Such corrections are also comparable to the SUSY QCD corrections given in Ref. [9].

In conclusion, we have presented calculations of the $O(\alpha m_t^2/m_W^2)$ corrections to the $H^i \to t\bar{t}(H^i = H, A)$ in the MSSM. The numerical examples show that such corrections typically imply a few percent reduction in the widths.

ACKNOWLEDGMENTS

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APPENDIX A

The vertex corrections for $H \to t\bar{t}$ are given by

$$\delta\Gamma^H = \sum_{i=1}^{7} \delta\Gamma_i^H , \tag{A1}$$

with

$$\delta\Gamma_1^H = -\frac{\sqrt{2}G_F}{16\pi^2} m_t^2 \left\{ \sum_{i=H,h} \eta_i^2[\Delta + m_H^2(c_{11}+c_{21}-c_{12}-c_{23}) + m_t^2(c_0+c_{22}-2c_{12}) + \bar{c}_{24} - \tfrac{1}{2}] \right.$$
$$\left. - \sum_{i=A,Z} \eta_i^2[\Delta + m_H^2(c_{11}+c_{21}-c_{12}-c_{23}) + m_t^2(c_0+c_{22}+2c_{12}) + \bar{c}_{24} - \tfrac{1}{2}] \right\} , \tag{A2}$$

$$\delta\Gamma_2^H = -\frac{\sqrt{2}G_F \sin\beta}{4\pi^2 \sin\alpha} m_t^2 \left\{ \frac{m_Z^2}{3} \sum_{i,j=H,h} \eta_i\eta_j\lambda_{ij}(c_0 - c_{11}) - \frac{m_Z^2}{3} \sum_{i,j=A,Z} \eta_i\eta_j\lambda_{ij}(c_0+c_{11}) - \frac{m_W}{2} \sum_{i,j=H^\pm,W^\pm} \eta_i\eta_j\lambda_{ij}c_{11} \right\} , \tag{A3}$$

$$\delta\Gamma_3^H = \frac{\sqrt{2}G_F m_W^3 \sin\beta}{2\pi^2 m_t \sin\alpha} \left\{ A_s c_0 + \tfrac{1}{2}(A_L T_{ij} + A_R T_{ji}^*)c_0 - m_t[B_s^0 c_{11} + \tfrac{1}{2}(B_s^1 T_{ij} + B_s^2 T_{ji}^*)c_{11}] \right.$$
$$\left. + E_S m_t^2 c_{21} + \tfrac{1}{2} m_t^2 (E_L T_{ji}^* + E_R T_{ij}) c_{21} \right\} , \tag{A4}$$

$$\delta\Gamma_4^H = -\frac{\sqrt{2}G_F m_W^3 \sin\beta}{4\pi^2 \sin\alpha} \frac{m_Z}{c_W} \lambda_t^2 V_{j2}^2 \cos(\alpha+\beta)(\tfrac{1}{2} - \tfrac{1}{3}s_W^2)c_{11} , \tag{A5}$$

$$\delta\Gamma_5^H = \frac{\sqrt{2}G_F m_W^3 \sin\beta}{2\pi^2 m_t \sin\alpha} \left\{ I_s c_0 + \tfrac{1}{2}(I_L b_{ij}^* + I_R b_{ij})c_0 - m_t(J_s^1 + \tfrac{1}{2}J_L^1)c_{11} - m_t\left[J_s^2 + \tfrac{1}{2}(J_L^2 + J_R)\right]c_{11} \right.$$
$$\left. + \tfrac{1}{2}(K_L^1 + K_R^1 + K_L^2 + K_R^2) m_t^2 c_{21} \right\} , \tag{A6}$$

$$\delta\Gamma_6^H=\delta\Gamma_5^H\Big|_{\lambda_i N_{j4}^*\to d_j, c_j\to\lambda_i N_{j4}}, \tag{A7}$$

$$\delta\Gamma_7^H=\frac{\sqrt{2}G_F m_W^3\sin\beta}{4\pi^2 m_t\sin\alpha}\alpha_{ij}[\tilde{M}_{0k}(\xi_{ki}\zeta_{kj}^*+\xi_{kj}^*\zeta_{ki})c_0-m_t(\xi_{ki}\xi_{kj}^*+\zeta_{kj}^*\zeta_{ki})c_{11}], \tag{A8}$$

where the sums over i,j,k are implied, and $c_{ij}(-p_H,p_t,m_t,m_t,m_i)$ in (A2), $c_{ij}(-p_t,p_H,m_t,m_i,m_j)$ in (A3), $c_{ij}(-p_t,p_H,\tilde{m}_b,\tilde{M}_i,\tilde{M}_j)$ in (A4), $c_{ij}(-p_t,p_H,\tilde{M}_j,\tilde{m}_b,\tilde{m}_b)$ in (A5), $c_{ij}(-p_t,p_H,\tilde{m}_t,\tilde{M}_{0i},\tilde{M}_{0j})$ in (A6) and (A7), $c_{ij}(-p_t,p_H,\tilde{M}_{0k},\tilde{m}_t,\tilde{m}_t)$ in (A8) are three-point Feynman integrals, definitions for which can be found in Ref. [15].

The vertex corrections for $A\to t\bar{t}$ are given by

$$\delta\Gamma^A=\sum_{i=1}^{6}\delta\Gamma_i^A, \tag{A9}$$

with

$$\delta\Gamma_1^A=-\frac{\sqrt{2}G_F}{16\pi^2}m_t^2\Bigg\{\sum_{i=H,h}\eta_i^2[-\Delta-m_A^2(c_{11}+c_{21}-c_{12}-c_{23})+m_t^2(3c_0-c_{22}-2c_{12})-4\bar{c}_{24}+\tfrac{1}{2}]$$
$$-\sum_{i=A,Z}\eta_i^2[-\Delta-m_A^2(c_{11}+c_{21}-c_{12}-c_{23})-m_t^2(c_0+c_{22}+2c_{12})-4\bar{c}_{24}+\tfrac{1}{2}]\Bigg\}, \tag{A10}$$

$$\delta\Gamma_2^A=-\frac{\sqrt{2}G_F m_W^3}{2\pi^2 m_t}\tan\beta[\tfrac{1}{2}(A_R T_{ji}'^*-A_L T_{ij}')c_0+m_t B_{ps}^0(2c_{12}-c_{11})$$
$$+m_t(B_{ps}^1 T_{ij}'+B_{ps}^2 T_{ji}'^*)(c_{12}-\tfrac{1}{2}c_{11})+m_t^2(E_R T_{ij}'-E_L T_{ji}'^*)c_{21}], \tag{A11}$$

$$\delta\Gamma_3^A=-\frac{\sqrt{2}G_F m_W^3}{2\pi^2 m_t}\tan\beta[I_{ps}c_0+\tfrac{1}{2}(I_R b_{ij}'-I_L b_{ij}'^*)c_0-\tfrac{1}{2}m_t(J_L^1-J_R+J_L^2)(2c_{12}-c_{11})+\tfrac{1}{2}m_t^2(K_R^1+K_R^2-K_L^1-K_L^2)c_{21}], \tag{A12}$$

$$\delta\Gamma_4^A=\delta\Gamma_3^A\Big|_{\lambda_t N_{j4}^*\to d_j, c_j\to\lambda_t N_{j4}, \tilde{t}_1\to\tilde{t}_2}, \tag{A13}$$

$$\delta\Gamma_5^A=-\frac{\sqrt{2}G_F m_W^3}{4\pi^2 m_t}\tan\beta\sigma_{ij}[\tilde{M}_{0k}(\zeta_{ki}\xi_{kj}^*-\xi_{ki}\zeta_{kj}^*)c_0+m_t(\xi_{ki}\xi_{kj}^*-\zeta_{kj}^*\zeta_{ki})(2c_{12}-c_{11})], \tag{A14}$$

$$\delta\Gamma_6^A=\sum_{i=1}^{6}\delta\Gamma_{6(i)}^A, \tag{A15}$$

with

$$\delta\Gamma_{6(1)}^A=\frac{\sqrt{2}G_F m_Z^2}{16\pi^2}\cos2\beta\cos(\alpha+\beta)m_t^2\eta_H[(c_0+c_{11}-2c_{12})(-p_t,p_A,m_t,m_A,m_h)$$
$$+(c_0-c_{11}+2c_{12})(-p_t,p_A,m_t,m_H,m_A)], \tag{A16}$$

$$\delta\Gamma_{6(2)}^A=-\delta\Gamma_{6(1)}^A\Big|_{H\to h,\cos(\alpha+\beta)\to\sin(\alpha+\beta)}, \tag{A17}$$

$$\delta\Gamma_{6(3)}^A=\delta\Gamma_{6(1)}^A|_{A\to Z,\cos2\beta\to\sin2\beta}, \tag{A18}$$

$$\delta\Gamma_{6(4)}^A=-\delta\Gamma_{6(1)}^a|_{H\to h,\, A\to Z,\cos(\alpha+\beta)\to\sin(\alpha+\beta),\cos2\beta\to\sin2\beta}, \tag{A19}$$

$$\delta\Gamma_{6(5)}^A=\frac{\sqrt{2}G_F m_t^2}{16\pi^2}(m_{H^+}^2-m_A^2)\tan\beta\eta_{H^+}\eta_{W^+}(2c_{12}-c_{11})(-p_t,p_A,m_b,m_W,m_{H^+}), \tag{A20}$$

$$\delta\Gamma_{6(6)}^A=-\delta\Gamma_{6(5)}^A|_{(H^+,W^+)\to(W^+,H^+)}. \tag{A21}$$

In the above the sums over i,j,k are implied, and $c_{ij}(-p_H,p_t,m_t,m_t,m_i)$ in (A10), $c_{ij}(-p_t,p_H,\tilde{m}_b,\tilde{M}_i,\tilde{M}_j)$, $c_{ij}(-p_t,p_H,\tilde{m}_t,\tilde{M}_{0i},\tilde{M}_{0j})$ in (A12), and $c_{ij}(-p_t,p_H,\tilde{M}_{0k},\tilde{m}_t,\tilde{m}_t)$ in (A14) are three-point Feynman integrals. In (A3) λ_{ij} are related to the H-i-j couplings by $(-igm_t/2\cos\theta_W)\lambda_{ij}$ for $i,j=H,h,A,G^0(Z)$ and by $-ig\lambda_{ij}$ for $i,j=H^\pm,G^\pm(W^\pm)$.

APPENDIX B

Here we present all the factors appearing in this paper. The factors appearing in the counterterms are

$$\lambda_t = \frac{m_t}{\sqrt{2}m_W \sin\beta}, \quad g_t = \frac{gm_t}{2\sqrt{2}m_W}\cot\beta, \tag{B1}$$

$$V_t^H = -\frac{gm_t \sin\alpha}{2m_W\sin\beta}, \quad V_t^h = -\frac{gm_t\cos\alpha}{2m_W\sin\beta}, \quad V_t^A = -\frac{gm_t\cos\beta}{2m_W\sin\beta}, \tag{B2}$$

$$\tilde{V}_{t11}^H = \tilde{V}_{t22}^H = -\frac{gm_t^2\sin\alpha}{m_W\sin\beta}, \quad \tilde{V}_{t12}^H = \tilde{V}_{t21}^H = -\frac{gm_t}{2m_W\sin\beta}(\mu\cos\alpha + A_t\sin\alpha), \tag{B3}$$

$$\tilde{V}_{t12}^h = \tilde{V}_{t21}^h = \frac{gm_t}{2m_W\sin\beta}(\mu\sin\alpha - A_t\cos\alpha), \quad \tilde{V}_{t11}^h = \tilde{V}_{t22}^h = \tilde{V}_{t11}^H(\sin\alpha \to \cos\alpha), \tag{B4}$$

$$\tilde{V}_{11}^{H^+} = \frac{gm_t^2}{\sqrt{2}m_W}\cot\beta, \quad \tilde{V}_{21}^{H^+} = \frac{gm_t}{\sqrt{2}m_W}(A_t\cot\beta - \mu), \quad \tilde{V}_{12}^{H^+} = \tilde{V}_{22}^{H^+} = 0, \tag{B5}$$

$$\tilde{V}_{t12}^A = -\tilde{V}_{t21}^A = \frac{gm_t}{2m_W}(\mu - A_t\cot\beta), \quad \tilde{V}_{t11}^A = \tilde{V}_{t22}^A = 0, \tag{B6}$$

$$\tilde{U}_{t11}^{H^iH^i} = \tilde{U}_{t22}^{H^iH^i} = -\frac{g^2m_t^2}{2m_W^2}\eta_i^2, \quad \tilde{U}_{t12}^{H^iH^i} = \tilde{U}_{t21}^{H^iH^i} = 0.$$

$$\tilde{U}_{t11}^{Hh} = \tilde{U}_{t22}^{Hh} = -\frac{g^2\sin2\alpha m_t^2}{4m_W^2\sin^2\beta}, \tag{B7}$$

$$\eta_H = \sin\alpha/\sin\beta, \quad \eta_h = \cos\alpha/\sin\beta, \quad \eta_A = \eta_{H^+} = \cot\beta, \quad \eta_Z = \eta_W = 1. \tag{B8}$$

The factors appearing the vertex corrections are given by

$$A_s = \frac{\tilde{M}_j\sin\alpha}{2m_W\sin\beta}(m_t^2\lambda_t U_{j1}V_{j2} - m_t\lambda_t^2\tilde{M}_jV_{j2}^2)\delta_{ij}, \quad A_L = m_t^2\lambda_t U_{j1}V_{j2} - m_t\lambda_t^2\tilde{M}_iV_{i2}V_{j2}, \quad A_R = m_t^2\lambda_t U_{j1}V_{i2} - m_t\lambda_t^2\tilde{M}_jV_{i2}V_{j2}, \tag{B9}$$

$$B_s^0 = \frac{\tilde{M}_j\sin\alpha}{2m_W\sin\beta}(\lambda_t^2\tilde{M}_jV_{j2}^2 - 2m_t\lambda_t U_{j1}V_{j2})\delta_{ij}, \quad B_s^1 = \lambda_t^2\tilde{M}_iV_{i2}V_{j2} - 2m_t\lambda_t U_{i1}V_{j2}, \quad B_s^2 = \lambda_t^2\tilde{M}_jV_{i2}V_{j2} - 2m_t\lambda_t U_{j1}V_{i2}, \tag{B10}$$

$$E_s = \frac{\tilde{M}_j\sin\alpha}{2m_W\sin\beta}\lambda_t U_{j1}V_{j2}\delta_{ij}, \quad E_L = \lambda_t U_{j1}V_{i2}, \quad E_R = \lambda_t U_{i1}V_{j2}, \tag{B11}$$

$$T_{ij} = Q_{ij}^*\frac{\sin(\beta-\alpha)}{\sin\beta} - R_{ij}^*\frac{\sin\alpha}{\sin\beta}, \quad T'_{ij} = T_{ij}(\sin(\beta-\alpha)\to\cos2\beta, \ \sin\alpha \to -\cos\beta), \tag{B12}$$

$$a_i = \frac{\tilde{M}_{0i}\sin\alpha}{2\sin\beta m_W}, \quad a'_i = a_i(\sin\alpha \to \cos\beta), \tag{B13}$$

$$b_{ij} = T_{ij}(Q_{ij} \to Q''_{ij}, R_{ij} \to R''_{ij}), \quad b'_{ij} = b_{ij}(\sin(\beta-\alpha) \to -\cos2\beta, \sin\alpha \to \cos\beta), \tag{B14}$$

$$c_j = \sqrt{2}[\tfrac{2}{3}s_W N'_{j1} + \frac{1}{c_W}(\tfrac{1}{2} - \tfrac{2}{3}s_W^2)N'_{j2}], \tag{B15}$$

$$d_j = \sqrt{2}\left[\frac{2s_W^2}{3c_W}N'_{j2} - \tfrac{2}{3}s_W N'_{j1}\right], \tag{B16}$$

$$\xi_{j1} = \lambda_t N_{j4}^*, \quad \xi_{j2} = d_j, \quad \zeta_{j1} = c_j, \quad \zeta_{j2} = \xi_{j1}^*, \tag{B17}$$

$$\alpha_{11} = \alpha_{22} = -\frac{m_t^2\sin\alpha}{m_W\sin\beta}, \quad \alpha_{12} = \alpha_{21} = -\frac{m_t}{2m_W\sin\beta}(\mu\cos\alpha + A_t\sin\alpha), \tag{B18}$$

$$\sigma_{11} = \sigma_{22} = 0, \quad \sigma_{12} = -\sigma_{21} = \frac{m_t}{2m_W}(A_t\cot\beta - \mu), \tag{B19}$$

$$I_s = -a_j m_t\lambda_t(\lambda_t\tilde{M}_{0j}N_{j4}^2 - m_tN_{j4}^2c_j)\delta_{ij}, \quad I_L = -m_t\lambda_tN_{j4}(m_tc_i - \lambda_t\tilde{M}_{0i}N_{i4}), \quad I_R = -m_t\lambda_tN_{i4}^*(m_tc_j + \lambda_t\tilde{M}_{0j}N_{j4}), \tag{B20}$$

$$J_s^1 = m_t\lambda_ta_jc_jN_{j4}\delta_{ij}, \quad J_L^1 = 0, J_L^2 = 0, \quad J_s^2 = m_t\lambda_t(b_{ij}c_j^*N_{i4}^* + b_{ij}^*c_iN_{j4}), \quad J_R = \lambda_t^2[2\tilde{M}_{0j}N_{j4}^2 + N_{j4}N_{i4}^*(b_{ij}\tilde{M}_{0j} + b_{ij}^*\tilde{M}_{0i})], \tag{B21}$$

$$K_L^1 = -\lambda_ta_jc_j^*N_{j4}^*, \quad K_R^1 = -\lambda_ta_jc_jN_{j4}, \quad K_L^2 = -\lambda_tb_{ij}c_j^*N_{i4}^*, \quad K_R^2 = -\lambda_tb_{ij}^*c_iN_{j4}. \tag{B22}$$

Here, Q_{ij}, R_{ij}, Q''_{ij}, and R''_{ij} are defined in Ref. [1]. In this paper, U_{ij} and V_{ij} are the elements of 2×2 matrices U and V which are given in Ref. [1]. The N_{ij} are the elements of the 4×4 matrix N which is defined in Ref. [1] and can be calculated numerically. The chargino mass $\tilde{M}_j$ and neutralino mass $\tilde{M}_{0i}$ depend on the parameters M, μ, and $\tan\beta$. $\tilde{M}_j$ is given in Ref. [1]. $\tilde{M}_{0i}$ can be obtained numerically.

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