"Hard" Pomeron approach to "soft" processes at high energy

E. Levin
Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, Illinois 60510
and St. Petersburg Nuclear Physics Institute, 188350, Gatchina, Russia
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An attempt is made to give a consistent description of high energy hadron interactions starting with the physical assumption that only "hard" processes contribute to the Pomeron structure. Using the general properties of a "hard" Pomeron in perturbative QCD an equation for shadowing corrections is suggested and solved. It allows one to extend the new approach to high energy hadron collisons. In doing so we generalize the so-called eikonal approximation widely used to describe the shadowing corrections for both hadron and nucleus scattering at high energy. New formulas are also suggested for the large rapidity gap survival probability which crucially differ from the eikonal ones.

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I. INTRODUCTION

Calculations of large-cross-section physics at high energy are usually regarded as dirty since there is a widespread delusion that it is impossible to develop any theoretical approach to such processes based on our microscopic theory—QCD. It is widely believed that the gap between current phenomenological models for high energy hadron and/or nucleus scattering and QCD is so big that it is difficult to see any interrelation between them. The main goal of this paper is to develop an approach that is based on QCD and establishes a very transparent relationship between high energy "soft" scattering and our microscopic theory.

Of course, in order to do this we need to make a hypothesis. Our key assumption is that only "hard" processes contribute to the Pomeron structure. It means that we can describe the Pomeron in the framework of perturbative QCD. I would like to develop a self-consistent approach based on some new physical assumption in comparison with all previous attempts to take into account both "soft" and "hard" processes [1]. The principal difference is the fact that I am going to use the leading log (energy) approximation (leading$[\ln(s)]A$, see Sec. II for details) of perturbative QCD for "hard" processes. In Refs. [1] two approaches have been used for the same purposes: either the factorization formula for the inclusive cross section to calculate the "hard" contribution to the total inelastic cross section (see Ref. [2] for the relevant criticism on this point) or/and the leading log (transverse momentum) approximation [leading$(\ln k_t^2)A$] of perturbative QCD to estimate the "hard" part of the total inelastic cross section (see [3]). We do not expect good accuracy for the second approach. This is the reason why we have to go back and try to develop a new approach for the "hard" part of the total inelastic cross section.

Let me list the arguments that show that the assumption that only "hard" processes contribute to the Pomeron structure is not so crazy as it seems to be at first sight.

(1) In any attempt to fit the experimental data, the slope of the Pomeron trajectory (α') turns out to be very small, at least not bigger than $\alpha'=0.25\ \mathrm{GeV}^{-2}$ [4]–[6].[1] We use the following notation for the Pomeron trajectory, $\alpha_P(t=-q_t^2)=1+\Delta+\alpha' t$.

(2) The experimental slope of diffractive dissociation in the system of secondary hadrons with large mass is approximately two times smaller than the slope for the elastic scattering. In terms of Pomeron phenomenology this fact results in the small proper size of the triple Pomeron vertex (G_{3P}). To a first approximation, we can assign a zero slope for the triple Pomeron vertex so as to describe the experimental data on diffraction dissociation.

(3) The idea that gluons inside a hadron are confined in the volume of smaller radius ($R_G \approx 0.1$ fm $\ll R_h \sim 1$ fm) is still a working hypothesis which helps to describe the experimental data (see Ref. [4] for the details).

(4) The introduction of "semihard" processes in QCD [7] which are responsible for the total inclusive cross section of hadron interaction at high energy leads to a value of the total cross section compatible with the geometrical size of the hadron. The assumption that "semihard" processes are responsible for the major part of the total cross section provides the most probable and natural way to describe the matching between "hard" and "soft" processes.

(5) Previous experience in multiperipheral models shows that one could describe the global features of the "soft" interaction at high energy providing the main transverse momentum of produced hadrons is large enough (of the order of 1 GeV).

(6) In the eikonal approach, the QCD Pomeron is able to describe the current experimental data on total and elastic cross section as well as the slope (see Ref. [12] for details).

I hope that the above arguments are convincing

[1]We will comment on this point in more detail at the end of Sec. II.

enough to consider a hard Pomeron as a first approximation to high energy scattering. This hypothesis has at least three big advantages: simplicity, natural matching with QCD at small distances, and the obvious possibility to check it experimentally.

Let me discuss the general strategy of this approach. The first step is a review of the main properties of the QCD Pomeron. It will be shown in Sec. II that the QCD Pomeron has no slope ($\alpha'=0$) and can be considered as an exchange with definite impact parameter b_t. Moreover, in the leading log approximation the interaction between Pomeron cannot change b_t. This fact allows us to regenerate the old Reggeon field theory [8] for the interaction of hard Pomerons, to be discussed in Sec. III. In this section the new equation for the shadowing (screening) corrections will be discussed as well as solutions to these new equations. Physical applications are collected in Sec. IV, where we discuss such important problems as the behavior of the inclusive cross section and the large rapidity gap survival probability. The results and their physical meaning will be further discussed in the Conclusions.

II. THE "HARD" POMERON IN QCD

As discussed in the Introduction, we assume that only hard processes contribute to the structure of the Pomeron. This means that we believe in some natural cutoff in momentum (Q_0) and that only the production of quarks and gluons with transverse momenta $k_t > Q_0$ is dominant in the Pomeron. Since we assume that the value of Q_0 is so large that $\alpha_s(Q_0^2) \ll 1$, we can use perturbative QCD to calculate Pomeron exchange in the leading log approximation (LLA) considering the following parameters as small ones:

$$\alpha_s(Q_0^2) \ll 1, \quad \alpha_s(Q_0^2)\ln\frac{k_t^2}{Q_0^2} \ll 1, \quad \text{but} \quad \alpha_s(Q_0^2)\ln s \gg 1 .$$

The scattering amplitude in the LLA is given by the summation of the perturbative series:

$$f(s,t;k^2,Q_0^2) = \sum_n C_n[\alpha_s(Q_0^2)\ln s]^n + O\left[\alpha_s(Q_0^2);\alpha_s(Q_0^2)\ln\frac{k^2}{Q_0^2}\right] , \quad (1)$$

where k^2 and Q_0^2 are the virtualities of the scattering partons (quarks or gluons).

During the last two decades the sum of Eq. (1) has been studied in great detail (see the original papers [9,10] or several reviews [7,11,13]). I would like to outline the solution to this problem, using a slightly different technique, which will be very convenient for further presentation.

A. The Pomeron in the LLA of perturbative QCD at $t=0$

In the leading log approximation [leading$(\ln s)A$] we can reduce the problem of summation of the perturbative series of Eq. (1) to the solution of the so-called ladder equation (see Fig. 1), namely,

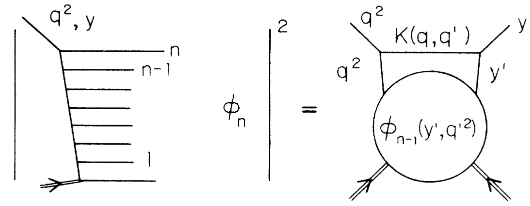

FIG. 1 The Pomeron as a QCD ladder.

$$\phi_n(y,q^2) = \frac{N_c\alpha_S}{\pi}\int dy'\int\frac{d^2q'}{\pi}K(q^2,q'^2)\phi_{n-1}(y',q'^2) , \quad (2)$$

where $y=\ln s/q^2$ ($y'=\ln s'/q'^2$), and q^2 and q'^2 are the virtualities of the two slowest particles (see Fig. 1). The function ϕ is closely related to the gluon structure function in deep inelastic scattering, since

$$\alpha_s(q^2)x_BG(x_B,q^2) = \int^{q^2} dq'^2\alpha_s(q'^2)\phi(y=\ln\frac{1}{x_B},q'^2) . \quad (3)$$

To solve Eq. (2) it is very convenient to introduce the auxiliary function

$$\Psi(y,r=\ln q^2,x) = \sqrt{q^2}\sum_n \phi_n(y,r)x^n . \quad (4)$$

It is easy to see from Eq. (4) that the amplitude of gluon-gluon scattering, or in other words the gluon structure function, can be reduced to

$$\phi(y,q^2) = \sum_n \phi_n(y,r) = \frac{1}{\sqrt{q^2}}\Psi(y,r,x=1) . \quad (5)$$

The partial cross section (ϕ_n) can be calculated as

$$\phi_n(y,r) = \frac{1}{\sqrt{y^2}}\frac{1}{n!}\frac{\partial^n\Psi(y,r,x)}{\partial^n x}\bigg|_{x=0} . \quad (6)$$

We can also write down simple expressions for different correlators through the function Ψ, such as

$$\langle n\rangle = \frac{\frac{\partial\Psi}{\partial x}}{\Psi}\bigg|_{x=1} , \qquad \langle n(n-1)\rangle = \frac{\frac{\partial^2\Psi}{\partial^2 x}}{\Psi}\bigg|_{x=1} . \quad (7)$$

We can rewrite Eq. (2) as the equation for Ψ, which is very simple if we assume that $r-r' \ll r$ and adopt the expansion (as was first done in Ref. [10])

$$\Psi(y,r',x) = \Psi(y,r,x) + \frac{\partial\Psi(y,r,x)}{\partial r}(r'-r) + \frac{1}{2}\frac{\partial^2\Psi(y,r,x)}{\partial r^2}(r'-r)^2 + \cdots . \quad (8)$$

Finally the equation for Ψ looks like

$$\frac{\partial\Psi(y,r,x)}{\partial y}=\omega_0 x\Psi(y,r,x)+\delta x\frac{\partial^2\Psi(y,r,x)}{\partial r^2}\ , \tag{9}$$

where (see Ref. [10] for details)

$$\omega_0=\frac{4N_c\alpha_s}{\pi}\ln 2;\quad \delta=\frac{N_c\alpha_s}{\pi}14\zeta(3)\ .$$

Equation (9) can be solved by going to a Laplace representation and noting that Ψ depends on $z=xy$,

$$\Psi(z,r)=\int\frac{d\omega\, df}{(2\pi)^2}e^{(\omega z+fr)}\psi(\omega,f)\ . \tag{10}$$

For ψ the equation reads

$$\omega=\omega_0+\delta f^2\ , \tag{11}$$

which leads to the answer

$$\Psi(y,r,x)=\int e^{[(\omega_0+\delta f^2)z+fr]}\psi(f)\frac{df}{2\pi i}\ . \tag{12}$$

Starting with the initial condition $\Psi(y=0,r,x)=\delta(r-r_0)$ we can easily get the famous diffusion solution of the equation: namely,

$$\Psi(y,r,x)=\frac{1}{2\sqrt{xy\delta}}\exp\left[\omega_0 xy-\frac{(r-r_0)^2}{4\delta xy}\right]\ . \tag{13}$$

Equation (13) gives the solution that allows one to calculate both the amplitude and the multiplicity distribution using Eqs. (5)–(7) and therefore it enlarges our possibility to study the Pomeron structure in perturbative QCD. However, the main reason why I gave this somewhat new derivation of a well known solution is to illustrate the new technique of the auxiliary function that I am going to use later on to get the solution to the more complicated problem of Pomeron interaction. To demonstrate how this technique works let us calculate the multiplicity distribution

$$\frac{\sigma_n}{\sigma_t}=\frac{1}{\Psi(y,r,x=1)}\int\frac{dx}{2\pi i}\frac{1}{x^{(n+1)}}\Psi(y,r,x)\ ,$$

where the integration contour over x is to the right of all the singularities of the function Ψ. In the saddle point approximation we can perform the above integration and the answer can be written in the form

$$\frac{\sigma_n}{\sigma_t}=\left[\frac{n+1}{\omega_0 y}\right]^{1/2}\exp\left[-(n+1)[\ln(n+1)-1]-\omega_0 y-(r-r_0)^2\left[\frac{\omega_0}{4\delta(n+1)}-\frac{1}{4\delta y}\right]\right]\ . \tag{14}$$

From Eq. (14) we can also calculate the mean $(r-r_0)^2$ at fixed multiplicity:

$$\langle(r-r_0)^2\rangle=\frac{\int dr(r-r_0)^2\sigma_n(y,r)}{\sigma_n(y,r)}$$

Using Eq. (14) we can find that

$$\langle(r-r_0)^2\rangle=2\frac{\delta}{\omega_0}(n+1)\ . \tag{15}$$

Equation (15) shows a very important property of the LLA structure of the Pomeron, namely the fact that the mean log of the transverse momentum increases after the emission of n gluons.

B. b_t dependence of the LLA Pomeron

The main property of the impact parameter motion of the parton could be understood directly from the uncertainty principle, since

$$\Delta b_t q_t\approx 1\ . \tag{16}$$

It means that $\Delta b_t\propto 1/q_t$ for each emission, where q_t is the typical transverse momentum of the parton. As we assumed $q_t>Q_0\gg 1$ GeV for all produced partons, the displacement of the parton in b_t can be considered as a small one. Moreover, because of the emission of gluons, the mean transverse momentum increases at high energy or after $n\gg 1$ emissions. I hope that this discussion illuminates the strict LLA result (see Refs. [9,13,14]) that the LLA Pomeron does not depend on the momentum transferred (t). Thus, in the LLA of perturbative QCD, we can consider the Pomeron as being frozen in b_t space, or in other words its exchange is proportional to $\delta(b_t)$.

For the Pomeron exchange $\delta(b_t)$ means that the slope of the Pomeron trajectory (α') is negligibly small. Of course, it is so only in the first rough approximation and perturbative QCD is able to describe b_t behavior in more details (see Refs. [9,14]). However, strictly speaking, in the LLA we have to restrict ourselves to $\delta(b_t)$ behavior (see, for example, Ref. [15] where this problem has been discussed).

C. QCD motivated Pomeron

Now we can formulate what model for the Pomeron structure we are going to discuss as the first approximation to the "hard" Pomeron, namely, we assume that the Pomeron can be reduced to the simple formula

$$P(y,b_t)=ie^{\omega_0 y}\delta(b_t)\ . \tag{17}$$

Since we consider the case when the initial and final virtuality are equal, the over-simplified formula (17) does not take into account the powerlike behavior on y in Eq. (13). Throughout the paper we will use this simplified version of Eq. (13), but it should be stressed that it is not hard to incorporate the correct behavior of Eq. (13) in all our calculations. This simple expression, i.e., Eq. (17), makes all our calculations so transparent that we prefer to use this form so as to clarify the main property of the screening (shadowing) corrections.

III. SHADOWING CORRECTIONS

In this section we are going to discuss how to incorporate the shadowing (screening) corrections in the framework of the simplified approach to the Pomeron structure given by Eq. (17). There are two origins of the shadowing (screening) corrections: the interaction between colliding hadrons due to multi-Pomeron exchanges and the interaction between Pomerons. The first one is usually taken into account by the eikonal approach which is presently the only method in the market for the description of the shadowing corrections. During the last decade the eikonal approach has resulted in a better understanding of the origin and nature of the shadowing (screening) corrections, so much so, that it has become a synonym of the shadowing correction in general. This happened partly as a reaction to the failed attempts to account for the Pomeron interaction in the framework of the Reggeon field theory (RFT) [8]. The main goal of this section, as well as the whole of this paper, is to revive the RFT and to suggest a more general approach than the eikonal one to the problem of the shadowing correction.

A. Eikonal approach

Let me start with a review of the main ideas and formulas of the eikonal approach, which are carried out most compactly in the impact parameter (b_t) representation. Our amplitudes are normalized as follows:

$$\frac{d\sigma}{dt}=\pi|f(s,t)|^2, \quad \sigma_{\rm tot}=4\pi\,{\rm Im}f(s,0)\ ,$$

where

$$f(s,t)=\frac{1}{2\pi}\int d\mathbf{b}_t e^{i\mathbf{q}\cdot\mathbf{b}_t}a(b_t,s) \tag{18}$$

and

$$a(s,b_t)=\frac{1}{2\pi}\int d\mathbf{q}e^{-i\mathbf{q}\cdot\mathbf{b}_t}f(s,t)\ ; \tag{19}$$

hence we have $\sigma_{\rm tot}=2\int d\mathbf{b}_t\,{\rm Im}a(s,b_t)$ and $\sigma_{\rm el}=\int d\mathbf{b}_t|a(s,b_t)|^2$.

Unitarity requires ${\rm Im}a(s,b_t)\le 1$. In order to satisfy the unitarity constraint it is convenient to express $a(s,b_t)$ in terms of the complex eikonal function $\chi(s,b_t)$ with (${\rm Im}\chi\ge 0$), i.e.,

$$a(s,b_t)=i[1-e^{i\chi(s,b_t)}]\ , \tag{20}$$

which ensures that unitarity is restored on summing up all the eikonal multiparticle exchange amplitudes.

All of the above formulas are general and the eikonal model starts with two assumptions.

(1) At high energies elastic scattering is essentially diffractive and therefore ${\rm Re}\chi$ is small. We assume ${\rm Re}\chi\approx 0$; then the amplitude $a(s,b_t)$ is purely imaginary and determined by the opaqueness $\Omega(s,b_t)\equiv{\rm Im}\chi$.

(2) The opaqueness

$$\Omega(s,b_t)=\frac{1}{4\pi}\int d^2b_t e^{-i\mathbf{q}\cdot\mathbf{b}_t}g^2(t){\rm Im}P(s,t)$$
$$=s^{\omega_0}\int\frac{d^2b_t'}{2\pi}g(b_t-b_t')g(b_t')\ , \tag{21}$$

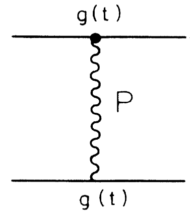

FIG. 2. Scattering amplitude with Pomeron exchange.

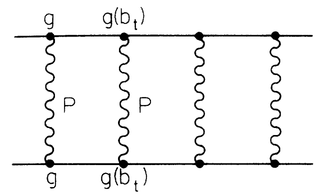

FIG. 3. Eikonal diagrams

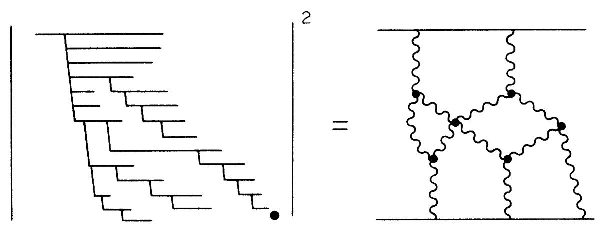

FIG 4. The structure of the parton cascade.

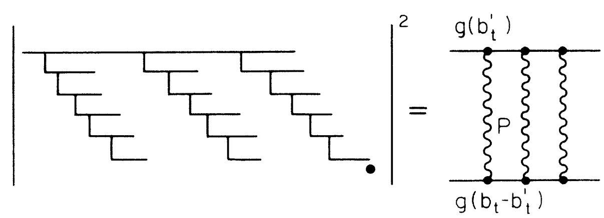

FIG. 5. Parton structure of eikonal diagrams.

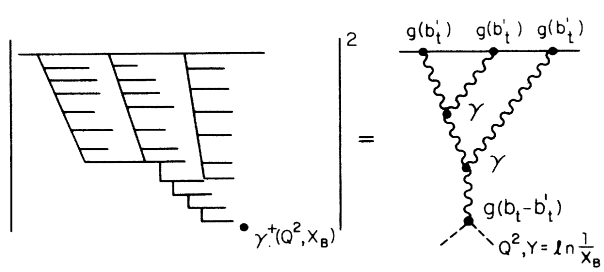

FIG. 6. "Fan" diagrams.

where all notation is obvious from Fig. 2 and $t=-q_t^2$. Here

$$g(b_t)=\frac{1}{2\pi}\int d^2b_t e^{-i\mathbf{q}\cdot\mathbf{b}_t}g(q_t^2)\ ,$$

where $g(t)$ is the vertex for the Pomeron-hadron interaction as seen from Fig. 2. Equation (21) establishes the direct relationship between the opaqueness and the Pomeron exchange. Within this assumption Eq. (20) sums up the diagrams of Fig. 3.

The advantages of the eikonal approach are evident: the exceptional simplicity of the approach and the fact that this approach takes into account the natural scale for the shadowing corrections. It makes this model very attractive and popular. However, it should be stressed that there are no theoretical arguments why this approach should work. The eikonal model looks extraordinarily strange from the point of view of the parton or QCD approach. Indeed, a slight glance at the QCD parton cascade (see Fig. 4) shows us that, in spite of the very complicated structure of this cascade, the number of partons drastically increases mostly due to the decay of each particular parton in its own chain of partons. No arguments have been found in QCD why this complicated structure of the parton cascade, which could in principle be described as the Pomeron interactions (see Fig. 4), could be reduced to eikonal diagrams. The parton cascade for the eikonal diagrams looks very simple, namely, it is only the production of the different parton chains by the fast hadron as is shown in Fig. 5. I would like to draw your attention to the fact that even in the simplest case of deep inelastic scattering the structure of the parton cascade can be described better by a "fan" diagram than by an eikonal one (see Fig. 6 and Ref. [7] for details).

B. Pomeron interaction ("fan" diagrams)

In this subsection I am going to discuss the "fan" diagram contribution to hadron-hadron scattering. I consider this problem as the next approximation to reality after the eikonal one. It certainly will teach us how Pomeron-Pomeron interaction results in the shadowing correction. However, I would first like to make some general remarks on the main features of Pomeron interactions in QCD.

1. Pomeron interactions in QCD

The main advantage of QCD in our problem is the fact that we can formulate what we are doing. Our QCD Pomeron is a well established object, namely, LLA "ladder" diagrams which lead to Eq. (17) in the first rough approximation. So in principle we can calculate in QCD the vertices of interaction between three, four, and so on, "ladders." In practice only triple "ladder" interactions have been calculated in specific kinematical regions where the virtualities of all interacting partons were large enough (see Refs. [16–18]) as well as the amplitude of the two "ladder" rescattering (see Refs. [19,20]). Let me summarize what we have learned from these calculations.

(1) In perturbative QCD we can introduce vertices for three and four Pomeron interaction (see Fig. 7), which

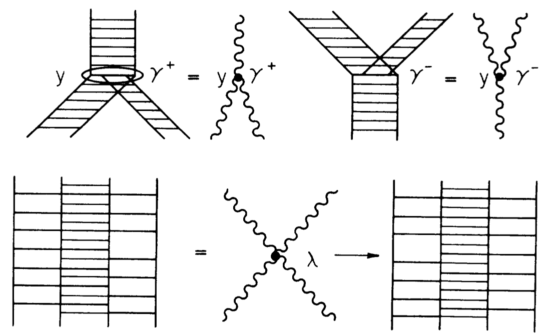

FIG. 7. Pomeron interactions.

are local in rapidity.

(2) All contributions with integration over small transverse momenta $k_t(k_t<Q_0)$ are canceled. This means that we can justify calculations in perturbative QCD.

(3) The vertices γ and λ in Fig. 7 have different orders of magnitude in α_s. Namely, it turns out that

$$\gamma\propto N_c\alpha_s^2,\quad \lambda\propto\alpha_s\ .$$

(4) The sign of the Pomeron-Pomeron scattering amplitude λ corresponds to the attractive forces [19,20] as was discussed many years ago by McCoy and Wu [21].

(5) Concerning the b_t dependence of the Pomeron-Pomeron interaction vertices we can also consider them as a δ function in b_t.

2. Strategy of approach

Based on this experience with QCD calculations, I would like to suggest the following strategy of approach.

(1) We start from the simplest formula of Eq. (17) for one Pomeron exchange.

(2) We introduce the vertices $g(b_t/R)$ for the Pomeron interaction with the hadron (see Fig. 2). In our approach this is the only vertex for which dependence on b_t is scaled by the hadron radius R.

(3) We describe the Pomeron-Pomeron interaction introducing the triple Pomeron vertex (γ) and four Pomeron amplitude (λ) which are local in rapidity and are proportional to $\delta(b_t)$ with respect to any impact parameter related to the interaction.

It is easy to understand that the above approach is an attempt to calculate the scattering amplitude within accuracy $O(\alpha'\ln s/R^2)$. In QCD the effective α' of the Pomeron trajectory depends on energy ($\alpha'\propto 1/\sqrt{\ln s}$, see Ref. [14]) and it is proportional to the extra power of the coupling constant α_s. Thus we can consider this approach as a legitimate try in QCD.

3. Summation of the "fan" diagrams

To demonstrate the problems that we face in finding the screening correction contributions let me discuss the simplest nontrivial case: summation of the "fan" diagrams of Fig. 6 only, neglecting even the Pomeron rescattering (vertex λ in Fig. 7).

To solve this problem we develop the same method of auxiliary function[2]

$$\Psi(y,x)=\sum_n C_n(y)x^n , \tag{22}$$

in which the coefficients $C_n(y)$ constitute the probability amplitude for finding n Pomerons at rapidity y. For $\Psi(y,x)$ it is very simple to write down the equation

$$-\frac{\partial\Psi(y,x)}{\partial y}=\omega_0 x\frac{\partial\Psi(y,x)}{\partial x}-\gamma x^2\frac{\partial\Psi}{\partial x} . \tag{23}$$

This equation is nothing more than a different form of the equation for C_n:

$$-\frac{dC_n(y)}{dy}=\omega_0 nC_n-\gamma(n-1)C_{n-1} . \tag{24}$$

The physical meaning of Eq. (24) is clear from Fig. 8, where the first term describes the propagation of Pomerons which do not interact with each other while the second one annihilates any Pomeron in the interval dy, replacing it by two others. The minus sign in front of this term reflects the shadowing (screening) character of the interaction or, in other words, the fact that our scattering amplitude is purely imaginary at high energy.

Equation (23) can be solved and the solution is an arbitrary function of one variable $\Psi(\kappa)$, where

$$\kappa=\omega_0(Y-y)+\ln\frac{x}{1-\frac{x}{\omega_0}x} . \tag{25}$$

The function $\Psi(\kappa)$ can be found from an initial condition, which for our problem is (see Fig. 6 for notation)

$$\Psi(\kappa)=xg(b_t-b_t') \quad \text{at } y=Y . \tag{26}$$

From Eq. (26) we can find that

$$x=\frac{e^\kappa}{1+\frac{\gamma}{\omega_0}e^\kappa} \quad \text{and} \quad \Psi=\frac{g(b_t-b_t')e^\kappa}{1+\frac{\gamma}{\omega_0}e^\kappa} . \tag{27}$$

Finally to get the answer for the scattering amplitude at fixed impact parameter b_t we need to substitute $y=0$ and $x=g(b_t')$ in the definition of κ and find $\Psi(\kappa)$ from the previous equation. Thus

$$a_{\rm FD}(Y=\ln s,b_t)=\int\frac{d^2b_t'}{2\pi}\frac{g(b_t-b_t')g(b_t')}{\frac{\gamma}{\omega_0}+e^{-\omega_0 Y}\left[1-\frac{\gamma}{\omega_0}g(b_t')\right]} . \tag{28}$$

4. *Eikonal+ "fan" diagrams*

It is very instructive to get now the formula for the amplitude that takes into account eikonal and "fan" diagrams (see Fig. 9) together. Such a formula can be written in terms of the opaqueness $\Omega(s,b_t)$ and Eq. (20) if

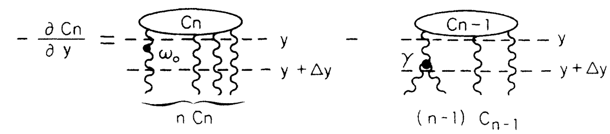

FIG. 8. Graphical representation of the equation for the sum of "fan" diagrams.

$$\Omega(Y=\ln s,b_t)=e^{\omega_0 Y}\int\frac{d^2b_t'}{2\pi}g(b_t-b_t')g(b_t') + 2[a_{\rm FD}(Y,b_t)-a_{\rm FD}(Y,b_t,\gamma=0)] . \tag{29}$$

The above expression for Ω takes into account in a correct way the fact that two sets of the "fan" diagrams with Pomeron interaction coupled to the top or bottom part of a Fig. 9–type have the same common part, the one Pomeron exchange.

C. Rescattering of Pomerons

I have considered a toy model for the origin of the shadowing correction in the previous subsection; here I would like to discuss a self-consistent approach in which the Pomeron-Pomeron rescattering will be taken into account, since this interaction seems to be the biggest one in QCD ($\lambda\propto\alpha_s$ while $\gamma\propto\alpha_s^2$). It means that I am going to sum up the diagrams of the type shown in Fig. 10, or in other words we include self-consistently the interaction of the Pomeron with the hadron, which is of the order of α_s in QCD, as well as the Pomeron-Pomeron rescattering ($\lambda\propto\alpha_s$).

The equation for the auxiliary function Ψ for this problem looks like a trivial generalization of Eq. (24): namely,

$$-\frac{\partial\Psi(y,x)}{\partial y}=\omega_0 x\frac{\partial\Psi(y,x)}{\partial x}+\lambda x^2\frac{\partial^2\Psi(y,x)}{\partial x^2} . \tag{30}$$

It should be stressed that Eq. (30) describes the attractive interaction between Pomerons ($\lambda>0$) as was discovered many years ago by McCoy and Wu [21] and has been recently rediscovered in QCD (see Refs. [19,20]).

However, this equation cannot be solved in such an easy way as Eq. (24). First let us simplify the equation a little bit, introducing a new variable $\eta=\ln x$ and going to the ω representation:

$$\Psi(y,\eta)=\int\frac{d\omega}{2\pi i}e^{\omega(Y-y)}\psi(\omega,\eta) . \tag{31}$$

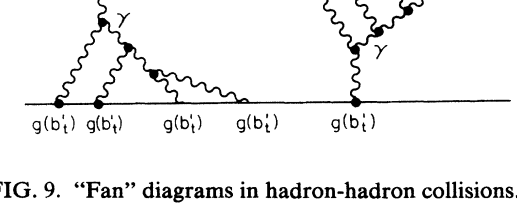

FIG. 9. "Fan" diagrams in hadron-hadron collisions.

[2]As far as I know this method was first applied to the problem of the shadowing correction in Ref. [22].

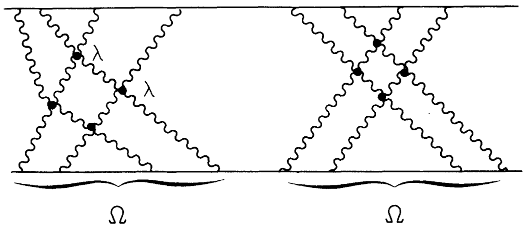

FIG. 10. Pomeron-Pomeron rescattering diagrams in hadron-hadron collisions.

For ψ the equation looks like

$$\omega\psi(\omega,\eta)=(\omega_0-\lambda)\frac{d\psi(\omega,\eta)}{d\eta}+\lambda\frac{d^2\psi(\omega,\eta)}{d\eta^2} . \quad (32)$$

Equation (32) can be easily solved by going to a Laplace representation with respect to η:

$$\psi(\omega,\eta)=\int\frac{df}{2\pi i}e^{f\eta}\phi(\omega,f) . \quad (33)$$

In the ω and η representation Eq. (32) reduces to

$$\omega=(\omega_0-\lambda)f+\lambda f^2 . \quad (34)$$

So the solution of Eq. (32) finally looks like

$$\Psi(Y-y,\eta)=\int\frac{df}{2\pi i}\phi(f)e^{[(\omega_0-\lambda)f+\lambda f^2](Y-y)+f\eta} . \quad (35)$$

The function $\phi(f)$ should be found from the initial condition

$$\Psi(Y-y,\eta)=\sum_{n=1}(-1)^{n+1}\frac{[g(b_t-b_t')]^n}{n!}e^{n\eta}$$
$$=1-\exp[-g(b_t-b_t')e^{\eta}] \text{ at } y=Y . \quad (36)$$

To satisfy the above condition we need to choose the function $\phi(f)$ equal to

$$\phi(f)=\Gamma(-f)$$

and the solution can be rewritten in the form

$$\Psi(Y-y,\eta)=\sum_{n=1}(-1)^{n+1}\frac{[g(b_t-b_t')e^{\eta}]^n}{n!}\times e^{[(\omega_0-\lambda)n+\lambda n^2](Y-y)} . \quad (37)$$

Using the obvious relation

$$e^{(\omega_0 f+\lambda f^2)(Y-y)}=\int e^{-f\eta'}d\eta'\frac{1}{2\sqrt{\pi\lambda(y-y)}}\times\exp\left[-\frac{[(\omega_0-\lambda)(Y-y)+\eta']^2}{4\lambda(Y-y)}\right] , \quad (38)$$

we can rewrite the answer in a form more convenient for further discussions:

$$\Omega(Y=\ln s,b_t)=\int\frac{d^2b_t'}{2\pi}\Psi(Y,\eta=\ln g(b_t'))$$
$$=\int\frac{d^2b_t'}{2\pi}\int d\eta'\{1-\exp[-g(b_t-b_t')g(b_t')e^{-\eta'}]\}\frac{1}{2\sqrt{\pi\lambda Y}}\exp\left[-\frac{[(\omega_0-\lambda)Y+\eta']^2}{4\lambda Y}\right] . \quad (39)$$

Equation (39) can be simplified assuming the Gaussian for b_t dependence of $g(b_t)$: namely,

$$g(b_t)=\frac{\sigma_0(s=s_0)}{\pi R_h^2}e^{-b_t^2/R_h^2} , \quad (40)$$

where R_h is the radius of the hadron while σ_0 is the value of the cross section of the hadron-hadron interaction at sufficiently small energy $s=s_0$. Performing integration over b_t we get the result

$$\Omega(Y,b_t)=\frac{R_h^2}{4\sigma_0}\int d\eta'\left\{\ln\left[\frac{\sigma_0^2}{(\pi R_h^2)^2}e^{-b_t^2/2R_h^2-\eta'}\right]+C-\text{Ei}\left[-\frac{\sigma_0^2}{(\pi R_h^2)^2}e^{-b_t^2/2R_h^2-\eta'}\right]\right\}$$
$$\times\frac{1}{2\sqrt{\pi\lambda Y}}\exp\left[-\frac{[(\omega_0-\lambda)Y+\eta]^2}{4\lambda Y}\right] . \quad (41)$$

At very high energy the dominant contribution in the integral over η' gives the value of $\eta'\sim-(\omega_0-\lambda)Y$ and for $b_t^2\ll 2R_h^2(\omega_0-\lambda)Y$ we get

$$\Omega(Y,b_t)=\frac{R_h^2}{4\sigma_0}\left[(\omega_0-\lambda)Y+\ln\frac{\sigma_0}{R_h^2}+C-\frac{b_t^2}{2R_h^2}\right] . \quad (42)$$

For $b_t^2\gg 2R_h^2(\omega_0-\lambda)Y$ we have

$$\Omega(Y,b_t)=\frac{\sigma_0}{\pi R_h^2}e^{\omega_0 Y-b_t^2/2R_h^2} . \quad (43)$$

Substituting this expression for Ω in Eq. (20) we can conclude that the total cross section increases logarithmically with energy:

$$\sigma_t\propto 2\pi R_h^2\ln s .$$

However, this result depends crucially on the Gaussian parametrization of function $g(b_t)$. If $g(b_t)\propto e^{-b_t/R_h}$, $\sigma_t \to \ln^2 s$ at high energy.

D. Pomeron interaction (general case)

In this subsection I am going to consider the general case and sum up diagrams of the type shown in Fig. 11, taking into account both the Pomeron-Pomeron rescattering (λ) and the Pomeron splitting into two Pomerons (γ^+) as well as the Pomeron annihilation (γ^-) (see Fig. 11 for notation). The first question that arises is why we can restrict ourselves to summing only diagrams of the type of Fig. 11, or in other words why we neglect the more complicated interactions among Pomerons, for example, the one Pomeron transition to three or even more Pomerons. To answer these questions we need to recall that in QCD we have the following order of the magnitudes for our basic interactions:

$$g\propto\alpha_s,\quad \omega_0\propto N_c\alpha_s,\quad \lambda\propto\alpha_s,\quad \gamma^-\sim\gamma^+\propto N_c\alpha_s^2\ . \tag{44}$$

Summing diagrams as shown in Fig. 11 we make an attempt to calculate the high energy amplitude within the accuracy of the order of $O((\alpha_s^3\ln s)^n)$. Indeed, if we consider the following set of small parameters,

$$\alpha_s \ll 1,\quad \alpha_s\ln s \gg 1,\quad \alpha_s^2\ln s\sim 1\ , \tag{45}$$

we can reduce our problem to the summation of the Fig. 11 set of the diagrams. The Reggeon diagrams give us the possibility to take into account in a constructive way the factorization property of QCD that is very general, at least more general than any leading log approximations.

The equation for the auxiliary function Ψ for the general set of the diagrams of Fig. 11 can be written in the form

$$-\frac{\partial\Psi(Y-y,x)}{\partial y}=\omega_0 x\frac{\partial\Psi(Y-y,x)}{\partial x}+\lambda x^2\frac{\partial^2\Psi(Y-y,x)}{\partial x^2}-\gamma^+x^2\frac{\partial\Psi(Y-y,x)}{\partial x}+\gamma^- x\frac{\partial^2\Psi(Y-y,x)}{\partial x^2}\ . \tag{46}$$

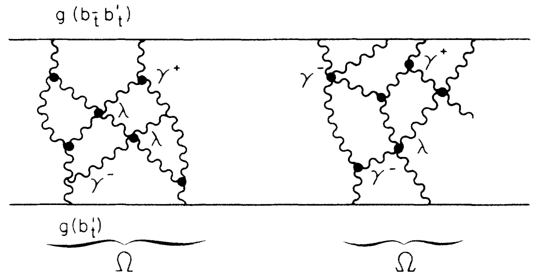

FIG. 11. Diagrams for the general case of Pomeron interactions.

This equation can be simplified by introducing the new variable $\eta=\ln x$ and going to the ω representation [see Eq. (31)]. For $\psi(\omega,\eta)$ the equation can be reduced to the form

$$\omega\psi(\omega,\eta)=(\omega_0-\lambda-\gamma^+e^{\eta}-\gamma^-e^{-\eta})\frac{d\psi(\omega,\eta)}{d\eta}+(\lambda+\gamma^-e^{-\eta})\frac{d^2\psi(\omega,\eta)}{d\eta^2}\ . \tag{47}$$

Let us find the semiclassical solution of the equation by substituting

$$\psi(\omega,\eta)=\exp[\chi(\omega,\eta)]$$

and assuming that

$$\frac{d^2\chi(\omega,\eta)}{d\eta^2}\ll\left[\frac{d\chi(\omega,\eta)}{d\eta}\right]^2 .$$

In this case Eq. (47) can be reduced to the algebraic one: namely,

$$\omega=[\omega_0-\lambda-\gamma^+e^{\eta}-\gamma^-e^{-\eta}]\frac{d\chi}{d\eta}+[\lambda+\gamma^-e^{-\eta}]\left[\frac{d\chi}{d\eta}\right]^2 . \tag{48}$$

Solving this equation we can get the answer

$$\psi(\omega,\eta)=\phi(\omega)\exp\left\{\int_{-\infty}^{\eta}d\eta'\frac{1}{2(\lambda+\gamma^-e^{-\eta'}}\times[\lambda+\gamma^+e^{\eta'}+\gamma^-e^{-\eta'}-\omega_0-\sqrt{(\lambda+\gamma^+e^{\eta'}+\gamma^-e^{-\eta'}-\omega_0)^2+4\omega(\lambda+\gamma^-e^{-\eta'})}]\right\}\ . \tag{49}$$

However solution (34) is reliable only at sufficiently large ω,

$$\omega\gg\gamma e^{\eta}\ ,$$

where $\chi''_{\eta\eta}\ll\chi'^2_{\eta}$. At very small ω, however, the last term in Eq. (32) turns out to be negligibly small, so solution (34) is able to describe this region too. The function $\phi(\omega)$ can be found from the initial condition of Eq. (26): namely,

$$\int\frac{d\omega}{2\pi i}\phi(\omega)\exp\left\{\int_{-\infty}^{\eta}d\eta'\frac{1}{2(\lambda+\gamma^-e^{-\eta'})}[\lambda+\gamma^+e^{\eta'}+\gamma^-e^{-\eta'}-\omega_0-\sqrt{(\lambda+\gamma^+e^{\eta'}+\gamma^-e^{-\eta'}-\omega_0)^2+4\omega(\lambda+\gamma^-e^{-\eta'})}]\right\}=\{1-\exp[-e^{\eta}g(b_t-b_t')]\}\ . \tag{50}$$

However the solution given by Eqs. (49) and (50) cannot be considered as transparent from a physical point of view. This is the reason why I would like to give another solution which has worse accuracy, but is simple enough to clarify the situation and to demonstrate the main property of the solution. Let us assume that the last term in Eq. (47) is small enough to be neglected. In this case the solution looks like the solution of Eq. (23), namely, that Ψ is a function of one variable $[\Psi(\kappa)]$,

$$\kappa = Y - y - \frac{1}{\Delta}\ln\frac{(x-x_+)x_-}{(x-x_-)x_+}\,, \tag{51}$$

where

$$x_\pm = \frac{-(\omega_0-\lambda)\pm\sqrt{(\omega_0-\lambda)^2-4\gamma^+\gamma^-}}{2\gamma^-}$$

and

$$\Delta = 2\gamma^-(x_+-x_-),\quad x=e^\eta\,.$$

The function $\Psi(\kappa)$ could be found from the initial condition (50) at $y=Y$. It is easy to see that $\Psi(\kappa)$ is given by

$$\Psi(\kappa) = 1-\exp\left\{-g(b_t-b_t')\frac{x_+-x_-(x-x_+)/(x-x_-)e^{-\Delta(Y-y)}}{1-(x-x_+)/(x-x_-)e^{-\Delta(Y-y)}}\right\}. \tag{52}$$

Using this solution we can calculate the last term of Eq. (47). One can see that this term is small enough even at $y=Y$ and becomes smaller as $y\to 0$.

Thus, to get the solution of our problem, namely, to sum up all diagrams of the type shown in Fig. 11, we need to (1) substitute $x=g(b_t')$ in Eq. (52), (2) integrate over b_t', (3) calculate $\Omega(s,b_t)$ as

$$\Omega(Y=\ln s,b_t) = \int\frac{d^2b_t'}{2\pi}\times\{\Psi(b_t,b_t',\kappa(\gamma^+=\gamma^-=0)) + 2[\Psi(b_t,b_t',\kappa) - \Psi(b_t,b_t',\kappa(\gamma^+=\gamma^-=0))]\}\,, \tag{53}$$

and (4) substitute the opaqueness $\Omega(s,b_t)$ in Eq. (20).

IV. LARGE RAPIDITY GAPS IN HADRON-HADRON COLLISIONS

Equation (47) gives us the possibility to discuss the behavior of the total cross section and elastic cross sections as well as inclusive observables in hadron-hadron collisions. Here I am going to discuss only two problems: the energy behavior of the inclusive cross section and the probability of a large rapidity gap.

A. Inclusive cross section

Accordingly to the Abramovski-Gribov-Kancheli (AGK) cutting rules [23] the inclusive cross section can be calculated as a sum of diagrams shown in Fig. 12. It is obvious that the formulas for the inclusive cross section looks like

$$\frac{d\sigma}{dy} = G_{2Ph}\int\frac{d^2b_t}{2\pi}\Psi_{\rm inc}(Y-y,x=g(b_t)) \times\int\frac{d^2b_t'}{2\pi}\Psi_{\rm inc}(y,x=g(b_t'))\,, \tag{54}$$

where all notation is clear from Fig. 12. Ψ is the solution of Eq. (47) but with a different initial condition as compared with Eq. (50), namely,

$$\Psi(Y-y=0,x)=x\,. \tag{55}$$

In the approximation that leads to solution (51) one can find the answer for $\Psi_{\rm inc}$:

$$\Psi_{\rm inc}(Y-y,x) = \frac{x_+-x_-(x-x_+)(x-x_-)e^{-\Delta(Y-y)}}{1-(x-x_+)(x-x_-)e^{-\Delta(Y-y)}}\,. \tag{56}$$

Equation (56) solves the problem, allowing one to calculate the inclusive cross section.

B. Large rapidity gap

Dokshitzer, Khoze, and Sjostrand and Bjorken suggested [24] that one consider not the inclusive cross section of hard processes such as Higgs boson production (see Fig. 13), but the cross section for an event with a very interesting signature, namely, such that no hadrons are produced with rapidities between y_1 and y_2, except for the Higgs boson and hadrons from its fragmentation. To obtain the formulas for the cross section of such an event, we need to multiply the expression for the inclusive cross section [see, Eq. (54)] by the probability that the partons with rapidity larger than y_1 do not interact

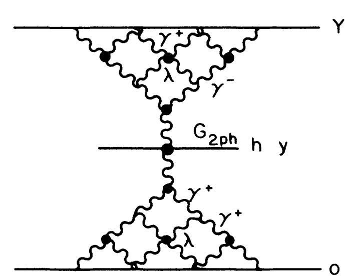

FIG. 12. Inclusive cross section.

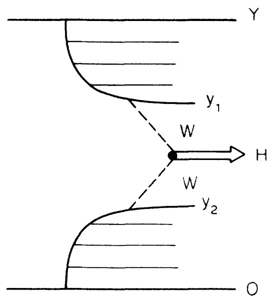

FIG. 13. Higgs boson production (inclusive cross section).

with partons that have rapidity less than y_2. Introducing the probability $P(y_1-y_2,s,b_t)$ that no inelastic interaction takes place at impact parameters b_t in the rapidity region y_1-y_2 at energy s, one can write the following formula for the survival probability of the rapidity gap:

$$\langle |S^2| \rangle = \frac{\int a_{\rm hard} P(y_1-y_2,s,b_t) d^2 b_t}{\int a_{\rm hard} d^2 b_t} , \tag{57}$$

where $a_{\rm hard}$ is the inclusive cross section [see Eq. (54) and Fig. 12]. To calculate $\langle |S^2| \rangle$ one needs to estimate $P(y_1-y_2,s,b_t)$.

1. *Eikonal approximation*

The above estimate is easy to do in the eikonal approach [25]. s-channel unitarity can be written in the diagonalized form, viz., in the b_t representation, as

$$2\,{\rm Im} a(s,b_t) = |a(s,b_t)|^2 + G_{\rm in}(s,b_t) , \tag{58}$$

where

$$\sigma_{\rm in} = \int d^2 b_t G_{\rm in}(s,b_t) . \tag{59}$$

In the eikonal approach one can see that from Eq. (20) it follows that

$$G_{\rm in}(s,b_t) = 1 - e^{-2\Omega(s,b_t)} . \tag{60}$$

From the above equation we can conclude that the factor

$$e^{-2\Omega(s,b_t)} \tag{61}$$

describes the probability $P(s,b_t)$ which does not depend on y_1-y_2 within the eikonal approximation. Finally we can obtain the Bjorken formula [24] for $\langle |S^2| \rangle$ by substituting Eq. (61) in the expression (57) for $\langle |S^2| \rangle$.

2. *General approach*

As has been discussed, the eikonal approximation oversimplifies the structure of the parton cascade, reducing the complicated parton cascade with a rich variety of different parton interactions to the simple picture of Fig. 5. Using the general equation (47) and taking into account the interaction between Pomerons we are able to write down a more general expression for the survival probability of a large rapidity gap than Eq. (61). We can also examine how important could be the parton interactions inside the parton cascade for the evaluation of the survival probability. To find the expression for $\langle |S^2| \rangle$ we can use Eq. (57) and a general expression for the opaqueness Ω [see Eq. (53)]. However, we need to take into account only inelastic interaction due to the Pomeron exchanges and the Pomeron interactions in the rapidity region y_1-y_2. The solution of this problem looks very simple and we can summarize the procedure of the solution as follows (see Fig. 14).

(1) One solves Eq. (47) with the initial condition of Eq. (50). Thus we found the function $\Psi(Y-y,x,b_t,b_t,b_t')$.

(2) The next step is to find the solution of Eq. (47) $\Psi(y_1-y)$ with the initial condition

$$\Psi(y_1-y,x,b_t,b_t')|_{y=y_1} = \Psi(Y-y_1,x,b_t,b_t') . \tag{62}$$

(3) A solution of Eq. (47) $\Psi(y,x,b_t')$ should be found which satisfies the initial condition of Eq. (50) but with $g(b_t')$ instead of $g(b_t-b_t')$.

(4) We specify the function $\Psi(y_1-y,x,b_t,b_t')$, extracting the value of $x=x_m$ from the matching condition

$$\Psi(Y-y_2,x_m,b_t,b_t') = \Psi(y_2,x_m,b_t') . \tag{63}$$

(5) To calculate the opaqueness $\Omega(y_1-y_2,s,b_t)$ we need to substitute in Eq. (53) $\Psi(y_1-y_2,x_m,b_t,b_t')$.

(6) Equation (57) with the opaqueness $\Omega(y_1-y_2,s,b_t)$ allows us to calculate the survival probability in the general case.

The described general procedure could be specified using one of the previous explicit solutions [see Eqs. (27), (39), or (52)]. We do not want to discuss the application of the above solution since it is better to do it together with some phenomenological estimates of the value of the vertices for Pomeron-Pomeron interaction (such as γ^+, γ^-, and λ). It should be stressed only that the resulting formula turns out to be much more complicated than the eikonal formula. I hope to publish a close investigation of the role of the Pomeron interaction elsewhere rather soon.

V. CONCLUSIONS

Concluding the paper I would like to repeat once more than an attempt was made in this paper to regenerate the Reggeon calculus as a way to take into account the Pomeron-Pomeron interaction to understand the origin and the main properties of shadowing (screening) corrections. The approach is based on two principal assumptions.

(1) Only "hard" processes with the typical scale of the transverse momentum of the order of $Q_0 \gg \mu$ such as $\alpha_s(Q_0^2) \ll 1$ contribute to the structure of the Pomeron.

(2) We can introduce vertices for Pomeron-Pomeron interactions which are local in rapidity and in the impact parameter.

Both assumptions look very natural from experience in perturbative QCD calculations as well as from current experimental information. However, we need a much more detailed study of the above assumptions.

In particular, we have to redo all description of the experimental data in the eikonal model to estimate the value of the triple Pomeron vertex as well as the Pomeron-Pomeron scattering amplitude. This task became even more urgent in connection with the new data from the Collider Detector at Fermilab (CDF) Collaboration on total, elastic, and diffraction dissociation cross sections [26] that cannot be fitted in the eikonal model in a natural way [27]. The first estimate of the value of the triple Pomeron vertex [27] shows that the correction described by Eq. (28) is not too small and the ratio γ/ω_0 is of the order of $\frac{1}{4}$. Unfortunately only after finishing this job will we be able to give a prediction that could be checked experimentally. However, the first qualitative prediction is obvious: the shadowing correction in our approach turns to be much stronger than in the eikonal model (see Fig. 14). This prediction is in perfect qualitative agreement with the new CDF data [26].

I would like to recall to you that the main goal of this paper for me was to convince the reader that the calculation of the shadowing corrections could be formulated as a theoretical problem with a restricted number of assumptions that could be checked experimentally. I will be happy if somebody will find arguments against my approach, since such a discussion will be able to promote a deeper understanding of the problem. It is worthwhile mentioning that the above approach is only the first step in the development of a self-consistent theoretical approach to high energy interaction based on QCD. The next step will be an attempt to take into account both

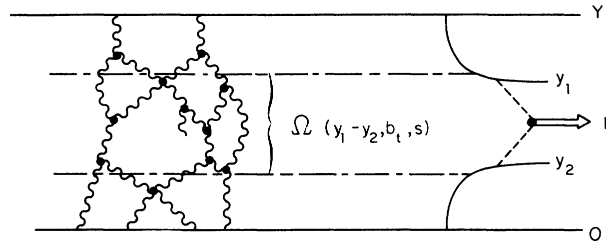

FIG. 14. Higgs boson production (shadowing corrections).

"hard" and "soft" processes (see Ref. [28] for a hint on how we could deal with this problem). At the very end of this paper I would like to note that the technique developed in Sec. III can be used also in a more phenomenological approach without any reference to QCD if we assume that the slope of the Pomeron trajectory (α') as well as the slope of all vertices for Pomeron-Pomeron interaction are much smaller than the hadron radius R.

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