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Twist-4 nuclear parton distributions from photoproduction

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We analyze anomalous nuclear enhancement in the photoproduction of jets as a higher-twist process in perturbative QCD. We use the Fermilab E683 data on dijet momentum imbalance to estimate the size of the relevant twist-4 parton distributions. We find that twist-4 matrix elements are of the order of 0.05–0.1 GeV^2 times typical twist-2 parton distributions. We discuss a physical interpretation of the nuclear enhancement as a measure of the average net transverse color force on an off-shell parton moving through nuclear matter, and give an order-of-magnitude estimate for the typical squared transverse field strengths encountered by a fast moving parton.

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I. INTRODUCTION

In this paper we continue our investigation of high-energy multiple scattering with nuclear targets [1,2]. Theoretical considerations [3] and experimental results [4] suggest that the process of hadronization is largely independent of the size of the target, and that it takes place for the most part outside the nucleus. If this is the case, effects that grow with nuclear size are directly sensitive to the propagation of partons, rather than hadrons, through nuclear matter. This analysis is consistent with a basic insight of the parton model, that the hadronization of high-energy outgoing partons is time dilated in the rest frames of the remaining participants in the hard process. A high-energy scattered quark, for instance, which is hadronizing at a normal rate in its own rest frame, propagates across the entire nucleus before this hadronization can begin. Thus, for high-energy hard scattering events, it is sensible to think of multiple scattering at the partonic level [5].

The field theoretic analogues of parton model formulas are factorization theorems [6–8]. In Ref. [1] we observed that factorization theorems at first-nonleading power in momentum transfer can naturally describe multiple scattering at large transverse momentum in photoproduction. (For related work, see [9].) In that paper, we discussed the cross section $d\sigma/d^3l$, with l the momentum of an outgoing parton, which, according to our discussion above, evolves into a jet of hadrons outside the target nucleus. The relevant factorization theorem is of the form

$$\sigma(l_T)=H^0\otimes f_2\otimes f_2+\left[\frac{1}{l_T^2}\right]H^1\otimes f_2\otimes f_4+O\left[\frac{1}{l_T^4}\right], \quad (1)$$

where $\otimes$ represents a convolution in the $m-1$ parton fractional momenta of f_m. Thus, at leading power, each hadron is represented by a single fractional momentum x, while at first nonleading power, the nucleus has, in the most general case, three fractional momenta over which to integrate.

The technical evaluation of Eq. (1) is described in detail in Refs. [2,8]. At lowest order it consists of an expansion ("collinear expansion") in transverse momenta of the partons and/or by the inclusion of more partons. We have found that it is convenient to carry out this procedure in the Feynman gauge [8].

The lowest-order corrections included in this analysis are shown schematically in Fig. 1, in which a quark scatters from a photon, radiating in the process a hard gluon. One of the outgoing partons later picks up an additional gluon of momentum k on its way out of the nucleus [Fig. 1(a)]. At the same order in the cross section, there are virtual contributions in which the outgoing quark exchanges two gluons on the way out [Fig. 1(b)]. The likelihood of an extra scattering is evidently proportional to the nuclear size, and hence to $A^{1/3}$. The leading effects of this scattering are found at low gluon momentum, but although the gluon itself is soft, it continues to *couple weakly to the outgoing quark,* which is effectively far off-shell before it evolves into a jet [3,8]. This observation is the basis for the application of perturbative QCD to multiple scattering processes in nuclei. We shall give expressions for such cross sections below, of the form of the second term in Eq. (1).

In the following section, we describe the "twist-four" matrix elements, corresponding to f_4 in Eq. (1), which arise in our analysis. In particular, we point out their relation to transverse color fields in the nucleus. In Sec. III

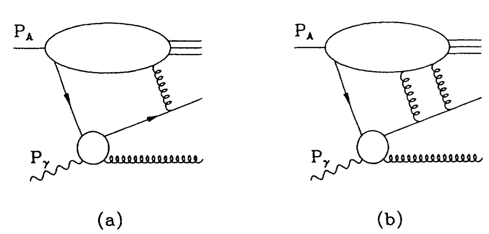

FIG. 1. (a) Lowest-order scattering. (b) Order g^2 scattering.

we give the lowest-order twist-4 contribution to the dijet transverse momentum imbalance, and compare our form for anomalous nuclear enhancement to the Fermilab E683 data [10]. Comparing the data with our theoretical expression, we estimate the size of the relevant twist-4 parton distributions. We also discuss an interpretation of the momentum imbalance in terms of transverse color forces, and determine the order of magnitude of the equivalent field strengths encountered by an outgoing parton. A brief derivation of the A-enhanced twist-4 corrections at lowest order, following the methods introduced in Refs. [1,2], is given in Sec. IV, along with a short discussion of the influence of higher orders. We conclude with a brief summary and suggestions for future work.

II. MATRIX ELEMENTS

The relevant matrix elements are proportional to the joint expectation value of a pair of hard parton (quark or gluon) fields with a squared line integral of the gluonic field strength:

$$T_q(x,A)=\int\frac{dy_1^-}{2\pi}e^{i(n\cdot p)xy_1^-}\int\frac{dy^-dy_2^-}{2\pi}\theta(y^- -y_1^-)\theta(y_2^-)\tfrac{1}{2}\langle p_A|\bar{q}(0)(n\cdot\gamma)n_\nu F^{\alpha\nu}(y_2^-)n_\mu F^{\mu}{}_{\alpha}(y^-)q(y_1^-)|p_A\rangle\ , \tag{2}$$

and

$$T_g(x,A)=\int\frac{dy_1^-}{2\pi}e^{i(n\cdot p)xy_1^-}\int\frac{dy^-dy_2^-}{2\pi}\theta(y^- -y_1^-)\theta(y_2^-)\frac{1}{x(n\cdot p)}\langle p_A|n_\xi F^{\beta\xi}(0)n_\nu F^{\alpha\nu}(y_2^-)n_\mu F^{\mu}{}_{\alpha}(y^-)n_\zeta F^{\zeta}{}_{\beta}(y_1^-)|p_A\rangle\ . \tag{3}$$

Here n^μ is a vector defined by its scalar product with the momentum p of a single nucleon in the nucleus, $n\cdot p=m$, with m the nucleon mass. At high energies, we may think of n^μ as a lightlike vector in the minus direction $[n^{\pm}=2^{-1/2}(n^0\pm n^3)]$,

$$n^\mu=\delta_{\mu-}\ , \tag{4}$$

defined in the projectile-nucleus center of mass system. Thus, we take the incident photon to be moving in the $-z$ direction. In this system, we may also take for the momentum of a nucleon $p^\mu=p^+v^\mu$, with

$$v^\mu=\delta^{\mu+}\ . \tag{5}$$

We note that (2) and (3) are exhibited as they would appear in the $n\cdot A=0$ gauge. In gauge-invariant form, they include ordered exponentials of $n\cdot A$ between the various fields.

The matrix elements T_g and T_q may also be written as squared matrix elements summed over hadronic final states c: for instance,

$$T_q(x,A)=\sum_c(2\pi)\delta(n\cdot p_c-n\cdot(p_A-xp))\langle p_A|\bar{q}(0)\int_0^\infty dy^- n^\mu F_{\alpha\mu}(y^-)|c\rangle\left\langle c\left|\int_0^\infty dy^- n_\nu F^{\nu\alpha}(y^-)\frac{1}{2}(n\cdot\gamma)q(0)\right|p_A\right\rangle\ , \tag{6}$$

where the index α may be $+$ or transverse. The largest contribution is for transverse components, $\alpha=\perp$, in which case the integral of the field strength has the interpretation of the transverse non-Abelian Lorentz force felt by a lightlike quark moving in the n^μ direction through the nucleus. The occurrence of this expectation value is natural for a cross section with measured transverse momentum.

From the general form of Eq. (1), we see that A enhancement must be a property of the relevant nuclear matrix elements, since all information on the identity of the target has been factorized into the matrix elements. Equations (2) and (3) show how such enhancement can occur, through integrals over the relative positions of the fields that appear in the expectation values. For the matrix elements at hand, these are the lightlike separations y_i. Such an enhancement can occur when the expectation of the fields at fixed y_i is essentially independent of A when A is large. We note, however, that this assumption is reasonable only if the fields pair off into color singlets that are separated by no more than a nucleon diameter in the y_i integrals. Otherwise, we would expect color confinement to imply sharp falloff as the fields separate.

These considerations reduce the possible nuclear enhancement in Eqs. (2) and (3) to growth that is linear with the radius. In T_q, for instance, enhancement can occur when the two quark fields are close together and the two gluon fields are close together, but the two pairs are separated by a distance that varies up to the nuclear size. In this manner, we anticipate an $A^{1/3}$ nuclear enhancement, aside from an overall factor of A, which reflects growth with the nuclear volume, even when all four operators are close together.

We may summarize the above considerations with the following representation of A enhancement in the matrix elements (2) and (3):

$$T_a(x,A)=\lambda^2 A^{4/3} f_a(x,A)\ , \tag{7}$$

where λ is a constant with dimensions of mass, and where $f_a(x,A)$ is a dimensionless function that depends on the identity of the "hard" parton, $a=g,q$. We shall assume that λ is the same for quarks and gluons and that $f_a(x,A)$ depends only weakly on A. In fact, because the origin of the A enhancement is in scatterings that occur far apart in the nucleus, we expect $f_a(x,A)$ to be essentially identical to a normal, leading-twist, distribution of parton a in the nucleus (normalized by $1/A$), in which A dependence for moderate x tends to saturate at relatively small values of A, becoming essentially independent of A for $A>10$–20 [11].

Of course, there are twist-4 matrix elements other than those in Eqs. (2) and (3). To produce a nuclear enhancement from the separation of two color-singlet combinations of fields, the relevant operators must include four physical fields. Of particular interest are those involving transverse derivatives on the quark fields. Each such derivative, however, promotes the twist by one. For a single such derivative, then, we can have only a single gluon field strength at twist four, which by itself cannot form a color singlet to generate the A enhancement identified above. (Note that fields like $n\cdot A$, which may be included at higher order without changing the twist, will be incorporated in the ordered exponentials that make the matrix elements gauge invariant. They will not combine with a field strength to produce a physical nuclear enhancement.) We shall, therefore, neglect higher-twist terms which involve derivatives on quark fields. Another possibility is a matrix element with four quark fields. As we shall see, however, the contributions associated with these matrix elements turn out to be even higher twist.

Finally, we note that for the matrix elements (2) and (3) that are relevant to nuclear enhancement, the twist-four convolution in Eq. (1) is in terms of a single momentum fraction for both target and projectile, just as in the leading twist case. This is a significant simplification. We are now ready to turn to the role of these matrix elements in dijet photoproduction cross sections.

III. NUCLEAR ENHANCEMENT FOR PHOTOPRODUCED DIJETS

In Ref. [1], we computed the first correction to $d\sigma^{\gamma A}/d^3l$, the inclusive single-jet cross section with jet momentum l. Here we shall consider a variation of this process, suggested by the experimental studies of Refs. [10,12]. This is the average squared transverse momentum imbalance, k_T^2, of a pair of observed jets. At lowest order, $k_T=0$. The computation of this cross section is actually simpler than that for the inclusive single-jet cross section [1], $d\sigma^{\gamma A}/d^3l$, because the extra factor of k_T^2 in the phase space integral eliminates the virtual double-scattering diagrams, Fig. 1(b), as well as the Born contribution. We will use the observed values of k_T^2 in photoproduction [10] to give a rough estimate for the constant λ in Eq. (7), and hence for the size of the nuclear matrix elements.

Consider the cross section for the observation of two outgoing partons (jets):

$$\gamma+A\to\text{jet}(l)+\text{jet}(l')+X\ . \tag{8}$$

We will study the role of rescattering in the dijet transverse momentum imbalance at lowest order. A general diagram is shown in Fig. 2. In second order in the strong coupling g, the outgoing partons scatter from two additional gluons of the target, labeled $k-k_1$ and $k-k_2$ in the figure.

The extra scatterings of Fig. 2 can produce a transverse momentum imbalance,

$$k_T=l_T+l_T'\ , \tag{9}$$

whose average value we shall calculate. We work in the laboratory frame. At lowest order (Born approximation), the two transverse momenta are equal and opposite:

$$d\sigma^{\gamma A}(l_T)\sim\delta^2(k_T)\ . \tag{10}$$

To compute any (higher-twist) nuclear enhancement, we perform a collinear expansion [7,8] in the momenta of the four external lines (two quark, two gluon) that connect the target nucleus with the order α_s^2 hard scattering, working to second order in the transverse momenta of these lines. The nuclear enhancement, as in Ref. [1], comes about only from the second derivative with respect to k_T, the transverse momentum that flows between the two gluons in Fig. 2. Expansion in terms quark transverse momenta, in particular, will produce higher twist matrix elements with only two *physical* fields (say, two transverse derivatives of the quark fields).

k_T defined above is slightly different from the momentum $k_{T\phi}$ quoted by Ref. [10], which is the projection of k_T out of the scattering plane defined by the beam axes and either of the two observed jets. The relation, assuming rotational symmetry, is simply (we always use rms averages)

$$\langle k_T^2\rangle=2\langle k_{T\phi}^2\rangle\ . \tag{11}$$

To be specific, we consider the average value of k_T^2 for fixed momentum of the less energetic (say) of the two observed jets, denoted l:

$$\langle k_T^2\rangle=\frac{\langle k_T^2E_l(d\sigma/d^3l)\rangle}{E_l(d\sigma/d^3l)}\ , \tag{12}$$

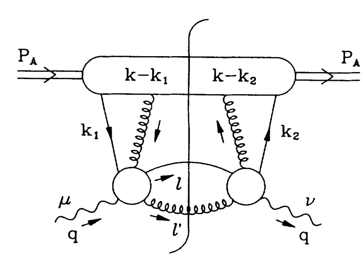

FIG. 2. Sample diagram for a multiple scattering process. The vertical line represents the final state.

where

$$\left\langle k_T^2 E_l \frac{d\sigma}{d^3 l}\right\rangle = \int d^3 k_T k_T^2 E_l \frac{d\sigma}{d^3 l d^2 k_T} \ . \tag{13}$$

Again, these averages are manifestly zero in the Born approximation.

The computation of $\langle k_T^2 E_l (d\sigma/d^3 l)\rangle$ at lowest order closely follows the method introduced in Refs. [1,2], and will be given in the following section. We may summarize the argument there as follows. Nuclear enhancement is found from the discontinuities of the hard scattering, which are computed by setting the lines adjacent to the final state in Fig. 2 on shell. We emphasize that taking the poles is only a calculational device, and that the hard scattering remains short distance and calculable in perturbation theory. The poles fix two of the fractional momenta flowing from the target into the hard scattering to zero, resulting in the matrix elements of Eqs. (2) and (3), which involve four fields but depend on a single fractional momentum only.

In this manner, the nuclear enhancement to $\langle k_T^2 E_l (d\sigma/d^3 l)\rangle$ at order $A^{4/3}$ is found to be

$$\begin{aligned}\left\langle k_T^2 E_l \frac{d\sigma}{d^3 l}\right\rangle_{4/3} &= \sum_{a=q,g} \int dx\, T_a(x,A) H^{\gamma a}(xp,p_\gamma,l) \\ &= \lambda^2 A^{4/3} \sum_{a=q,g} \int dx f_a(x,A) H^{\gamma a}(xp,p_\gamma,l) \ ,\end{aligned} \tag{14}$$

where in the second form we have used Eq. (7), and where $H^{\gamma a}$ is a hard-scattering function. The form shown here is dependent only on the assumption of no long-distance color correlations in the nucleus. We will make an identification of $f_a(x,A)$ with a nuclear twist-two parton distribution. This is a separate assumption, however, upon which the form of Eq. (7) does not depend.

The hard scattering functions $H^{\gamma a}$ that appear in Eq. (14) depend on the particular cross section being computed. We note, however, that all double-scattering cross sections of this type are proportional to the dimensional constant λ^2 at lowest order in the expansion in twist, from the matrix elements Eqs. (2) and (3). Of course, when A becomes large, the $A^{1/3}$ enhancement from the matrix elements may become correspondingly large, and it may be necessary to incorporate higher powers of A, that is, even higher twist. In this paper, however, we will content ourselves with the first correction, which may be appropriate to the moderate nuclear effects found in photoproduction [10].

At lowest order, explicit calculation gives the strikingly simple result [see Eq. (45) below]

$$H^{\gamma a}(xp,p_\gamma,l) = \frac{8\pi^2 \alpha_s(l_T^2)}{N^2-1} C_a E_l \frac{d\hat{\sigma}^{\gamma a}}{d^3 l}(xp,p_\gamma,l) \ , \tag{15}$$

where the $\hat{\sigma}^{\gamma a}$ are partonic jet production cross sections, and C_a are the usual group factors, $C_q=(N^2-1)/2N$, $C_g=N=3$. $\alpha_s(l_T^2)$ is evaluated with the running coupling at the scale of the hard scattering. This gives, from Eq. (14),

$$\left\langle k_T^2 E_l \frac{d\sigma}{d^3 l}\right\rangle_{4/3} = \lambda^2 A^{4/3} \frac{8\pi^2\alpha_s(l_T^2)}{N^2-1}\left[C_q \int dx\, f_q(x,A) E_l \frac{d\hat{\sigma}^{\gamma q}}{d^3 l}(xp,p_\gamma,l) + C_g \int dx\, f_g(x,A) E_l \frac{d\hat{\sigma}^{\gamma g}}{d^3 l}(xp,p_\gamma,l)\right] , \tag{16}$$

with $d\hat{\sigma}_{\gamma a}$ the leading-twist cross section. We suppress the evolution of the distributions, which we assume to be moderate in the experimental range of l_T. Equation (16) is our basic result for the momentum imbalance in photoproduction.

The average value of k_T^2 for all events in a region R of dijet phase space is found from Eq. (16) by integrating over that region and dividing by the corresponding unweighted cross section:

$$\sigma^{\gamma A}(R,p_\gamma) = \int_R \frac{d^3 l}{E_l (2\pi)^3} E_l \frac{d\sigma^{\gamma A}}{d^3 l} \ . \tag{17}$$

In this calculation, we shall evaluate the running coupling $\alpha_s(l_T^2)$ in Eq. (15) at a typical value, Q, of the momentum transfer. We assume that $\alpha_s(l_T^2)$ is approximately constant within R. We then find

$$\begin{aligned}\langle k_T^2(R)\rangle_{4/3} &= \frac{8\pi^2\alpha_s(Q^2)/(N^2-1)}{\sigma^{\gamma A}(R,p_\gamma)} \\ &\quad\times \lambda^2 A^{1/3}[C_q \sigma_q^{\gamma A}(R,p_\gamma) + C_g \sigma_g^{\gamma A}(R,p_\gamma)] \ ,\end{aligned} \tag{18}$$

where $\sigma_g^{\gamma A}(R,p_\gamma)$, for instance, is the gluonic contribution to the nuclear cross section. We have absorbed a factor of A into these cross sections, in accordance with our definitions of the f_a above. In our approximation, linear in twist-4 matrix elements, we may take $\sigma^{\gamma A}(R,p_\gamma)$ as the leading-twist total cross section:

$$\sigma^{\gamma A}(R,p_\gamma) = \sigma_q^{\gamma A}(R,p_\gamma) + \sigma_g^{\gamma A}(R,p_\gamma) \ . \tag{19}$$

If we take, as indicated above, quark and gluon nuclear distributions for the functions $f_a(x,A)$, we can compute the right-hand side of Eq. (18) and, comparing to the experimental A dependence measured for the left-hand side, determine λ^2 and hence, by Eq. (7), the size of the relevant twist-4 matrix elements. In the following, we shall give such an estimate.

Most of the dijet events measured by the Fermilab E683 Collaboration [10] are in the central region of the c.m. frame. As a result, the typical initial parton momentum fraction x satisfies

$$x \geq (2 l_T)^2/s \ , \tag{20}$$

where $s=(21.5)^2$ GeV^2. The data was cut for $l_T > 3$

GeV, so that the initial parton momentum fraction typically obeys $x > 0.08$ for the sampled events. Similarly, in the dijet phase space, R in Eq. (17), if we take $\langle l_T \rangle \sim 4$ GeV, we will have $\langle x \rangle \sim 0.14$ for the Fermilab E683 data. Since gluons concentrate with momentum fraction $x < 0.1$, we may make a simplifying assumption that valence quark contributions dominate over gluonic contributions in Eqs. (18) and (19), at least for the purposes of a rough estimate for the size of the twist-4 matrix elements. In the other extreme, we could assume that the gluonic contribution dominates. In this case, our estimate in Eqs. (21) and (23) below will differ only by replacing C_q by C_g. If we assume that quarks and gluons contribute to a comparable extent, we find an intermediate value. More accurate estimates can be given, and our assumptions about the matrix elements in Eq. (7) checked, when $\langle k_T^2 \rangle$ is given as a function of l_T. It is already clear from Eq. (18), however, that $\langle k_T^2 \rangle$ will be enhanced in a region of phase space, R, where gluons dominate, simply because $C_g > C_q$. Thus, at fixed l_T, we may expect the momentum imbalance to increase with increasing energy.

To be specific, let us return to the approximation in which only valence quarks initiate the hard process [$f_q \gg f_g$ in Eq. (14)]. In this case, $\sigma_q^{\gamma A}$, Eq. (19), cancels between numerator and denominator on the right-hand side of (18), to give

$$\langle k_T^2(R) \rangle_{4/3} = \lambda^2 A^{1/3} \frac{8\pi^2 \alpha_s(Q^2)}{N^2-1} C_q = \tfrac{4}{3}\pi^2 \alpha_s(Q) \lambda^2 A^{1/3} , \tag{21}$$

where we have set, for QCD, $N=3$. Thus, in view of Eq. (6), A enhancement in the average squared momentum imbalance is a direct measure of the squared color Lorentz force experienced by the scattered partons as they leave the nucleus. Note that the only remaining R dependence in Eq. (21) is implicit in Q^2.

From Ref. [10] we have the approximate relation (in terms of the natural logarithm of A)

$$\langle k_T^2 \rangle \sim 2(1.2 + 0.09 \ln A)^2 \sim 2(1.44 + 0.216 \ln A) , \tag{22}$$

in units of GeV^2, where, in the second form we have dropped the term quadratic in $\ln A$ in view of its rather small coefficient. This average includes a set of high l_T dijet events that satisfy criteria that are described in Ref. [10]. In the valence quark approximation of Eq. (21), the dependence on the region R that corresponds to this criteria is relatively weak. We now observe that for $10 < A < 200$, the relation $\ln A \sim A^{1/3}$ is good to 10%, so that the $\ln A$ dependence of (22) has the direct interpretation of dependence on the nuclear radius.

The approximately linear dependence of the average of $\langle k_T^2 \rangle$ on the nuclear radius implies that the average of the absolute value of k_T grows as the square root of the number of scattering centers, identified with nucleons. This is what we would expect if the parton is scattered in a random, independent manner by each nucleus in its path. This implies, in each transverse direction, an average net transverse momentum transfer to the outgoing partons of roughly 0.47 GeV per internucleon distance, or a net transverse force of roughly 0.39 GeV per fm, *within each nucleon*. We have just seen that this interpretation arises naturally in the formalism of higher-twist factorization.

In the approximation $A^{1/3} \sim \ln A$, we can also identify the coefficients of $A^{1/3}$ in Eq. (21) with the coefficient of $\ln A$ in Eq. (22). We then find

$$\lambda^2 \sim \frac{2(0.216)}{C_q \pi^2 \alpha_s(Q^2)} \mathrm{GeV}^2 \sim 0.1 \ \mathrm{GeV}^2 . \tag{23}$$

Recalling the discussion that led to Eq. (7), and estimating $\pi\alpha_s(Q^2) \sim 1$, we see that Eq. (23) suggests that the twist-4 matrix elements T_q and T_g are of order 0.1 GeV^2 times twist-2 parton distribution functions. Of course, we emphasize that the specific number depends on the valence quark approximation. If the gluonic contribution is important, we expect the value of λ^2 to be smaller than the 0.1 GeV^2 estimated in Eq. (23). It should, however, be larger than 0.046 GeV^2, the value that is obtained by the extreme assumption the *quarks* may be neglected. As mentioned above, in this case C_q in Eq. (23) is replaced by C_g. In any case, the experimentally determined result for the λ^2, given here, will be slightly larger, even up to a factor of 2 larger, than the guess $\lambda^2 \sim \Lambda_{\mathrm{QCD}}^2 \sim 0.04$ GeV^2 given in Ref. [1].

Going a step further, we estimate a set of equivalent classical field strengths [13] that would produce a λ^2 of this size in T_q, Eq. (2). We do this by identifying λ^2 in Eq. (7) with the integrals of the squared field strengths, which we factor out, as if they were classical quantities:

$$T_q(x) \sim \int \frac{dy^- dy_2^-}{2\pi} (F^{\perp +})^2 A f_q$$
$$\sim \frac{2R_0(A^{1/3}R_0)}{2\pi} (F^{\perp +})^2 A f_q . \tag{24}$$

To get the first expression, we note that the matrix element that is left in Eq. (2) once the field strengths are removed is just the quark distribution in the nucleus, which we write as Af_q. In the second expression, we have taken the squared fields strengths as constants, and have replaced the y^- and y_2^- integrals by twice the nucleon radius times the nuclear radius. This reflects our intuition that $F(y^-)F(y_2^-)$ contributes to T_q only when the F's are within a nucleon diameter of each other, although their average position may be anywhere within the nucleus in the path of the scattered parton. Given the (7) and (24) for the matrix element and (23) for λ^2, we find the following estimate for the average squared transverse field strength encountered by the outgoing partons:

$$(F^{\perp +})^2 \sim \frac{\pi \lambda^2}{R_0^2}$$
$$\sim 0.22 \ \mathrm{GeV}^2 \ \mathrm{fm}^{-2} \sim 0.85 \times 10^{-2} \ \mathrm{GeV}^4 . \tag{25}$$

Such a semiclassical estimate, in which we have removed the integrals of the field strengths from the matrix element, can have at best an order of magnitude significance. Our central result is the estimate of $\lambda^2 \sim 0.05 - 0.1$ GeV^2, and hence of the twist-4 matrix elements. The connection with field strengths is strongly dependent on our assumptions.

Of course, we also made a number of approximations to reach λ^2, but we believe it to be a reasonable guide to the scale of the nuclear matrix elements that we set out to study. In the next section, we shall give some technical details of the calculation of the twist-4 hard-scattering function, Eq. (15).

IV. FACTORIZATION AT LOWEST ORDER

In this section we discuss the lowest-order computation that leads to the result of Eq. (16), and discuss the effect of higher orders in α_s on this result.

Equation (16) has the factorized form given in Eq. (1), which is a direct consequence of the higher-twist factorization theorem [7,8]. We should note that a general Feynman diagram contributes to almost every term in the power expansion of cross sections illustrated by Eq. (1). To disentangle leading and nonleading powers, we employ an expansion ("collinear expansion") in the transverse momenta of the partons linking the matrix elements and the corresponding partonic hard-scattering diagram. The detailed structure of the hard-scattering functions, the H^i in Eq. (1), depends on the choices made for the matrix elements at leading and nonleading twist.

A. Collinear expansion

The collinear expansion is the first step in separating the short-distance scattering from the long-distance matrix elements. Consider a typical diagram contributing to momentum imbalance of two jets, as shown in Fig. 2. It has three parton momenta, k, k_1, and k_2, linking the partonic diagram with the matrix element.

The collinear expansion is an expansion in transverse momenta of the partons. At leading power, all partons are represented by their longitudinal momentum fractions, and consequently, integrations over k, k_1, and k_2 are reduced into three one-dimensional integrals over the momentum fractions x, x_1, and x_2, respectively. As we will show later, because of the pole structure of the Feynman diagrams the x_1 and x_2 integrals can be done explicitly by contour integration, for that part of the cross section that is enhanced in A. Therefore, the final result presented in Eq. (16) has only one x integration linking the matrix elements and hard parts.

We can write the contribution of Fig. 2 to the two-jet cross section as

$$E_l \frac{d\sigma}{d^3 l} = \int \frac{d^4k}{(2\pi)^4}\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4} \times \mathrm{Tr}[M^{\alpha\beta}(k,k_1,k_2,p)\hat{H}_{\alpha\beta}(k,k_1,k_2,q,l,l')] , \tag{26}$$

where $M^{\alpha\beta}$ is the two-quark, two-gluon matrix element, and $\hat{H}$ includes all corresponding diagrams for photon-parton scattering to produce two jets with momentum l and l'. We now expand the parton momenta k, k_1, and k_2 in $\hat{H}$ about the values,

$$k_1^\mu = x_1 p^\mu,\ k_2^\mu = x_2 p^\mu,\ k^\mu = x p^\mu + k_T^\mu , \tag{27}$$

and keep only the leading term of $\hat{H}$. We then obtain

$$E_l \frac{d\sigma}{d^3 l} = \int dx\, dx_1\, dx_2 \int d^2k_T \mathrm{Tr}[\hat{T}^{\alpha\beta}(x,x_1,x_2,k_T,p)\hat{H}_{\alpha\beta}(xp,x_1p,x_2p,k_T,q,l,l')] , \tag{28}$$

where the matrix element $\hat{T}$ is given by

$$\hat{T}^{\alpha\beta}(x,x_1,x_2,k_T,p) = \int \frac{p^+dy^-}{2\pi}\frac{p^+dy_1^-}{2\pi}\frac{p^+dy_2^-}{2\pi}\int \frac{d^2y_T}{(2\pi)^2} e^{ix_1p^+y_1^-} e^{i(x-x_1)p^+y^-} e^{-i(x-x_2)p^+y_2^-} e^{-ik_T\cdot y_T} \times \langle p_A|\bar{q}(0)A^\alpha(y_2^-,0_T)A^\beta(y^-,y_T)q(y_1^-,0_T)|p_A\rangle . \tag{29}$$

At the lowest order, k_T and the jet momenta l and l' in Eq. (28) are not independent, but are related by

$$k_T^\mu = l^\mu + l'^\mu . \tag{30}$$

Therefore, in the following, we will suppress l' dependence in the partonic part H. Note that we have fixed all transverse positions to zero except for y_T (we treat y_T as a transverse vector, so that $k_T\cdot y_T \equiv \mathbf{y}_T\cdot\mathbf{k}_T$). This corresponds to the rule, discussed in Sec. II above, that we neglect quark transverse momenta in the hard scattering. The transverse momenta flowing between the quark fields, and between the quark and gluon fields, is thus implicitly integrated over, which sets three of the four fields in $\hat{T}^{\alpha\beta}$ to zero transverse separation.

The matrix element $\hat{T}$ and the partonic part $\hat{H}$ are still linked by a spinor trace and by sums over vector Lorentz indices, in addition to the integrals over parton momentum fractions. The spinor trace can be separated in the standard manner by using a Fierz transformation. The leading power contribution is given by contracting $\gamma^+/(2p^+)$ to the quark operators $\bar{q}(0)$ and $q(y_1^-)$ in matrix element $\hat{T}$, and contracting $(\gamma\cdot p)/2$ to the partonic part $\hat{H}$ [8]. In Feynman gauge, the leading contribution of the gluon field operators in the matrix element $\hat{T}$ is given by

$$A^\alpha A^\beta \Longrightarrow A^+A^+\left[\frac{p^\alpha}{p^+}\right]\left[\frac{p^\beta}{p^+}\right] , \tag{31}$$

where $A^+=n\cdot A$. By absorbing the factor $p^\alpha p^\beta$ into the hard-scattering function, the sums over Lorentz indices are also separated between the short- and long-distance functions. We now have a somewhat simpler factorized expression for the cross section,

$$E_l\frac{d\sigma}{d^3l}=\int dx\, dx_1\, dx_2 \int d^2k_T \overline{T}(x,x_1,x_2,k_T,p) \times\overline{H}(x,x_1,x_2,p,k_T,q,l)\ , \tag{32}$$

where the modified matrix element $\overline{T}$ is

$$\overline{T}(x,x_1,x_2,k_T,p)=\int \frac{dy^-}{2\pi}\frac{dy_1^-}{2\pi}\frac{dy_2^-}{2\pi}\int\frac{d^2y_T}{(2\pi)^2}e^{ix_1p^+y_1^-}e^{i(x-x_1)p^+y^-}e^{-i(x-x_2)p^+y_2^-}e^{-ik_T\cdot y_T} \times\frac{1}{2}\langle p_A|\bar{q}(0)(n\cdot\gamma)A^+(y_2^-,0_T)A^+(y^-,y_T)q(y_1^-)|p_A\rangle\ . \tag{33}$$

The partonic part $\overline{H}$ is given the diagrams shown in Fig. 3, with quark lines traced with $(\gamma\cdot p)/2$, and gluon lines contracted with $p^\alpha p^\beta$.

The matrix element $\overline{T}$ given in Eq. (33) has gluon field operators A^+, which are not gauge covariant, so that the resulting matrix element cannot be made gauge invariant by the inclusion of ordered exponentials of gauge fields [8]. We can, however, convert the A^+'s into field strengths, $F^{\perp+}$. To relate A^+'s to the field strength, we rewrite Eq. (32) as

$$\begin{aligned}E_l\frac{d\sigma}{d^3l}&=\int dx\, dx_1\, dx_2 \int d^2k_T \overline{T}(x,x_1,x_2,k_T,p)k_T^2\overline{H}(x,x_1,x_2,p,k_T,q,l)/k_T^2\\&\equiv\int dx\, dx_1\, dx_2 \int d^2k_T \hat{T}_F(x,x_1,x_2,k_T,p)\overline{H}(x,x_1,x_2,k_T,p,q,l)/k_T^2\ ,\end{aligned} \tag{34}$$

where $\hat{T}_F(x,x_1,x_2,k_T,p)$ is given by

$$\hat{T}_F(x,x_1,x_2,k_T,p)=\int \frac{dy^-}{2\pi}\frac{dy_1^-}{2\pi}\frac{dy_2^-}{2\pi}\int\frac{d^2y_T}{(2\pi)^2}e^{ix_1p^+y_1^-}e^{i(x-x_1)p^+y^-}e^{-i(x-x_2)p^+y_2^-}e^{-ik_T\cdot y_T} \times\frac{1}{2}\langle p_A|\bar{q}(0)(n\cdot\gamma)F_\alpha{}^+(y_2^-,0_T)F^{+\alpha}(y^-,y_T)q(y_1^-)|p_A\rangle\ . \tag{35}$$

The two factors of k_T have been converted into transverse derivatives on the two fields A^+, by integration by parts. Derivatives of quark fields are neglected in identifying the A-enhanced matrix elements, following the comments of Sec. II above. Had $\hat{T}$ included four quark fields, the collinear expansion would have produced a matrix element of four quark fields and two transverse derivatives, a twist-6 contribution. Dimensional counting requires that the hard part associated with such a matrix element be suppressed by a power of l_T^2 relative to those associated with twist-4 operators. This justifies our neglect of these operators at this order.

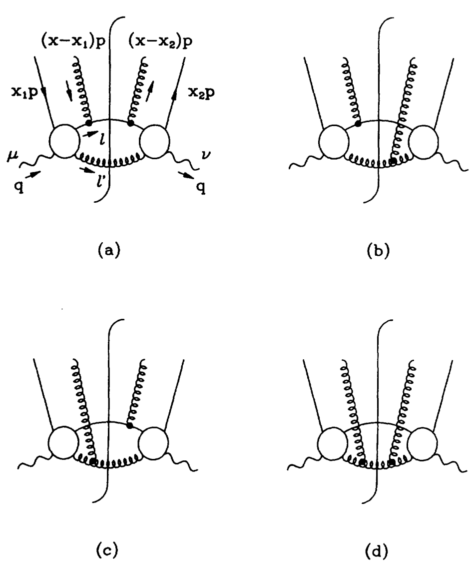

FIG. 3. Lowest-order diagrams in quark photoproduction which give rise to nuclear enhancement proportional to the matrix element of Eq. (2).

The explicit $1/k_T^2$ factor in the modified partonic part in Eq. (34) requires that $\overline{H}$ vanish at $k_T=0$ for the higher-twist contribution to be well defined. This is certainly the case for the averaged squared momentum imbalance of two jets, $\langle k_T^2 d\sigma\rangle$. Therefore, we may usefully introduce a new hard-scattering function H, by

$$\overline{H}(x,x_1,x_2,k_T,p,q,l)/k_T^2=H(x,x_1,x_2,p,q,l)\ , \tag{36}$$

where we have neglected higher orders of k_T in $\overline{H}$, which contribute at yet higher twist.

At leading power, the momentum imbalance is now

$$\left\langle k_T^2E_l\frac{d\sigma}{d^3l}\right\rangle=\int dx\, dx_1\, dx_2\int d^2k_T\hat{T}_F(x,x_1,x_2,k_T,p) \times H(x,x_1,x_2,p,q,l)\ . \tag{37}$$

Now we can integrate over k_T in the matrix element, to get

$$\left\langle k_T^2 E_l \frac{d\sigma}{d^3l}\right\rangle = \int dx\, dx_1\, dx_2\, T_F(x,x_1,x_2,p) \times H(x,x_1,x_2,p,q,l)\ , \tag{38}$$

where T_F is defined by

$$T_F(x,x_1,x_2,p)=\int \frac{dy^-}{2\pi}\frac{dy_1^-}{2\pi}\frac{dy_2^-}{2\pi} e^{ix_1p^+y_1^-} e^{i(x-x_1)p^+y^-} e^{i(x-x_2)p^+y_2^-} \frac{1}{2}\langle p_A|\bar{q}(0)(n\cdot\gamma)F_\alpha{}^+(y_2^-)F^{+\alpha}(y^-)q(y_1^-)|p_A\rangle\ . \tag{39}$$

In T_F, as in Eq. (2), all transverse position vectors are equal to zero.

In deriving Eq. (38), we have so far neglected the color sums that link the hard scattering and the matrix element. In line with our comments of Sec. II, we always assume that color has no long-distance correlations in the nucleus. This means that A enhancements in the matrix elements can only be derived when the quark and gluon fields in T_F separately form color singlets. Thus, in the following, we shall assume that quark and gluon color indices are traced separately in both matrix elements and hard-scattering functions. This involves normalizing the hard-scattering function by a factor

$$C_{\rm color}=\frac{1}{N}\frac{1}{N^2-1}\ , \tag{40}$$

with $N=3$. As usual, higher order at the same twist will produce ordered exponentials between the various fields, which make these matrix elements gauge invariant.

With the color traces understood in both H and T_F, and the color factor $C_{\rm color}$ absorbed into H, Eq. (38) gives the physically measured quantity $\langle k_T^2 E_l d\sigma/d^3l\rangle$ in the general factorized form of Eq. (1).

B. Leading pole contributions

The averaged momentum imbalance given in Eq. (38) depends on integrations over three parton momentum fractions. For fixed, nonzero fractions $x-x_1$ and $x-x_2$, oscillations of the exponentials in T_F, Eq. (39), destroy any A enhancement that could come from the y integrals (see Sec. II above). At low order in the hard-scattering diagrams of Fig. 2, however, the x_1 and x_2 integrals can be done explicitly by contour integration. We will find that certain diagrams have poles precisely at $x=x_1=x_2$, which results in the matrix elements $T_a, a=q,g$ of Eqs. (2) and (3), without these exponentials, and hence *with* A enhancement. By the same token, the A-enhanced contribution to the cross section depends on a single x integral only, as claimed above.

To see how this comes about, consider the specific diagram shown in Fig. 3(a), in which the incoming photon and target quark undergo a "Compton" scattering to produce a quark and a gluon, and the quark interacts with another gluon on its way out to form a jet of momentum l. There are two poles in this diagram corresponding to vanishing momenta $(x-x_1)p$ and $(x-x_2)p$. The partonic part H for the diagram shown in Fig. 3(a) is given by simply

$$H(x,x_1,x_2,q,l)=g^2\frac{C_q}{N^2-1}\left[\frac{1}{x_1-x+i\epsilon}\right]\left[\frac{1}{x_2-x-i\epsilon}\right] \times E_l\frac{d\hat{\sigma}^{\gamma q}}{d^3l}(xp,q,l)\ , \tag{41}$$

where $C_q=(N^2-1)/(2N)$ and where $E_l(d\hat{\sigma}^{\gamma q}/d^3l)(xp,q,l)$ is the lowest-order photon-quark "Compton" cross section. Substituting Eq. (41) into Eq. (38), and using the integrals

$$\int dx_1 \frac{1}{x_1-x+i\epsilon}e^{x_1p^+(y_1^- - y^-)} = -2\pi i\,\theta(y^- - y_1^-) \times e^{ixp^+(y_1^- - y^-)}\ ,$$
$$\int dx_2 \frac{1}{x_2-x-i\epsilon}e^{ix_2p^+y_2^-} = 2\pi i\,\theta(y_2^-)e^{ixp^+y_2^-}\ , \tag{42}$$

we obtain

$$\left\langle k_T^2 E_l \frac{d\sigma^q}{d^3l}\right\rangle = \frac{8\pi^2\alpha_s}{N^2-1}C_q\int dx T_q(x,p)E_l\frac{d\hat{\sigma}^{\gamma q}}{d^3l}(xp,q,l)\ , \tag{43}$$

where $T_q(x,p)$ is given in Eq. (2). The remaining lowest-order two-quark and two-gluon diagrams with poles at $x=x_1=x_2$ are given in Figs. 3(b)–3(d). A brief calculation such as the one above shows that these three diagrams just cancel, and give no net contribution.

Similarly, for two-photon and four-gluon diagrams, of the form of Fig. 4, we have, at lowest order,

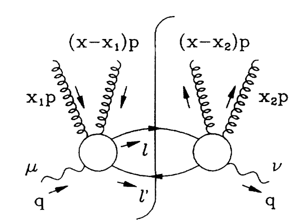

FIG. 4. Diagrams in gluon photoproduction which give rise to nuclear enhancement proportional to the matrix element of Eq. (3).

$$\left\langle k_T^2 E_l \frac{d\sigma^g}{d^3l} \right\rangle = \frac{8\pi^2\alpha_3}{N^2-1} C_g \int dx\, T_g(x,p) E_l \frac{d\hat{\sigma}^{\gamma g}}{d^3l}(xp,q,l) \,, \tag{44}$$

where $C_g = N = 3$ and $T_g(x,p)$ is given in Eq. (3).

In summary, our complete result is of the form

$$\left\langle k_T^2 E_l \frac{d\sigma}{d^3l} \right\rangle = \frac{8\pi^2\alpha_s}{N^2-1} \int dx \left[C_q T_q(x,p) E_l \frac{d\hat{\sigma}^{\gamma q}}{d^3l}(xp,q,l) + C_g T_g(x,p) E_l \frac{d\hat{\sigma}^{\gamma g}}{d^3l}(xp,q,l) \right] , \tag{45}$$

as given in Eqs. (14)–(16), with p_γ for q.

C. Higher-order corrections

The lowest-order calculation given above neglects the evolution of the final state partons into jets. Although this evolution may be expected to take place outside of the nucleus, ideally this feature should be a result of the formalism, rather than an input. In fact, this is the case, as we now show. Consider the diagram of Fig. 5, which is a generalization of Fig. 1. J represents graphical contributions to the evolution of an outgoing parton. We include (any number) of soft interactions with this jet. The generalization of the following argument to the remaining outgoing parton(s) is trivial.

As above, at lowest order in the momentum imbalance k_T^2, the momenta of the soft gluons, k_i, may be replaced by their projections in the p^μ direction, $x_i p$, as in Eq. (27). In addition, only the polarization in the p direction contributes to leading order in the collinear expansion. In the collinear expansion, then, each soft gluon is replaced, in the jet to which it attaches, by a "scalar" polarized gluon, whose polarization is proportional to its momentum. This is a standard situation in factorization arguments [6], and in this case the soft gluon decouples from the jet by the use of Ward identities. Without going into the details [6], we may summarize the result: the leading contributions of soft rescattering factorize from the internal structure of the jet, and the cross section may be written as the product of two factors, one that describes the evolution of the jet, and the other the coupling of soft gluons to a single "eikonal" line, whose direction and color are defined by the outgoing jet. Thus, the evolution of the jet is unaffected by soft rescattering, although the soft gluons may still transfer momentum from the nucleus to the jet.

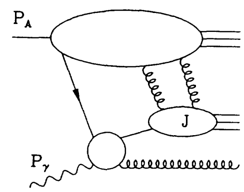

FIG. 5. Typical higher-order contributions to jet photoproduction.

V. CONCLUSIONS

In this paper we have applied the twist-4 formalism for nuclear enhancement [1,2] to dijet momentum imbalance in photoproduction. Because of the simplicity of the expression for this quantity Eq. (18), we were able to use experimental results to estimate the size of the relevant two-parton matrix elements, such as Eq. (2), as the order of $A^{1/3}$ times $\lambda^2 \sim 0.05-0.1$ GeV^2 times twist-2, nuclear single-parton distributions. This is a sizable matrix element, and should give significant contributions to nuclear enhancement in other processes.

Our estimates employed approximations in which the result is essentially independent of the directions of the outgoing jets. Measurements of "differential" averages $\langle k_T^2 \rangle$ as a function of l_T, or other kinetic jet variables, would make it possible to determine λ^2 separately for the matrix elements T_g and T_q. In addition, it remains to study the evolution of these matrix elements. It is possible that this evolution is highly nontrivial, since they involve vanishing partonic momentum fractions.

Finally, in this paper and Refs. [1,2], we have developed the formalism of nuclear enhancement only for the electromagnetic-induced processes of deeply inelastic scattering and photoproduction. Data on dijet momentum imbalance in hadroproduction [10,12], in which substantial nuclear enhancement has been seen, should provide direct tests of the formalism, given that we now know the scale of the relevant matrix elements. At the same time, the much smaller nuclear dependence seen in Drell-Yan production [14] suggests a complementary test for these methods. We hope that these and related issues will be the subject of future work.

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