Parton interaction rates in the quark-gluon plasma

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The transport interaction rates of elastic scattering processes of thermal partons in the quark-gluon plasma are calculated beyond the leading logarithm approximation using the effective perturbation theory for QCD at finite temperatures developed by Braaten and Pisarski. The results for the ordinary and transport interaction rates obtained from the effective perturbation theory are compared to perturbative approximations based on an infrared cutoff by the Debye screening mass. The relevance of those interaction rates for a quark-gluon plasma possibly formed in ultrarelativistic heavy ion collisions is discussed.

I. INTRODUCTION

Interaction or damping rates of parton scattering processes in a thermalized quark-gluon plasma (QGP) are of great physical significance. For example, the inverse rates related to the elastic scattering of thermal partons ($gg \to gg$, $gq \to gq$, $qq \to qq$) give the mean free paths and typical interaction times, i.e., the relaxation times used in the collision term of the Boltzmann equation. Hence they can be used for an estimate of the thermalization time of a preequilibrium parton gas in ultrarelativistic heavy ion collisions [1] and the maintenance of the thermal equilibrium by comparing the interaction rates versus the cooling rate. Of course thermalization times calculated in this way at finite temperature should be valid only for situations which do not start too far from equilibrium. On the other hand, this thermalization time does not depend on the somewhat ambiguous definition of the beginning and the end of the preequilibrium stage as the one deduced from numerical studies of ultrarelativistic heavy ion collisions (HIJING [2], parton cascade [3]). After all, both methods lead to similar results, namely a fast thermalization of the gluons, i.e., an isotropic and exponential momentum distribution, after about 0.2 fm/c for the CERN Large Hadron Collider (LHC) and 0.3 fm/c for the BNL Relativistic Heavy Ion Collider (RHIC) [4]. The full equilibration of the phase space density can be investigated, on the other hand, using the inelastic interaction rates, e.g., $gg \to ggg$ and $gg \to q\bar{q}$, describing the chemical equilibration process of the QGP [4].

Furthermore the elastic rates are the basic inputs for the energy loss of a parton in the QGP [5–12] and the viscosity of the QGP [13]. The first quantity is related to a possible signature of the QGP in ultrarelativistic heavy ion collisions (jet quenching) [7,14,15], while the latter provides information about the role of dissipation in the hydrodynamic expansion phase [16].

Finally, damping rates are especially suited for studying problems of quantum field theory at finite temperature such as gauge dependences and infrared singularities in perturbation theory [17]. In particular, the puz-

zle of the gauge dependence of the gluon damping rate at rest (plasmon damping) in naive perturbation theory started a recent development in finite temperature field theory, which led to a powerful effective perturbation theory [18]. Assuming the weak coupling limit ($g \ll 1$) effective propagators and vertices are obtained by resumming self-energies and vertices in the high temperature limit (hard thermal loops), which are used for soft external momenta of the order gT, while bare Green's functions are sufficient for hard momenta of the order T. In this way gauge-independent results for physical quantities are found, which are complete to leading order in the coupling constant. (The gauge independence of this method is still under discussion by evaluating the damping rates in a general covariant gauge. However, using an appropriate infrared regularization for the gauge-dependent part, independence of the gauge-fixing parameter can be shown [19–22].) In addition, screening effects are included yielding an improved infrared behavior. In summary, the Braaten-Pisarski method provides a consistent treatment for quantities which are sensitive to the momentum scale gT, meaning a crucial improvement compared to the naive perturbation theory at finite temperature.

Here, we will present calculations of the interaction rates based on the Braaten-Pisarski technique and study their physical relevance for the QGP. Two different kinds of interaction rates are considered. In the next section we will discuss the ordinary interaction rate Γ, in the following simply called interaction rate, of a parton with hard momentum, defined by $\Gamma \equiv n\,\sigma$, where n is the particle density of the QGP and σ the cross section of the scattering process under consideration. In naive perturbation theory without resummation $\Gamma \sim \alpha_s^2 T$ is found ($\alpha_s = g^2/4\pi$). However, Γ turns out to be quadratically infrared divergent. Using the Braaten-Pisarski method, $\Gamma \sim \alpha_s T \ln(1/\alpha_s)$ follows, which is only logarithmically infrared divergent [7,22–29]. The interaction rate is enhanced by a factor $1/\alpha_s$ compared to the one obtained from naive perturbation theory due to the infrared regularization inherent in the resummation method. The logarithmic term, reflecting the logarithmic infrared di-

vergence, arises from the sensitivity of the rate to the momentum scale $g^2 T$, which cannot be treated within the Braaten-Pisarski method.

In Sec. III, we will discuss the transport interaction rate defined by $\Gamma_{\text{trans}} = n\,\sigma_{\text{trans}}$, where the transport cross section $\sigma_{\text{trans}} = \int d\sigma\,(1 - \cos\theta)$ enters the collision term of the transport equation in the case of a plasma with long range interactions, for which the interaction rate is dominated by distant collisions [30]. Here θ denotes the scattering angle in the center of mass system. Hence the mean free path as well as the thermalization time may rather be given by the inverse of the transport interaction rate [30]. The relevance of this rate for the QGP has been pointed out in Refs. [23,31,32]. The transport interaction rate is also closely connected to the shear viscosity beyond the relaxation time approximation [13,33].

The factor $1 - \cos\theta$ cuts off small scattering angles and therefore small momentum transfers, too. Thus the transport interaction rate is only logarithmically infrared divergent in naive perturbation theory (Coulomb logarithm [30]), but finite using the Braaten-Pisarski method since it is sensitive to the scale gT only. Hence dynamical screening of the magnetic interaction, which is included in the effective gluon propagator, is sufficient [34] leading to $\Gamma_{\text{trans}} \sim \alpha_s^2 T \ln(1/\alpha_s)$ [13,29,31]. Note the α_s^2 dependence as expected from naive perturbation theory since there is no enhancement by a factor $1/\alpha_s$ caused by a

quadratic infrared divergence. Also the logarthmic term stems from the sensitivity to the scale gT instead of $g^2 T$ as is the case for Γ [29].

Up to now the transport interaction rates based on the Braaten-Pisarski method have been calculated only within the leading logarithm approximation [13]; i.e., the coefficient of $1/\alpha_s$ under the logarithm has not been determined. However, if we want to extrapolate the result for the transport interaction rate in the weak coupling limit to realistic values of $\alpha_s = 0.2 - 0.5$, we should not neglect this coefficient.

One aim of the present paper is the calculation of the transport interaction rate beyond the leading logarithm approximation using the Braaten-Pisarski method. Furthermore we will discuss the consequences of Γ and Γ_{trans} for realistic values of α_s. Finally, we will compare the results of the Braaten-Pisarski method with the widely used approximation of the naive perturbation theory regularizing the infrared singularities simply by using the Debye screening mass as an infrared cutoff.

II. INTERACTION RATES

There are two equivalent ways of computing the interaction rates either using matrix elements or self-energies [35]. Considering for example elastic quark-quark scattering we may find the corresponding interaction rate Γ_{qq} from the matrix element via [8]

$$\Gamma_{qq} = \frac{1}{2p} \int \frac{d^3 p'}{(2\pi)^3 2p'}\,[1 - n_F(p')] \int \frac{d^3 k}{(2\pi)^3 2k}\,n_F(k)$$

$$\times \int \frac{d^3 k'}{(2\pi)^3 2k'}\,[1 - n_F(k')]\,(2\pi)^4\,\delta^4(P + K - P' - K')\,6N_f\,\langle|\mathcal{M}(qq \to qq)|^2\rangle. \tag{1}$$

The four-momentum of the incoming particle is denoted by $P = (p, \mathbf{p})$, where quark masses are neglected and $p = |\mathbf{p}|$. (We consider only the case of a QGP containing up and down quarks for which the bare masses are negligible compared to the temperature of the QGP.) The quark of the heat bath from which the quark under consideration is scattered off has the momentum $K = (k, \mathbf{k})$. Momenta with a prime belong to the outgoing particles. The Fermi-Dirac distribution functions are given by $n_F(k) = 1/[\exp(k/T) + 1]$, while N_f denotes the number of thermalized flavors in the QGP. The matrix element is averaged over the spin and color degrees of freedom of the incoming particles and summed over the ones of the final state. The factor $6N_f$ comes from summing over the possible spin, color, and flavor states of the quark with momentum K. The diagrams which enter the matrix element to lowest order are shown in Fig. 1(a).

In the case of quark-antiquark scattering we have to replace the diagrams of Fig. 1(a) by the ones of Fig. 1(b). In the case of quark-gluon scattering we have to substitute the Fermi distributions by the Bose-Einstein distributions $n_B(k) = 1/[\exp(k/T) - 1]$, the Pauli blocking factor $1 - n_F(k')$ by the Bose enhancement factor $1 + n_B(k')$,
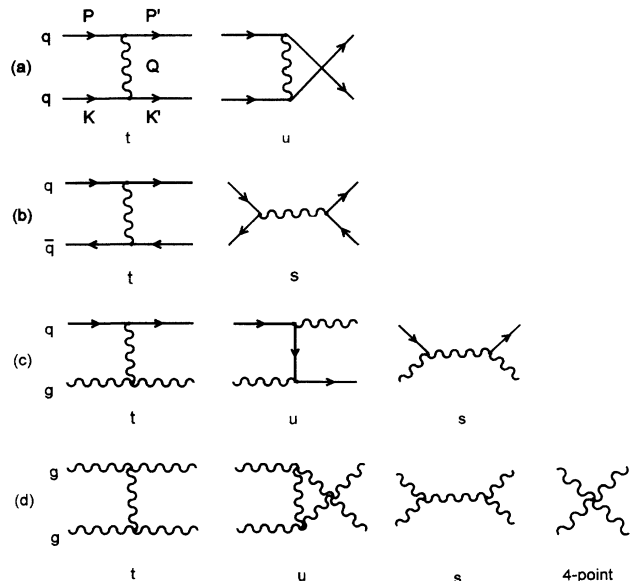

FIG. 1. Lowest order Feynman diagrams for $qq \to qq$ (a), $q\bar{q} \to q\bar{q}$ (b), $qg \to qg$ (c), and $gg \to gg$ (d) scattering.

and the factor $6N_f$ by 16 (number of gluonic degrees of freedom). If we wish to consider the scattering of an incoming gluon by the partons of the QGP we have to deal with the diagrams of Figs. 1(c) and 1(d).

The second possibility to determine the quark interaction rate is given by the imaginary part of the quark self-energy on mass shell [8]:

$$\Gamma_q(p) = -\frac{1}{2p} \left[1 - n_F(p)\right] \text{tr} \left[\gamma^\mu P_\mu \, \text{Im}\Sigma(p, \mathbf{p})\right]. \tag{2}$$

The equivalence of the expressions (1) and (2) can be seen from cutting the self-energy of Fig. 2(a) through the fermion lines. Equation (2) is the starting point for applying the Braaten-Pisarski method by considering the quark self-energy of Fig. 2(b), where the effective gluon propagator contains the resummed one-loop gluon self-energy in the high temperature limit [36,37]. [Note that Fig. 2(b) contains the diagram of Fig. 2(a) if the high temperature limit is used for the fermion loop, called hard thermal loop approximation [18], in the latter.] It is not necessary to take an effective quark-gluon vertex into account because the external quark with a momentum of the order of the temperature is hard. Also because the interaction rate falls off rapidly for large momentum
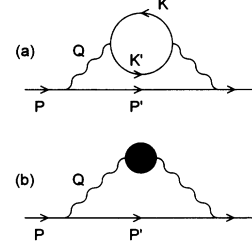

FIG. 2. Lowest order quark self-energy contributions to the quark interaction rate using naive perturbation theory (a) and using the Braaten-Pisarski method (b).

transfers $q \equiv |\mathbf{p} - \mathbf{p}'|$, $\Gamma \sim \int dq/q^3$, i.e., only small momentum transfers contribute, a bare quark propagator is sufficient. An effective quark propagator and quark-gluon vertex would contribute to higher order in α_s only.

Since the final result for observables using the Braaten-Pisarski method is gauge independent we are free of choosing any gauge. Using Coulomb gauge and the approximation $p, k \simeq 3T \gg q \sim gT$ the quark interaction can be written as [8]

$$\Gamma_q = \frac{C_F g^2 T}{2\pi} \int_0^\infty dq \, q \int_{-q}^q \frac{d\omega}{\omega} \left[\rho_l(\omega, q) + \left(1 - \frac{\omega^2}{q^2}\right) \rho_t(\omega, q)\right], \tag{3}$$

where $C_F = 4/3$ is the Casimir invariant of the fundamental representation. The interaction rate of a hard gluon is simply obtained by replacing C_F by the Casimir invariant of the adjoint representation $C_A = 3$ [26]. The four-momentum of the exchanged gluon is denoted by $P - P' = K' - K \equiv Q = (\omega, \mathbf{q})$, and the discontinuous parts of the longitudinal and transverse spectral functions, related to the effective gluon propagator $\Delta_{l,t}$ through $\rho_{l,t}(\omega, q) = \text{Im}\Delta_{l,t}(\omega, q)/\pi$, are given by $(-q \leq \omega \leq q)$ [38]

$$\rho_l(\omega, q) = \frac{3m_g^2 \omega}{2q} \left[\left(q^2 + 3m_g^2 - \frac{3m_g^2 \omega}{2q} \ln \frac{q + \omega}{q - \omega}\right)^2 + \left(\frac{3\pi m_g^2 \omega}{2q}\right)^2\right]^{-1},$$

$$\rho_t(\omega, q) = \frac{3m_g^2 \omega(q^2 - \omega^2)}{4q^3} \left\{\left[q^2 - \omega^2 + \frac{3m_g^2 \omega^2}{2q^2} \left(1 + \frac{q^2 - \omega^2}{2\omega q} \ln \frac{q + \omega}{q - \omega}\right)\right]^2\right.$$
$$\left. + \left(\frac{3\pi m_g^2 \omega(q^2 - \omega^2)}{4q^3}\right)^2\right\}^{-1}, \tag{4}$$

where $m_g^2 \equiv (1 + N_f/6) g^2 T^2/3$ may be interpreted as an effective gluon mass generated by the interaction with the thermal ensemble of the QGP. This expression was derived assuming the high temperature approximation ω, $q \ll T$.

The integration over q can be performed analytically, whereas the one over ω has to be done numerically. The result for the longitudinal part of the interaction rate corresponding to the exchange of a longitudinal gluon is given by [7]

$$\Gamma_q^l = 1.098 \, C_F \, \alpha_s \, T. \tag{5}$$

Note that the interaction rate is independent of the number of flavors N_f due to a cancellation of the factors m_g^2 in (4) after integration. Although a larger number of

thermal flavors corresponds to an increase of the number of scattering partners enlarging the interaction rate, the screening mass is also increased cancelling this enlargement. Also the interaction rate does not depend on the external momentum p in the $p \gg \omega, q$ limit.

The transverse part of the interaction rate Γ_q^t, on the other hand, is still infrared divergent, although the infrared behavior has been improved from a quadratic singularity in the bare two-loop case [Fig. 2(a)] to a logarithmic one due to dynamical screening of magnetic fields. On the other hand, there is no static magnetic screening in the transverse part of the spectral function (or the effective propagator); i.e., the denominator of ρ_t vanishes in the static limit $\omega = 0$, $q \to 0$, while the denominator of ρ_l is given by $\mu_D^2 \equiv 3m_g^2$ in this limit. In other words, the high temperature limit of the perturbatively calcu-

lated gluon self-energy contains static electric screening (Debye mass μ_D) but no static magnetic screening. In QCD, however, static magnetic screening may arise non-perturbatively from monopole configurations due to the gluon self-interaction. Indeed, lattice as well as semiclassical calculations show the existence of a magnetic screening mass $m_{\mathrm{mag}}^2 \simeq 15\,\alpha_s^2\,T^2$ [39,40]. Thus static magnetic screening is provided on the momentum scale g^2T.

After all, in order to obtain an estimate of the transverse interaction rate we consider the nearly static limit ($\omega \ll q$) of (4) [25]

$$\rho_l(\omega \ll q) = \frac{3m_g^2\omega}{2q(q^2 + 3m_g^2)^2},$$

$$\rho_t(\omega \ll q) = \frac{3m_g^2\omega q}{4q^6 + (3\pi m_g^2\omega/2)^2}. \tag{6}$$

Inserting (6) into (3) the integrations can be done exactly, leading to

$$\Gamma_q^l = C_F\,\alpha_s\,T,$$

$$\Gamma_q^t = C_F\,\alpha_s\,T\,\ln\frac{\kappa}{\alpha_s}. \tag{7}$$

We observe that the nearly static limit is a good approximation (within 10%) for the longitudinal rate. The logarithmic term of the transverse rate comes from assuming a hypothetical infrared cutoff of the order g^2T. For example, the magnetic screening mass [25] or the interaction rate itself, i.e., the imaginary part of the quark propagator [22,23], have been suggested as such an infrared regulator. The coefficient κ cannot be calculated using the Braaten-Pisarski technique but must await the development of nonperturbative methods at finite temperature for dynamical quantities [41].

For a rough estimate we propose

$$\Gamma_q = \Gamma_q^l + \Gamma_q^t \simeq (2 \pm 1)\,C_F\alpha_s\,T, \tag{8}$$

assuming $\alpha_s \simeq 0.3$ under the logarithm and $\kappa \overset{<}{\sim} 2$. A justification for the latter assumption may come from choosing μ_D as an upper limit and m_{mag} as a lower limit for the integration over q. The gluon interaction rate, corresponding to the scattering processes shown in Figs. 1(c) and 1(d), is given by $\Gamma_g = (C_A/C_F)\,\Gamma_q = (9/4)\,\Gamma_q$. Because it is also independent of the number of flavors, it holds for the QGP as well as for the pure gluon gas.

Compared to the real part of the dispersion relation for thermal partons, $\omega \simeq k \simeq 3T$, the damping rate $\gamma = \Gamma/2$ defined by the imaginary part of the self-energy, is not really small, $\gamma_g \simeq 3\alpha_s T$ for realistic values of α_s. This anomalously large damping [22,23] indicates that interactions in the QGP are important, at least for temperatures not too far above the phase transition in accordance with the recently emerging picture of the QGP [42].

An alternative method to the Braaten-Pisarski method based on (2) is given by inserting the t-channel diagrams of Fig. 1 in the $-t \equiv -(P - P')^2 \ll s \equiv (P + K)^2$ approximation into (1), where the effective gluon propagator $\Delta_{l,t}$ is used instead of the bare one [43]. This method has been shown to be equivalent to the self-energy calculation [Eqs. (2)-(4)] in the case of the energy loss of a

massive fermion in a hot plasma [8].

Recently, Pisarski proposed an empirical way of including the magnetic mass and an imaginary part of the fermion self-energy in the parton damping rates [44]. In the case of a hard, massless quark, using the magnetic mass of [39,40], $\Gamma_q \simeq 0.8\,C_F\,\alpha_s\,T$ follows from his investigation assuming $\alpha_s = 0.3$ under the logarithm of the transverse part. This result is smaller than the estimate (8), since the transverse part of it turns out to be negative for realistic values of the coupling constant.

Next we discuss a much simpler, widely used approximation (see, for example, Refs. [4-6]) based on the naive perturbation theory, i.e., bare propagators and vertices, where the Debye mass $\mu_D^2 = 3m_g^2$ is simply introduced by hand as an infrared regulator into the gluon propagator. It should be noted that in this case the Debye mass also cuts off the magnetic divergence without justification. Then the interaction rate is easily calculated from

$$\Gamma = n\,\sigma = \int \frac{d^3k}{(2\pi)^3}\,\rho(k)\int dt\,\frac{d\sigma}{dt}, \tag{9}$$

where the parton momentum densities are given by $\rho_q(k) = 12\,N_f\,n_F(k)$ for quarks plus antiquarks and $\rho_g(k) = 16\,n_B(k)$ for gluons, respectively. The cross sections in the small momentum transfer limit ($-t \ll s$) containing the Debye mass read

$$\frac{d\sigma}{dt} = \zeta\,\frac{2\pi\alpha_s^2}{(t + \mu_D^2)^2}, \tag{10}$$

where the color factor $\zeta = 4/9$ for quark-quark scattering, $\zeta = 1$ for quark-gluon scattering, and $\zeta = 9/4$ for gluon-gluon scattering. In contrast with the complete calculation (to leading order in the coupling constant) using the Braaten-Pisarski method, the result depends weakly on the number of flavors. In the case of two flavors ($N_f = 2$) we find

$$\Gamma_q(N_f = 2) \simeq 1.1\,\alpha_s\,T,$$

$$\Gamma_g(N_f = 2) \simeq 2.5\,\alpha_s\,T \tag{11}$$

and, in the pure gluonic case,

$$\Gamma_g(N_f = 0) \simeq 2.2\,\alpha_s\,T. \tag{12}$$

Comparing with (8) the simple approach leading to (11) or (12) appears to underestimate the interaction rates by about a factor of 2. However, it agrees with Pisarski's result [44].

Finally, we discuss the consequences of the present results for typical values, $T = 300$ MeV and $\alpha_s = 0.3$, expected at RHIC and LHC. Using (8) we arrive at relaxation times

$$\tau_g \simeq (0.5 \pm 0.3)\ \mathrm{fm}/c,$$

$$\tau_q \simeq (1.0 \pm 0.5)\ \mathrm{fm}/c \tag{13}$$

indicating a rapid thermalization of the QGP and the maintenance of the local thermal equilibrium during the expansion phase by comparing with a typical expansion time $\tau_{\mathrm{expan}} > 1$ fm/c [4]. Similar results have been found

from Monte Carlo simulations of ultrarelativistic heavy ion collisions [2–4,45–47]. Furthermore the results (13) support the prediction of a two-stage equilibration; i.e., there is first a thermal equilibrium of the gluonic component before a complete thermalization is achieved later on because of the stronger interaction of the gluons compared to the quarks [1,4].

III. TRANSPORT INTERACTION RATES

The relevant physical quantities, such as mean free path and equilibration time, in a plasma with long range interactions are described rather by the transport interaction rates than by the ordinary ones [23,30–32]. The transport rates are obtained from the latter by introducing a weight $1 - \cos\theta$ under the integrals of (1) or (2), defining $\Gamma_{\mathrm{trans}} \equiv \int d\Gamma\,(1 - \cos\theta)$. Here θ denotes the scattering angle in the center of mass system: $\cos\theta = (\mathbf{p}\cdot\mathbf{p'})/(pp') = 1 + 2t/s$. Thus the transport factor $1 - \cos\theta = -2t/s = 2q^2\,(1 - \omega^2/q^2)/s$ is proportional to the square of the momentum transfer q^2. This additional factor q^2 changes the infrared and ultraviolet behavior of the interaction rate completely. The transport interaction rate behaves like $\Gamma_{\mathrm{trans}} \sim \int dq/q$ in naive perturbation theory. Therefore the soft as well as the hard momentum transfer regimes contribute to Γ_{trans}. This is

very similar to the energy loss of a charged particle in a relativistic plasma, where an additional factor ω^2 appears compared to the interaction rate [7,8]. Quantities which are logarithmically infrared divergent in naive perturbation theory turn out to be finite using the Braaten-Pisarski method for soft momentum transfers (dynamical screening). Such quantities can be calculated using the method proposed by Braaten and Yuan [48]. According to this, introducing a separation scale $q^\star$, the soft and hard contributions are calculated separately. For the soft contribution $(q < q^\star)$ resummed propagators and vertices have to be used, whereas bare Green's functions are sufficient for the hard one $(q > q^\star)$. Assuming $gT \ll q^\star \ll T$ the otherwise arbitrary scale $q^\star$ drops out at the end by adding the soft and the hard contributions, reflecting the completeness of the effective perturbation theory. In the following the Braaten-Yuan method will be used for computing the transport interaction rate following the example of the energy loss [8] as close as possible.

We will present the calculation of the gluon transport rate in a pure gluon plasma $(N_f = 0)$ in detail, quoting only the results for the quark and gluon transport rates in a QGP of two active flavors afterwards. The soft contribution follows from (3) introducing the transport weight $1 - \cos\theta$ under the integral and using $q^\star$ as an upper limit for the q integration:

$$\Gamma_{g,\mathrm{trans}}^{\mathrm{soft}} = \frac{C_A g^2 T}{\pi s} \int_0^{q^\star} dq\, q^3 \int_{-q}^{q} \frac{d\omega}{\omega} \left(1 - \frac{\omega^2}{q^2}\right) \left[\rho_l(\omega,q) + \left(1 - \frac{\omega^2}{q^2}\right) \rho_t(\omega,q)\right]. \tag{14}$$

The integral over q can be done exactly, while the ω integral has to be evaluated numerically. This has been done already in Ref. [13] yielding $(m_g^2 = 4\pi\alpha_s T^2/3)$

$$\Gamma_{g,\mathrm{trans}}^{\mathrm{soft}} = \frac{3C_A g^2 T}{2\pi s} m_g^2 \left[\ln\left(\frac{q^{\star 2}}{m_g^2}\right) - 1.379\right]$$

$$= \frac{24\pi\alpha_s^2 T^3}{s} \left[\ln\left(\frac{q^{\star 2}}{\alpha_s T^2}\right) - 2.811\right]. \tag{15}$$

In Ref. [13] the hard contribution has not been computed. However, from general arguments ($q^\star$ cancellation) [48] we know that the hard contribution has to be of the form $\Gamma_{g,\mathrm{trans}}^{\mathrm{hard}} = B\left[\ln(T^2/q^{\star 2}) + A_{\mathrm{hard}}\right]$, where $B = 24\pi\alpha_s^2 T^3/s$ and the constant A_{hard} has to be determined from a detailed calculation of the hard contribution. Thus, if we are only interested in a logarithmic accuracy (leading logarithm approximation) valid in the weak coupling limit, we end up with [13]

$$\Gamma_{g,\mathrm{trans}} = \frac{24\pi\alpha_s^2 T^3}{s} \ln\frac{1}{\alpha_s}. \tag{16}$$

However, since the strong coupling constant is not small for realistic values expected in ultrarelativistic heavy ion collisions, the knowledge of the coefficient under the logarithm is essential. Therefore the constant A_{hard} of the hard contribution has to be determined. Before we turn to this, let us note that the $\ln(1/\alpha_s)$ term in (16) arises

from the sensitivity of Γ_{trans} to the momentum scale gT, while the logarithmic term in the interaction rate (7) comes from a sensitivity to the scale $g^2 T$ [29]. Thus the transport interaction rate can be calculated completely to leading order in the coupling constant using the Braaten-Pisarski method in contrast with Γ. These entirely different properties of the two different kinds of rates originate, of course, from the additional factor q^2 in Γ_{trans}.

For the hard contribution $(q > q^\star)$ it is sufficient to use the bare gluon propagator. However, in contrast to the soft contribution the $-t \ll s$ approximation does not hold any more and the u- and s-channel diagrams of Fig. 1 cannot be neglected. The corresponding amplitudes have been calculated a long time ago in Feynman gauge [49,50]. Since the on-shell matrix elements are gauge invariant, regardless of the momenta integrated over, there is no problem in adopting those results although the soft contribution has been evaluated in Coulomb gauge [8,51].

However, a new difficulty arises now from the divergence of the u-channel contribution for $u = (P - K')^2 \to 0$, i.e., $-t \to s$. (For massless particles $s+t+u = 0$ holds.) The u-channel divergence can be regulated in the same way as the t-channel singularity, if we choose the transport factor $(\sin\theta)^2/2 = 2tu/s^2$ instead of $1 - \cos\theta$. This choice is justified because the transport weight $1 - \cos\theta$ has been introduced only for small scattering angles [30] for which it is identical to $(\sin\theta)^2/2$. The latter

also restores the t-u-channel symmetry in the transport cross sections of quark-quark and gluon-gluon scattering. Furthermore, the shear viscosity coefficient beyond the relaxation time approximation also contains a factor $\sin^2\theta$ [13,16,32,33]. Finally, parton collisions with scattering angles near $0°$ as well as $180°$ are less important for achieving an isotropic momentum distribution (ther-

mal equilibrium) indicating the physical significance of a transport rate defined by a weight proportional to $\sin^2\theta$ instead of $1 - \cos\theta$ for the equilibration process.

For calculating the hard contribution of the transport interaction rate in a gluon gas we start from (1) modified to gluon-gluon scattering and including the factor $(\sin\theta)^2/2$:

$$\Gamma_{g,\text{trans}}(N_f = 0) = \frac{1}{2p} \int \frac{d^3p'}{(2\pi)^3 2p'} \left[1 + n_B(p')\right] \int \frac{d^3k}{(2\pi)^3 2k} \, n_B(k)$$

$$\times \int \frac{d^3k'}{(2\pi)^3 2k'} \left[1 + n_B(k')\right] (2\pi)^4 \, \delta^4(P + K - P' - K') \, 16 \, \langle |\mathcal{M}(gg \to gg)|^2 \rangle \, \frac{\sin^2\theta}{2}, \quad (17)$$

where the matrix element contains the scattering diagrams of Fig. 1(d). While the hard contribution of the energy loss of a heavy fermion with mass $M \gg T$ could be calculated exactly [8,9], this is not possible for (17). Therefore we propose the following approximations: First we assume $n_B(p') \simeq n_B(p)$ and $n_B(k') \simeq n_B(k)$. These simplifications hold as long as $q = |\mathbf{p} - \mathbf{p}'| = |\mathbf{k}' - \mathbf{k}|$ is not too large or $-t$ is not of the order of s. This assumption may be justified because the transport factor $(\sin\theta)^2/2$ cuts off those momenta effectively. While we will neglect $n_B(p)$ because of $\langle p \rangle \simeq 3T$, we do not set $n_B(k) = 0$. As a matter of fact, the Bose enhancement factor $1 + n_B(k)$ is important for the exact matching of the soft and hard parts, i.e., for the cancellation of $q^\star$.

Using the definition of the differential cross section [52] (17) may now be written as

$$\Gamma_{g,\text{trans}}(N_f = 0) = \int \frac{d^3k}{(2\pi)^3} \, 16 \, n_B(k) \left[1 + n_B(k)\right] \int dt \left(\frac{d\sigma}{dt}\right)_{gg} \frac{2tu}{s^2}. \quad (18)$$

The k integration over the Bose distribution functions gives a factor $8T^3/3$, compared to $16\zeta(3)T^3/\pi^2 = 1.95\,T^3$ neglecting the Bose enhancement factor. The differential cross section for $gg \to gg$ scattering according to the diagrams of Fig. 1(d) are taken from Ref. [49]:

$$\left(\frac{d\sigma}{dt}\right)_{gg} = \frac{9g^4}{64\pi s^2} \left(-\frac{us}{t^2} - \frac{st}{u^2} - \frac{tu}{s^2} + 3\right), \quad (19)$$

where a factor $1/2$ has been included to account for the identical particles in the final state [29]. The hard contribution follows from (18) by restricting the t integration from $-s$ to $-q^{\star 2}$. Assuming $s \gg q^{\star 2}$, (18) together with (19) results in

$$\Gamma_{g,\text{trans}}^{\text{hard}}(N_f = 0)$$

$$= \frac{24\pi\alpha_s^2 T^3}{s} \left[\ln\left(\frac{T^2}{q^{\star 2}}\right) + \ln\left(\frac{s}{T^2}\right) - \frac{19}{15}\right]. \quad (20)$$

In order to proceed with the calculation we replace the Mandelstam variable s under the logarithm by its average thermal value $\langle s \rangle = 2\langle p \rangle \langle k \rangle = 14.59\,T^2$ for gluon momenta $\langle p \rangle = \langle k \rangle = 2.701\,T$. Then we arrive at

$$\Gamma_{g,\text{trans}}^{\text{hard}}(N_f = 0) = \frac{24\pi\alpha_s^2 T^3}{s} \left[\ln\left(\frac{T^2}{q^{\star 2}}\right) + 1.414\right]. \quad (21)$$

Adding up the soft and hard contributions, (15) and (21), the separation scale $q^\star$ drops out as required:

$$\Gamma_{g,\text{trans}}(N_f = 0) = \frac{24\pi\alpha_s^2 T^3}{s} \left[\ln\left(\frac{1}{\alpha_s}\right) - 1.397\right]$$

$$= \frac{24\pi\alpha_s^2 T^3}{s} \ln\frac{0.25}{\alpha_s}$$

$$\simeq 5.2\,\alpha_s^2\,T\,\ln\frac{0.25}{\alpha_s}, \quad (22)$$

where we have used $s \simeq \langle s \rangle$ in the last equation.

The result (22) may be compared to the simple approximation of using bare propagators with the Debye mass as infrared regulator. Since the degree of the infrared singularity is only logarithmic this amounts approximately to using (18), where the separation scale $q^{\star 2}$ is replaced by $\mu_D^2 = 4\pi\alpha_s T^2$ in the upper limit of the t integration. In this way we find

$$\Gamma_{g,\text{trans}} \simeq \frac{24\pi\alpha_s^2 T^3}{s} \ln\frac{0.33}{\alpha_s}. \quad (23)$$

Comparing to (22), one realizes that the, to leading order in α_s exact, result (22) may be obtained by using an effective infrared cutoff of $1.32\,\mu_D^2$ instead of μ_D^2.

In the case of a QGP with two flavors we have to consider the diagrams of Figs. 1(a)–1(c) in addition. Modifying (18) to these processes, the corresponding calculations are a little bit more involved than in the purely gluonic case. For example, we have to be careful about the flavors of the final state, e.g., the flavors of the final state quarks of the u- and s-channel diagrams of Figs. 1(a) and 1(b) have to be identical. Furthermore there is no t-u-channel symmetry in the quark-gluon scattering process. After all the u-channel singularity is cancelled by the transport factor $(\sin\theta)^2/2$ because it is only logarithmic

in this case, rendering the use of an effective quark propagator unnecessary. Also the gluon "mass" m_g depends now on N_f and, finally, in the case of quarks (or antiquarks) with momenta K and K' we have to replace in (18) the factor 16 by $6N_f$, $n_B(k)$ by $n_F(k)$, and $1+n_B(k)$ by $1 - n_F(k)$. The soft contribution for the quark rate follows from (14) by replacing C_A by C_F. Putting everything together we end up with

$$\Gamma_{g,\text{trans}}(N_f = 2) = \left(1 + \frac{N_f}{6}\right) \frac{24\pi\alpha_s T^3}{s'} \ln\frac{0.19}{\alpha_s}$$

$$\simeq 6.6\,\alpha_s^2\, T\, \ln\frac{0.19}{\alpha_s},$$

$$\Gamma_{q,\text{trans}}(N_f = 2) = \left(1 + \frac{N_f}{6}\right) \frac{32\pi\alpha_s T^3}{3s''} \ln\frac{0.21}{\alpha_s}$$

$$\simeq 2.5\,\alpha_s^2\, T\, \ln\frac{0.21}{\alpha_s}, \quad (24)$$

where $s' = (1 + N_f/6)/(1/s_{gg} + N_f/6s_{gq})$ and $s'' = (1 + N_f/6)/(1/s_{gq} + N_f/6s_{qq})$. Here s_{gg} is the Mandelstam variable for two gluons, s_{gq} for one gluon and one quark, and s_{qq} for two quarks in the initial state, leading to $\langle s'\rangle = 15.13\,T^2$ and $\langle s''\rangle = 17.65\,T^2$.

Applying the transport interaction rates (22) and (24) to ultrarelativistic heavy ion collisions we encounter a serious problem: The results (22) and (24), obtained to lowest order perturbation theory, become negative for realistic values of $\alpha_s > 0.2$. Hence we cannot draw any conclusions regarding thermalization times at RHIC and LHC from (24). However, the validity of the Landau collision integral containing the transport cross section beyond the logarithmic approximation depends on the condition that the characteristic length l over which the distribution function varies significantly must be large compared with the screening length $1/\mu_D$, i.e., $l\,\mu_D \gg 1$ [30], which is not satisfied for $1/l \simeq T$ and $\mu_D \simeq 4\sqrt{\alpha_s}T$. Thus the physical significance of the transport interaction rate is somewhat obscure in the QGP.

The unphysical, negative results for the transport rates for realistic values of α_s arise from the separate calculation of the soft and hard contributions, which works only in the weak coupling limit $g \ll 1$. In the soft and hard contributions the assumption $T \gg q^\star \gg gT$ is essential for achieving the cancellation of $q^\star$. Of course, this assumption cannot be satisfied for $g \gtrsim 1$ rendering (15) and (21) negative, although each contribution is positive by itself without any restriction on $q^\star$, since they are equivalent to integrals over squares of magnitudes of matrix elements. A similar problem occurred in (23) by using μ_D as an upper limit for the t integration instead of a regulator in the gluon propagator and assuming $s \gg \mu_D$, which does not hold for $g \gtrsim 1$ any more.

The problem of an unphysical, negative result already appeared in the computation of the energy loss of a heavy quark in the QGP [9]. There it was argued that by using the effective perturbation theory for the entire momentum range this problem may be circumvented, increasing, however, the complexity of the calculation drastically. Gauge invariance and completeness in the order of the coupling constant then demand the use of effective

vertices and quark propagators in addition, corresponding to including higher orders in g [9,48]. Thus, in order to guarantee positive results and gauge invariance at the same time for values of the coupling constant $g \gtrsim 1$, one has to go beyond the lowest order in g. In the weak coupling limit, on the other hand, the lowest order contribution, here $\alpha_s^2 \ln(1/\alpha_s)$, is sufficient. Furthermore such an unphysical behavior for realistic values of the coupling constant has also been observed in the calculation of the gluon plasma frequency beyond leading order [53]. In all of these cases, a negative result shows up, if g exceeds a critical value of about 1.

Finally, we will discuss the implications of the transport interaction rates for the viscosity of the QGP [13,16,32,43,54–56]. In Refs. [13,16] the shear viscosity coefficient η has been calculated from

$$\eta_i \simeq \frac{4}{15} \frac{\epsilon_i}{\Gamma_i^\eta}, \quad (25)$$

where $i = q, g$, ϵ_i is the energy density of the quarks and gluons in the QGP, and $\Gamma_i^\eta = 2\,\Gamma_{i,\text{trans}}$ [13]. [The factor of 2 comes from using the weight $\sin^2\theta$ instead of $(\sin\theta)^2/2$ in the definition of the shear viscosity coefficient [33].] Inserting (24) into (25) we find

$$\eta = \eta_g + \eta_q = \frac{T^3}{\alpha_s^2}\left[\frac{0.11}{\ln(0.19/\alpha_s)} + \frac{0.37}{\ln(0.21/\alpha_s)}\right]. \quad (26)$$

This should be compared to the most elaborate calculation of η (albeit in the leading logarithm approximation) using a variational method for the Boltzmann equation resulting in $\eta = 1.16\,T^3/[\alpha_s^2\ln(1/\alpha_s)]$ [43]. For $\alpha_s < 0.1$ both results are comparable.

IV. CONCLUSIONS

Before presenting our conclusions, we discuss the validity of the approximations used here. The distinction between soft and hard momenta and the cancellation of the separation scale $q^\star$, which are the foundations of the Braaten-Pisarski and the Braaten-Yuan method [18,48], rely on the weak coupling limit assumption $g \ll 1$. In contrast, realistic values of $\alpha_s > 0.2$ imply $g > 1.5$. Since α_s is expected to decrease only logarithmically with increasing temperature, $g \ll 1$ is not even satisfied at the extreme temperature of the Planck scale. On the other hand, the Braaten-Pisarski method is nothing but an improvement of the usual perturbation theory at finite temperature which should work at temperatures above twice the critical according to comparisons with lattice QCD [57,58]. Furthermore, comparing the effective gluon "mass" m_g, calculated using the nonperturbative Hartree approximation, with perturbative results shows a difference between both results by less than 30% at $g = 1.5$ [41]. Also in the case of the energy loss of an energetic quark in the QGP it has been shown that the result depends only weakly on the assumption $gT \ll q^\star \ll T$ [11].

Those observations indicate that the assumption $g \ll 1$ should be merely regarded as a mathematical trick and not as a physical restriction. Therefore we believe that the Braaten-Pisarski method not only provides a consistent treatment of QCD at high temperatures taking into account at the same time important physics as collective effects of the nonideal relativistic plasma, e.g., screening, but also gives results for realistic situations which are correct within about a factor of 2 [59], as long as logarithmic factors $\ln(\text{const}/\alpha_s)$ are not too close to zero or negative as is the case for the transport interaction rate.

We have estimated the ordinary interaction rate of thermal quarks and gluons by using the effective perturbation theory of Braaten and Pisarski, for which the use of an effective gluon propagator is sufficient in the case of thermal partons. Because of the missing static magnetic screening in the transverse part of the effective gluon propagator and the absence of an imaginary part of the quark propagator we still encounter a logarithmic infrared singularity. Assuming a reasonable cutoff, a rough estimate has been obtained, $\Gamma_g = (6.0 \pm 3.0)\,\alpha_s\,T$ for gluons and $\Gamma_q = (2.7 \pm 1.3)\,\alpha_s\,T$ for quarks, which corresponds to relaxation times of the order $\tau = (0.5 \pm 0.3)$ fm/c for gluons and $\tau = (1.0 \pm 0.5)$ fm/c for quarks. This indicates a rapid thermalization of the gluon component (two-stage equilibration [1]) and a maintenance of the local thermal equilibrium during the expansion phase of the possibly formed QGP at RHIC and LHC in accordance with computer simulations of ultrarelativistic heavy ion collisions [2,3,45].

On the other hand, in a plasma with long range interactions as in QCD the physically relevant quantity should be the transport rather than the ordinary interaction rate. The transport rate follows from the ordinary one by introducing a transport weight containing the scattering angle in the center of mass system. Because of this factor the infrared behavior of the interaction rate is completely changed. The transport rate turns out to be finite using the Braaten-Pisarski method because dynamical screening suffices now. We have calculated the transport interaction rate for thermal quarks and gluons beyond the leading logarithm approximation by decomposing it into soft and hard parts according to the prescription of Braaten and Yuan [48]. The soft contribution has been computed by using the effective gluon propagator of the Braaten-Pisarski method, while the hard contribution has been treated using bare propagators and vertices.

Compared to the ordinary interaction rate the transport rate is reduced by a factor of α_s caused by the improved infrared behavior due to the transport weight. For a QGP of two active flavors $\Gamma_{g,\text{trans}} \simeq 6.6\,\alpha_s^2 T\,\ln(0.19/\alpha_s)$ for gluons and $\Gamma_{q,\text{trans}} \simeq 2.5\,\alpha_s^2 T\,\ln(0.21/\alpha_s)$ for quarks have been found. The surprisingly small values of the coefficients under the logarithm show that Γ_{trans} is only meaningful for $\alpha_s \lesssim 0.1$. Improving the calculation by using the Braaten-Pisarski method over the entire momentum range increases the complexity of the calculation enormously, including higher orders of g. Hence no statement about the consequences (e.g., thermalization times, mean free paths) for realistic values of the coupling constant can be given here. For this purpose, at least a calculation beyond the lowest order perturbation theory is required. However, the transport interaction rate obtained here suggests that it may be much smaller than the ordinary rate. Thus the realization of a local thermal equilibrium in relativistic heavy ion collisions seems to be questionable assuming the transport rates to be responsible for thermalization, in contrast to computer simulations. However, neither in HIJING [2] nor in the parton cascade [3] are transport cross sections for the fundamental parton interactions used.

Furthermore the shear viscosity, which is proportional to the inverse of the transport interaction rate [13], has been obtained for the first time beyond the leading logarithm approximation. For values of $\alpha_s < 0.1$, for which the result is well defined, it is large and comparable to the ones obtained by using the leading logarithm approximation [13,16,43]. This supports the idea that dissipation cannot be neglected in hydrodynamic descriptions of the expansion phase of the QGP in ultrarelativistic heavy ion collisions [16].

Finally, we have compared the rates obtained from the Braaten-Pisarski method, which are complete to leading order in the coupling constant, with the widely used approach of using bare propagators including the Debye mass as an infrared regulator. While the latter approximation works well for transport rates and energy losses [8], it seems to underestimate the ordinary rates. This observation suggests that the use of the Debye regulator is justified for quantities which are logarithmically infrared divergent in naive perturbation theory as the transport rates or the energy loss, but might be questionable for quadratically infrared divergent quantities as the ordinary interaction rate.

ACKNOWLEDGMENTS

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