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Quasi-two-body decays of nonstrange baryons

Simon Capstick*
Continuous Electron Beam Accelerator Facility, 12000 Jefferson Avenue, Newport News, Virginia 23606

Winston Roberts
Department of Physics, Old Dominion University, Norfolk, Virginia 23529
and Continuous Electron Beam Accelerator Facility, 12000 Jefferson Avenue, Newport News, Virginia 23606
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We examine the decays of nonstrange baryons to the final states $\Delta\pi$, $N\rho$, $N\eta$, $N\eta'$, $N\omega$, $N\frac{1}{2}^+(1440)\pi$, and $\Delta\frac{3}{2}^+(1600)\pi$, in a relativized pair-creation ($^3P_0$) model which has been developed in a previous study of the $N\pi$ decays of the same baryon states. As it is our goal to provide a guide for the possible discovery of new baryon states at CEBAF and elsewhere, we examine the decays of resonances which have already been seen in the partial-wave analyses, along with those of states which are predicted by the quark model but which remain undiscovered. The level of agreement between our calculation and the available widths from the partial-wave analyses is encouraging.

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I. INTRODUCTION

The N^* program at the Continuous Electron Beam Accelerator Facility (CEBAF) is designed to study many interesting aspects of the spectrum and decays of baryon states. It contains, among others, experiments to search for the so-called missing nonstrange baryons, to examine the structure of the Roper resonance [$N^*(1440)$], the $\Lambda(1405)$ and other low-lying resonances, to search for exotic states, and to study the couplings of nucleons to strange matter.

Most of these experiments are planned for CEBAF's Hall B, where the CEBAF Large Acceptance Spectrometer (CLAS) will be located. The design goals of the CLAS include good momentum resolution (typically $\delta p/p \approx 1\%$), which is necessary for missing mass measurement of particles such as π^0, η, and ω. This detector also allows good particle identification so that electrons, pions, kaons, protons, and deuterons can be separated, as well as neutrals such as photons and neutrons.

Some of these experiments, in particular [1], will examine the process $\gamma p \to p\pi^+\pi^-$, with the analysis focusing on $\gamma p \to \Delta^{++}\pi^-$, $\gamma p \to \Delta^0\pi^+$, and $\gamma p \to p\rho^0$. This experiment proposes to search for new (missing and undiscovered [2]) resonances with masses between the $\Delta\pi$ threshold (1.3 GeV) and 2.3 GeV. There are other experiments that propose to search in other channels. These include one [3] which will focus on the decays of such states to $N\eta$, one [4] which will examine the $N\eta$ and $N\eta'$ channels, and one [5] which proposes to measure $N\omega$ decays. The three channels $N\eta$, $N\eta'$, and $N\omega$ offer the advantage of being isospin selective, in that only $I = \frac{1}{2}$ N^* resonances (as opposed to $I = \frac{3}{2}$ Δ^* resonances) can couple to these final states. Channels with the $\Delta(1232)$ substituted for the final-state nucleon may, in principle, also be investigated, but the increased multiplicity of daughter hadrons makes the analysis of such experiments more difficult. However, in [1], for instance, the plan is to trigger on any event containing one or more charged particles. This means that multiparticle decays such as $\Delta\eta$ and $\Delta\omega$ will be present in the data and can, in principle, be analyzed.

A detailed understanding of baryon physics will also be required for interpretation of the results of many other CEBAF experiments. Some examples are experiments to study the electroproduction of the $\Delta(1232)$ [6], the production of baryon resonances at high momentum transfer [7], and experiments which will determine the polarized structure functions of the nucleon by electroproduction of $\Delta(1232)$ and $N\frac{1}{2}^+(1440)$ [8]. While this list is by no means representative or exhaustive, it should illustrate the pivotal role that baryon physics will play in the CEBAF experimental program.

In view of this proposed program, we believe it is important to focus some attention on models of baryon physics which are capable of providing detailed information pertinent to these experiments. In an earlier article [9] we reported on a study of the baryon spectrum which focused on the $N\pi$ couplings of baryon states. Our purpose there was twofold: to test an existing model of the baryon spectrum [10] by looking more carefully at its predictions for the strong decays, and to investigate a previously proposed solution to the problem of the missing baryons [11].

Our conclusions were (i) that the model works quite well in describing the $N\pi$ decays of baryon resonances, and (ii) that, as suggested in [11], the missing baryons consistently couple weakly to the $N\pi$ channel, and so should be sought elsewhere. We were also able to identify aspects of our model which could be improved.

One possibility for producing the missing baryons is to use photons or electrons incident on nucleon targets. The photo- and electroproduction amplitudes of baryon resonances have been recently examined in this model framework by one of us [12]. Complementary to the work of Refs. [9] and [12] is an examination of the strong cou-

*Present address: Supercomputer Computations Research Institute and Department of Physics, Florida State University, Tallahassee, FL 32306.

plings of the nonstrange baryons to decay channels other than $N\pi$. The motivation for such a study should be clear from the above description of some aspects of the CEBAF N^* program; baryon resonances produced in the photo- or electroexcitation of a nucleon must decay, and while some of these states will decay to $N\pi$, other channels must be explored for the missing resonances. In particular, channels with more than a single daughter pion may offer some of the best opportunities for their discovery.

The multipion final states we study are those in which the extra pion(s) result from decay of a baryon resonance to the so-called quasi-two-body final states $N\rho$, $\Delta\pi$, $N\frac{1}{2}^+(1440)\pi$, and $\Delta\frac{3}{2}^+(1600)\pi$. In addition, we examine the $N\eta$, $N\eta'$, and $N\omega$ channels, which will also receive experimental attention in the CEBAF program.

The study of baryon spectroscopy and decays has a long history; it is clear that we cannot refer to every article in the vast literature on these topics. Instead, we mention a few of the more recent papers that deal with the decays which we discuss here. A more complete list of references can be found in Refs. [9] and [10]. Using

TABLE I. Results for nucleons in the $N=1$ and $N=2$ bands in the $N\pi$, $N\eta$, $N\eta'$, and $N\omega$ channels. Notation for model states is $[J^P]_n$(mass[MeV]), where J^P is the spin/parity of the state and n its principal quantum number. The first row gives our model results, while the second row lists the corresponding numbers obtained by Manley and Saleski in their partial-wave analysis, as well as the Particle Data Group name, $N\pi$ partial wave, star rating, and $N\pi$ amplitude for the state.

Model state $N\pi$ state/rating	$N\pi$	$N\eta$	$N\eta'$	$N\omega$	$N\omega$	$N\omega$	$\sqrt{\Gamma^{\rm tot}_{N\omega}}$
$[N\frac{1}{2}^-]_1(1460)$	14.7 ± 0.5	$+14.6^{+0.7}_{-1.3}$					
$N(1535)S_{11}$****	8.0 ± 2.8	$+8.1\pm0.8$					
$[N\frac{1}{2}^-]_2(1535)$	12.2 ± 0.8	$-7.8^{+0.1}_{-0.0}$					
$N(1650)S_{11}$****	8.7 ± 1.9	-2.4 ± 1.6					
				$d_{\frac{1}{2}}$	$s_{\frac{3}{2}}$	$d_{\frac{3}{2}}$	
$[N\frac{3}{2}^-]_1(1495)$	8.6 ± 0.3	$+0.4^{+2.9}_{-0.4}$					
$N(1520)D_{13}$****	8.3 ± 0.9						
$[N\frac{3}{2}^-]_2(1625)$	5.8 ± 0.6	-0.2 ± 0.1		$0.0^{+0.0}_{-0.3}$	$0.0^{+0.0}_{-16.2}$	$0.0^{+0.3}_{-0.0}$	$0.0^{+16.2}_{-0.0}$
$N(1700)D_{13}$***	3.2 ± 1.3						
$[N\frac{5}{2}^-]_1(1630)$	5.3 ± 0.1	-2.5 ± 0.2					
$N(1675)D_{15}$****	7.7 ± 0.7						
				$p_{\frac{1}{2}}$	$p_{\frac{3}{2}}$		
$[N\frac{1}{2}^+]_2(1540)$	$20.3^{+0.8}_{-0.9}$	$+0.0^{+1.0}_{-0.0}$					
$N(1440)P_{11}$****	19.9 ± 3.0[a]						
$[N\frac{1}{2}^+]_3(1770)$	4.2 ± 0.1	$+5.7\pm0.3$		$0.0^{+0.0}_{-2.3}$	$0.0^{+0.0}_{-0.4}$		$0.0^{+2.3}_{-0.0}$
$N(1710)P_{11}$***	4.7 ± 1.2						
$[N\frac{1}{2}^+]_4(1880)$	$2.7^{+0.6}_{-0.9}$	$-3.7^{+0.5}_{-0.0}$	$+0.0^{+0.0}_{-2.5}$	$-4.3^{+3.6}_{-1.1}$	$-1.6^{+1.3}_{-0.2}$		$4.6^{+1.1}_{-3.9}$
$[N\frac{1}{2}^+]_5(1975)$	$2.0^{+0.2}_{-0.3}$	$+0.1^{+0.2}_{-0.1}$		$-3.1^{+1.1}_{-0.3}$	-0.8 ± 0.1		$3.1^{+0.3}_{-1.1}$
				$p_{\frac{1}{2}}$	$p_{\frac{3}{2}}$	$f_{\frac{3}{2}}$	
$[N\frac{3}{2}^+]_1(1795)$	14.1 ± 0.1	$+5.7\pm0.3$		$0.0^{+0.0}_{-0.2}$	$0.0^{+1.2}_{-0.0}$	$0.0^{+0.1}_{-0.0}$	$0.0^{+1.3}_{-0.0}$
$N(1720)P_{13}$****	5.5 ± 1.6						
$[N\frac{3}{2}^+]_2(1870)$	$6.1^{+0.6}_{-1.2}$	-4.6 ± 0.3	$+0.0^{+0.0}_{-3.3}$	0.0	$+4.4^{+1.2}_{-4.4}$	$+0.6^{+1.2}_{-0.6}$	$4.5^{+1.4}_{-4.4}$
	11.4 ± 1.6[b]						
$[N\frac{3}{2}^+]_3(1910)$	$1.0^{+0.1}_{-0.2}$	-0.9 ± 0.1	$-0.2^{+0.2}_{-0.6}$	$-5.8^{+3.5}_{-0.9}$	$+5.7^{+0.9}_{-3.4}$	$-0.5^{+0.4}_{-0.7}$	$8.2^{+1.3}_{-4.9}$
$[N\frac{3}{2}^+]_4(1950)$	$4.1^{+0.4}_{-0.7}$	0.0	0.0	$-5.4^{+2.2}_{-0.4}$	$-3.2^{+1.3}_{-0.2}$	$+0.7^{+0.9}_{-0.6}$	$6.3^{+0.7}_{-2.6}$
$[N\frac{3}{2}^+]_5(2030)$	1.8 ± 0.2	$+0.4\pm0.1$	$+0.2^{+0.1}_{-0.2}$	$-1.6^{+0.2}_{-0.2}$	$-2.9^{+0.3}_{-0.3}$	$+0.7^{+0.6}_{-0.4}$	$3.3^{+0.5}_{-0.4}$
				$f_{\frac{1}{2}}$	$p_{\frac{3}{2}}$	$f_{\frac{3}{2}}$	
$[N\frac{5}{2}^+]_1(1770)$	6.6 ± 0.2	$+0.6\pm0.1$	0.0				
$N(1680)F_{15}$****	8.7 ± 0.9						
$[N\frac{5}{2}^+]_2(1980)]$	1.3 ± 0.3	-0.8 ± 0.2	$-0.1^{+0.1}_{-0.2}$	$+2.1^{+1.3}_{-1.1}$	$-1.7^{+0.3}_{-0.1}$	$-1.1^{+0.6}_{-0.7}$	$2.9^{+1.3}_{-1.1}$
$[N\frac{5}{2}^+]_3(1995)]$	0.9 ± 0.2	$+1.9\pm0.8$	$+0.2^{+0.6}_{-0.2}$	$-0.3^{+0.2}_{-0.3}$	$+3.1^{+0.5}_{-0.5}$	$-1.6^{+1.1}_{-1.5}$	$3.5^{+1.3}_{-0.8}$
$N(2000)F_{15}$**	2.0 ± 1.2						
				$f_{\frac{1}{2}}$	$f_{\frac{3}{2}}$	$h_{\frac{3}{2}}$	
$[N\frac{7}{2}^+]_1(1980)]$	2.4 ± 0.4	$-2.2^{+0.6}_{-0.7}$	$-0.2^{+0.2}_{-0.5}$	$-0.8^{+0.4}_{-0.5}$	$+1.4^{+0.9}_{-0.7}$	$0.0^{+0.0}_{-0.0}$	$1.6^{+1.0}_{-0.9}$
$N(1990)F_{17}$**	4.6 ± 1.9						

[a]From Ref. [25].

[b]Second P_{13} found in Ref. [19].

an elementary pseudoscalar emission model, Koniuk and Isgur [11] discuss the $N\pi$, $\Delta\pi$, and $N\eta$ decay channels, as well as the ΛK and ΣK channels. Similar work has been done by Blask *et al.* [13]. Koniuk [14] has also examined $N\rho$ and $N\omega$ decays in the approximation of treating the ρ in the narrow-width limit. In a series of articles, Stancu and Stassart use a flux-tube-breaking model to discuss decays to the $N\pi$ [15], $N\rho$ [16], and $N\omega$ [17] channels. We have chosen not to examine the channel in which the $\pi\pi$ pair are in a relative S wave, with total isospin zero; Stassart [18] has modeled such decays by treating the $\pi\pi$ pair as resulting from a σ pseudoresonance with a mass of about 600 MeV.

This article is organized as follows. In the next section we briefly recap the model used to describe the strong decays, and discuss our treatment of quasi-two-body de-

TABLE II. Results for nucleons in the $N = 1$ and $N = 2$ bands in the $\Delta\pi$ and $N\rho$ channels. Notation for model states is $[J^P]_n$(mass[MeV]), where J^P is the spin/parity of the state and n its principal quantum number. The first row gives our model results, while the second row lists the numbers obtained by Manley and Saleski in their analysis, as well as the Particle Data Group name, $N\pi$ partial wave, and star rating for the state.

Model state $N\pi$ state/rating	$\Delta\pi$	$\Delta\pi$	$\sqrt{\Gamma^{\rm tot}_{\Delta\pi}}$	$N\rho$	$N\rho$	$N\rho$	$\sqrt{\Gamma^{\rm tot}_{N\rho}}$
	d			$s_{\frac{1}{2}}$	$d_{\frac{3}{2}}$		
$[N\frac{1}{2}^-]_1(1460)$	$+1.4 \pm 0.3$		1.4 ± 0.3	-0.7 ± 0.1	$+0.4 \pm 0.1$		$0.8^{+0.2}_{-0.1}$
$N(1535)S_{11}$****	0.0		0.0	-1.7 ± 0.6	-1.4 ± 0.7		2.2 ± 0.6
$[N\frac{1}{2}^-]_2(1535)$	$+3.6^{+0.8}_{-0.6}$		$3.6^{+0.8}_{-0.6}$	$+0.9^{+0.3}_{-0.2}$	$+0.4 \pm 0.1$		$1.0^{+0.3}_{-0.2}$
$N(1650)S_{11}$****	$+1.7 \pm 0.6$		1.7 ± 0.6	0.0	$+2.2 \pm 0.9$		2.2 ± 0.9
	s	d		$d_{\frac{1}{2}}$	$s_{\frac{3}{2}}$	$d_{\frac{3}{2}}$	
$[N\frac{3}{2}^-]_1(1495)$	$-5.7^{+3.6}_{-1.6}$	$-1.5^{+1.3}_{-3.0}$	$5.9^{+2.6}_{-3.8}$	$-0.1^{+0.1}_{-0.3}$	$-2.4^{+1.9}_{-6.4}$	$-0.3^{+0.2}_{-1.0}$	$2.5^{+6.5}_{-1.9}$
$N(1520)D_{13}$****	-2.6 ± 0.8	-4.2 ± 0.6	5.0 ± 0.6		-5.1 ± 0.6		5.1 ± 0.6
$[N\frac{3}{2}^-]_2(1625)$	-27.5 ± 1.6	$+4.6^{+1.6}_{-1.3}$	$27.9^{+1.9}_{-1.8}$	0.0	$\pm 0.1 \pm 0.0$	$-0.9^{+0.3}_{-0.6}$	$0.9^{+0.6}_{-0.4}$
$N(1700)D_{13}$***	$+3.5 \pm 3.6$	$+14.1 \pm 5.6$	14.5 ± 5.5		-5.6 ± 5.7		5.6 ± 5.7
	d	g		$d_{\frac{1}{2}}$	$d_{\frac{3}{2}}$	$g_{\frac{3}{2}}$	
$[N\frac{5}{2}^-]_1(1630)$	$+5.7 \pm 0.4$	0.0	5.7 ± 0.4	$+0.2 \pm 0.0$	-0.4 ± 0.0	0.0	0.5 ± 0.0
$N(1675)D_{15}$****	$+9.2 \pm 0.3$		9.2 ± 0.3	0.8 ± 0.4		-0.5 ± 0.5	1.0 ± 0.4
	p			$p_{\frac{1}{2}}$	$p_{\frac{3}{2}}$		
$[N\frac{1}{2}^+]_2(1540)$	$+3.3^{+2.3}_{-1.8}$		$3.3^{+2.3}_{-1.8}$	$-0.3^{+0.2}_{-0.3}$	$-0.5^{+0.3}_{-0.5}$		$0.6^{+0.5}_{-0.3}$
$N(1440)P_{11}$****	$+9.4 \pm 0.8$		9.4 ± 0.8				
$[N\frac{1}{2}^+]_3(1770)$	-13.9 ± 1.5		13.9 ± 1.5	$+0.3 \pm 0.1$	$-3.7^{+0.9}_{-1.2}$		$3.7^{+1.2}_{-1.0}$
$N(1710)P_{11}$***	-15.3 ± 3.6		15.3 ± 3.6				
$[N\frac{1}{2}^+]_4(1880)$	$-8.7^{+2.1}_{-0.4}$		$8.7^{+0.4}_{-2.1}$	$+2.3^{+1.7}_{-1.4}$	$\pm 0.3^{+0.0}_{-0.1}$		$2.3^{+1.7}_{-1.4}$
$[N\frac{1}{2}^+]_5(1975)$	$-4.6^{+0.4}_{-0.2}$		$4.6^{+0.2}_{-0.4}$	$+0.7^{+0.1}_{-0.3}$	$-2.4^{+1.0}_{-0.4}$		$2.5^{+0.4}_{-1.1}$
	p	f		$p_{\frac{1}{2}}$	$p_{\frac{3}{2}}$	$f_{\frac{3}{2}}$	
$[N\frac{3}{2}^+]_1(1795)$	-1.7 ± 0.2	$-1.0^{+0.2}_{-0.3}$	2.0 ± 0.3	$-2.6^{+0.7}_{-0.8}$	$+1.8^{+0.6}_{-0.5}$	$+0.7^{+0.3}_{-0.2}$	$3.3^{+1.0}_{-0.8}$
$N(1720)P_{13}$****				$+18.2 \pm 4.6$			18.2 ± 4.6
$[N\frac{3}{2}^+]_2(1870)$	$+3.8 \pm 0.5$	$-2.2^{+1.2}_{-1.5}$	$4.4^{+1.3}_{-1.0}$	$-1.4^{+0.9}_{-1.0}$	-1.0 ± 0.6	$+0.2^{+0.5}_{-0.2}$	$1.8^{+1.2}_{-1.1}$
$N(1880)P_{13}$[a]				-14.7 ± 2.9			14.7 ± 2.9
$[N\frac{3}{2}^+]_3(1910)$	$+16.8^{+0.2}_{-2.9}$	$+2.9^{+1.9}_{-1.4}$	$17.0^{+0.6}_{-3.0}$	$+2.5 \pm 1.3$	$-2.2^{+1.1}_{-1.2}$	$+0.5^{+0.9}_{-0.4}$	$3.3^{+1.9}_{-1.7}$
$[N\frac{3}{2}^+]_4(1950)$	$-5.9^{+0.7}_{-0.3}$	$+5.1^{+3.4}_{-2.3}$	$7.8^{+2.7}_{-1.9}$	$+1.0^{+0.2}_{-0.4}$	$+3.6^{+1.1}_{-1.6}$	$+1.2^{+1.3}_{-0.8}$	$3.9^{+1.6}_{-1.8}$
$[N\frac{3}{2}^+]_5(2030)$	-6.6 ± 0.4	$+2.5^{+1.7}_{-1.0}$	$7.1^{+1.1}_{-0.7}$	$\pm 0.1^{+0.1}_{-0.0}$	$+2.8 \pm 0.7$	$+0.5^{+0.3}_{-0.4}$	2.9 ± 0.8
	p	f		$f_{\frac{1}{2}}$	$p_{\frac{3}{2}}$	$f_{\frac{3}{2}}$	
$[N\frac{5}{2}^+]_1(1770)$	$+1.6 \pm 0.1$	$+0.5 \pm 0.1$	1.7 ± 0.1	-0.2 ± 0.0	$-3.0^{+0.4}_{-0.5}$	-0.3 ± 0.1	$3.0^{+0.5}_{-0.4}$
$N(1680)F_{15}$****	-3.6 ± 0.6	$+1.0 \pm 0.5$	3.7 ± 0.6		-2.8 ± 0.7	-1.7 ± 0.6	3.3 ± 0.7
$[N\frac{5}{2}^+]_2(1980)]$	$+15.0^{+0.1}_{-0.3}$	$+3.7^{+1.5}_{-1.1}$	15.5 ± 0.5	$-1.8^{+1.0}_{-0.8}$	$+0.8 \pm 0.1$	$+0.8^{+0.3}_{-0.4}$	$2.2^{+0.9}_{-1.0}$
$[N\frac{5}{2}^+]_3(1995)]$	$+7.8^{+0.4}_{-0.6}$	$-5.8^{+2.4}_{-3.9}$	$9.8^{+3.0}_{-1.7}$	-0.4 ± 0.3	$-7.8^{+3.1}_{-0.2}$	-0.2 ± 0.1	$7.8^{+0.2}_{-3.1}$
$N(2000)F_{15}$**	$+7.7 \pm 5.8$	$+1.4 \pm 9.2$	7.9 ± 5.9		-17.2 ± 6.2	$+8.5 \pm 5.8$	19.2 ± 6.1
	f	h		$f_{\frac{1}{2}}$	$f_{\frac{3}{2}}$	$h_{\frac{3}{2}}$	
$[N\frac{7}{2}^+]_1(1980)]$	$+5.0^{+2.0}_{-1.4}$	0.0	$5.0^{+2.0}_{-1.4}$	$+0.6 \pm 0.3$	$-1.0^{+0.6}_{-0.5}$	0.0	$1.2^{+0.6}_{-0.7}$
$N(1990)F_{17}$**							

[a]Second P_{13} found in Ref. [19].

cays. In Sec. III we present our strong decay amplitudes for the $\Delta\pi$, $N\rho$, $N\frac{1}{2}^+(1440)\pi$, $\Delta\frac{3}{2}^+(1600)\pi$, $N\eta$, $N\eta'$, and $N\omega$ channels. To explain our assignment of model states to states seen in analyses of the experimental data, we reproduce our model predictions for the $N\pi$ couplings of all states considered here. Our primary source of partial widths with which to compare our model predictions is the Manley-Saleski recent partial-wave analysis of the $N\pi\pi$ final state [19]; we have also taken some widths from the Particle Data Group (PDG) compilation [20]. In Sec. IV we present our conclusions and outlook.

II. DECAY AMPLITUDES

A. The model

The models we use to obtain the baryon spectrum and strong decay amplitudes are described in some detail elsewhere [9, 10, 21]. For completeness we outline these very briefly here. The baryon spectrum is obtained by solving a Schrödinger-like equation in a Fock space consisting solely of valence quarks. The Hamiltonian used is

$$H = \sum_i \sqrt{\mathbf{p}_i^2 + m_i^2} + V, \tag{1}$$

where V is a relative-position and relative-momentum-dependent potential which includes the usual confining, Coulomb, hyperfine, and spin-orbit terms. Wave functions are expanded in a large harmonic oscillator basis, and a diagonalization procedure is used to find the energy eigenvalues. One consequence of the procedure is that none of the terms in the Hamiltonian are treated as perturbations. As discussed in [9], this has a profound effect on the wave functions obtained.

The spectrum that results is comparable to those obtained using other methods, including the simpler, nonrelativistic approaches, but the wave functions we use here have been obtained using a more consistent treatment of all the terms in the Hamiltonian. Furthermore,

TABLE III. Results for Δ states in the $N = 0$, $N = 1$, and $N = 2$ bands in the $N\pi$ and $\Delta\pi$ channels. Notation as in Table I.

Model state $N\pi$ state/rating	$N\pi$	$\Delta\pi$	$\Delta\pi$	$\sqrt{\Gamma^{\rm tot}_{\Delta\pi}}$
		p		
$[\Delta\frac{1}{2}^-]_1(1555)$	5.1 ± 0.7	$-4.2^{+1.3}_{-1.8}$		$4.2^{+1.8}_{-1.3}$
$\Delta(1620)S_{31}$****	6.5 ± 1.0	-9.7 ± 1.3		9.7 ± 1.3
		s	d	
$[\Delta\frac{3}{2}^-]_1(1620)$	4.9 ± 0.7	$+15.4^{+0.9}_{-1.0}$	$+5.0^{+2.4}_{-1.8}$	$16.2^{+1.7}_{-1.5}$
$\Delta(1700)D_{33}$****	6.5 ± 2.0	$+21.1 \pm 4.7$	$+5.1 \pm 2.2$	21.7 ± 4.6
		p		
$[\Delta\frac{1}{2}^+]_1(1835)$	$3.9^{+0.4}_{-0.7}$	$+14.1^{+0.7}_{-4.5}$		$14.1^{+0.7}_{-4.5}$
$\Delta(1740)P_{31}$[a]	4.9 ± 1.3			
$[\Delta\frac{1}{2}^+]_2(1875)$	9.4 ± 0.4	$-8.4^{+0.2}_{-0.1}$		$8.4^{+0.1}_{-0.2}$
$\Delta(1910)P_{31}$****	6.6 ± 1.6			
		p	f	
$[\Delta\frac{3}{2}^+]_1(1230)$	10.4 ± 0.1			
$\Delta(1232)P_{33}$****	10.7 ± 0.3			
$[\Delta\frac{3}{2}^+]_2(1795)$	8.7 ± 0.2	$+8.4^{+3.6}_{-3.5}$	0.0	$8.4^{+3.6}_{-3.5}$
$\Delta(1600)P_{33}$***	7.6 ± 2.3	$+17.0 \pm 1.6$		17.0 ± 1.6
$[\Delta\frac{3}{2}^+]_3(1915)$	4.2 ± 0.3	$-8.9^{+0.3}_{-0.2}$	$+4.4^{+0.8}_{-0.7}$	10.0 ± 0.5
$\Delta(1920)P_{33}$***	7.7 ± 2.3	-11.2 ± 1.7		11.2 ± 1.7
$[\Delta\frac{3}{2}^+]_4(1985)$	$3.3^{+0.8}_{-1.1}$	$-9.2^{+0.8}_{-0.6}$	$-3.2^{+1.4}_{-2.2}$	$9.7^{+1.4}_{-1.2}$
		p	f	
$[\Delta\frac{5}{2}^+]_1(1910)$	3.4 ± 0.3	-1.5 ± 0.0	$+4.7 \pm 0.6$	$4.9^{+0.6}_{-0.5}$
$\Delta(1750)F_{35}$[b]	2.0 ± 0.8	$+8.4 \pm 3.6$	$+11.0 \pm 2.9$	13.9 ± 3.2
$\Delta(1905)F_{35}$****	5.5 ± 2.7	-2.0 ± 2.5	$+1.4 \pm 1.4$	2.4 ± 2.2
$[\Delta\frac{5}{2}^+]_2(1990)$	1.2 ± 0.3	$-14.0^{+1.6}_{-0.1}$	$+1.5^{+1.5}_{-0.8}$	$14.1^{+0.4}_{-1.6}$
$\Delta(2000)F_{35}$**	5.3 ± 2.3			
		f	h	
$[\Delta\frac{7}{2}^+]_1(1940)]$	7.1 ± 0.1	$+4.8 \pm 0.2$	0.0	4.8 ± 0.2
$\Delta(1950)F_{37}$****	9.8 ± 2.7	$+7.4 \pm 0.7$		7.4 ± 0.7

[a]First P_{31} state found in Ref. [19].

[b]Reference [19] finds two F_{35} states; this one and $\Delta(1905)F_{35}$.

the model is extended, with perhaps surprising success, to the description of many of the higher lying states.

The strong decays of baryons are treated in a version of the pair-creation model, specifically the $^3P_0$ model. Our ansatz for the pair creation operator is

$$T = -3\gamma \sum_{i,j} \int d\mathbf{p}_i d\mathbf{p}_j \delta(\mathbf{p}_i + \mathbf{p}_j) C_{ij} F_{ij} \times \sum_m \langle 1, m; 1, -m | 0, 0 \rangle \chi_{ij}^m \mathcal{Y}_1^{-m}(\mathbf{p}_i - \mathbf{p}_j) \times b_i^\dagger(\mathbf{p}_i) d_j^\dagger(\mathbf{p}_j). \tag{2}$$

Here, C_{ij} and F_{ij} are the color and flavor wave functions of the created pair, both assumed to be singlet, χ_{ij} is the spin-triplet wave function of the pair, and $\mathcal{Y}_1(\mathbf{p}_i - \mathbf{p}_j)$ is the vector harmonic indicating that the pair is in a relative p wave. Using the wave functions described above, we evaluate the decay amplitude for the process $A \to BC$ as $M = \langle BC|T|A\rangle$. In this form the model has one parameter, which is the pair creation constant γ.

B. Quasi-two-body decays

In our previous treatment of the two-body decays $A \to BC$, the decay widths were obtained from the amplitudes calculated in the $^3P_0$ model by using

$$\Gamma = 2\pi \int dk k^2 \left|M(k)\right|^2 \delta(M_a - E_b(k) - E_c(k)), \tag{3}$$

where k is the momentum of either daughter hadron in the rest frame of the parent, and $M(k)$ is the decay amplitude calculated in the $^3P_0$ model. This eventually leads to

$$\Gamma = 2\pi \frac{\left|M(k_0)\right|^2 E_b(k_0) E_c(k_0)}{M_a}, \tag{4}$$

and in our version of the model we make the replacements $E_b \to \tilde{M}_b$, $E_c \to \tilde{M}_c$, and $M_a \to \tilde{M}_a$, where $\tilde{M}_a$, $\tilde{M}_b$ and $\tilde{M}_c$ are the masses of these states in the weak-binding limit.

We now turn our attention to the quasi-two-body de-

TABLE IV. Results for Δ states in the $N = 0$, $N = 1$, and $N = 2$ bands in the $N\rho$ channel. Notation as in Table II.

Model state $N\pi$ state/rating	$N\rho$	$N\rho$	$N\rho$	$\sqrt{\Gamma_{N\rho}^{\rm tot}}$
	$s_{\frac{1}{2}}$	$d_{\frac{3}{2}}$		
$[\Delta\frac{1}{2}^-]_1(1555)$	$-3.6^{+1.3}_{-2.5}$	$-0.3^{+0.1}_{-0.2}$		$3.6^{+2.5}_{-1.3}$
$\Delta(1620)S_{31}$****	$+6.2 \pm 0.9$	-2.4 ± 0.2		6.6 ± 0.8
	$d_{\frac{1}{2}}$	$s_{\frac{3}{2}}$	$d_{\frac{3}{2}}$	
$[\Delta\frac{3}{2}^-]_1(1620)$	$-1.2^{+0.6}_{-1.2}$	$+3.4^{+2.2}_{-1.7}$	$+0.5^{+0.5}_{-0.2}$	$3.6^{+2.5}_{-1.8}$
$\Delta(1700)D_{33}$****		$+6.8 \pm 2.3$		6.8 ± 2.3
	$p_{\frac{1}{2}}$	$p_{\frac{3}{2}}$		
$[\Delta\frac{1}{2}^+]_1(1835)$	$-6.5^{+4.6}_{-4.1}$	$+4.7^{+3.1}_{-3.3}$		$8.0^{+5.1}_{-5.7}$
$\Delta(1740)P_{31}$[a]	-13.8 ± 1.9			13.8 ± 1.9
$[\Delta\frac{1}{2}^+]_2(1875)$	$+5.6^{+0.9}_{-0.4}$	$+2.6^{+0.4}_{-0.2}$		$6.1^{+1.0}_{-0.5}$
$\Delta(1910)P_{31}$****	$+4.9 \pm 1.1$			4.9 ± 1.1
	$p_{\frac{1}{2}}$	$p_{\frac{3}{2}}$	$f_{\frac{3}{2}}$	
$[\Delta\frac{3}{2}^+]_1(1230)$				
$\Delta(1232)P_{33}$****				
$[\Delta\frac{3}{2}^+]_2(1795)$	$+0.4^{+0.7}_{-0.3}$	$-0.9^{+0.6}_{-1.4}$	0.0	$1.0^{+1.6}_{-0.6}$
$\Delta(1600)P_{33}$***				
$[\Delta\frac{3}{2}^+]_3(1915)$	$+5.3^{+1.3}_{-0.5}$	$+6.6^{+1.6}_{-0.7}$	$-0.7^{+0.2}_{-0.4}$	$8.5^{+2.0}_{-0.8}$
$\Delta(1920)P_{33}$***				
$[\Delta\frac{3}{2}^+]_4(1985)$	$-6.3^{+2.6}_{-0.5}$	$+3.2^{+0.5}_{-1.4}$	$-2.2^{+1.5}_{-1.6}$	$7.4^{+1.2}_{-3.2}$
	$f_{\frac{1}{2}}$	$p_{\frac{3}{2}}$	$f_{\frac{3}{2}}$	
$[\Delta\frac{5}{2}^+]_1(1910)$	-0.7 ± 0.2	$+6.3^{+0.8}_{-0.4}$	$-0.7^{+0.1}_{-0.2}$	$6.4^{+0.8}_{-0.4}$
$\Delta(1750)F_{35}$[b]		-7.4 ± 1.9		7.4 ± 1.9
$\Delta(1905)F_{35}$****		$+16.8 \pm 1.3$		16.8 ± 1.3
$[\Delta\frac{5}{2}^+]_2(1990)$	$+2.6^{+2.8}_{-2.1}$	$+3.1 \pm 1.2$	$-3.1^{+2.4}_{-3.2}$	$5.1^{+4.2}_{-3.0}$
$\Delta(2000)F_{35}$**				
	$f_{\frac{1}{2}}$	$f_{\frac{3}{2}}$	$h_{\frac{3}{2}}$	
$[\Delta\frac{7}{2}^+]_1(1940)]$	$+1.3 \pm 0.1$	-2.3 ± 0.2	0.0	2.6 ± 0.2
$\Delta(1950)F_{37}$****		$+11.4 \pm 0.5$		11.4 ± 0.5

[a]First P_{31} state found in Ref. [19].

[b]Reference [19] finds two F_{35} states; this one and $\Delta(1905)F_{35}$.

cays of the baryon resonances, i.e., the decays of the type $A \to BC$, with the subsequent decay of one of the daughter hadrons, B, say, as illustrated in Fig. 1. One could use the prescription described above, which amounts to the narrow-width treatment of both daughter hadrons. Indeed, this has been done by Koniuk and Isgur [11] in their discussion of $\Delta\pi$ decays, as well as by Koniuk [14] in his treatment of $N\rho$ final states. The treatment of some final states in the narrow-width limit may, however, paint a somewhat inaccurate picture of couplings, particularly for states with masses near the threshold of the decay channel under consideration. For instance, the nominal mass of the $N\frac{1}{2}^+(1710)$ resonance lies very close to the threshold for decays into $N\rho$, if the ρ is treated as a narrow resonance, so that the result obtained would be very dependent on the mass chosen for the $N\frac{1}{2}^+(1710)$. Phase space is very limited and the decay width obtained in this way would be very small. Similarly, misleading results may be obtained for other states that are close to threshold.

Our approach is to take the width of the daughter

TABLE V. Results for the lightest few negative-parity nucleon resonances of each J in the $N = 3$ band in the $N\pi$, $N\eta$, and $N\omega$ channels. Notation as in Table I.

Model state $N\pi$ state/rating	$N\pi$	$N\eta$	$N\eta'$	$N\omega$	$N\omega$	$N\omega$	$\sqrt{\Gamma^{\rm tot}_{N\omega}}$
				$s_{\frac{1}{2}}$	$d_{\frac{3}{2}}$		
$[N\frac{1}{2}^-]_3(1945)$	$5.7^{+0.5}_{-1.6}$	$+2.4^{+1.5}_{-2.3}$	$-3.6^{+3.3}_{-2.3}$	$-0.9^{+3.0}_{-2.2}$	$-5.6^{+1.8}_{-0.7}$		5.7 ± 1.3
$N\frac{1}{2}^-(2090)S_{11}$*	7.9 ± 3.8						
$[N\frac{1}{2}^-]_4(2030)$	$3.7^{+0.5}_{-1.1}$	$-1.0^{+1.5}_{-1.1}$	$+1.3 \pm 1.3$	$-0.1^{+2.4}_{-1.9}$	$-2.8^{+1.2}_{-0.6}$		$2.8^{+1.1}_{-0.0}$
$[N\frac{1}{2}^-]_5(2070)$	$2.1^{+0.8}_{-1.5}$	$+0.1^{+0.3}_{-0.5}$	$-1.0^{+0.6}_{-0.2}$	$-1.4^{+1.9}_{-2.1}$	$-6.3^{+2.4}_{-1.3}$		$6.5^{+1.9}_{-2.5}$
$[N\frac{1}{2}^-]_6(2145)$	0.4 ± 0.1	$-0.4^{+0.4}_{-0.3}$	$+0.6 \pm 0.6$	$+0.2 \pm 0.0$	$-0.8^{+0.2}_{-0.1}$		$0.8^{+0.1}_{-0.2}$
$[N\frac{1}{2}^-]_7(2195)$	0.1 ± 0.1	$-0.9^{+0.5}_{-0.3}$	$+0.5^{+1.0}_{-0.8}$	$+0.3^{+0.4}_{-0.6}$	$-0.2^{+0.1}_{-0.0}$		$0.4^{+0.4}_{-0.0}$
				$d_{\frac{1}{2}}$	$s_{\frac{3}{2}}$	$d_{\frac{3}{2}}$	
$[N\frac{3}{2}^-]_3(1960)$	$8.2^{+0.7}_{-1.7}$	$+4.0 \pm 0.2$	$+2.3^{+1.3}_{-1.9}$	$-4.3^{+1.4}_{-0.4}$	$-0.2^{+2.0}_{-2.6}$	$-4.6^{+1.5}_{-0.5}$	$6.3^{+1.2}_{-1.7}$
$N\frac{3}{2}^-(2080)D_{13}$**	5.0 ± 2.5						
$[N\frac{3}{2}^-]_4(2055)$	$6.2^{+0.1}_{-0.6}$	$+0.4^{+0.0}_{-0.1}$	$+0.1 \pm 0.1$	$+2.0^{+0.6}_{-0.9}$	$-1.3^{+3.1}_{-3.8}$	$-2.7^{+1.1}_{-0.7}$	$3.6^{+3.0}_{-0.9}$
$[N\frac{3}{2}^-]_5(2095)$	$0.2^{+0.1}_{-0.2}$	$-0.2^{+0.1}_{-0.0}$	-0.2 ± 0.1	$-3.2^{+1.1}_{-0.5}$	$+1.9^{+1.3}_{-1.9}$	$+3.8^{+0.4}_{-1.2}$	$5.3^{+1.1}_{-1.9}$
$[N\frac{3}{2}^-]_6(2165)$	$1.5^{+0.1}_{-0.2}$	-2.4 ± 0.1	$-1.6^{+0.9}_{-0.5}$	$-1.0^{+0.2}_{-0.1}$	$+0.3 \pm 0.0$	$-1.1^{+0.3}_{-0.1}$	$1.5^{+0.1}_{-0.3}$
$[N\frac{3}{2}^-]_7(2180)$	$1.7^{+0.1}_{-0.2}$	-1.7 ± 0.1	$-1.3^{+0.6}_{-0.4}$	$-1.9^{+0.5}_{-0.2}$	0.0	$-1.8^{+0.4}_{-0.2}$	$2.6^{+0.3}_{-0.6}$
				$d_{\frac{1}{2}}$	$d_{\frac{3}{2}}$	$g_{\frac{3}{2}}$	
$[N\frac{5}{2}^-]_2(2080)$	$5.1^{+0.2}_{-0.8}$	$+3.5 \pm 0.4$	$+3.1^{+0.7}_{-1.9}$	$-2.2^{+0.7}_{-0.3}$	$-0.3^{+0.4}_{-0.1}$	$+2.0^{+1.9}_{-1.3}$	$2.9^{+1.6}_{-1.4}$
$[N\frac{5}{2}^-]_3(2095)$	$5.2^{+0.4}_{-1.0}$	$+0.0^{+0.4}_{-0.2}$	-0.2 ± 0.1	$-3.1^{+1.0}_{-0.3}$	$+3.3^{+0.2}_{-0.9}$	$+0.8^{+0.8}_{-0.5}$	$4.6^{+0.5}_{-1.4}$
$N\frac{5}{2}^-(2200)D_{15}$**	4.5 ± 2.3						
$[N\frac{5}{2}^-]_4(2180)$	$1.9^{+0.1}_{-0.3}$	-1.1 ± 0.0	$-0.8^{+0.4}_{-0.2}$	$-1.3^{+0.3}_{-0.1}$	$-1.8^{+0.4}_{-0.1}$	$+1.0^{+0.7}_{-0.5}$	$2.4^{+0.5}_{-0.6}$
$[N\frac{5}{2}^-]_5(2235)$	$2.0^{+0.1}_{-0.3}$	$+0.6^{+0.0}_{-0.1}$	$+0.4^{+0.1}_{-0.2}$	$+2.5^{+0.0}_{-0.4}$	$+3.1^{+0.1}_{-0.5}$	$+0.9^{+0.5}_{-0.4}$	$4.1^{+0.2}_{-0.7}$
$[N\frac{5}{2}^-]_6(2260)$	0.4 ± 0.1	$+0.1^{+0.1}_{-0.0}$	0.0	$+2.0^{+0.0}_{-0.3}$	$-1.5^{+0.2}_{-0.1}$	$+1.0^{+0.6}_{-0.5}$	$2.7^{+0.3}_{-0.5}$
$[N\frac{5}{2}^-]_7(2295)$	0.2 ± 0.1	$-1.6^{+0.3}_{-0.1}$	$-1.5^{+0.4}_{-0.1}$	$+0.1 \pm 0.0$	$-1.5^{+0.2}_{-0.0}$	$+0.4 \pm 0.2$	$1.6^{+0.1}_{-0.2}$
$[N\frac{5}{2}^-]_8(2305)$	0.3 ± 0.1	$-0.6^{+0.1}_{-0.0}$	$-0.6^{+0.2}_{-0.1}$	$+1.5^{+0.0}_{-0.2}$	$-0.5^{+0.1}_{-0.0}$	$+0.8 \pm 0.4$	$1.8^{+0.2}_{-0.3}$
				$g_{\frac{1}{2}}$	$d_{\frac{3}{2}}$	$g_{\frac{3}{2}}$	
$[N\frac{7}{2}^-]_1(2090)$	6.9 ± 1.3	$+2.5 \pm 0.7$	$+0.6^{+0.5}_{-0.4}$	$-1.5^{+0.6}_{-0.7}$	$-3.7^{+0.4}_{-0.2}$	$-1.7^{+0.7}_{-0.8}$	4.4 ± 0.8
$N\frac{7}{2}^-(2190)G_{17}$****	7.0 ± 3.0						
$[N\frac{7}{2}^-]_2(2205)$	4.0 ± 1.1	-0.1 ± 0.0	0.0	$-0.2^{+0.1}_{-0.2}$	$-5.1^{+1.0}_{-0.2}$	$+0.3 \pm 0.2$	$5.1^{+0.2}_{-1.0}$
$[N\frac{7}{2}^-]_3(2255)$	0.8 ± 0.2	0.0	0.0	$+1.8^{+1.1}_{-0.9}$	-0.3 ± 0.0	$-1.6^{+0.8}_{-0.9}$	$2.5^{+1.4}_{-1.2}$
$[N\frac{7}{2}^-]_4(2305)$	0.4 ± 0.1	-0.8 ± 0.3	-0.3 ± 0.2	$+0.6 \pm 0.3$	-1.4 ± 0.1	$+0.6 \pm 0.3$	$1.6^{+0.3}_{-0.2}$
$[N\frac{7}{2}^-]_5(2355)$	1.1 ± 0.3	$+0.4 \pm 0.1$	$+0.2 \pm 0.1$	$+0.1 \pm 0.0$	$-0.8^{+0.1}_{-0.0}$	$-0.6^{+0.2}_{-0.3}$	1.0 ± 0.2
			$g_{\frac{1}{2}}$	$g_{\frac{3}{2}}$	$i_{\frac{3}{2}}$		
$[N\frac{9}{2}^-]_1(2215)$	2.5 ± 0.3	-2.1 ± 0.4	-0.7 ± 0.3	-1.0 ± 0.3	$+1.7 \pm 0.5$	0.0	2.0 ± 0.6
$N\frac{9}{2}^-(2250)G_{19}$****	5.9 ± 1.9						

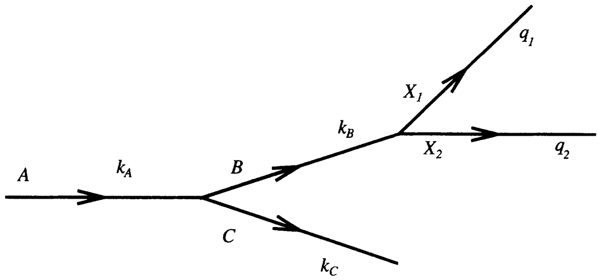

FIG. 1. The quasi-two-body decay $A \to (X_1 X_2)_B C$, showing the kinematic variables.

hadron into account by replacing the Dirac δ function in Eq. (3). One may regard the δ-function as arising from the narrow-width limit of the energy denominator:

$$\frac{1}{M_a - E_b - E_c - i\varepsilon}$$

$$= P\frac{1}{M_a - E_b - E_c} + i\pi\delta\left(M_a - E_b - E_c\right), \qquad (5)$$

where ε is related to the total width of the "unstable" final state. For daughter hadrons that are broad, the energy denominator becomes

TABLE VI. Results for the lightest few negative-parity nucleon resonances of each J in the $N = 3$ band in the $\Delta\pi$ and $N\rho$ channels. Notation as in Table II.

Model state $N\pi$ state/rating	$\Delta\pi$	$\Delta\pi$	$\sqrt{\Gamma^{\rm tot}_{\Delta\pi}}$	$N\rho$	$N\rho$	$N\rho$	$\sqrt{\Gamma^{\rm tot}_{N\rho}}$
	d			$s_{\frac{1}{2}}$	$d_{\frac{3}{2}}$		
$[N\frac{1}{2}^-]_3(1945)$	$-6.7^{+1.3}_{-1.5}$		$6.7^{+1.5}_{-1.3}$	$+2.3 \pm 0.6$	$-17.9^{+7.3}_{-3.8}$		$18.1^{+3.9}_{-7.3}$
$N\frac{1}{2}^-(2090)S_{11}$*	-5.1 ± 5.9		5.1 ± 5.9	$+14.2 \pm 4.3$	0.0 ± 2.0		14.2 ± 4.3
$[N\frac{1}{2}^-]_4(2030)$	$-5.7^{+1.1}_{-1.3}$		$5.7^{+1.3}_{-1.1}$	-0.5 ± 0.2	-0.9 ± 0.3		1.1 ± 0.4
$[N\frac{1}{2}^-]_5(2070)$	$+13.1^{+3.3}_{-2.7}$		$13.1^{+3.3}_{-2.7}$	$+3.6^{+1.3}_{-1.2}$	$-6.9^{+3.3}_{-1.3}$		$7.8^{+1.7}_{-3.5}$
$[N\frac{1}{2}^-]_6(2145)$	$+1.0 \pm 0.2$		1.0 ± 0.2	$-0.1^{+0.0}_{-0.1}$	$+2.3^{+1.4}_{-0.6}$		$2.3^{+1.4}_{-0.6}$
$[N\frac{1}{2}^-]_7(2195)$	$+2.1 \pm 0.1$		2.1 ± 0.1	$-0.4^{+0.1}_{-0.3}$	$+3.5^{+2.0}_{-0.5}$		$3.5^{+2.0}_{-0.5}$
	s	d		$d_{\frac{1}{2}}$	$s_{\frac{3}{2}}$	$d_{\frac{3}{2}}$	
$[N\frac{3}{2}^-]_3(1960)$	$-1.4^{+0.5}_{-1.2}$	$-5.3^{+0.9}_{-0.8}$	$5.5^{+1.2}_{-1.0}$	$-4.4^{+1.9}_{-0.7}$	-6.2 ± 2.4	$-11.3^{+4.9}_{-1.6}$	$13.6^{+2.7}_{-5.8}$
$N\frac{3}{2}^-(2080)D_{13}$**	-3.9 ± 4.0	$+9.7 \pm 4.2$	10.5 ± 4.2		-10.7 ± 3.5		10.7 ± 3.5
$[N\frac{3}{2}^-]_4(2055)$	$+2.9 \pm 0.7$	$+10.7^{+2.5}_{-2.1}$	$11.1^{+2.6}_{-2.2}$	$-5.1^{+2.7}_{-0.8}$	$-2.2^{+1.0}_{-0.5}$	$-5.3^{+2.7}_{-0.6}$	$7.7^{+1.1}_{-3.9}$
$[N\frac{3}{2}^-]_5(2095)$	$-6.1^{+1.0}_{-4.9}$	$+3.3^{+0.4}_{-0.5}$	$6.9^{+4.7}_{-1.1}$	$+2.1^{+0.6}_{-0.8}$	$+1.0 \pm 0.2$	$-2.8^{+1.1}_{-0.5}$	$3.6^{+0.8}_{-1.4}$
$[N\frac{3}{2}^-]_6(2165)$	-0.3 ± 0.1	$-3.1^{+0.7}_{-0.9}$	$3.1^{+0.9}_{-0.7}$	$+1.3^{+0.7}_{-0.2}$	$+0.2 \pm 0.1$	$+1.1^{+0.6}_{-0.2}$	$1.7^{+0.9}_{-0.3}$
$[N\frac{3}{2}^-]_7(2180)$	$+0.1 \pm 0.1$	$-5.0^{+1.1}_{-1.3}$	$5.0^{+1.3}_{-1.1}$	$+0.7^{+0.4}_{-0.1}$	$+0.1 \pm 0.0$	$+1.5^{+0.8}_{-0.2}$	$1.6^{+0.9}_{-0.2}$
	d	g		$d_{\frac{1}{2}}$	$d_{\frac{3}{2}}$	$g_{\frac{3}{2}}$	
$[N\frac{5}{2}^-]_2(2080)$	-4.2 ± 0.4	$+2.1^{+2.4}_{-1.2}$	$4.7^{+1.8}_{-0.8}$	$-3.9^{+0.9}_{-2.1}$	$+5.9^{+3.1}_{-1.4}$	$+5.3^{+9.3}_{-3.3}$	$8.8^{+9.3}_{-3.0}$
$[N\frac{5}{2}^-]_3(2095)$	$+7.8^{+1.1}_{-1.3}$	$+0.9^{+0.8}_{-0.4}$	$7.9^{+1.2}_{-1.3}$	$-0.8^{+0.3}_{-0.4}$	$+0.8^{+0.6}_{-0.4}$	$+2.1^{+2.4}_{-1.2}$	$2.3^{+2.4}_{-1.3}$
$N\frac{5}{2}^-(2200)D_{15}$**							
$[N\frac{5}{2}^-]_4(2180)$	$-5.8^{+1.2}_{-1.3}$	$+2.4^{+2.0}_{-1.2}$	$6.2^{+2.1}_{-1.5}$	$+0.1^{+0.1}_{-0.0}$	$+2.2^{+1.2}_{-0.3}$	$+0.2^{+0.2}_{-0.1}$	$2.2^{+1.2}_{-0.3}$
$[N\frac{5}{2}^-]_5(2235)$	$+1.8^{+0.3}_{-0.4}$	$-6.7^{+3.2}_{-5.1}$	$7.0^{+5.1}_{-3.2}$	$-3.5^{+0.7}_{-1.6}$	$-0.5^{+0.1}_{-0.2}$	$+1.7^{+2.0}_{-0.9}$	$4.0^{+2.4}_{-1.0}$
$[N\frac{5}{2}^-]_6(2260)$	$-6.7^{+1.3}_{-0.8}$	$-2.5^{+1.2}_{-1.8}$	$7.1^{+1.5}_{-1.6}$	$-2.2^{+0.6}_{-0.9}$	$+1.8^{+0.7}_{-0.5}$	$-0.3^{+0.2}_{-0.3}$	$2.9^{+1.1}_{-0.8}$
$[N\frac{5}{2}^-]_7(2295)$	$-4.8^{+1.1}_{-0.9}$	$+0.9^{+0.6}_{-0.4}$	$4.9^{+1.0}_{-1.2}$	$+1.2^{+0.2}_{-0.4}$	-0.4 ± 0.1	$-2.5^{+1.4}_{-1.8}$	$2.8^{+1.7}_{-1.4}$
$[N\frac{5}{2}^-]_8(2305)$	$-4.4^{+0.9}_{-0.6}$	$-2.8^{+1.3}_{-1.8}$	$5.2^{+1.5}_{-1.4}$	$-1.4^{+0.5}_{-0.2}$	$+0.6^{+0.1}_{-0.2}$	$-0.9^{+0.5}_{-0.6}$	$1.8^{+0.6}_{-0.7}$
	d	g		$g_{\frac{1}{2}}$	$d_{\frac{3}{2}}$	$g_{\frac{3}{2}}$	
$[N\frac{7}{2}^-]_1(2090)$	-1.3 ± 0.2	$-2.6^{+0.9}_{-1.3}$	$2.9^{+1.3}_{-0.9}$	$-1.9^{+0.7}_{-1.5}$	$-11.4^{+1.0}_{-3.8}$	$-3.7^{+1.4}_{-3.0}$	$12.1^{+4.8}_{-1.4}$
$N\frac{7}{2}^-(2190)G_{17}$****					-12.5 ± 1.2		12.5 ± 1.2
$[N\frac{7}{2}^-]_2(2205)$	$-3.7^{+0.8}_{-0.7}$	$+5.4^{+4.3}_{-2.6}$	$6.5^{+4.1}_{-2.5}$	$-1.1^{+0.5}_{-1.4}$	$-1.9^{+0.3}_{-1.0}$	$-2.3^{+1.1}_{-3.0}$	$3.2^{+3.3}_{-1.1}$
$[N\frac{7}{2}^-]_3(2255)$	$+11.8^{+1.5}_{-2.3}$	$+1.1^{+0.7}_{-0.5}$	$11.9^{+1.6}_{-2.3}$	$-2.0^{+1.1}_{-1.9}$	$-1.2^{+0.2}_{-0.4}$	$+0.8^{+0.8}_{-0.4}$	$2.4^{+2.1}_{-1.1}$
$[N\frac{7}{2}^-]_4(2305)$	-0.3 ± 0.1	$+4.3^{+2.8}_{-2.0}$	$4.3^{+2.8}_{-2.0}$	-0.1 ± 0.1	$+1.9^{+0.3}_{-0.6}$	-0.3 ± 0.2	$2.0^{+0.3}_{-0.7}$
$[N\frac{7}{2}^-]_5(2355)$	$+0.9 \pm 0.1$	$-0.1^{+0.0}_{-0.1}$	0.9 ± 0.1	$-1.0^{+0.6}_{-0.3}$	-2.0 ± 0.5	$-0.8^{+0.5}_{-0.3}$	$2.4^{+0.6}_{-0.7}$
	g	i		$g_{\frac{1}{2}}$	$g_{\frac{3}{2}}$	$i_{\frac{3}{2}}$	
$[N\frac{9}{2}^-]_1(2215)$	$+6.3^{+2.3}_{-1.8}$	0.0	$6.3^{+2.3}_{-1.8}$	$+0.9^{+0.5}_{-0.4}$	$-1.5^{+0.6}_{-0.9}$	0.0	$1.8^{+1.0}_{-0.7}$
$N\frac{9}{2}^-(2250)G_{19}$****							

$$\frac{1}{M_a - E_b - E_c - i\frac{\Gamma_t}{2}} = \frac{M_a - E_b - E_c + i\frac{\Gamma_t}{2}}{(M_a - E_b - E_c)^2 + \frac{\Gamma_t^2}{4}}, \quad (6)$$

implying the replacement

$$\delta\left(M_a - E_b - E_c\right) \to \frac{\Gamma_t}{\pi\left[(M_a - E_b - E_c)^2 + \frac{\Gamma_t^2}{4}\right]}. \quad (7)$$

The decay rate for $A \to (X_1X_2)_B C$ then generalizes to [22]

$$\Gamma = \int_0^{k_{\max}} dk \frac{k^2\,|M(k)|^2\,\Gamma_t(k)}{[M_a - E_b(k) - E_c(k)]^2 + \frac{\Gamma_t(k)^2}{4}}, \quad (8)$$

where $\Gamma_t(k)$ is the energy-dependent total width of the unstable daughter hadron, B in this case. Our prescription for this quantity depends on the daughter hadron being studied. For states such as the Δ and the ρ, where the hadron decays with a branching fraction of close to 100% into a single two-body final state, we use the energy-dependent width as calculated in our version of the ${}^3P_0$ model. For a state such as the Roper resonance $N\frac{1}{2}^+(1440)$, which has a branching fraction of about 70% to $N\pi$, we write

$$\Gamma_t(k) = \Gamma_{N\pi}(k) + 0.3\Gamma_0\left(\frac{k}{k_0}\right)^{2\ell+1}\frac{k^2+\kappa^2}{k_0^2+\kappa^2}, \quad (9)$$

where Γ_0 is the total width of the Roper at the pole position. The first term is the energy-dependent $N\pi$ width of the Roper, calculated in the ${}^3P_0$ model. The second term is the width of the Roper for decays into other final states such as $N\pi\pi$. Here k_0 is a reference momentum, chosen to be the momentum of the Δ in the $\Delta\pi$ decay of the Roper measured in its rest frame, and κ is a phenomenological constant, chosen to be 0.35 MeV. The

TABLE VII. Results for nucleons in the $N = 3$ band in the $\Delta(1600)\pi$ and $N(1440)\pi$ channels. Notation as in Table II.

Model state $N\pi$ state/rating	$\Delta(1600)\pi$	$\Delta(1600)\pi$	$\sqrt{\Gamma^{\rm tot}_{\Delta(1600)\pi}}$	$N(1440)\pi$
	d			
$[N\frac{1}{2}^-]_3(1945)$	$6.4^{+3.4}_{-2.8}$		$6.4^{+3.4}_{-2.8}$	2.9 ± 0.4
$N\frac{1}{2}^-(2090)S_{11}$*				-11.1 ± 4.4
$[N\frac{1}{2}^-]_4(2030)$	$4.2^{+2.5}_{-2.0}$		$4.2^{+2.5}_{-2.0}$	$2.6^{+0.5}_{-0.3}$
$[N\frac{1}{2}^-]_5(2070)$	$-1.2^{+0.3}_{-0.1}$		$1.2^{+0.1}_{-0.3}$	-5.4 ± 0.2
$[N\frac{1}{2}^-]_6(2145)$	-0.1 ± 0.0		0.1 ± 0.0	-0.3 ± 0.0
$[N\frac{1}{2}^-]_7(2195)$	$2.4^{+1.6}_{-1.1}$		$2.4^{+1.6}_{-1.1}$	0.4 ± 0.1
	s	d		
$[N\frac{3}{2}^-]_3(1960)$	3.8 ± 0.3	$5.9^{+3.2}_{-2.6}$	$7.0^{+2.9}_{-2.2}$	$-6.3^{+1.8}_{-1.3}$
$N\frac{3}{2}^-(2080)D_{13}$**				
$[N\frac{3}{2}^-]_4(2055)$	$4.6^{+0.2}_{-0.8}$	-3.3 ± 1.3	$5.6^{+0.9}_{-1.4}$	$2.4^{+2.0}_{-1.3}$
$[N\frac{3}{2}^-]_5(2095)$	$7.7^{+0.5}_{-1.7}$	$-4.6^{+2.0}_{-2.5}$	$9.0^{+1.9}_{-2.4}$	$-4.2^{+1.6}_{-1.7}$
$[N\frac{3}{2}^-]_6(2165)$	0.6 ± 0.1	$0.2^{+0.1}_{-0.0}$	0.6 ± 0.1	-0.3 ± 0.0
$[N\frac{3}{2}^-]_7(2180)$	-0.2 ± 0.0	1.8 ± 0.6	1.9 ± 0.6	$-0.6^{+0.1}_{-0.0}$
	d	g		
$[N\frac{5}{2}^-]_2(2080)$	5.0 ± 2.3	$-0.6^{+0.4}_{-0.7}$	$5.0^{+2.4}_{-2.3}$	$-1.1^{+0.8}_{-1.6}$
$[N\frac{5}{2}^-]_3(2095)$	$-6.4^{+2.7}_{-3.0}$	$-0.3^{+0.1}_{-0.3}$	$6.4^{+3.0}_{-2.7}$	$-3.7^{+1.1}_{-0.8}$
$N\frac{5}{2}^-(2200)D_{15}$**				
$[N\frac{5}{2}^-]_4(2180)$	$2.3^{+0.7}_{-0.8}$	$-0.7^{+0.4}_{-0.6}$	2.4 ± 0.8	$-0.3^{+0.2}_{-0.4}$
$[N\frac{5}{2}^-]_5(2235)$	$1.7^{+1.0}_{-0.7}$	$2.0^{+1.5}_{-1.0}$	$2.6^{+1.8}_{-1.2}$	$-0.4^{+0.3}_{-0.4}$
$[N\frac{5}{2}^-]_6(2260)$	$-3.2^{+1.5}_{-2.1}$	$0.8^{+0.5}_{-0.4}$	$3.3^{+2.2}_{-1.5}$	$-1.7^{+0.5}_{-0.4}$
$[N\frac{5}{2}^-]_7(2295)$	$2.6^{+0.6}_{-0.7}$	$-0.3^{+0.1}_{-0.2}$	$2.6^{+0.7}_{-0.8}$	0.4 ± 0.1
$[N\frac{5}{2}^-]_8(2305)$	$0.4^{+0.1}_{-0.0}$	$0.8^{+0.5}_{-0.4}$	$0.9^{+0.5}_{-0.3}$	$-0.1^{+0.0}_{-0.1}$
	d	g		
$[N\frac{7}{2}^-]_1(2090)$	0.2 ± 0.0	$0.8^{+0.4}_{-0.3}$	$0.8^{+0.4}_{-0.3}$	$-2.5^{+0.8}_{-1.0}$
$N\frac{7}{2}^-(2190)G_{17}$****				
$[N\frac{7}{2}^-]_2(2205)$	0.6 ± 0.1	$-1.6^{+0.8}_{-1.3}$	$1.7^{+1.3}_{-0.8}$	$-1.4^{+0.6}_{-0.9}$
$[N\frac{7}{2}^-]_3(2255)$	-1.7 ± 0.1	-0.3 ± 0.2	$1.8^{+0.2}_{-0.1}$	-0.3 ± 0.1
$[N\frac{7}{2}^-]_4(2305)$	$0.0^{+0.1}_{-0.0}$	$-1.2^{+0.6}_{-0.8}$	$1.2^{+0.8}_{-0.6}$	-0.1 ± 0.1
$[N\frac{7}{2}^-]_5(2355)$	-0.2 ± 0.0	0.0 ± 0.0	0.2 ± 0.1	$-0.4^{+0.1}_{-0.2}$
	g	i		
$[N\frac{9}{2}^-]_1(2215)$	$-1.9^{+0.6}_{-0.7}$	0.0 ± 0.0	$1.9^{+0.7}_{-0.6}$	-1.0 ± 0.3
$N\frac{9}{2}^-(2250)G_{19}$****				

power $2\ell + 1$ represents the dependence of the energy-dependent width on angular momentum, and is chosen here to be unity. We note that the results we present are largely insensitive to changes in κ, ℓ, and k_0; nor are they overly sensitive to the inclusion of the second term in Eq. (9.)

The variable of integration in the expression above is k, the magnitude of the three-momentum of the daughter hadron B. In the rest frame of A, this ranges from $k = 0$ (X_1 and X_2 back to back, with $\mathbf{q}_1 = -\mathbf{q}_2$) to $k_{\max}$ (X_1 and X_2 collinear). These limits correspond to $M_b(k) = M_a - M_c$, and $M_b(k) = M_{X_1} + M_{X_2}$, respectively, where $M_b(k)$ is the momentum-dependent effective mass of daughter hadron B.

TABLE VIII. Results in the $N\pi$ and $\Delta\pi$ channels for the lightest few negative-parity Δ resonances of each J in the $N = 3$ band, and for the lightest few Δ resonances for J^P values which first appear in the $N = 4$, 5, and 6 bands. Notation as in Table I.

Model state $N\pi$ state/rating	$N\pi$	$\Delta\pi$	$\Delta\pi$	$\sqrt{\Gamma^{\rm tot}_{\Delta\pi}}$
		d		
$[\Delta\frac{1}{2}^-]_2(2035)$	1.2 ± 0.2	$+8.2^{+0.6}_{-0.7}$		$8.2^{+0.6}_{-0.7}$
$\Delta\frac{1}{2}^-(1900)S_{31}$***	4.1 ± 2.2	$+6.4 \pm 1.9$		6.4 ± 1.9
$[\Delta\frac{1}{2}^-]_3(2140)$	$3.1^{+0.4}_{-1.1}$	$-4.4^{+0.8}_{-0.7}$		$4.4^{+0.7}_{-0.8}$
$\Delta\frac{1}{2}^-(2150)S_{31}$*	4.0 ± 1.5			
		s	d	
$[\Delta\frac{3}{2}^-]_2(2080)$	$2.2^{+0.1}_{-0.2}$	$+3.1 \pm 0.7$	$+6.6^{+1.4}_{-1.9}$	$7.4^{+1.6}_{-2.0}$
$\Delta\frac{3}{2}^-(1940)D_{33}$*	3.2 ± 1.4	$+5.5 \pm 6.3$	$+13.7 \pm 7.8$	14.7 ± 7.6
$[\Delta\frac{3}{2}^-]_3(2145)$	$2.2^{+0.1}_{-0.3}$	-3.4 ± 0.8	$+6.3^{+1.0}_{-1.1}$	$7.1^{+1.3}_{-1.4}$
		d	g	
$[\Delta\frac{5}{2}^-]_1(2155)$	5.2 ± 0.1	$+3.9 \pm 0.2$	-0.7 ± 0.1	$4.0^{+0.2}_{-0.3}$
$\Delta\frac{5}{2}^-(1930)D_{35}$***	5.0 ± 2.3			
$[\Delta\frac{5}{2}^-]_2(2165)$	0.6 ± 0.1	$+7.0^{+1.3}_{-1.4}$	$+4.1^{+3.5}_{-2.0}$	$8.1^{+3.1}_{-2.1}$
$[\Delta\frac{5}{2}^-]_3(2265)$	$2.4^{+0.5}_{-0.7}$	$-5.4^{+0.7}_{-0.5}$	$+3.6^{+1.4}_{-1.2}$	6.5 ± 1.2
$\Delta\frac{5}{2}^-(2350)D_{35}$*	7.7 ± 5.3			
$[\Delta\frac{5}{2}^-]_4(2325)$	0.1 ± 0.1	$-4.6^{+0.8}_{-0.4}$	$+3.2^{+2.0}_{-1.5}$	$5.6^{+1.6}_{-1.5}$
$[\Delta\frac{7}{2}^-]_1(2230)$	2.1 ± 0.6	$+6.9^{+1.2}_{-1.3}$	$+3.8^{+3.1}_{-1.8}$	$7.8^{+2.7}_{-2.0}$
$\Delta\frac{7}{2}^-(2200)G_{37}$*	5.2 ± 1.9			
$[\Delta\frac{7}{2}^-]_2(2295)$	1.8 ± 0.4	$-7.4^{+1.4}_{-0.9}$	$+4.4^{+2.9}_{-2.1}$	$8.7^{+2.4}_{-2.2}$
		g	i	
$[\Delta\frac{9}{2}^-]_1(2295)$	4.8 ± 0.9	$+7.0^{+4.4}_{-3.7}$	0.0	$7.0^{+4.4}_{-3.7}$
$\Delta\frac{9}{2}^-(2400)G_{39}$**	4.1 ± 2.1			
		f	h	
$[\Delta\frac{7}{2}^+]_2(2370)$	$1.5^{+0.6}_{-0.9}$	$+5.7^{+0.4}_{-1.3}$	0.0	$5.7^{+0.4}_{-1.3}$
$\Delta\frac{7}{2}^+(2390)F_{37}$*	4.9 ± 2.0			
$[\Delta\frac{7}{2}^+]_3(2460)$	1.1 ± 0.1	$-3.9^{+1.1}_{-0.8}$	$+3.7^{+2.5}_{-1.9}$	$5.4^{+2.4}_{-2.0}$
$[\Delta\frac{9}{2}^+]_1(2420)$	$1.2^{+0.5}_{-0.4}$	-0.4 ± 0.1	$+2.6^{+2.2}_{-1.4}$	$2.6^{+2.2}_{-1.4}$
$\Delta\frac{9}{2}^+(2300)H_{39}$**	5.1 ± 2.2			
$[\Delta\frac{9}{2}^+]_2(2505)$	0.4 ± 0.1	$-7.6^{+1.4}_{-0.3}$	$+1.1^{+0.5}_{-0.5}$	$7.6^{+0.4}_{-1.4}$
		h	j	
$[\Delta\frac{11}{2}^+]_1(2450)$	2.9 ± 0.7	$+3.7^{+1.8}_{-1.5}$	0.0	$3.7^{+1.8}_{-1.5}$
$\Delta\frac{11}{2}^+(2420)H_{3\ 11}$****	6.7 ± 2.8			
$[\Delta\frac{13}{2}^+]_1(2880)$	0.8 ± 0.2	-0.1 ± 0.0	$+2.8^{+1.1}_{-0.9}$	$2.8^{+1.1}_{-0.9}$
$[\Delta\frac{13}{2}^+]_2(2955)$	0.2 ± 0.1	-3.3 ± 0.1	$+0.5 \pm 0.2$	3.4 ± 0.1
		i	k	
$[\Delta\frac{13}{2}^-]_1(2750)$	2.2 ± 0.4	$+3.8^{+0.9}_{-0.8}$	0.0	$3.8^{+0.9}_{-0.8}$
$\Delta\frac{13}{2}^-(2750)I_{3\ 13}$**	3.7 ± 1.5			
		j	l	
$[\Delta\frac{15}{2}^+]_1(2920)$	1.6 ± 0.3	$+2.8^{+0.7}_{-0.6}$	0.0	$2.8^{+0.7}_{-0.6}$
$\Delta\frac{15}{2}^+(2950)K_{3\ 15}$**	3.6 ± 1.5			
$[\Delta\frac{15}{2}^+]_2(3085)$	0.4 ± 0.1	$+0.8 \pm 0.2$	0.0	0.8 ± 0.2

TABLE IX. Results in the $N\rho$ channel for the lightest few negative-parity Δ resonances of each J in the $N=3$ band, and for the lightest few Δ resonances for J^P values which first appear in the $N=4$, 5, and 6 bands. Notation as in Table II.

Model state $N\pi$ state/rating	$N\rho$	$N\rho$	$N\rho$	$\sqrt{\Gamma^{\rm tot}_{N\rho}}$
	$s_{\frac{1}{2}}$	$d_{\frac{3}{2}}$		
$[\Delta\frac{1}{2}^-]_2(2035)$	$+2.5\pm0.6$	$+1.5^{+0.5}_{-0.3}$		$2.9^{+0.8}_{-0.6}$
$\Delta\frac{1}{2}^-(1900)S_{31}$***	-3.5 ± 2.7	-9.3 ± 1.7		9.9 ± 1.9
$[\Delta\frac{1}{2}^-]_3(2140)$	-2.2 ± 0.6	$+2.3^{+1.0}_{-0.4}$		$3.2^{+1.2}_{-0.7}$
$\Delta\frac{1}{2}^-(2150)S_{31}$*				
	$d_{\frac{1}{2}}$	$s_{\frac{3}{2}}$	$d_{\frac{3}{2}}$	
$[\Delta\frac{3}{2}^-]_2(2080)$	$-3.8^{+2.3}_{-2.5}$	$+1.0\pm0.3$	$+1.4^{+0.9}_{-0.8}$	$4.2^{+2.7}_{-2.4}$
$\Delta\frac{3}{2}^-(1940)D_{33}$*				
$[\Delta\frac{3}{2}^-]_3(2145)$	$+0.4^{+0.2}_{-0.1}$	$+2.5\pm0.7$	$-3.9^{+0.7}_{-1.7}$	$4.7^{+1.8}_{-1.0}$
	$d_{\frac{1}{2}}$	$d_{\frac{3}{2}}$	$g_{\frac{3}{2}}$	
$[\Delta\frac{5}{2}^-]_1(2155)$	$+0.1\pm0.0$	$-2.9^{+0.5}_{-0.8}$	$-0.1^{+0.0}_{-0.1}$	$2.9^{+0.8}_{-0.5}$
$\Delta\frac{5}{2}^-(1930)D_{35}$***				
$[\Delta\frac{5}{2}^-]_2(2165)$	$+6.2^{+3.1}_{-0.9}$	$-1.7^{+0.2}_{-0.8}$	$+1.1^{+1.5}_{-0.5}$	$6.6^{+3.5}_{-1.0}$
$[\Delta\frac{5}{2}^-]_3(2265)$	$+2.5^{+0.0}_{-0.6}$	$+3.8^{+0.2}_{-1.0}$	$-2.8^{+1.2}_{-0.8}$	$5.3^{+0.6}_{-1.5}$
$\Delta\frac{5}{2}^-(2350)D_{35}$*				
$[\Delta\frac{5}{2}^-]_4(2325)$	$+1.5^{+0.1}_{-0.5}$	$+5.0^{+0.4}_{-1.8}$	$-1.0^{+0.6}_{-0.5}$	$5.3^{+0.5}_{-1.9}$
	$g_{\frac{1}{2}}$	$d_{\frac{3}{2}}$	$g_{\frac{3}{2}}$	
$[\Delta\frac{7}{2}^-]_1(2230)$	$-2.8^{+1.4}_{-3.7}$	$+3.0^{+1.6}_{-0.4}$	$+0.7^{+1.0}_{-0.4}$	$4.2^{+4.0}_{-1.2}$
$\Delta\frac{7}{2}^-(2200)G_{37}$*				
$[\Delta\frac{7}{2}^-]_2(2295)$	$+1.4^{+1.0}_{-0.8}$	$+4.8^{+0.9}_{-1.5}$	$-3.3^{+1.9}_{-2.3}$	$6.0^{+2.3}_{-2.4}$
	$g_{\frac{1}{2}}$	$g_{\frac{3}{2}}$	$i_{\frac{3}{2}}$	
$[\Delta\frac{9}{2}^-]_1(2295)$	$+3.0^{+0.9}_{-1.9}$	$-4.9^{+3.1}_{-1.4}$	0.0	$5.7^{+1.6}_{-3.6}$
$\Delta\frac{9}{2}^-(2400)G_{39}$**				
	$f_{\frac{1}{2}}$	$f_{\frac{3}{2}}$	$h_{\frac{3}{2}}$	
$[\Delta\frac{7}{2}^+]_2(2370)$	$+1.7\pm0.4$	-3.0 ± 0.7	0.0	3.5 ± 0.8
$\Delta\frac{7}{2}^+(2390)F_{37}$*				
$[\Delta\frac{7}{2}^+]_3(2460)$	$+2.9^{+0.0}_{-1.2}$	$+4.5^{+0.1}_{-1.9}$	$-1.1^{+0.6}_{-0.4}$	$5.4^{+0.2}_{-2.3}$
	$h_{\frac{1}{2}}$	$f_{\frac{3}{2}}$	$h_{\frac{3}{2}}$	
$[\Delta\frac{9}{2}^+]_1(2420)$	$-0.6^{+0.4}_{-0.5}$	$+3.1^{+1.1}_{-1.4}$	$-0.7^{+0.4}_{-0.6}$	$3.2^{+1.3}_{-1.5}$
$\Delta\frac{9}{2}^+(2300)H_{39}$**				
$[\Delta\frac{9}{2}^+]_2(2505)$	$+2.9\pm1.0$	$+1.3\pm0.1$	-2.8 ± 1.0	$4.2^{+1.4}_{-1.3}$
	$h_{\frac{1}{2}}$	$h_{\frac{3}{2}}$	$j_{\frac{3}{2}}$	
$[\Delta\frac{11}{2}^+]_1(2450)$	$+1.5^{+0.4}_{-0.6}$	$-2.4^{+1.0}_{-0.6}$	0.0	$2.8^{+0.7}_{-1.2}$
$\Delta\frac{11}{2}^+(2420)H_{3\ 11}$****				
	$j_{\frac{1}{2}}$	$h_{\frac{3}{2}}$	$j_{\frac{3}{2}}$	
$[\Delta\frac{13}{2}^+]_1(2880)$	$-0.6^{+0.3}_{-0.5}$	$+1.6^{+0.6}_{-0.5}$	$-0.8^{+0.4}_{-0.7}$	$1.9^{+1.0}_{-0.6}$
$[\Delta\frac{13}{2}^+]_2(2955)$	$+2.4^{+1.8}_{-1.2}$	$+0.6\pm0.2$	$-2.2^{+1.0}_{-1.6}$	$3.3^{+2.4}_{-1.6}$
	$i_{\frac{1}{2}}$	$i_{\frac{3}{2}}$	$k_{\frac{3}{2}}$	
$[\Delta\frac{13}{2}^-]_1(2750)$	$+1.3^{+0.6}_{-0.4}$	$-2.0^{+0.6}_{-0.9}$	0.0	$2.4^{+1.1}_{-0.7}$
$\Delta\frac{13}{2}^-(2750)I_{3\ 13}$**				
	$j_{\frac{1}{2}}$	$j_{\frac{3}{2}}$	$l_{\frac{3}{2}}$	
$[\Delta\frac{15}{2}^+]_1(2920)$	$+1.4^{+0.6}_{-0.5}$	$-2.1^{+0.7}_{-1.0}$	0.0	$2.5^{+1.2}_{-0.9}$
$\Delta\frac{15}{2}^+(2950)K_{3\ 15}$**				
$[\Delta\frac{15}{2}^+]_2(3085)$	$+0.5\pm0.2$	-0.7 ± 0.3	0.0	0.8 ± 0.4

In what follows we present decay amplitudes for decays to $\Delta\pi$, $N\rho$, $N\eta$, $N\eta'$, $N\omega$, $N\frac{1}{2}^+(1440)\pi$, and $\Delta\frac{3}{2}^+(1600)\pi$. For most of these channels we use the prescription described above. For $N\eta$ and $N\omega$, the unstable daughter hadrons are sufficiently narrow that we ignore their widths. This is because a total width of less than 10 MeV is well within the expected accuracy of the model, so that such states may be safely treated as being narrow.

We end this subsection with a comparison of the method we use for treating quasi-two-body decays with other prescriptions found in the literature. In their treatment of $N\rho$ decays, Stancu and Stassart [16] use the pre-

TABLE X. Results in the $\Delta(1600)\pi$ and $N(1440)\pi$ channels for the lightest few negative-parity Δ resonances in the $N = 3$ band, and for the lightest few Δ resonances for J^P values that first appear in the $N = 4, 5$, and 6 bands. Notation as in Table II.

Model state $N\pi$ state/rating	$\Delta(1600)\pi$	$\Delta(1600)\pi$	$\sqrt{\Gamma^{\rm tot}_{\Delta(1600)\pi}}$	$N(1440)\pi$
	d			
$[\Delta\frac{1}{2}^-]_2(2035)$	$-4.0^{+0.9}_{-1.0}$		$4.0^{+1.0}_{-0.9}$	1.8 ± 0.1
$\Delta\frac{1}{2}^-(1900)S_{31}$***				-4.1 ± 2.8
$[\Delta\frac{1}{2}^-]_3(2140)$	$-1.4^{+0.8}_{-1.3}$		$1.4^{+1.3}_{-0.8}$	$-5.1^{+0.7}_{-0.1}$
$\Delta\frac{1}{2}^-(2150)S_{31}$*				
	s	d		
$[\Delta\frac{3}{2}^-]_2(2080)$	-5.0 ± 0.5	$-3.3^{+1.7}_{-2.4}$	$6.0^{+1.9}_{-1.3}$	$-2.0^{+0.9}_{-1.0}$
$\Delta\frac{3}{2}^-(1940)D_{33}$*				
$[\Delta\frac{3}{2}^-]_3(2145)$	$-3.6^{+0.8}_{-0.4}$	$2.9^{+2.4}_{-1.5}$	$4.7^{+2.0}_{-1.5}$	$1.7^{+1.0}_{-0.7}$
	d	g		
$[\Delta\frac{5}{2}^-]_1(2155)$	$0.6^{+0.2}_{-0.1}$	0.2 ± 0.0	$0.6^{+0.2}_{-0.1}$	1.0 ± 0.2
$\Delta\frac{5}{2}^-(1930)D_{35}$***				
$[\Delta\frac{5}{2}^-]_2(2165)$	$-0.6^{+0.4}_{-1.0}$	$-1.2^{+0.6}_{-1.1}$	$1.3^{+1.4}_{-0.8}$	$1.8^{+0.8}_{-0.7}$
$[\Delta\frac{5}{2}^-]_3(2265)$	$1.5^{+0.1}_{-0.2}$	$-1.1^{+0.3}_{-0.4}$	1.8 ± 0.3	$-0.4^{+0.1}_{-0.3}$
$\Delta\frac{5}{2}^-(2350)D_{35}$*				
$[\Delta\frac{5}{2}^-]_4(2325)$	$0.7^{+0.7}_{-0.4}$	$-0.9^{+0.4}_{-0.6}$	$1.1^{+0.9}_{-0.6}$	-0.9 ± 0.3
$[\Delta\frac{7}{2}^-]_1(2230)$	-1.0 ± 0.1	$-1.1^{+0.6}_{-0.9}$	$1.5^{+0.8}_{-0.5}$	$-0.7^{+0.3}_{-0.5}$
$\Delta\frac{7}{2}^-(2200)G_{37}$*				
$[\Delta\frac{7}{2}^-]_2(2295)$	1.0 ± 0.0	$-1.3^{+0.6}_{-0.9}$	$1.7^{+0.8}_{-0.5}$	$-0.6^{+0.2}_{-0.3}$
	g	i		
$[\Delta\frac{9}{2}^-]_1(2295)$	$-2.1^{+1.1}_{-1.5}$	0.0 ± 0.0	$2.1^{+1.5}_{-1.1}$	$-1.9^{+0.9}_{-1.1}$
$\Delta\frac{9}{2}^-(2400)G_{39}$**				
	f	h		
$[\Delta\frac{7}{2}^+]_2(2370)$	$-5.8^{+2.1}_{-2.1}$	0.0 ± 0.0	$5.8^{+2.1}_{-2.1}$	$-5.0^{+1.4}_{-1.3}$
$\Delta\frac{7}{2}^+(2390)F_{37}$*				
$[\Delta\frac{7}{2}^+]_3(2460)$	$0.8^{+0.0}_{-0.2}$	$-1.1^{+0.6}_{-0.9}$	$1.4^{+0.8}_{-0.6}$	-0.4 ± 0.0
$[\Delta\frac{9}{2}^+]_1(2420)$	0.1 ± 0.0	$-0.8^{+0.4}_{-0.7}$	$0.8^{+0.7}_{-0.4}$	-0.3 ± 0.2
$\Delta\frac{9}{2}^+(2300)H_{39}$**				
$[\Delta\frac{9}{2}^+]_2(2505)$	1.3 ± 0.1	$-0.3^{+0.1}_{-0.2}$	1.3 ± 0.1	$-0.3^{+0.1}_{-0.2}$
	h	j		
$[\Delta\frac{11}{2}^+]_1(2450)$	$-1.2^{+0.5}_{-0.7}$	0.0 ± 0.0	$1.2^{+0.7}_{-0.5}$	$-1.2^{+0.4}_{-0.6}$
$\Delta\frac{11}{2}^+(2420)H_{3\ 11}$****				
$[\Delta\frac{13}{2}^+]_1(2880)$	0.0 ± 0.0	$-1.6^{+0.7}_{-0.9}$	$1.6^{+0.9}_{-0.7}$	$-0.6^{+0.2}_{-0.3}$
$[\Delta\frac{13}{2}^+]_2(2955)$	1.2 ± 0.1	$-0.3^{+0.1}_{-0.2}$	1.2 ± 0.1	-0.2 ± 0.1
	i	k		
$[\Delta\frac{13}{2}^-]_1(2750)$	$-1.7^{+0.5}_{-0.6}$	0.0 ± 0.0	$1.7^{+0.6}_{-0.5}$	$-1.5^{+0.4}_{-0.5}$
$\Delta\frac{13}{2}^-(2750)I_{3\ 13}$**				
		j	l	
$[\Delta\frac{15}{2}^+]_1(2920)$	$-1.7^{+0.5}_{-0.6}$	0.0 ± 0.0	$1.7^{+0.6}_{-0.5}$	$-1.5^{+0.4}_{-0.5}$
$\Delta\frac{15}{2}^+(2950)K_{3\ 15}$**				
$[\Delta\frac{15}{2}^+]_2(3085)$	$-0.6^{+0.2}_{-0.3}$	0.0 ± 0.0	$0.6^{+0.3}_{-0.2}$	$-0.5^{+0.1}_{-0.2}$

scription of Cutkosky *et al.* [23] by including a relativistic Breit-Wigner mass distribution σ, and integrating over the mass of the ρ. Their width to $N\rho$ is then

$$\Gamma = \int_{(2m_\pi)^2}^{(M_R-M_N)^2} dm_\rho^2 \sigma(m_\rho^2)\Gamma_{R\to N\rho}(m_\rho^2), \qquad (10)$$

with

$$\sigma(m_\rho^2) = \frac{\Gamma_\rho(m_\rho^2)m_\rho/\pi}{(M_\rho^2 - m_\rho^2)^2 + \Gamma_\rho^2(m_\rho^2)m_\rho^2} \qquad (11)$$

and where

$$\Gamma_\rho(m_\rho^2) = \Gamma_0 \left(\frac{k}{k_0}\right)^3 \frac{M_\rho}{m_\rho}\frac{2k_0^2}{k^2 + k_0^2} \qquad (12)$$

is the energy-dependent width of the ρ. This last form is suggested by Gottfried and Jackson [24], with $M_\rho = 770$ MeV, $\Gamma_\rho(M_\rho^2) = 153$ MeV, $k_0 = k(M_\rho^2)$, and where $k(m_\rho^2)$ is the relative momentum of the pions in the decay $\rho \to \pi\pi$. We have compared the results of using this prescription to our results, and find no significant differences.

TABLE XI. Results in the $\Delta\pi$ and $N\rho$ channels for the lightest few nucleon resonances for J^P values which first appear in the $N = 4$, 5 and 6 bands. Notation as in Table II.

Model state $N\pi$ state/rating	$\Delta\pi$	$\Delta\pi$	$\sqrt{\Gamma_{\Delta\pi}^{\rm tot}}$	$N\rho$	$N\rho$	$N\rho$	$\sqrt{\Gamma_{N\rho}^{\rm tot}}$
	p			$p_{\frac{1}{2}}$	$p_{\frac{3}{2}}$		
$[N\frac{1}{2}^+]_6(2065)$	$-1.9^{+1.0}_{-2.8}$		$1.9^{+2.8}_{-1.0}$	$+1.4^{+3.1}_{-0.1}$	$+5.7\pm2.9$		$5.9^{+3.9}_{-2.8}$
$N\frac{1}{2}^+(2100)P_{11}$*							
$[N\frac{1}{2}^+]_7(2210)$	$-7.0^{+4.3}_{-6.4}$		$7.0^{+6.4}_{-4.3}$	$-2.0^{+0.7}_{-1.8}$	-2.6 ± 1.2		$3.3^{+2.1}_{-1.3}$
	f	h		$f_{\frac{1}{2}}$	$f_{\frac{3}{2}}$	$h_{\frac{3}{2}}$	
$[N\frac{7}{2}^+]_2(2390)$	$+0.1\pm0.0$	$-1.8^{+0.9}_{-1.2}$	$1.8^{+1.2}_{-0.9}$	$-5.2^{+2.1}_{-0.0}$	$+4.7^{+0.0}_{-1.9}$	$+6.4^{+2.5}_{-3.7}$	$9.5^{+1.8}_{-4.6}$
$[N\frac{7}{2}^+]_3(2410)$	$+4.9\pm0.5$	-0.2 ± 0.1	4.9 ± 0.5	$+0.7\pm0.1$	-1.4 ± 0.2	$-0.3^{+0.2}_{-0.1}$	1.6 ± 0.3
$[N\frac{7}{2}^+]_4(2455)$	$-5.5^{+1.2}_{-0.4}$	$-4.0^{+1.8}_{-2.2}$	$6.8^{+1.8}_{-2.0}$	-1.4 ± 0.2	$+0.1\pm0.0$	$-1.0^{+0.5}_{-0.3}$	1.8 ± 0.4
	f	h		$h_{\frac{1}{2}}$	$f_{\frac{3}{2}}$	$h_{\frac{3}{2}}$	
$[N\frac{9}{2}^+]_1(2345)$	$+0.4\pm0.1$	$+0.5^{+0.3}_{-0.2}$	$0.7^{+0.3}_{-0.2}$	$-1.0^{+0.5}_{-1.0}$	$-5.4^{+1.5}_{-2.9}$	$-1.6^{+0.7}_{-1.5}$	$5.8^{+3.4}_{-1.7}$
$N\frac{9}{2}^+(2220)H_{19}$****							
$[N\frac{9}{2}^+]_2(2500)$	-0.6 ± 0.0	$-7.4^{+3.2}_{-3.6}$	$7.4^{+3.6}_{-3.1}$	$+0.1\pm0.1$	-4.3 ± 0.2	-0.7 ± 0.2	4.3 ± 0.3
$[N\frac{9}{2}^+]_3(2490)$	$+9.5^{+0.6}_{-1.9}$	$+0.5\pm0.2$	$9.5^{+0.6}_{-1.9}$	$-1.8^{+0.7}_{-0.6}$	-1.0 ± 0.1	$+0.7\pm0.2$	$2.2^{+0.6}_{-0.7}$
	h	j		$h_{\frac{1}{2}}$	$h_{\frac{3}{2}}$	$j_{\frac{3}{2}}$	
$[N\frac{11}{2}^+]_1(2490)$	$+5.2^{+2.6}_{-2.2}$	0.0	$5.2^{+2.6}_{-2.2}$	$+0.7^{+0.2}_{-0.3}$	-1.1 ± 0.4	0.0	1.4 ± 0.5
$[N\frac{11}{2}^+]_2(2600)$	$+2.4\pm0.9$	0.0	2.4 ± 0.9	$+0.4^{+0.2}_{-0.1}$	$-0.6^{+0.1}_{-0.3}$	0.0	$0.7^{+0.3}_{-0.2}$
	g	i		$i_{\frac{1}{2}}$	$g_{\frac{3}{2}}$	$i_{\frac{3}{2}}$	
$[N\frac{11}{2}^-]_1(2600)$	$-0.4^{+0.1}_{-0.0}$	$-1.2^{+0.5}_{-0.6}$	$1.3^{+0.6}_{-0.5}$	$-1.5^{+0.5}_{-0.9}$	$-5.9^{+0.3}_{-0.8}$	$-2.5^{+0.8}_{-1.6}$	$6.6^{+1.6}_{-0.7}$
$N\frac{11}{2}^-(2600)I_{1\ 11}$***							
$[N\frac{11}{2}^-]_2(2670)$	$-1.0^{+0.2}_{-0.1}$	$+4.6^{+1.9}_{-1.7}$	$4.7^{+1.9}_{-1.7}$	$-0.9^{+0.3}_{-0.6}$	-0.9 ± 0.1	$-1.5^{+0.5}_{-1.1}$	$2.0^{+1.2}_{-0.5}$
$[N\frac{11}{2}^-]_3(2700)$	$+6.8^{+0.1}_{-0.8}$	$+0.5\pm0.2$	$6.8^{+0.1}_{-0.8}$	$-1.3^{+0.4}_{-0.9}$	$-0.4^{+0.0}_{-0.1}$	$+0.6^{+0.4}_{-0.2}$	$1.4^{+1.0}_{-0.5}$
$[N\frac{11}{2}^-]_4(2770)$	$+0.3\pm0.0$	$+3.3^{+1.2}_{-1.0}$	$3.3^{+1.2}_{-1.0}$	$-0.1^{+0.0}_{-0.1}$	$+1.0^{+0.2}_{-0.1}$	-0.1 ± 0.1	$1.0^{+0.3}_{-0.2}$
$[N\frac{11}{2}^-]_5(2855)$	$+0.5^{+0.0}_{-0.1}$	$+0.5^{+0.2}_{-0.1}$	$0.7^{+0.1}_{-0.2}$	$-0.6^{+0.3}_{-0.5}$	$-0.8^{+0.2}_{-0.3}$	$-0.5^{+0.2}_{-0.4}$	$1.1^{+0.6}_{-0.4}$
	h	j		$j_{\frac{1}{2}}$	$h_{\frac{3}{2}}$	$j_{\frac{3}{2}}$	
$[N\frac{13}{2}^+]_1(2820)$	$+0.2\pm0.0$	$+0.3^{+0.2}_{-0.1}$	$0.4^{+0.2}_{-0.1}$	$-1.0^{+0.4}_{-0.9}$	$-3.2^{+0.6}_{-1.2}$	$-1.5^{+0.6}_{-1.3}$	$3.7^{+1.9}_{-0.9}$
$N\frac{13}{2}^+(2700)K_{1\ 13}$**							
$[N\frac{13}{2}^+]_2(2930)$	-0.1 ± 0.0	$-4.2^{+1.3}_{-1.6}$	$4.2^{+1.6}_{-1.3}$	-0.1 ± 0.1	$-2.2^{+0.6}_{-0.7}$	$-0.4^{+0.2}_{-0.3}$	2.3 ± 0.7
$[N\frac{13}{2}^+]_3(2955)$	$+4.4\pm0.1$	$+0.9\pm0.3$	4.4 ± 0.2	$-1.5^{+0.7}_{-1.1}$	-0.1 ± 0.1	$+0.7^{+0.5}_{-0.3}$	$1.6^{+1.2}_{-0.8}$
	i	k		$i_{\frac{1}{2}}$	$i_{\frac{3}{2}}$	$k_{\frac{3}{2}}$	
$[N\frac{13}{2}^-]_1(2715)$	$+4.6^{+1.8}_{-1.6}$	0.0	$4.6^{+1.8}_{-1.6}$	$+0.6^{+0.5}_{-0.2}$	$-0.9^{+0.3}_{-0.7}$	0.0	$1.1^{+0.8}_{-0.4}$
$[N\frac{13}{2}^-]_2(2845)$	$+1.3\pm0.4$	0.0	1.3 ± 0.4	$+0.2\pm0.1$	$-0.3^{+0.1}_{-0.2}$	0.0	$0.3^{+0.2}_{-0.1}$
	j	l		$j_{\frac{1}{2}}$	$j_{\frac{3}{2}}$	$l_{\frac{3}{2}}$	
$[N\frac{15}{2}^+]_1(2940)$	$+3.3^{+1.2}_{-1.0}$	0.0	$3.3^{+1.2}_{-1.0}$	$+0.6^{+0.4}_{-0.3}$	$-0.9^{+0.4}_{-0.7}$	0.0	$1.1^{+0.8}_{-0.5}$
$[N\frac{15}{2}^+]_2(3005)$	$+1.4^{+0.5}_{-0.4}$	0.0	$1.4^{+0.5}_{-0.4}$	$+0.3\pm0.2$	$-0.5^{+0.2}_{-0.3}$	0.0	$0.6^{+0.4}_{-0.3}$

C. Parameters

The parameters of the model are the pair-creation strength γ, and the Gaussian parameters of the meson and baryon wave functions. For consistency, the parameters that this work has in common with Ref. [9] have not been changed from the values used there, so that $\gamma = 2.6$, and the Gaussian parameters α of all baryon wave functions are set to 0.5 GeV (a common value is necessary in a decay calculation in order to maintain orthogonality of the initial and final baryon wave functions). In addition the corresponding parameters for all the mesons are set to β=0.4 GeV.

An additional parameter for this work is κ, discussed in the previous subsection. However, as mentioned there, the results that we present are largely independent of κ; we use a value of 0.35 GeV.

III. RESULTS

Our results are presented in the form of several tables. In order to list as many of our results as possible in a form that makes various comparisons relatively easy to carry out, these tables are necessarily dense; accordingly we have included the following guide to their organization.

TABLE XII. Results in the $N\pi$, $N\eta$, and $N\omega$ channels for the lightest few nucleon resonances for J^P values which first appear in the $N = 4$, 5, and 6 bands. Notation as in Table I.

Model state $N\pi$ state/rating	$N\pi$	$N\eta$	$N\eta'$	$N\omega$	$N\omega$	$N\omega$	$\sqrt{\Gamma^{\rm tot}_{N\omega}}$
				$p_{\frac{1}{2}}$	$p_{\frac{3}{2}}$		
$[N\frac{1}{2}^+]_6(2065)$	$7.7^{+2.4}_{-2.9}$	$+1.1\pm1.1$	-0.6 ± 0.5	$+0.6^{+0.1}_{-0.2}$	$-0.4^{+2.9}_{-3.7}$		$0.7^{+3.5}_{-0.7}$
$N\frac{1}{2}^+(2100)P_{11}$*	5.0 ± 2.0						
$[N\frac{1}{2}^+]_7(2210)$	$0.3^{+0.2}_{-0.1}$	$-1.1^{+0.8}_{-0.7}$	$+0.6^{+0.5}_{-0.9}$	$+2.1\pm1.8$	-1.5 ± 0.3		$2.5^{+1.7}_{-1.3}$
				$f_{\frac{1}{2}}$	$f_{\frac{3}{2}}$	$h_{\frac{3}{2}}$	
$[N\frac{7}{2}^+]_2(2390)$	$4.9^{+0.1}_{-0.4}$	$+1.7\pm0.1$	$+1.3^{+0.3}_{-0.4}$	-0.8 ± 0.1	$+2.1^{+0.2}_{-0.4}$	$+2.0^{+1.2}_{-0.9}$	$3.0^{+1.0}_{-0.9}$
$[N\frac{7}{2}^+]_3(2410)$	$0.4^{+0.2}_{-0.4}$	$-1.1^{+0.4}_{-0.1}$	$-1.1^{+0.3}_{-0.1}$	-0.7 ± 0.1	$+1.3\pm0.1$	0.0	1.5 ± 0.1
$[N\frac{7}{2}^+]_4(2455)$	0.5 ± 0.1	$-0.7^{+0.1}_{-0.0}$	$-0.5^{+0.2}_{-0.1}$	$+1.7^{+0.0}_{-0.2}$	-0.3 ± 0.0	$+0.7^{+0.4}_{-0.3}$	$1.9^{+0.2}_{-0.3}$
				$h_{\frac{1}{2}}$	$f_{\frac{3}{2}}$	$h_{\frac{3}{2}}$	
$[N\frac{9}{2}^+]_1(2345)$	$3.6^{+0.9}_{-0.8}$	$+0.8\pm0.3$	$+0.2^{+0.2}_{-0.1}$	$-0.3^{+0.1}_{-0.2}$	$-2.9^{+0.6}_{-0.5}$	-0.6 ± 0.3	$2.9^{+0.6}_{-0.7}$
$N\frac{9}{2}^+(2220)H_{19}$****	8.5 ± 2.0						
$[N\frac{9}{2}^+]_2(2500)$	0.4 ± 0.1	$+1.3^{+0.5}_{-0.4}$	$+0.6^{+0.4}_{-0.3}$	$-1.1^{+0.4}_{-0.5}$	$+1.3^{+0.0}_{-0.1}$	$-1.0^{+0.4}_{-0.5}$	$1.9^{+0.6}_{-0.5}$
$[N\frac{9}{2}^+]_3(2490)$	0.6 ± 0.2	$+0.1\pm0.0$	0.0	$+1.4^{+0.7}_{-0.6}$	-0.1 ± 0.0	$-1.2^{+0.5}_{-0.6}$	$1.9^{+0.9}_{-0.8}$
				$h_{\frac{1}{2}}$	$h_{\frac{3}{2}}$	$j_{\frac{3}{2}}$	
$[N\frac{11}{2}^+]_1(2490)$	1.3 ± 0.4	$-1.3^{+0.4}_{-0.5}$	$-0.6^{+0.3}_{-0.4}$	$-0.7^{+0.3}_{-0.4}$	$+1.2^{+0.6}_{-0.5}$	0.0	$1.4^{+0.7}_{-0.6}$
$[N\frac{11}{2}^+]_2(2600)$	0.7 ± 0.2	-0.5 ± 0.1	-0.3 ± -0.1	-0.3 ± 0.1	$+0.5\pm0.2$	0.0	0.6 ± 0.2
				$i_{\frac{1}{2}}$	$g_{\frac{3}{2}}$	$i_{\frac{3}{2}}$	
$[N\frac{11}{2}^-]_1(2600)$	$3.3^{+1.1}_{-0.9}$	$+1.2^{+0.5}_{-0.4}$	$+0.5^{+0.4}_{-0.3}$	$-0.7^{+0.3}_{-0.4}$	$-1.7^{+0.3}_{-0.1}$	$-1.0^{+0.4}_{-0.5}$	2.1 ± 0.5
$N\frac{11}{2}^-(2600)I_{1\ 11}$***	4.5 ± 1.5						
$[N\frac{11}{2}^-]_2(2670)$	1.8 ± 0.5	-0.1 ± 0.0	0.0	0.0	$-1.8^{+0.2}_{-0.1}$	$+0.1\pm0.1$	$1.8^{+0.1}_{-0.2}$
$[N\frac{11}{2}^-]_3(2700)$	0.3 ± 0.1	0.0	0.0	$+1.1^{+0.5}_{-0.4}$	-0.1 ± 0.0	$-0.9^{+0.3}_{-0.4}$	$1.5^{+0.7}_{-0.5}$
$[N\frac{11}{2}^-]_4(2770)$	0.2 ± 0.1	$-0.5^{+0.1}_{-0.2}$	-0.3 ± 0.1	$+0.4^{+0.2}_{-0.1}$	-0.5 ± 0.0	$+0.4^{+0.2}_{-0.1}$	$0.8^{+0.2}_{-0.1}$
$[N\frac{11}{2}^-]_5(2855)$	0.6 ± 0.1	$+0.1\pm0.0$	$+0.1\pm0.0$	$+0.1\pm0.0$	-0.3 ± 0.0	-0.2 ± 0.1	0.4 ± 0.1
				$j_{\frac{1}{2}}$	$h_{\frac{3}{2}}$	$j_{\frac{3}{2}}$	
$[N\frac{13}{2}^+]_1(2820)$	$2.0^{+1.0}_{-0.8}$	$+0.6^{+0.3}_{-0.2}$	$+0.2^{+0.2}_{-0.1}$	$-0.3^{+0.1}_{-0.2}$	$-1.3^{+0.3}_{-0.2}$	$-0.5^{+0.2}_{-0.3}$	$1.4^{+0.3}_{-0.4}$
$N\frac{13}{2}^+(2700)K_{1\ 13}$**	3.7 ± 1.2						
$[N\frac{13}{2}^+]_2(2930)$	0.2 ± 0.1	$+0.7\pm0.2$	$+0.4\pm0.2$	-0.5 ± 0.2	$+0.5\pm0.0$	$-0.6^{+0.2}_{-0.3}$	0.9 ± 0.3
$[N\frac{13}{2}^+]_3(2955)$	0.2 ± 0.1	-0.1 ± 0.0	0.0	$+1.0^{+0.4}_{-0.3}$	-0.2 ± 0.0	$-0.7^{+0.2}_{-0.3}$	$1.2^{+0.5}_{-0.4}$
				$i_{\frac{1}{2}}$	$i_{\frac{3}{2}}$	$k_{\frac{3}{2}}$	
$[N\frac{13}{2}^-]_1(2715)$	1.1 ± 0.3	$-1.0^{+0.3}_{-0.4}$	$-0.5^{+0.2}_{-0.3}$	$-0.6^{+0.2}_{-0.3}$	$+1.0\pm0.4$	0.0	$1.1^{+0.5}_{-0.4}$
$[N\frac{13}{2}^-]_2(2845)$	0.2 ± 0.1	-0.3 ± 0.1	-0.2 ± 0.1	-0.2 ± 0.1	$+0.3\pm0.1$	0.0	0.4 ± 0.1
		$j_{\frac{1}{2}}$	$j_{\frac{3}{2}}$	$l_{\frac{3}{2}}$			
$[N\frac{15}{2}^+]_1(2940)$	0.7 ± 0.1	$-0.7^{+0.2}_{-0.3}$	-0.4 ± 0.2	-0.5 ± 0.2	$+0.7^{+0.3}_{-0.2}$	0.0	0.8 ± 0.3
$[N\frac{15}{2}^+]_2(3005)$	0.4 ± 0.1	-0.3 ± 0.1	-0.2 ± 0.1	-0.2 ± 0.1	$+0.3\pm0.1$	0.0	0.3 ± 0.1

For each set of nucleon resonances, there are three tables. The first of these lists the model state, its decay amplitudes into the $N\pi$, $N\eta$, $N\eta'$ channels, and its helicity partial-wave and total decay amplitudes for the $N\omega$ channel. The second table lists the helicity partial-wave and total decay amplitudes for each state in the $\Delta\pi$ and $N\rho$ channels, while for the nucleons beyond the $N=2$ band, the third table lists the corresponding quantities for the $\Delta(1600)\pi$ and $N(1440)\pi$ channels (we do not present the latter type of table for nucleons in the $N \leq 2$ bands). The format for presentation of the decay amplitudes for the Δ resonances is similar, although we have split some of the larger tables. In addition to this division among the tables, for each state there are two rows of entries. The first row lists our model predictions, the second the values published in the recent $N\pi\pi$ partial-wave analysis of Manley and Saleski [19], along with the Particle Data Group [20] name, $N\pi$ partial wave, star rating, and $N\pi$ amplitude for this state. In all cases, the predicted $N\pi$ amplitudes in the first row are the same as in Ref. [9], reproduced here for ease of comparison. The association of a given state from the partial-wave analyses with a model state in our tables is designed to make explicit our assigment of model states to established states.

For all but the $N\pi$ channels, we give the sign of the amplitude along with its magnitude. This sign must be understood as the sign relative to the unmeasurable sign of the $N\pi$ production amplitude. In other words, the signs presented with our model amplitudes are the product $\zeta_{N\pi}\zeta_{BM}$ of the sign for the $N\pi$ entrance channel

TABLE XIII. Results in the $\Delta(1600)\pi$ and $N(1440)\pi$ channels for the lightest few nucleon resonances for J^P values which first appear in the $N=4$, 5, and 6 bands. Notation as in Table II.

Model state $N\pi$ state/rating	$\Delta(1600)\pi$	$\Delta(1600)\pi$	$\sqrt{\Gamma^{\rm tot}_{\Delta(1600)\pi}}$	$N(1440)\pi$
	p			
$[N\frac{1}{2}^+]_6(2065)$	$9.1^{+1.5}_{-2.5}$		$9.1^{+1.5}_{-2.5}$	7.9 ± 0.4
$N\frac{1}{2}^+(2100)P_{11}$*				
$[N\frac{1}{2}^+]_7(2210)$	8.1 ± 0.5		8.1 ± 0.5	$2.1^{+0.0}_{-0.3}$
	f	h		
$[N\frac{7}{2}^+]_2(2390)$	-0.1 ± 0.0	$0.5^{+0.4}_{-0.3}$	$0.6^{+0.4}_{-0.3}$	-1.6 ± 0.1
$[N\frac{7}{2}^+]_3(2410)$	$-4.4^{+1.3}_{-1.1}$	0.1 ± 0.0	$4.4^{+1.1}_{-1.3}$	-0.4 ± 0.0
$[N\frac{7}{2}^+]_4(2455)$	$1.8^{+0.2}_{-0.4}$	$1.3^{+0.9}_{-0.6}$	$2.2^{+0.7}_{-0.6}$	-0.2 ± 0.1
	f	h		
$[N\frac{9}{2}^+]_1(2345)$	-0.1 ± 0.0	-0.2 ± 0.1	0.2 ± 0.1	$-1.3^{+0.5}_{-0.7}$
$N\frac{9}{2}^+(2220)H_{19}$****				
$[N\frac{9}{2}^+]_2(2500)$	0.1 ± 0.0	$2.3^{+1.5}_{-1.1}$	$2.3^{+1.5}_{-1.1}$	$-0.4^{+0.1}_{-0.2}$
$[N\frac{9}{2}^+]_3(2490)$	-1.6 ± 0.1	-0.2 ± 0.1	1.6 ± 0.1	-0.4 ± 0.2
	h	j		
$[N\frac{11}{2}^+]_1(2490)$	$-1.7^{+0.8}_{-1.1}$	0.0 ± 0.0	$1.7^{+1.1}_{-0.8}$	-0.4 ± 0.2
$[N\frac{11}{2}^+]_2(2600)$	$-0.8^{+0.3}_{-0.5}$	0.0 ± 0.0	$0.8^{+0.5}_{-0.3}$	-0.7 ± 0.3
	g	i		
$[N\frac{11}{2}^-]_1(2600)$	0.1 ± 0.0	$0.5^{+0.3}_{-0.2}$	$0.5^{+0.3}_{-0.2}$	$-2.4^{+1.0}_{-1.4}$
$N\frac{11}{2}^-(2600)I_{1\,11}$***				
$[N\frac{11}{2}^-]_2(2670)$	0.2 ± 0.0	$-1.9^{+0.8}_{-1.2}$	$1.9^{+1.2}_{-0.8}$	$-1.3^{+0.5}_{-0.7}$
$[N\frac{11}{2}^-]_3(2700)$	-1.7 ± 0.1	-0.2 ± 0.1	1.7 ± 0.1	-0.2 ± 0.1
$[N\frac{11}{2}^-]_4(2770)$	0.0 ± 0.0	$-1.5^{+0.6}_{-0.8}$	$1.5^{+0.8}_{-0.6}$	-0.1 ± 0.0
$[N\frac{11}{2}^-]_5(2855)$	-0.2 ± 0.0	-0.2 ± 0.1	0.3 ± 0.1	$-0.4^{+0.1}_{-0.2}$
	h	j		
$[N\frac{13}{2}^+]_1(2820)$	-0.1 ± 0.0	-0.2 ± 0.1	0.2 ± 0.1	$-1.7^{+0.8}_{-1.1}$
$N\frac{13}{2}^+(2700)K_{1\,13}$**				
$[N\frac{13}{2}^+]_2(2930)$	0.0 ± 0.0	$2.5^{+1.4}_{-1.0}$	$2.5^{+1.4}_{-1.0}$	-0.5 ± 0.2
$[N\frac{13}{2}^+]_3(2955)$	-1.5 ± 0.1	$-0.6^{+0.2}_{-0.3}$	1.6 ± 0.2	$-0.3^{+0.1}_{-0.2}$
	i	k		
$[N\frac{13}{2}^-]_1(2715)$	$-2.0^{+0.8}_{-1.2}$	0.0 ± 0.0	$2.0^{+1.2}_{-0.8}$	$-0.9^{+0.3}_{-0.4}$
$[N\frac{13}{2}^-]_2(2845)$	$-0.7^{+0.3}_{-0.4}$	0.0 ± 0.0	$0.7^{+0.4}_{-0.3}$	-0.1 ± 0.0
	j	l		
$[N\frac{15}{2}^+]_1(2940)$	$-2.0^{+0.8}_{-1.1}$	0.0 ± 0.0	$2.0^{+1.1}_{-0.8}$	-0.5 ± 0.2
$[N\frac{15}{2}^+]_2(3005)$	$-0.9^{+0.4}_{-0.5}$	0.0 ± 0.0	$0.9^{+0.5}_{-0.4}$	$-0.6^{+0.2}_{-0.3}$

times that of the decay channel being considered (one can extract the sign for the amplitude with the γN entrance channel by taking the product of the sign $\zeta_{\gamma N}\zeta_{N\pi}$ for the photocoupling amplitudes from Ref. [12] and the signs in the tables in this paper).

All theoretical amplitudes are also given with upper and lower limits, along with the central value, in order to convey the uncertainty in our results due to the uncertainty in the resonance's mass. These correspond to our predictions for the amplitudes for a resonance whose mass is set to the upper and lower limits, and to the central value, of the experimentally determined mass. For states as yet unseen in the analyses of the data, we have adopted a "standard" uncertainty in the mass of 150 MeV and used the model predictions for the state's mass as the central value.

By examining the tables one can see the significant mass dependence in some of the decay amplitudes. One example is the $N\rho$ decay of the $N(1520)D_{13}$ (Table II). At its central mass value, the amplitude for this decay is 2.5 $\text{MeV}^{\frac{1}{2}}$, while if the mass is increased by the uncertainty from the partial-wave analyses (150 MeV in this case), the amplitude becomes 9.0 $\text{MeV}^{\frac{1}{2}}$, corresponding to a factor of more than 12 in the decay width. Although this is an extreme example, many amplitudes exhibit similar dependencies.

A. N^* resonances in the $N \leq 2$ bands

The model predictions for nucleon resonances in the $N \leq 2$ bands (Tables I and II) are in generally good agreement with the analyses of the experimental data. The larger predicted amplitudes correspond to larger measured amplitudes, although the magnitudes of the theoretical and measured amplitudes may differ. Predicted signs of amplitudes are also mostly in agreement with the experimentally reported signs; most of those that are in disagreement with their experimental counterparts correspond to small measured amplitudes. Notable exceptions are the s- and d-wave $\Delta\pi$ partial amplitudes of $D_{13}(1700)$ (although we successfully predict a relatively large total width to $\Delta\pi$ for this state), and the $N\rho$ amplitudes of the $P_{13}(1720)$ and $F_{15}(2000)$ resonances. In addition, Manley and Saleski [19] also find a second light P_{13} resonance, $P_{13}(1880)$, in the $N\rho$ channel. In our model, none of the missing P_{13} states in the $N = 2$ band (whose masses are in this region) are predicted to have large $N\rho$ widths.

As far as the missing states are concerned, we predict sizable amplitudes in the $\Delta\pi$ channel for the predicted states $[N\frac{1}{2}^+]_4(1880)$, $[N\frac{1}{2}^+]_5(1975)$, $[N\frac{3}{2}^+]_3(1910)$, $[N\frac{3}{2}^+]_4(1950)$, $[N\frac{3}{2}^+]_5(2030)$, and $[N\frac{5}{2}^+]_3(1995)$. Thus, our model predicts that these states should be clearly seen in the experiment proposed in Ref. [1], and can be viewed as a guide to which channels and partial waves show the greatest potential for their discovery. Note in Tables I and II we have also reassigned the two-star state $N(2000)F_{15}$ from the model state $[N\frac{5}{2}^+]_2(1980)$ to $[N\frac{5}{2}^+]_3(1995)$, on the basis of its $\Delta\pi$ and $N\rho$ decays; the assignment from Ref. [9] based on the (small) $N\pi$ amplitudes of these model states was at best tentative.

All of the $\Delta(1600)\pi$ and $N(1440)\pi$ amplitudes for these states are small, the largest corresponding to a width of about 16 MeV. Thus we do not show tables for these channels in this sector. This result implies little hope of discovering missing baryon resonances in either of these two channels. We note, however, that the small amplitudes in the $N(1440)\pi$ channel are in contradiction with some large amplitudes reported in Ref. [19], and may point to possible shortcomings in the model. In particular, all of our predictions, but especially the couplings to broad final states such as Roper π, may be modified by the inclusion of decay-channel-coupling effects in the spectrum and wave functions, which we plan to address in a later study.

In the $N\eta$, $N\eta'$, and $N\omega$ channels, there are not many extracted amplitudes published. Our prediction for the $S_{11}(1535)$ is compatible with the measured amplitude, but that for its heavier counterpart $S_{11}(1650)$ is too large. Note, however, that there is recent controversy about existing analyses of the $N\eta$ final state [26]. Most states in this sector have little or no phase space for decays to these three channels, and so it is not surprising to find small amplitudes. The exceptions to this are the states $[N\frac{3}{2}^+]_2(1870)$, $[N\frac{3}{2}^+]_3(1910)$, and $[N\frac{3}{2}^+]_4(1950)$, all of which couple strongly to $N\omega$, and slightly less strongly to $N\eta$. This suggests that these channels offer good opportunities for the discovery or confirmation of these states; they should therefore be seen in the experiments proposed in Refs. [3–5].

B. Δ resonances in the $N \leq 2$ bands

The magnitudes and signs of most $\Delta\pi$ and $N\rho$ amplitudes are well reproduced in this sector (see Tables III and IV). The most notable exceptions are the signs of the $s_{\frac{1}{2}}$ $N\rho$ partial amplitude of the $S_{31}(1620)$ and of the $f_{\frac{3}{2}}$ $N\rho$ partial amplitude of $F_{37}(1950)$. Our calculation suggests that the new state $\Delta(1740)P_{31}$ found by Manley and Saleski [19] is the formerly missing first $\Delta\frac{1}{2}^+$ state; we predict a sizable $N\rho$ amplitude for this state in the partial wave in which it was discovered. However, we also predict a sizable $\Delta\pi$ amplitude for this state, whereas the Manley-Saleski analysis shows no evidence for such a state in this channel. Our results also suggest that the other missing state here, the fourth $\Delta\frac{3}{2}^+$, should be visible in both the $\Delta\pi$ and $N\rho$ channels.

This sector also contains a puzzle, which is the $F_{35}(1750)$ state reported in Ref. [19]. The lowest-lying model state in this sector is the $[\Delta\frac{5}{2}^+]_1$ state at 1910 MeV. On the basis of our calculated $N\pi$, $\Delta\pi$, and $N\rho$ amplitudes, this state corresponds most closely to $F_{35}(1905)$. The three-quark spectrum therefore cannot easily accommodate the state $F_{35}(1750)$, unless all of the $J^P = \frac{5}{2}^+$ Δ's are reassigned. This is also problematic since there are just two states in the $N = 2$ band in the nonrelativistic quark model, and one of these three states would have to be reassigned to the $N = 4$ band, for which it is significantly too light.

As is the case with the nucleons in these bands, the couplings of the Δ's to the $\Delta(1600)\pi$ and $N(1440)\pi$ channels are all small, with the largest partial width into one of these channels being around 16 MeV.

C. N^* resonances in the $N = 3$ band

The number of widths extracted from the data with which we can compare our model diminishes significantly as we increase the masses of the baryon resonances we consider, and this begins to be apparent in this band. There are few extracted widths for these nucleons in the $\Delta\pi$ and $N\rho$ channels (Table VI) and none in the $N\eta$, $N\eta'$, and $N\omega$ channels (Table V). In the $\Delta\pi$ and $N\rho$ channels, the agreement of the model with the partial-wave analysis of Manley and Saleski is comparable to that obtained in other sectors. Discrepancies occur in the description of the $N\rho$ partial amplitudes of $S_{11}(2090)$ and in the signs of the amplitudes of the $\Delta\pi$ and $N\rho$ amplitudes of the $D_{13}(2080)$, although in both cases the total widths are in reasonable agreement with experiment.

Many of the as-yet-unseen states couple strongly to the $\Delta\pi$ and $N\rho$ channels (Table VI). The most noticeable of these is the predicted state $[N\frac{1}{2}^-]_5(2070)$ which has a $\Delta\pi$ amplitude in excess of 13 MeV$^{\frac{1}{2}}$. Thus, these channels offer good opportunities for discovery of many of these missing resonances.

In the $N\eta$, $N\eta'$, and $N\omega$ channels (Table V), some undiscovered states have appreciable couplings, but few of these will be likely to yield what may be termed "smoking-gun" signals, with the possible exceptions of the states $[N\frac{1}{2}^-]_5(2070)$ and $[N\frac{7}{2}^-]_2(2205)$ in the $N\omega$ channel. Our results (Table VII) indicate that weak evidence for the existence of the lowest-lying $N = 3$ band resonances $N\frac{1}{2}^-(2090)S_{11}$, $N\frac{3}{2}^-(2080)D_{13}$, and $N\frac{3}{2}^-(2200)D_{15}$ could be strengthened with $\Delta(1600)\pi$ or $N(1440)\pi$ experiments in this mass range. In the $\Delta(1600)\pi$ channel, a few undiscovered states have appreciable couplings, most noticeably $[N\frac{3}{2}^-]_5(2095)$, $[N\frac{5}{2}^-]_2(2080)$, and $[N\frac{3}{2}^-]_4(2055)$. In the $N(1440)\pi$ channel, only the model states $[N\frac{1}{2}^-]_5(2070)$ and $[N\frac{5}{2}]_3^-(2095)$ have appreciable couplings.

D. Δ resonances in the $N \geq 3$ bands

In this sector, there is general agreement between our model and the partial-wave analyses in the $\Delta\pi$ channel (Table VIII), although there are few extracted widths with which to compare. However, the $N\rho$ couplings (Table IX) of $S_{31}(1900)$ are not well reproduced. Most of the states in this sector may be classified as undiscovered, and most of these have sizable partial widths in the $\Delta\pi$ and $N\rho$ channels. These channels should, therefore, allow for the discovery of many new baryon states. In the $\Delta(1600)\pi$ channel (Table X), only the $[\Delta\frac{3}{2}^-]_3(2145)$ offers a good possibility for discovery, although it should be possible to confirm the existence of the state $\Delta\frac{1}{2}^-(1900)S_{31}$, and the weakly established states $\Delta\frac{3}{2}^-(1940)D_{33}$ and $\Delta\frac{7}{2}^+(2390)F_{37}$ which appear in the $N\pi$ analyses [20]. In the $N(1440)\pi$ channel, we predict no real possibilities for discovering new baryons, although confirmation of the tentatively established states $\Delta\frac{1}{2}^-(2150)S_{31}$ and $\Delta\frac{7}{2}^+(2390)F_{37}$ should be possible.

E. N^* resonances in the $N \geq 4$ bands

Very few of this group of states predicted by the quark model have been seen; our results (Table XI) indicate that many of these states should be discovered in the $\Delta\pi$ and $N\rho$ channels. Partial widths into the $N\eta$, $N\eta'$, and $N\omega$ channels (Table XII) are generally small. The two lightest $N = 4$ band $N\frac{1}{2}^+$ states both have sizable couplings to the $\Delta(1600)\pi$ channel (Table XIII); we have assigned the lightest of these to the tentative state $N\frac{1}{2}^+(2100)P_{11}$ from the $N\pi$ analyses. Verification of the existence of this state and discovery of the second state ($[N\frac{1}{2}^+]_7(2210)$ in our model) appear to be possible in this channel. None of the states we have considered in this sector, with the exception of the $N\frac{1}{2}^+(2100)P_{11}$, have appreciable couplings to $N(1440)\pi$.

IV. CONCLUSIONS AND OUTLOOK

Our results indicate that many of the baryon states predicted by the quark model but as yet unseen in the partial-wave analyses should appear first in analyses of decays to the $N\pi\pi$ channel through the $\Delta\pi$ and $N\rho$ quasi-two-body channels. The rough agreement of the signs and magnitudes of our predicted amplitudes with the majority of the existing data in these channels also suggests that our predictions can act as a rough guide to the specific channel, partial wave, and mass range in which to look for these new states. The same is also true for the lighter missing and undiscovered N ($I = \frac{1}{2}$) states in the $N\eta$, $N\eta'$, and $N\omega$ channels. These amplitudes tend to be quite small for the higher-mass states studied here.

As mentioned above, our model tends to predict small amplitudes for the lighter ($N \leq 2$) baryons to decay to $N(1440)\pi$ and $\Delta(1600)\pi$. This can be traced back to the node in the radial part of the momentum-space wave functions of these final baryon states, which are described as radial excitations in our model. We expect that this aspect of our model, which disagrees with some of the Manley-Saleski results [19], will be affected by including decay-channel couplings into the spectrum and wave functions [note that $N(1440)$ is an exceptionally broad state in our model and in the recent analyses]. We intend to address this problem in a later study. We have, however, displayed some results from our calculation for decays of heavier baryons into these final states, as in this case there can be appreciable amplitudes; it is possible that the larger amplitudes will remain large after the necessary corrections are applied.

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