QCD demonstration for the color string structures of $e^+e^- \rightarrow q\bar{q}g$ and $\Upsilon \rightarrow 3g$ systems

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The $e^+e^- \rightarrow q\bar{q}g \rightarrow 3$ jets and $\Upsilon \rightarrow 3g \rightarrow$hadrons processes are well suited to the study of the gluon hadronization mechanism. In the LUND model, these two processes are treated by applying the string fragmentation model to the assumed color string structure of $q\bar{q}g$ and $3g$ systems. In this paper, the color string structure of the $q\bar{q}g$ and ggg systems is given by directly analyzing their color wave functions in the context of perturbative QCD. In addition, the reasonableness and accuracy of the LUND string pictures are discussed.

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I. INTRODUCTION

How do quarks and gluons fragment into various hadrons? Are quark and gluon fragmentations two independent questions or different aspects of one question? These are still the fundamental problems in present particle physics and can only be understood gradually from comparing different models with experiments. The $e^+e^- \rightarrow q\bar{q} \rightarrow 2$ jet process is well suited to the study of quark fragmentation, while the $e^+e^- \rightarrow q\bar{q}g$ process can be used to study the differences and connections between quark and gluon fragmentations. As the strong Υ decay can only proceed via a $3g$ intermediate state, it has many advantages in the study of gluon fragmentation over the high energy multijet events, such as (A) there is no need to distinguish the gluon jet from a quark jet and to measure the angle between jets as in the $e^+e^- \rightarrow q\bar{q}g$ events, and (B) the energy of each gluon in $\Upsilon \rightarrow 3g$ is so low that the contribution from parton cascade processes can be neglected. Hence the hadrons in the final state carry more directly the gluon hadronization information.

In QCD the fundamental difference between the quark and gluon is that the quark is a color triplet, whereas the gluon is a color octet. This difference is treated in very different ways in the independent fragmentation [1] (IF) and string fragmentation [2] (SF) models. In the IF model, the fragmentations of partons are assumed to be independent of each other. The average hadron multiplicity $\langle n \rangle_g$ from gluon fragmentation is then much larger than that $\langle n \rangle_q$ from a quark jet at the same energy. At a nonasymptotic energy, their ratio is given by [3]

$$\frac{\langle n \rangle_g}{\langle n \rangle_q} = \tfrac{9}{4}[1 - 0.27\sqrt{\alpha_s} - 0.07\alpha_s] \ , \tag{1}$$

where α_s is the ordinary running coupling constant at that energy. In this case, both the gluon and quark produce jets with symmetrical particle density. In the SF model, much attention is paid to the essential connections between quarks and gluons. Since $3 \otimes \bar{3} = 8 \oplus 1$, a color octet gluon can be regarded as a bicolor system of which the color interaction is equivalent to that of a $q\bar{q}$ color octet system. In string language, the gluon in three-jet events is introduced naturally as a kink carrying energy and momentum on the string stretched between a q and a $\bar{q}$ end. Thus the gluon has two string pieces attached to it, whereas the q or $\bar{q}$ only has one. In particular, there is no string directly between q and $\bar{q}$ [Fig. 1(a)]. When the string pieces between q (or $\bar{q}$) and g fragment, the transverse motion of these strings tends to boost the particles produced away from the central region, i.e., to deplete the region between the q and $\bar{q}$ of particles. This so-called "string effect" has been shown by Azimov *et al.* [4] within the framework of perturbative QCD. In their perturbative calculations, the asymmetry in the particle populations between jets in three-jet events arises from interference among the soft gluons radiated from the q, $\bar{q}$, and g. This string effect [Fig. 1(b)], contradicting the prediction of Eq. (1), has been verified by the studies at energies reached at the SLAC e^+e^- storage ring PEP, the DESY e^+e^- collider PETRA, and the CERN e^+e^- collider [5]. In 1985 the LUND group extended the above hypothesis to $\Upsilon \rightarrow 3g$ decay. They assumed that three gluons form a closed triangle consisting of three attractive strings (Fig. 2). This picture qualitatively explained the fact that the baryon yields in the Υ decay are significantly higher than that in the adjacent $e^+e^- \rightarrow q\bar{q}$ continuum [6]. From the above discussion, we conclude that the hypothesis regarding a gluon as a bicolor and each color charge stretching the string with other partons is reasonable. However, it was shown recently that, regardless of how we adjust the free parameters, the LUND model cannot quantitatively reproduce the precise ARGUS results on various particle yields and other

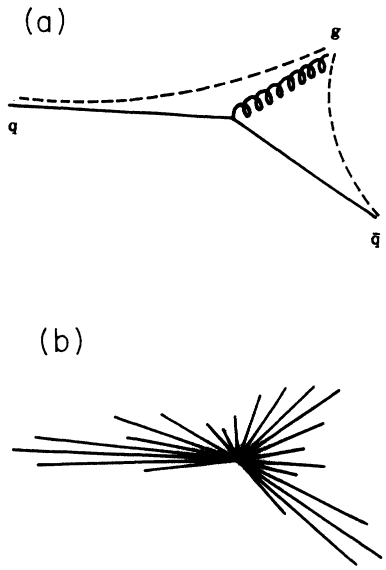

FIG. 1. (a) Color attractive string stretched between g and q (or $\bar{q}$) in the $q\bar{q}g$ system. The LUND model assumes that both of them are fragmentized independently as a $e^+e^-\to q\bar{q}$ color singlet string. This is a close approximation with an accuracy of 90%, in fact. (b) In fragmentation along the two strings shown in (a), the "string effect" appears in the final particle density distribution.

features of Υ decay [7]. This shows that the $\Upsilon\to 3g\to$hadrons process is not perfectly described by the LUND model.

In the LUND model, the treatments for $e^+e^-\to q\bar{q}g\to 3$ jets and $\Upsilon\to 3g\to$ hadrons include two parts: The first is the assumption of the color string structures for $q\bar{q}g$ and $3g$ systems; the second is the application of the LUND string fragmentation model to the above string structures. Does the $q\bar{q}g$ system always form two independent (i.e., color singlet) attractive strings? Does the ggg system always form three attractive strings, and why are they not independent? These questions remain to be answered in the LUND model. In this paper we present an approach to these questions. Our analysis can be conveniently extended to the other multiparton systems, such as $q\bar{q}gg$, $q\bar{q}Q\bar{Q}$, etc. The plan of this paper is as follows: In Sec. II we write out explicitly the color singlet wave functions of $q\bar{q}g$ and ggg systems. According to perturbative QCD approximations, in Sec.

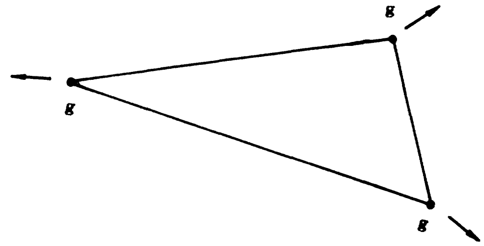

FIG. 2. In $\Upsilon\to ggg$ decays, the LUND model assumes that three gluons always form a closed triangle string connected by three attractive strings between them; our QCD demonstration shows that it is a very close approximation with an accuracy of 97%.

III we study the configurations between all color charges to determine which configuration is in attractive field and can form a string. Then we study the color configurations and their color string structure of $q\bar{q}g$ and ggg systems from their wave functions in Secs. IV and V. Finally, in Sec. VI a brief summary is given.

II. COLOR SINGLET WAVE FUNCTIONS FOR $q\bar{q}g$ AND ggg SYSTEMS

Obviously, the intermediate state $q\bar{q}g$ in $e^+e^-\to q\bar{q}g\to 3$ jets can only be the color singlet obtained from $8\otimes 8$. The singlet wave function W, i.e., the scalar product of A and B, is

$$W=\mathrm{Tr}(A\cdot B)\ , \tag{2}$$

where A and B are traceless irreducible tensors of eight dimensions:

$$A_i^j, B_i^j,\ A_i^i=B_i^i=0\ . \tag{3}$$

The matrix forms of A and B are given by

$$\begin{bmatrix} \frac{g_7}{\sqrt{2}}+\frac{g_8}{\sqrt{6}} & g_1 & g_2 \\ g_4 & \frac{-g_7}{\sqrt{2}}+\frac{g_8}{\sqrt{6}} & g_3 \\ g_5 & g_6 & -\frac{2g_8}{\sqrt{6}} \end{bmatrix}, \tag{4}$$

where

$$\begin{aligned} &g_1=R\bar{G},\ \ g_2=R\bar{B},\ \ g_3=G\bar{B},\ \ g_4=G\bar{R},\ \ g_5=B\bar{R},\ \ g_6=B\bar{G}\ , \\ &g_7=\frac{1}{\sqrt{2}}(R\bar{R}-G\bar{G}),\ \ g_8=\frac{1}{\sqrt{6}}(R\bar{R}+G\bar{G}-2B\bar{B})\ . \end{aligned} \tag{5}$$

Substituting Eqs. (4) and (5) into Eq. (2), one has

$$\begin{aligned} W=&[R_q\bar{B}_{\bar{q}}(B\bar{R})_g+R_q\bar{G}_{\bar{q}}(G\bar{R})_g+B_q\bar{R}_{\bar{q}}(R\bar{B})_g+B_q\bar{G}_{\bar{q}}(G\bar{B})_g+G_q\bar{R}_{\bar{q}}(R\bar{G})_g+G_q\bar{B}_{\bar{q}}(B\bar{G})_g] \\ &+\tfrac{2}{3}[R_q\bar{R}_{\bar{q}}(R\bar{R})_g+B_q\bar{B}_{\bar{q}}(B\bar{B})_g+G_q\bar{G}_{\bar{q}}(G\bar{G})_g] \\ &-\tfrac{1}{3}[R_q\bar{R}_{\bar{q}}(B\bar{B})_g+R_q\bar{R}_{\bar{q}}(G\bar{G})_g+B_q\bar{B}_{\bar{q}}(R\bar{R})_g+B_q\bar{B}_{\bar{q}}(G\bar{G})_g+G_q\bar{G}_{\bar{q}}(R\bar{R})_g+G_q\bar{G}_{\bar{q}}(B\bar{B})_g]\ . \end{aligned} \tag{6}$$

It was well known that the strong decay of the quarkonium state with $J^{PC}=1^{--}$, such as Υ, J/ψ, etc., can only proceed via the $3g$ intermediate state, e.g., $\Upsilon \rightarrow 3g \rightarrow$ hadrons. Apparently, the $3g$ intermediate state can only be a color singlet. We denote the three gluons by A, B, C, respectively. A, B, and C satisfy Eqs. (3), (4), and (5). There are two independent ways for A, B, and C to form a color singlet state:

$$W_{+}=\mathrm{Tr}(A\cdot B\cdot C-B\cdot A\cdot C)\ , \tag{7}$$

$$W_{-}=\mathrm{Tr}(A\cdot B\cdot C+B\cdot A\cdot C)\ . \tag{8}$$

The intermediate state $3g$ must have negative charge conjugation parity. Under the $\hat{C}$ transformation, each gluon wave function (e.g., A) transforms like

$$\hat{C}A=-A^{T}\ . \tag{9}$$

Thus the W_{+} has even C_{+} parity, while the W_{-} has the C_{-} parity, i.e., $\hat{C}W_{-}=-W_{-}$, $\hat{C}W_{+}=+W_{+}$. So the wave function of the $3g$ intermediate state can only be W_{-}. Putting Eq. (4) into (8), one can get the color wave function of the $3g$ intermediate state:

$$\begin{aligned}
W_{-}=&\left[\frac{g_7}{\sqrt{2}}+\frac{g_8}{\sqrt{6}}\right]_A\left[\left[\frac{g_7}{\sqrt{2}}+\frac{g_8}{\sqrt{6}}\right]_B\left[\frac{g_7}{\sqrt{2}}+\frac{g_8}{\sqrt{6}}\right]_C+g_{1B}g_{4C}+g_{2B}g_{5C}\right]\\
&+g_{1A}\left[g_{AB}\left[\frac{g_7}{\sqrt{2}}+\frac{g_8}{\sqrt{6}}\right]_C+\left[-\frac{g_7}{\sqrt{2}}+\frac{g_8}{\sqrt{6}}\right]_B g_{4C}+g_{3B}g_{5C}\right]\\
&+g_{2A}\left[g_{5B}\left[\frac{g_7}{\sqrt{2}}+\frac{g_8}{\sqrt{6}}\right]_C+\left[-2\frac{g_8}{\sqrt{6}}\right]_B g_{5C}+g_{4C}g_{6B}\right]\\
&+g_{4A}\left[\left[\frac{g_7}{\sqrt{2}}+\frac{g_8}{\sqrt{6}}\right]_B g_{1C}+g_{1B}\left[-\frac{g_7}{\sqrt{2}}+\frac{g_8}{\sqrt{6}}\right]_C+g_{2B}g_{6C}\right]\\
&+\left[-\frac{g_7}{\sqrt{2}}+\frac{g_8}{\sqrt{6}}\right]_A\left[g_{4B}g_{1C}+\left[-\frac{g_7}{\sqrt{2}}+\frac{g_8}{\sqrt{6}}\right]_B\left[-\frac{g_7}{\sqrt{2}}+\frac{g_8}{\sqrt{6}}\right]_C+g_{3B}g_{6C}\right]\\
&+g_{3A}\left[g_{5B}g_{1C}+g_{6B}\left[-\frac{g_7}{\sqrt{2}}+\frac{g_8}{\sqrt{6}}\right]_C+\left[-\frac{2g_8}{\sqrt{6}}\right]_B g_{6C}\right]\\
&+g_{5A}\left[\left[\frac{g_7}{\sqrt{2}}+\frac{g_8}{\sqrt{6}}\right]_B g_{2C}+g_{1B}g_{3C}+g_{2B}\left[-\frac{2g_8}{\sqrt{6}}\right]_C\right]\\
&+g_{6A}\left[g_{4B}g_{2C}+\left[-\frac{g_7}{\sqrt{2}}+\frac{g_8}{\sqrt{6}}\right]_B g_{3C}+g_{3B}\left[-\frac{2g_8}{\sqrt{6}}\right]_C\right]\\
&+\left[-\frac{2g_8}{\sqrt{6}}\right]_A\left[g_{5B}g_{2C}+g_{6B}g_{3C}+\left[-\frac{2g_8}{\sqrt{6}}\right]_B\left[\frac{-2g_8}{\sqrt{6}}\right]_C\right]+(A\leftrightarrow B)\ .
\end{aligned} \tag{10}$$

The W_{-} can be expressed in terms of the bicolors of A, B, and C by substituting Eq. (5) into (10). This lengthy expression is omitted here.

III. ATTRACTIVE STRINGS BETWEEN VARIOUS COLOR CHARGES

As mentioned in Sec. I, the extension of the LUND SF model to the cases containing gluons, i.e., $q\bar{q}g$ or ggg, is based on the following understanding of QCD: The color interaction of the gluon bicolor is equivalent to that of the color octet of $q\bar{q}$; each color charge of a gluon is similar to the single color charge of q or $\bar{q}$ and may interact independently with the color charges of other partons. When the interaction is an attractive one, we say that there is a string between them.

There are six kinds of color charges R, B, G, $\bar{R}$, $\bar{B}$, $\bar{G}$, and $6+(6\times5)/2=21$ combinations, such as $R-R$, $R-G$, $R-\bar{G}$, $\bar{R}-\bar{G}$, etc. Though we cannot calculate their interaction strength quantitatively, the study of hadron spectra indicates that even for these typical nonperturbative QCD phenomena the qualitative judgments that the force between color charges is attractive or repulsive, given by the perturbative QCD approximation

$$V\propto\sum_{k=1}^{8}F_c^k(1)F_c^k(2)=\tfrac{1}{2}(C^2-\tfrac{8}{3})\ , \tag{11}$$

are correct [8]. In Eq. (11), $F_c^k=\lambda^k/2$, where λ^k is Gell-Mann matrix of $\mathrm{SU}_c(3)$; C is the eigenvalue of the Casimir operator for the $\mathrm{SU}_c(3)$ eigenstate for the pair of color charges. Table I shows the value of C^2 and

TABLE I. Value of C^2 and $\sum F_c^k(1)F_c^k(2)$ in different states of the color configurations between two partons together with the conclusion that each state is attractive or repulsive.

The state of color configurations	1	3 or $\overline{3}$	6 or $\overline{6}$	8
The value of C^2	0	$\frac{4}{3}$	$\frac{1}{3}$	3
$\sum F_c^k(1)F_c^k(2)$	$-\frac{4}{3}$	$-\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{6}$
Repulsive (R) or attractive (A)	A	A	R	R

$\sum_{k=1}^{8} F_c^k(1)F_c^k(2)$ in different states. Whether the force between two color charges of different partons is attractive or repulsive (i.e., whether or not a string can be stretched) is determined by the configuration of the two color charges in the wave function of the whole system [e.g., Eq. (6) or (10)].

(1) For the nine color-anticolor combinations $R-\overline{R}$, $B-\overline{B}$, $G-\overline{G}$, and $R-\overline{G}$, $R-\overline{B}$, $B-\overline{G}$, $B-\overline{R}$, $G-\overline{R}$, $G-\overline{B}$, as $3\otimes 3=8\oplus 1$, there is an attractive string only when the combination is a color singlet,

$$\frac{R\overline{R}+B\overline{B}+G\overline{G}}{\sqrt{3}}\ , \tag{12}$$

but not a color octet,

$$\begin{aligned}&R\overline{G},\ R\overline{B},\ B\overline{G},\ B\overline{R},\ G\overline{R},\ G\overline{B},\\ &\frac{R\overline{R}-B\overline{B}}{\sqrt{2}},\ \frac{R\overline{R}+B\overline{B}-2G\overline{G}}{\sqrt{6}}\ .\end{aligned} \tag{13}$$

(2) For 12 color-color and anticolor-anticolor combinations $R-B$, $B-G$, $G-R$, $\overline{R}-\overline{B}$, $\overline{B}-\overline{G}$, $\overline{G}-\overline{R}$, and $R-R$, $B-B$, $G-G$, $\overline{R}-\overline{R}$, $\overline{B}-\overline{B}$, $\overline{G}-\overline{G}$, there is an attractive string only when the combination is in asymmetry states $\mathbf{3}$ or $\overline{\mathbf{3}}$,

$$\begin{aligned}&\frac{RB-BR}{\sqrt{2}},\ \frac{BG-GB}{\sqrt{2}},\ \frac{GR-RG}{\sqrt{2}}\ ,\\ &\frac{\overline{R}\overline{B}-\overline{B}\overline{R}}{\sqrt{2}},\ \frac{\overline{B}\overline{G}-\overline{G}\overline{B}}{\sqrt{2}},\ \frac{\overline{G}\overline{R}-\overline{R}\overline{G}}{\sqrt{2}}\ ,\end{aligned} \tag{14}$$

but not in symmetry state $\mathbf{6}$ and $\overline{\mathbf{6}}$,

$$\begin{aligned}&RR,\ BB,\ GG,\ \frac{BG+GB}{\sqrt{2}},\ \frac{RG+GR}{\sqrt{2}},\ \frac{GB+BG}{\sqrt{2}}\ ,\\ &\overline{R}\overline{R},\ \overline{B}\overline{B},\ \overline{G}\overline{G},\ \frac{\overline{R}\overline{B}+\overline{B}\overline{R}}{\sqrt{2}},\ \frac{\overline{R}\overline{G}+\overline{G}\overline{R}}{\sqrt{2}},\ \frac{\overline{G}\overline{B}+\overline{B}\overline{G}}{\sqrt{2}}\ .\end{aligned} \tag{15}$$

From the above discussion, we conclude that the combinations of the color and its noncomplementary anticolor (e.g., $R-\overline{B}$) or two of the same colors (or anticolors) (e.g., $R-R$, $\overline{B}-\overline{B}$) can never form strings because they can only be in $\mathbf{8}$, $\mathbf{6}$, or $\overline{\mathbf{6}}$ states; for the rest of the combinations, whether or not they can stretch a string depends on their states in the color wave function of the whole system.

IV. COLOR STRING STRUCTURE FOR THE $e^+e^-\to q\overline{q}g$ SYSTEM

From the above sections, we see that the color configurations for the $q\overline{q}g$ and $3g$ systems are treated in similar ways in our approach. However, the color singlet wave function of $q\overline{q}g$ is more simple than that of the $3g$ system. Unlike $\Upsilon\to 3g$, the center-of-mass energies of qg and qg systems in the $e^+e^-\to q\overline{q}g$ process can be so high that the α_s is significantly less than 1 (e.g., in the present LEP energy region). This makes the analysis of a force string based on perturbative QCD in Sec. III more reliable. Moreover, according to the recent study by the LUND group for parton-hadron duality [9], the multiplicity and momentum distribution of final state hadrons is mainly determined by the color string structure of $q\overline{q}g$ before hadronization, and so the color string structure of $q\overline{q}g$ may even be tested directly by using three-jet events. For a color singlet of the $q\overline{q}g$ system, the interaction between g and q (or $\overline{q}$) may come from that between the color charge of q (or anticolor charge of $\overline{q}$) and the anticolor (or color) charge of g. According to this combination, we rewrite Eq. (6) as

$$\begin{aligned}W=&[(R_q\overline{R}_g+B_q\overline{B}_g+G_q\overline{G}_g)(\overline{R}_{\overline{q}}R_g+\overline{B}_{\overline{q}}B_g+\overline{G}_{\overline{q}}G_g)]\\ &-\tfrac{1}{3}[(R_q\overline{G}_g)(\overline{R}_{\overline{q}}G_g)+(R_q\overline{B}_g)(\overline{R}_{\overline{q}}B_g)+(B_q\overline{R}_g)(\overline{B}_{\overline{q}}R_g)+(B_q\overline{G}_g)(\overline{B}_{\overline{q}}G_g)+(G_q\overline{R}_g)(\overline{G}_{\overline{q}}R_g)\\ &+(G_q\overline{B}_g)(\overline{G}_{\overline{q}}B_g)+(R_q\overline{R}_g)(\overline{R}_{\overline{q}}R_g)+(B_q\overline{B}_g)(\overline{B}_{\overline{q}}B_g)+(G_q\overline{G}_g)(\overline{G}_{\overline{q}}G_g)]\ .\end{aligned} \tag{16a}$$

Obviously, in the first term of W,

$$W_1=[(R_q\overline{R}_g+B_q\overline{B}_g+G_q\overline{G}_g)(\overline{R}_{\overline{q}}R_g+\overline{B}_{\overline{q}}B_g+\overline{G}_{\overline{q}}G_g)]\ , \tag{16b}$$

both $q-g$ and $\overline{q}-g$ form color singlet strings. We denote this normalized string configuration by $[\mathbf{1},\mathbf{1}]$:

$$[\mathbf{1},\mathbf{1}]=\frac{1}{\sqrt{3}}(R_q\overline{R}_g+B_q\overline{B}_g+G_q\overline{G}_g)\frac{1}{\sqrt{3}}(\overline{R}_{\overline{q}}R_g+\overline{B}_{\overline{q}}B_g+\overline{G}_{\overline{q}}G_g)\ .$$

On the other hand, the interaction between g and q (or $\overline{q}$) may also come from that between the color charge of q (or anticolor charge of $\overline{q}$) and the color (or anticolor) charge of g, i.e., $3\otimes 3$ (or $\overline{3}\otimes\overline{3}$). According to this combination, we rewrite Eq. (6) as

$$W=\tfrac{2}{3}\{(R_qR_g)(\bar{R}_{\bar{q}}\bar{R}_g)+(B_qB_g)(\bar{B}_{\bar{q}}\bar{B}_g)+(G_qG_g)(\bar{G}_{\bar{q}}\bar{G}_g)+[(R_qB_g)(\bar{B}_{\bar{q}}\bar{R}_g)+(B_qR_g)(\bar{R}_{\bar{q}}\bar{B}_q)]$$
$$+[(R_qG_g)(\bar{G}_{\bar{q}}\bar{R}_g)+(G_qR_g)(\bar{R}_{\bar{q}}\bar{G}_g)]+[(B_qG_g)(\bar{G}_{\bar{q}}\bar{B}_g)+(G_qB_g)(\bar{B}_{\bar{q}}\bar{G}_g)]\}$$
$$+\tfrac{1}{3}\{(R_qB_g-B_qR_g)(\bar{B}_{\bar{q}}\bar{R}_g-\bar{R}_{\bar{q}}\bar{B}_g)+(R_qG_g-G_qR_g)(\bar{G}_{\bar{q}}\bar{R}_g-\bar{R}_{\bar{q}}\bar{G}_g)+(G_qB_g-B_qG_g)(\bar{B}_{\bar{q}}\bar{G}_g-\bar{G}_{\bar{q}}\bar{B}_g)\}\ . \tag{17a}$$

In the second term

$$W_2=\{(R_qB_g-B_qR_g)(\bar{B}_{\bar{q}}\bar{R}_g-\bar{R}_{\bar{q}}\bar{B}_g)+(R_qG_g-G_qR_g)(\bar{G}_{\bar{q}}\bar{R}_g-\bar{R}_{\bar{q}}\bar{G}_g)+(G_qB_g-B_qG_g)(\bar{B}_{\bar{q}}\bar{G}_g-\bar{G}_{\bar{q}}\bar{B}_g)\}\ , \tag{17b}$$

both $g-q$ and $g-\bar{q}$ are an asymmetry state (3 or $\bar{3}$) and can form strings. We denote this normalized string configuration by $[3,\bar{3}]$:

$$[3,\bar{3}]=\frac{1}{\sqrt{2}}(R_qB_g-B_qR_g)\frac{1}{\sqrt{2}}(\bar{B}_{\bar{q}}\bar{R}_g-\bar{R}_{\bar{q}}\bar{B}_g)+\frac{1}{\sqrt{2}}(R_qG_g-G_qR_g)\frac{1}{\sqrt{2}}(\bar{G}_{\bar{q}}\bar{R}_g-\bar{R}_{\bar{q}}\bar{G}_g)$$
$$+\frac{1}{\sqrt{2}}(G_qB_g-B_qG_g)\frac{1}{\sqrt{2}}(\bar{B}_{\bar{q}}\bar{G}_g-\bar{G}_{\bar{q}}\bar{B}_g)\ .$$

Therefore the above study shows that the color singlet wave function of the $q\bar{q}g$ system cannot completely form configurations $[1,1]$ or $[3,\bar{3}]$. After tedious recombination and rearrangement, we find that

$$W=\tfrac{2}{3}W_1+\tfrac{1}{3}W_2=2[1,1]+\tfrac{2}{3}[3,\bar{3}]\ . \tag{18}$$

A simple calculation shows that the configuration $[3,\bar{3}]$ accounts for 10% of the $q\bar{q}g$ singlet.

From the comparison of the color string structure of the $q\bar{q}g$ system obtained directly from QCD with that of the LUND model, we conclude that (a) in the LUND model, the assumption that q-g and $\bar{q}$-g can always stretch two strings is reasonable, and (b) the assumption that both of the two color strings come from the interaction between color and anticolor, and each of them is an independent color singlet string (as the string of $e^+e^-\to q\bar{q}$ system); i.e., the string configuration $[1,1]$, is correct only when the 10% of $[3,\bar{3}]$ configurations in Eq. (18) is neglected.

V. COLOR STRING STRUCTURE FOR THE $\Upsilon\to 3g$ SYSTEM

Following the same procedure as in Sec. IV, we start from the color wave function of the $3g$ intermediate state in $\Upsilon\to 3g\to$ hadrons given by Eq. (10) to study its color string structure. Since Eq. (10) is more complicated than Eq. (6), here we only give the final results.

Substituting Eq. (5) into (10) and then recombining and classifying the terms, we get

$$W_-=W_1+W_2+W_3\ , \tag{19}$$

where W_1, W_2, and W_3 are defined as

$$W_1=\tfrac{2}{9}[(R_A\bar{R}_B+B_A\bar{B}_B+G_A\bar{G}_B)(R_B\bar{R}_C+B_B\bar{B}_C+G_B\bar{G}_C)(R_C\bar{R}_A+B_C\bar{B}_A+G_C\bar{G}_A)]$$
$$+(\text{terms with each color changed into its complementary color})\ , \tag{20}$$

where the subscripts A, B, and C, respectively, denote the three gluons (e.g., R_A represents the red color charge carried by gluon A, etc.). Obviously, the color and anticolor of the three pairs of gluons A-B, B-C, and A-C in W_1 can form color singlet strings. We symbolize this kind of color string configuration by $[1,1,1]$:

$$W_2=D+(A\leftrightarrow B\text{ in terms of }D)+(B\leftrightarrow C\text{ in terms of }D)\ , \tag{21}$$

where

$$D=\tfrac{11}{81}(R_A\bar{R}_B+B_A\bar{B}_B+G_A\bar{G}_B)[(R_BB_C-B_BR_C)(\bar{R}_C\bar{B}_A-\bar{B}_C\bar{R}_A)$$
$$+(B_BG_C-G_BB_C)(\bar{B}_C\bar{G}_A-\bar{G}_C\bar{B}_A)+(G_BR_C-R_BG_C)(\bar{G}_C\bar{R}_A-\bar{R}_C\bar{G}_A)]$$
$$+(\text{terms with each color changed into its complementary color})\ . \tag{22}$$

It is easy to see that there are three strings between gluons in W_2. One of them is always a color singlet; the other two are in 3 and $\bar{3}$. We represent this string configuration with $[1,3,\bar{3}]$:

$$W_3=E+(A\leftrightarrow C\text{ in terms of }E)+(B\leftrightarrow C\text{ interms of }E)\ , \tag{23}$$

where

$$
\begin{aligned}
E=\tfrac{2}{27}\{&(R_A\bar{B}_B)(B_BG_C-G_BB_C)(\bar{R}_A\bar{G}_C-\bar{G}_A\bar{R}_C)+(R_A\bar{G}_B)(G_BB_C-B_BG_C)(\bar{R}_A\bar{B}_C-\bar{B}_A\bar{R}_C)\\
&+(G_A\bar{R}_B)(R_BB_C-B_B\bar{R}_B)(\bar{G}_A\bar{B}_C-\bar{B}_A\bar{G}_C)+(G_A\bar{B}_B)(B_BR_C-R_BB_C)(\bar{G}_A\bar{R}_C-\bar{R}_A\bar{G}_C)\\
&+(B_A\bar{R}_B)(R_BG_C-G_BR_C)(\bar{B}_A\bar{G}_C-\bar{G}_A\bar{B}_C)+(B_AG_B)(G_BR_C-R_BG_C)(\bar{B}_A\bar{R}_C-\bar{R}_A\bar{B}_C)\\
&+\tfrac{1}{6}(R_A\bar{R}_B+B_A\bar{B}_B-2G_A\bar{G}_B)[(G_BR_C-R_BG_C)(\bar{R}_A\bar{G}_C-\bar{G}_A\bar{R}_C)+(B_BG_C-G_BB_C)(\bar{G}_A\bar{B}_C-\bar{B}_A\bar{G}_C)\\
&\qquad\qquad-2(B_BR_C-R_BB_C)(\bar{R}_A\bar{B}_C-\bar{B}_A\bar{R}_C)]\\
&+\tfrac{1}{2}(R_A\bar{R}_B-B_A\bar{B}_B)[(B_BG_C-G_BR_C)(\bar{G}_A\bar{B}_C-\bar{B}_A\bar{G}_C)-(G_BR_C-R_BG_C)(\bar{R}_A\bar{G}_C-\bar{G}_A\bar{R}_C)]\}\\
&+(\text{terms with each color changed into its complementary color})\,.
\end{aligned}
\tag{24}
$$

Comparing W_2 with W_3, we note that there is a fundamental difference between these two terms. In both W_2 and W_3, two of the three pairs of gluons A-B, B-C, and A-C can form 3 and $\bar{3}$ strings. The remaining pair in W_2 is a color singlet and can form a string, whereas it is a color octet and cannot form string in W_3. That is, the three gluons A, B, and C can only connect each other with 3 and $\bar{3}$ color strings, but cannot form a closed triangular string as assumed in the LUND model. We denote this structure as $[3,\bar{3},8]$. This case is not considered in the LUND model.

Using Eqs. (19)–(24) to express the W_- with $[1,1,1]$, $[1,3,\bar{3}]$, and $[3,\bar{3},8]$, the probabilities of the three kinds of string structure can be calculated to be 65%, 32%, and 3%, respectively. The 3% of $[3,\bar{3},8]$ is neglected in the LUND model. Because $[1,3,\bar{3}]$ still accounts for 32%, the color field between two gluons may be in 3, $\bar{3}$, or 1; only the whole system connected by the three strings is surely a color singlet. This feature has been reflected by the LUND model: In the LUND model, the three strings between three gluons are not independent and must fragment as a closed triangular string which is a color singlet.

VI. CONCLUSION

In the LUND model, the explanation for $e^+e^-\to 3$ jets and Υ strong decay is based on the color string structure of the $q\bar{q}g$ and $3g$ systems shown in Figs. 1 and 2 (see Sec. I for details). In this paper we start directly from the color wave function of $q\bar{q}g$ and $3g$ systems to study their color string structure in QCD. For $e^+e^-\to q\bar{q}g$ process, 10% configurations form 3 and $\bar{3}$ strings of the $q\bar{q}g$ system that cannot fragment independently; the remaining 90% forms two color singlet strings. In this case the LUND model is an approximation neglecting 10% of the $[3,\bar{3}]$ configuration. For the $\Upsilon\to 3g$ process, 3% configurations of the $3g$ system form two 3 and $\bar{3}$ color strings; the remaining 97% form three color strings, and each of them is a mixture of 1, 3, and $\bar{3}$ color states; only the closed triangular string formed by the three strings is a color singlet. In this case the LUND model is equivalent to the case neglecting the 3% of the $[3,\bar{3},8]$ configurations.

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