Chiral phases of QCD at finite density and temperature

A. Barducci, R. Casalbuoni, and G. Pettini
Dipartimento di Fisica, Università di Firenze, I-50125 Firenze, Italy
and Istituto Nazionale di Fisica Nucleare, Sezione di Firenze, I-50125 Firenze, Italy

R. Gatto
Département de Physique Théorique, Université de Genève, CH-1211 Genève 4, Switzerland
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Chiral transitions in QCD at finite temperature and density are discussed within a composite operator formalism. For massless quarks the phase diagram in temperature and chemical potentials presents a tricritical point at the intersection of the critical line for first-order phase transitions and second-order transitions. The overall picture is not sensibly affected by small quark masses, except that the quark condensate no longer vanishes for large temperatures and/or chemical potentials, and for first-order transitions it remains discontinuous only for small quark mass values. Such effects are discussed by Clausius-Clapeyron-like relations. The Landau expansion is used around the tricritical point and for second-order transitions.

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I. INTRODUCTION

Strong-interaction physics at low energy is well described by flavor chiral symmetry spontaneously broken with the related appearance of pseudo Goldstone bosons. On the other hand, at a sufficiently high temperature and/or density it is generally believed that this spontaneously broken symmetry is restored much like as in ferromagnets where condensed states (broken symmetry) become normal (symmetric) above a critical temperature [1]. For instance, in QCD-like gauge theories one expects that by increasing the temperature and/or the quark density the interaction among quarks and gluons becomes less strong and chiral symmetry is restored. Most of the theoretical insight into this phase transition comes from Monte Carlo simulations for a pure gauge theory (deconfinement) and from model calculations with massless quarks.

At this stage it is therefore relevant to investigate the temperature and density effects on hadronic matter by using methods alternative to lattice simulations. In this approach one tries to incorporate the main known features of QCD and to evaluate analytically quantities such as the fermionic condensate as a function of (μ, T) where μ is the chemical potential. Such a dependence might be tested in heavy-ion collisions at high energy. In fact, the existence of a critical temperature and chemical potentials may already be suggested by certain high-energy cosmic-ray events [2] and it might be indirectly tested at future accelerators for heavy ions [3].

We shall refer here to previous attempts based on phenomenological chiral models, indicating the possible phase diagram of QCD in the (μ, T) plane of the chemical potential and temperature [4]. In the more realistic case of quarks with finite current masses, chiral symmetry is already broken explicitly and, strictly speaking, there is no transition, as happens in a ferromagnetic case in the presence of an external magnetic field. We have thus to better specify to what extent one can still speak of a phase transition.

In this work we extend the analysis previously performed in a QCD model considered in the chiral limit [4,5] to the case of massive quarks. A preliminary study of the model for massive quarks, but taking into account only the effects of the temperature, has been described in Ref. [6].

This paper is organized as follows. To make it self-contained we give a short review of the composite operator formalism and we derive the effective potential at finite temperature and density in Sec. II. In Sec. III we briefly discuss the phase diagram in the chiral case. We find that, moving along the critical line for increasing chemical potential, one encounters a tricritical point dividing the critical line in two parts L_{II} and L_I, by crossing which chiral symmetry is restored through a second-order or a first-order phase transition, respectively.

Section IV is devoted to the general study of the massive case. Here chiral symmetry is already broken in the Lagrangian, but we can still retain, for small masses at least, the notion of phase transition, by looking at the region of (μ, T) where the condensate has a rapid variation. On the contrary, for a quark mass of about 100 MeV the temperature or density evolution is quite smooth on large intervals of (μ, T). A mean field expansion in the manner of Landau of the effective potential allows us to evaluate the various critical exponents describing how quantities such as the chiral condensates approach the critical points, both in the region of the second-order phase transition and near the tricritical point. We study the region of the first-order phase transition and therein we recall briefly the generalized Clausius-Clapeyron-like relations. In principle, these equations allow us to relate discontinuities of the thermal averages of different observables at the critical points of a first-order phase transition, for

instance, the values of the condensate for different quarks. At this stage of the model, where the effective potential turns out to be factorized into independent flavor contributions we cannot use the Clausius-Clapeyron equations in its general content; for this it would be necessary to include in the theory some mechanism of flavor mixing, such as, for instance, the fermionic 't Hooft determinant arising from the anomaly. However, we can try to get some useful information from these equations by relating the discontinuity of the order parameter to some other discontinuity of physical quantities (for instance, the entropy density) through the variation of the critical points with the external parameter conjugate to the condensate, i.e., the current mass m_0.

Finally, in Sec. V, we study in some detail the limiting situation $T=0$, $\mu\neq 0$. Here the quark condensate shows the typical behavior for a first-order phase transition; i.e., it has a jump at some critical chemical potential; then it approaches a constant value depending on the current quark mass. We shall also give estimated values of the critical points.

II. EFFECTIVE ACTION FOR COMPOSITE OPERATORS

Following Ref. [7] the zero temperature and zero quark density Euclidean effective action for an SU(N) QCD-like gauge theory is

$$\Gamma[S]=-\Gamma_2[S]+\mathrm{Tr}\left[S\frac{\delta\Gamma_2}{\delta S}\right]-\mathrm{Tr}\ln\left[S_0^{-1}+\frac{\delta\Gamma_2}{\delta S}\right], \tag{2.1}$$

where S_0 and S are the free and the full quark propagator, respectively. Γ_2 is the sum of all the two-particle irreducible vacuum diagrams of the theory evaluated with a fermionic propagator equal to S and S_0 is given by

$$S_0(p)^{-1}=i\hat{p}-m_0 . \tag{2.2}$$

The dynamical variable of the theory is defined by the equation

$$\Sigma=-\delta\Gamma_2/\delta S \tag{2.3}$$

and at the physical points Eq. (2.3) is nothing but the Schwinger-Dyson equation for the fermionic propagator with Σ the fermion self-energy. By taking Γ_2 at the two-loop level (single gluon exchange) and parametrizing the fermionic propagator as

$$S(p)=iA(p)\hat{p}+B(p) , \tag{2.4}$$

one finds that in the Landau gauge no renormalization of the wave function is required at this order [Γ_2 does not depend on $A(p)$] and the effective action is completely expressed in terms of Σ [7]:

$$\Gamma[\Sigma]=-\mathrm{Tr}\ln(S_0^{-1}-\Sigma)-\frac{1}{2}\mathrm{Tr}(S\Sigma)$$
$$=-\mathrm{Tr}\ln(S_0^{-1}-\Sigma)+\Gamma_2[S] \tag{2.5}$$

with B related to Σ by

$$\Sigma(q)=-3g^2C_2\int\frac{d^4p}{(2\pi)^4}\frac{B(p)}{(p-q)^2} . \tag{2.6}$$

In Eq. (2.6) g is the gauge coupling constant and C_2 is the quadratic Casimir of the gauge group. The variational method consists in choosing a parameter-dependent test function for Σ to investigate the stability of the theory; then we have to invert the relation in Eq. (2.6) to get $B(p)$ in terms of $\Sigma(p)$ just to express the effective action as a functional of the self-energy Σ only.

To extend the zero-temperature theory to finite chemical potential μ, we have to perform the integrations over shifted Euclidean energies, $p_0\to p_0+i\mu$. This procedure modifies the structure of the singularities of the propagators, giving rise to typical additional contributions with respect to the case of zero chemical potential. The additional terms will contain step functions since they appear just beyond the Fermi energy.

As far as the temperature is concerned, in the imaginary time formalism, we can still work with continuous energies by substituting for the sum over discrete energies $\omega_n=(2n+1)\pi/\beta$ (where $\beta=1/T$) a sum of integrals over continuous energies by means of the Poisson's formula

$$\frac{1}{\beta}\sum_{n=-\infty}^{\infty}\int\frac{d^3p}{(2\pi)^3}f(\omega_n,\mathbf{p})$$
$$=\sum_{n=-\infty}^{\infty}(-)^n\int\frac{d^4p}{(2\pi)^4}f(p_0,\mathbf{p})e^{in\beta p_0} . \tag{2.7}$$

Thus, introducing the four-vectors $\bar{k}^{\nu}=(k_0+i\mu,\mathbf{k})$, at finite μ and T Eq. (2.6) must be read as

$$\Sigma(\bar{q})=-3g^2C_2\sum_{n=-\infty}^{\infty}(-)^n\int\frac{d^4p}{(2\pi)^4}\frac{B(\bar{p})}{(p-q)^2}e^{in\beta p_0} . \tag{2.8}$$

The factor $1/(p-q)^2$ on the right-hand side (RHS) of the previous equation, which comes from the gluon propagator, is not affected by the introduction of the chemical potential since it depends only on the difference $(p-q)$. This reflects the fact that gluons do not carry a chemical potential. Then the inversion of Eq. (2.8) is obtained, as in the case $T=\mu=0$, by applying the differential operators $\Box_q$ getting

$$\sum_n(-)^nB(\bar{p})e^{in\beta p_0}=\frac{4\pi^2}{3g^2C_2}\Box_p\Sigma(\bar{p}) . \tag{2.9}$$

In conclusion, the final form for $\Gamma[\Sigma]$ is (see also Refs. [4,6])

$$\Gamma[\Sigma]=-\Omega\left[2N\sum_{n=-\infty}^{\infty}(-)^n\int\frac{d^4p}{(2\pi)^4}\ln\det\{\bar{p}^2+[m_0+\Sigma(\bar{p}^2)]^2\}e^{in\beta p_0}+\frac{8N\pi^2}{3g^2C_2}\int\frac{d^4p}{(2\pi)^4}\mathrm{tr}[\Sigma(\bar{p}^2)\Box_p\Sigma(\bar{p}^2)]\right] . \tag{2.10}$$

We recall that both the current mass m_0 and Σ are, in general, matrices in flavor space. Nevertheless, as discussed in Refs. [6,7], if we neglect the mixing between the different flavors originating, for instance, from terms such as the 't Hooft determinant, it follows that only the flavor diagonal elements of the fermion self-energy and mass can be different from zero at the minimum. Therefore, the effective potential decomposes into the sum of n_f contributions, one for each flavor, and to study the minima it is formally sufficient to consider a single contribution. Of course, the choice of a given flavor number will reflect in the particular parameters assumed. In the present paper, as in Refs. [6,7], we will take $n_f=3$ and a number of colors $N=3$. The value of the parameters will be specified later on.

As far as the dynamical variable Σ is concerned we will adopt here the variational ansatz

$$\Sigma(\bar{p})=\chi\frac{M^3}{M^2+\bar{p}^2} . \tag{2.11}$$

In Eq. (2.11) M is a momentum scale (see Ref. [4]) and χ is a constant field to be taken as the variational parameter. Its value at the minimum of the effective potential is related to the fermion condensate, renormalized at the scale M:

$$\langle\bar{\psi}\psi\rangle_{\mu,T}=\frac{3M^3}{g^2(\mu,T)}\bar{\chi}(\mu,T) , \tag{2.12}$$

which generalizes in an obvious way the corresponding $\mu=T=0$ relation [4,7]. We have also assumed the momentum dependence of Σ in the ultraviolet region as predicted by the operator product expansion (OPE) analysis (up to the logarithmic corrections coming from renormalization group) and a constant behavior for $p\to 0$.

Let us now comment on the choice for the gauge coupling constant. As suggested by asymptotic freedom and renormalization-group considerations, we expect the strong forces to weaken at high temperatures and/or densities [8]. We shall then assume that in the UV region the coupling constant g depends logarithmically on the temperature T and on the chemical potential μ. We take into account this assumption by writing

$$\frac{g^2(\mu,T)}{2\pi^2}\equiv\frac{1}{c(\mu,T)}\equiv\frac{1}{c_0+c_1(\mu,T)}$$
$$=\frac{1}{c_0+(\pi^2/b)\ln(1+\xi T^2/M^2+\zeta\mu^2/M^2)} , \tag{2.13}$$

where $b=24\pi^2/(11N-2n_f)$, $c_0=0.554$, $M\simeq 280$ meV (see Ref. [4] where these values for c_0 and M are obtained by fitting at $T=\mu=0$ the mass spectrum and the decay constants of the pseudoscalar mesons), and the two parameters ξ and ζ have to be determined on phenomenological grounds. If we compare our model in the low-T regime (with $\mu=0$) with the results of Ref. [9] we find $\zeta=0.44\ (n_f^2-1)/n_f$, for n_f flavors, which gives $\xi\simeq 1$ for $n_f=3$ [5].

Because of the fact that the variational parameter χ does not depend on the quark momentum, we can study the various phases of the theory by minimizing the effective potential $V=\Gamma/\Omega$ with respect to χ. Its evaluation comes from Eq. (2.10), after having inserted the ansatz (2.11). We also define $\mathcal{V}=4\pi^2V/Nn_fM^4$, and, as we said, we consider the case $N=n_f=3$.

With these assumptions the final expression for the effective potential is

$$\mathcal{V}(\chi;\alpha,s,r)=\mathcal{V}_0^{(2)}+\mathcal{V}_0^{\log}+\mathcal{V}_{s,r}^{(2)}+\mathcal{V}_{s,r}^{\log}$$
$$+\alpha\chi z(s,r)+A(s,r) , \tag{2.14}$$

where $\alpha=m_0/M$, $r=T/M$, and $s=\mu/M$. The RHS of Eq. (2.14) is composed of six terms: the first one comes from the evaluation of Γ_2 at $T=\mu=0$ and is given by

$$\mathcal{V}_0^{(2)}=\frac{c_0}{3}\chi^2 , \tag{2.15}$$

whereas the second is the one-loop contribution again at $T=\mu=0$:

$$\mathcal{V}_0^{\log}=-\frac{1}{2}\int_0^1 dy\left[\frac{1-y}{y^3}\ln\left[1+\frac{\chi^2y^3+2\alpha\chi y^2}{1-y+\alpha^2y}\right]-\frac{2\alpha\chi}{y}\right] . \tag{2.16}$$

The corrections which arise from the introduction of chemical potential and temperature are

$$\mathcal{V}_{s,r}^{(2)}=\frac{c(s,r)}{3}\chi^2\left[\frac{c_1(s,r)}{c(s,r)}+\theta(s^2-1)\frac{(2s^2+3)\sqrt{(s^2-1)^3}}{2s^5}\right] , \tag{2.17}$$

which comes from the two-loop term, with

$$c_1(s,r)=\tfrac{9}{8}\ln(1+r^2+\zeta s^2) ,$$
$$c(s,r)=c_0+c_1(s,r) , \tag{2.18}$$

and

$$\mathcal{V}_{s,r}^{\log}=-4r\sum_{k=1}^{3}\int_0^\infty dy\, y^2\ln(1+e^{-(\sqrt{y^2+x_k}-s)/r})+(s\to -s) , \tag{2.19}$$

which comes from the one-loop term and where the z_k's are defined through the equation

$$x^3+(2+\alpha^2)x^2+(1+2\alpha^2+2\alpha\chi)x+(\alpha+\chi)^2=\prod_{k=1}^{3}(x+z_k)=0\ . \tag{2.20}$$

Notice that the first term in square brackets of $\mathcal{V}_0^{\log}$ in Eq. (2.16) is UV divergent for $\alpha\neq 0$ and it is regularized by the second one. The renormalization at $T=\mu=0$ can be performed by adding a counterterm and requiring that the derivative of the effective potential with respect to the term which breaks explicitly the chiral symmetry, evaluated at the minimum, satisfies for each flavor the normalization condition [7]

$$\lim_{m_0\to 0}\left.\frac{\partial V}{\partial(m_0\langle\bar{\psi}\psi\rangle_0)}\right|_{\min}=1\ . \tag{2.21}$$

At finite T and μ we do not have any additional divergence with respect to the case $T=\mu=0$; nevertheless, we have to add a finite counterterm $\alpha\chi z(s,r)$, in order to satisfy the generalization of this normalization condition at finite T and μ. This term turns out to be

$$z(s,r)=2c(s,r)-\int_0^1 dt\,\frac{\hat{\chi}^2(s,r)t^2}{1-t+\hat{\chi}^2(s,r)t^3}-\left[4\sum_{k=1}^{3}\frac{1}{(1-3z_k^0)}\int_0^\infty dy\,\frac{y^2}{\sqrt{y^2+z_k^0}\{1+\exp[(\sqrt{y^2+z_k^0}-s)/r]\}}+(s\to -s)\right], \tag{2.22}$$

where $\hat{\chi}(s,r)$ is the minimum of the effective potential at $\alpha=0$, and consequently the z_k^0's are the z_k's of Eq. (2.20) evaluated at $\alpha=0$, i.e.,

$$x^3+2x^2+x+\hat{\chi}^2=\prod_{k=1}^{3}(x+z_k^0)=0\ .$$

Finally, in the expression (2.14) for the effective potential a term $A(s,r)$ is also present (depending only on T and μ but not on the field χ) coming directly from Eq. (2.10). This term is irrelevant for the study of the symmetry breaking but it has a thermodynamical meaning. In fact, we recall that the effective potential evaluated on the minimum is related to the pressure of the system through the relation

$$p(s,r)=-\left.V\right|_{\chi=\bar{\chi}(s,r)}\ . \tag{2.23}$$

The term $A(s,r)$ has the expression

$$A(s,r)=8r\int_0^\infty dy\,y^2\ln(1+e^{-(\sqrt{y^2+1}-s)/r})+(s\to -s)\ . \tag{2.24}$$

Because of Eq. (2.23) the normalization condition for the effective potential can also be written as

$$-\lim_{m_0\to 0}\frac{\partial p(T,\mu)}{\partial m_0}=\langle\bar{\psi}\psi\rangle_{T,\mu}\ . \tag{2.25}$$

We want to stress that a thermodynamical relation of this kind between conjugate variables such as the fermionic condensate and the mass m_0 has to be verified in QCD for any finite quark mass [10,11]. However, because of the fact that we are using an effective potential approach (suitable for light quarks) we can satisfy this relation only in the limit of vanishing quark masses.

Finally, we recall that with an appropriate normalization for the pion field [7] Eq. (2.21) [or (2.25)], is also equivalent to the Adler-Dashen formula

$$m_\pi^2(T,\mu)f_\pi^2(T,\mu)=-2m_0\langle\bar{\psi}\psi\rangle_{T,\mu}\ . \tag{2.26}$$

III. FINITE TEMPERATURES AND DENSITIES: MASSLESS QUARKS

Let us recall that at $T=\mu=0$ chiral symmetry (CS) is spontaneously broken by a vacuum expectation value (VEV) of the condensate different from zero. We first discuss the massless case $m_0=0$ ($\alpha=0$) where the effective potential (2.14) is a function of χ^2 only. By choosing the negative solution (for continuity with the case $\alpha\neq 0$), we find that the absolute minimum is $\chi_0=-4.06$ and it corresponds, by Eqs. (2.12) and (2.13), with $M\simeq 280$ MeV and $c_0=\simeq 0.554$, to a condensate

$$\langle\bar{\psi}\psi\rangle_0\simeq(-197\ \text{MeV})^3\ . \tag{3.1}$$

By studying the evolution of the absolute minimum $\bar{\chi}(T,\mu)$ for growing temperatures and/or chemical potentials, we find that the condensate vanishes and that the phase diagram for chiral symmetry restoration in the (μ,T) plane shows a tricritical point $t=(\mu_t,T_t)$ where the line L_{II} of second-order phase transitions starting from the point $(0,T_c)$ merges with the line of the first-order phase transition L_{I} ending at the point $(\mu_c,0)$ (see Fig. 1 which is obtained with $\zeta=0.3$). The structure of the phase diagram is qualitatively the same for other values of $\zeta\neq 0$ that we have also considered. The line γ from $(0,T_c)$ to $(\mu_\gamma,0)$ is defined by the changing of the curvature at the origin (i.e., by the appearance of a minimum in the origin) and it is sufficient to describe CS restoration as long as the phase transition is second order. For $\mu\geq\mu_t$, even when the origin has turned into a minimum, another minimum survives deeper than the one at $\chi=0$ and they become degenerate along L_{I}, the line of the first-order phase transitions. Finally, the line δ is the borderline of the metastable phases, beyond which the effective potential is an increasing function of χ. This phase diagram has already been discussed in Ref. [4]. In

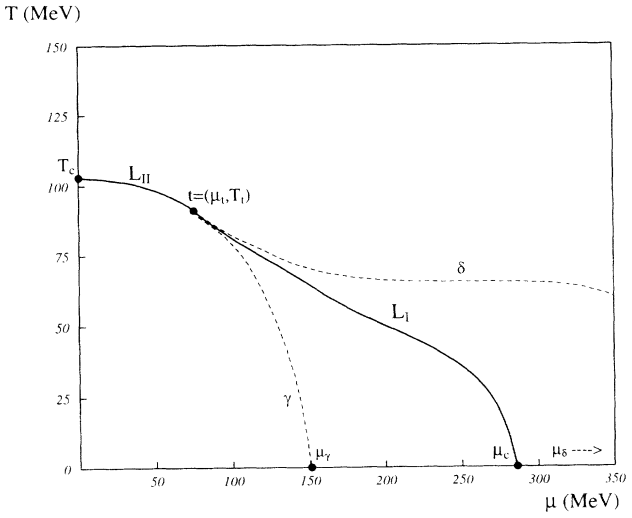

FIG. 1. The phase diagram in the (μ,T) plane for $m_0=0$ and $\zeta=0.3$. The tricritical point t has coordinates $(\mu_t,T_t)=(75,91)$ MeV while $T_c=103$ MeV, $\mu_\gamma=151$ MeV, $\mu_c=286$ MeV, and $\mu_\delta=555$ MeV.

that work the term

$$\frac{c(r,s)}{3}\chi^2\theta(s^2-1)\frac{(2s^2+3)\sqrt{(s^2-q)^3}}{2s^5} \tag{3.2}$$

appearing in Eq. (2.17) was lost in inverting the relation (2.8) by means of the Poisson formula involving both T and μ. The general structure for CS restoration is, however, not modified by this term, which is active only above the mass scale value $M\simeq 280$ MeV and thus it rarely has an effect since most of the critical values of the chemical potentials in the (μ,T) plane are well below the mass scale value. Whenever $\mu_c\geq M$, i.e., only for $T\simeq 0$ and $\zeta\leq 0.3$, the presence of the term (3.2) lowers the critical value μ_c. Under the same conditions it also makes smoother the function $\mu_c(\zeta)$. As to the critical points in the phase diagram, only the points $(\mu_c,0)$ and $(\mu_\delta,0)$ vs ζ for $\zeta\leq 0.3$ are modified (see Sec. IV), since all the others are unchanged with respect to Ref. [14].

IV. FINITE TEMPERATURES AND DENSITIES: MASSIVE QUARKS

By introducing a bare mass term m_0 in the model, we can try to study a more realistic situation, where the symmetry is explicitly broken from the beginning. Nevertheless, for small bare masses, as is the case for the u and d quarks, the mechanisms of dynamical mass generation and of CS restoration continue to play a basic role. Actually, the insertion of the mass term produces effects on the condensate below and beyond the transition; for $m_0\ll M$, they are both of order m_0, and consequently negligible with respect to the dynamical mass. Thus the jump of the condensate from the low to the high T (and/or μ) region is still a very strong effect. In this case m_0/M is a perturbative parameter which mainly affects the curvature of the condensate function going across the phase transition point, or in other words, the "speed" of the transition.

To study these effects it is convenient to separately discuss the case of second-order phase transitions where they are well described in terms of critical exponents, and the case of first-order phase transitions, where eventually one can be helped by considering Clausius-Clapeyron-like relations, as we shall see in the following.

A. Continuous transitions and critical exponents

At $m_0=0$, if we fix a value for the chemical potential $\bar{\mu}\leq\mu_t$ (and respectively for the temperature $\bar{T}\geq T_t$) and we let T (respectively μ) grow, we always cross the line L_{II}. In this case, as has already been shown in Ref. [5] for the chiral case, and in Ref. [6] (for $m_0\neq 0$ but $\mu=0$), we can expand the effective potential around $\chi=0$. Thus we can now consider the situation with the full set of parameters (μ,T,m_0). We get, for small $\alpha=m_0/M$, around any critical point along L_{II}, including $t=(\mu_t,T_t)$,

$$\mathcal{V}\simeq a_0(T_c,\mu_c)+a_2(T,\mu)\chi^2+a_4(T,\mu)\chi^4+a_6(T_c,\mu_c)\chi^6 + \cdots + b_1(T_c,\mu_c)\alpha\chi+\cdots , \tag{4.1}$$

where $\mathcal{V}=4\pi^2/Nn_fM^4V$. The coefficients of this expansion are evaluated at $\alpha=0$, and, as already discussed in Ref. [5], IR finite. The coefficients a_0, a_4, and a_6 are the same as in Ref. [5]. The coefficient a_2 is, however, different due to the presence of the new term which comes from Γ_2 and that modifies the coefficient of χ^2 in the effective potential [see Eq. (3.2)]. However, because of the fact that this term is active only for $s\geq 1$ (i.e., $\mu\geq 280$ MeV) and that μ_t is always ≤ 100 MeV [5], we can easily understand that the addition of this new term does not play any role as far as the Landau expansion of the effective potential is concerned. In view of Eqs. (2.12), (2.23), and (2.25), which specify the normalization condition we have chosen at finite T and μ, and due to Eq. (2.12), we get, for b_1,

$$b_1(\mu_c,T_c)=2c(\mu_c,T_c) \tag{4.2}$$

for any point lying on L_{II} and where c has been defined in Eq. (2.13).

The equation for the line L_{II} is

$$a_2(T_c,\mu_c)=0 \tag{4.3}$$

and $a_4(T_c,\mu_c)>0$ so that we can neglect in this case the term $a_6\chi^6$. The tricritical point t is defined by the equations

$$\begin{aligned} a_2(T_t,\mu_t)&=0 ,\\ a_4(T_t,\mu_t)&=0 , \end{aligned} \tag{4.4}$$

and $a_6(T_t,\mu_t)>0$. Let us notice that the parameter α plays here a role analogous to a small external magnetic field h in a magnetic system, and the order parameter $\langle\bar{\psi}\psi\rangle$ that of the spontaneous magnetization.

Excluding the point t and working at fixed $\mu=\mu_c$, the critical behavior is described exactly in the same way of Ref. [6] which corresponds to the particular case $\mu_c=0$; actually only the values of the coefficients of the expan-

sion change, since they depend now on μ_c. Of course, we can approach the critical line following a generic direction, namely, by expanding

$$a_2 \simeq a_T \left| \frac{T-T_c}{T_c} \right| + a_\mu \left| \frac{\mu-\mu_c}{\mu_c} \right| + \cdots . \tag{4.5}$$

Anyway, if we limit ourselves to consider orthogonal directions in the (μ, T) plane, we can now fix $T=T_c$, and so we find the same critical exponents in the chemical potential as in the temperature, since they come out from the powers of the field χ present in the expansion (4.1), and not from the (μ, T) dependence of the coefficients. We assign the classical definitions for the critical exponents: the exponent β by the behavior of the order parameter at $m_0=0$ and $\theta \to \theta_C$ (where θ can be either T at μ fixed or μ at T fixed),

$$\bar{\chi}(\theta) = a_\chi \left| 1 - \frac{\theta}{\theta_C} \right|^\beta ;$$

the δ exponent by the behavior of the order parameter at $\theta=\theta_C$ and $m_0 \to 0$,

$$\bar{\chi}(\theta_C) = b_\chi m_0^{1/\delta} ;$$

the γ exponent in analogy with the magnetic susceptibility at $m_0=0$ and $\theta \to \theta_C$,

$$\left. \frac{\partial \bar{\chi}(\theta)}{\partial m_0} \right|_{m_0=0} = d_\chi \left| 1 - \frac{\theta}{\theta_C} \right|^{-\gamma} .$$

The results, along L_{II} excluding point t, are $\beta=\frac{1}{2}$, $\delta=3$, and $\gamma=1$. It is easy to find that approaching the tricritical point again $\gamma=1$, but the other coefficients change to $\beta=\frac{1}{4}$ and $\delta=5$.

The critical behavior is therefore in our model the same as in any classical mean field theory, extensively studied in many textbooks. We want to stress that we are particularly interested in looking up to what extent we can retain the notion of a second-order phase transition once the current quark mass is introduced in the model. In other words, even if in this case chiral symmetry is broken explicitly, for the lightest quarks u and d we still expect abrupt changes of the condensates across some critical point, whereas the same behavior is no longer expected for the s quark. Actually, by considering $m_0 \ll M$, both the low- and the high-θ values of the condensate will change for an amount of order $\alpha = m_0/M$ with respect to the massless case. Anyway, not only the ratio of the condensate from the low-θ to the high-θ regimes will reduce for increasing m_0, but also the difference between them. This last feature, which can be observed directly in the model from the numerical results or by evaluating the coefficients of the α expansion of the condensate, is better clarified by looking at the coefficient d_χ of the susceptibility before and after $\theta=\theta_C$. This coefficient doubles in value by approaching a point of L_{II} from $\theta > \theta_C$ instead of $\theta < \theta_C$, and it quadruplicates by approaching the tricritical point. Thus, for example, if we fix $T \geq T_t$ (respectively, $\mu \leq \mu_T$) so that $\theta \equiv \mu$ (respectively, $\theta \equiv T$), and we consider two points symmetrical

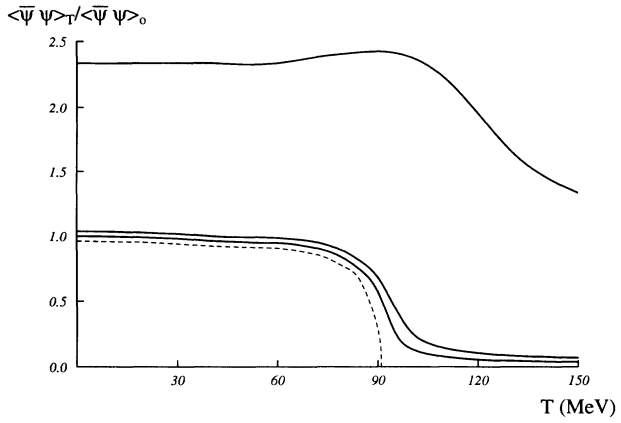

FIG. 2. Normalized fermion condensate vs temperature at $\mu=\mu_t=75$ MeV and $\zeta=0.3$ for $m_0=5$, 9.5, and 180 MeV (from the lowest to the highest full curves, respectively) normalized to $\langle \bar{\psi}\psi \rangle_0 = (-197 \text{ MeV})^3$. The dashed line is the normalized condensate in the chiral limit $m_0=0$.

and close to θ_C, we find that the difference between the values of the condensate evaluated at these points decreases (and consequently the transition becomes less and less steep) as α increases. This situation is shown in Fig. 2, where we have plotted the condensate vs temperature at $\mu=\mu_t=75$ MeV, $\zeta=0.3$ for different values of the mass parameter. It is also evident that the picture emerging from this approach is that we can retain the notion of the second-order phase transition for a light quark and not for a very massive quark. In Fig. 3 we show the behavior of the condensate for

$$m_0 = (m_u + m_d)/2 = 9.5 \text{ MeV}$$

for growing temperatures at different value of the chemical potential for $\zeta=0.3$.

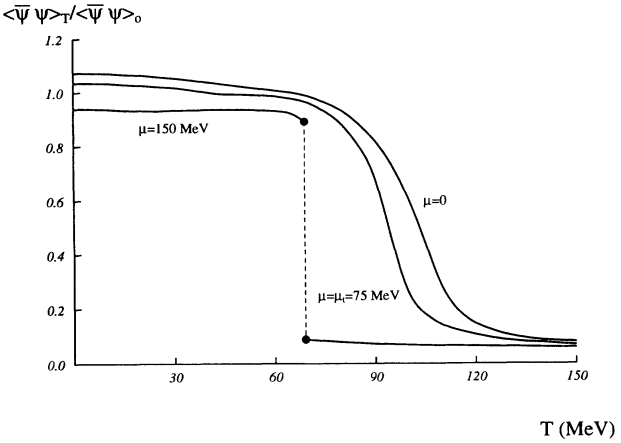

FIG. 3. Behavior of the normalized condensate (as in Fig. 2) at finite T and μ for $\zeta=0.3$ and $m_0=(m_u+m_d)/2=9.5$ MeV. The curves are found for increasing temperatures at different chemical potentials, i.e., $\mu=0$, $\mu=\mu_t=75$ MeV, and $\mu=150$ MeV.

B. Discontinuous transitions and Clausius-Clapeyron relations

Let us now study what happens by crossing L_I when a small quark mass parameter is present. What one finds is that there still exists a finite jump in the quark condensate, even if at different values of T and μ with respect to the chiral case. In practice, we can again define critical temperatures and chemical potentials and consequently critical lines $L_I^{m_0}$ (L_I will continue to indicate the curve of the case $m_0=0$). We have to distinguish roughly two regions of first-order phase transitions, one near the tricritical point and the other far from it (low temperatures and high chemical potentials). In fact, differently from the case of continuous transitions, where the critical features are nearly invariant along L_{II} (apart from the tricritical point), the situation is different when we turn our attention to the study of first-order phase transitions. Actually, in the first region, because of the expansion (4.1) of the effective potential, we still have some analytical control on the dependence of the discontinuities of the various physical quantities (condensate, energy density, etc.) on the parameters (mass, temperatures, etc.), whereas this is no longer true far from t.

Thus close to t let us consider again the expansion (4.1) written with the simplified notation

$$\mathcal{V}\simeq a_2\chi^2-|a_4|\chi^4+a_6\chi^6+b_1\alpha\chi+\cdots \tag{4.6}$$

since we are interested in evaluating the two degenerate absolute minima and thus $a_2>0$ and $a_4<0$ near the critical point, whereas $a_6,b_1>0$. If $\alpha=m_0/M=0$ there are two minima (from the negative side of χ):

$$\bar{\chi}_1^0=0\ ,\qquad \bar{\chi}_2^0=-\left[\frac{|a_4|+\sqrt{|a_4|^2-3a_2a_6}}{3a_6}\right]^{1/2}\ , \tag{4.7}$$

which become degenerate when the critical condition is satisfied,

$$a_2=|a_4|^2/4a_6\ , \tag{4.8}$$

and thus the discontinuity of $\bar{\chi}$ for $\alpha=0$ is

$$\mathrm{disc}\bar{\chi}=\bar{\chi}_2^0-\bar{\chi}_1^0=-\sqrt{|a_4|/2a_6}\ . \tag{4.9}$$

With $\alpha\ll 1$, one can easily evaluate the corrections at the leading order both to Eqs. (4.7) and (4.8):

$$\bar{\chi}_1=-\alpha\frac{b_1}{2a_2}\ ,\qquad \bar{\chi}_2=\bar{\chi}_2^0-\alpha b_1\frac{|a_4|-\sqrt{|a_4|^2-3a_2a_6}}{8a_2\sqrt{|a_4|^2-3a_2a_6}}\ , \tag{4.10}$$

$$a_2=|a_4|^2/4a_6+\alpha b_1\sqrt{2a_6/|a_4|}\ , \tag{4.11}$$

and consequently to Eq. (4.9) for the dependence of the discontinuity of $\bar{\chi}$ on α at the new critical point, which turns out to be

$$\mathrm{disc}\bar{\chi}=\bar{\chi}_2-\bar{\chi}_1=-\left[\frac{|a_4|}{2a_6}\right]^{1/2}\left[1-\alpha\frac{b_1}{\sqrt{2}}\left[\frac{a_6^3}{|a_4|^5}\right]^{1/2}\right]+\alpha b_1\frac{2a_6}{|a_4|^2}$$
$$=-\left[\frac{|a_4|}{2a_6}\right]^{1/2}+\frac{5}{2}\frac{a_6b_1}{|a_4|^2}\alpha\ . \tag{4.12}$$

Thus the absolute value of the discontinuity decreases for increasing α. The jump of $\langle\bar{\psi}\psi\rangle$ is then simply related to that of $\bar{\chi}$ by Eq. (2.12) and so these arguments are valid for the condensate as well.

In practice, if we do not move too far from the point t, the dependence of the so-called critical features of the condensate on (T,μ,m_0) are very well described in terms of the simple expansion (4.1).

On the other hand, far from t, even with a small m_0 we can only rely on the numerical study, which shows that the jump of the condensate grows by approaching the point $(\mu_c,0)$. As far as the dependence on m_0 is concerned, we find that in this region the discontinuities are so large that even a quite heavy quark suffers a big discontinuity crossing $L_I^{m_0}$, although the ratio of the condensate from the "broken" to the "restored" regions still decreases for increasing masses. We have to remember that our effective potential is reliable for light quarks only, and also that without a mixing between the flavors u, d, and s our model is equivalent to studying a single flavor theory. In any case, it does not make much sense to speak about chiral symmetry breaking and restoration for a relatively massive quark.

Let us now conclude this section by discussing Clausius-Clapeyron-like relations, as derived in the framework of the model.

At any critical point along $L_I^{m_0}$ the following condition on the effective potential (2.14) holds

$$V\{\bar{\chi}[T_c(m_0),\mu_c(m_0),m_0];T_c(m_0),\mu_c(m_0);m_0\}|_{\bar{\chi}=\bar{\chi}_1}$$
$$=[\mathrm{same}]_{\bar{\chi}=\bar{\chi}_2}\ , \tag{4.13}$$

where $\bar{\chi}_1$ and $\bar{\chi}_2$ are the two degenerate minima of the effective potential. Generally speaking, this constraint, in view of Eq. (2.23), is nothing but the condition of continuity of the pressure at the boundary of two different phases of a system. Thus, from Eq. (4.13), one could in theory follow the same procedure described in Refs. [10,11] to derive Clausius-Clapeyron-like relations in a general way.

However, in the present case we have to limit ourselves to deriving these relations only in the limit $m_0\ll M$.

Indeed in our context the relation (2.25) holds only for vanishing quark masses, differently from realistic QCD in which it should be verified for any finite m_0 as we have already discussed.

So, let us briefly describe the procedure to derive Clausius-Clapeyron-like relations in the present case by taking into account the leading order of the expansion of Eq. (4.13) for $m_0 \to 0$:

$$V\Big|_{c,m_0=0}^{\bar{\chi}_1} + m_0 \left[\frac{\partial V}{\partial m_0}\Big|_c^{\bar{\chi}_1} + \frac{\partial V}{\partial T}\Big|_c^{\bar{\chi}_1} \frac{dT_c}{dm_0} + \frac{\partial V}{\partial \mu}\Big|_c^{\bar{\chi}_1} \frac{d\mu_c}{dm_0} + \frac{\partial V}{\partial \chi}\Big|_c^{\bar{\chi}_1} \frac{d\bar{\chi}_1}{dm_0} \right] \Big|_{m_0=0} = [\text{same}]\Big|^{\bar{\chi}_2}, \tag{4.14}$$

where the subscript $|_c$ reminds us of the evaluation at a critical point. One has to notice that the first term in the left-hand side (LHS) is canceled by the analogous one in the RHS and also that the terms which contain the derivatives of V with respect to χ vanish since they are evaluated at the two minima of the effective potential. Thus, by taking into account again Eqs. (2.23) and (2.25) one gets

$$\text{disc}\langle \bar{\psi}\psi \rangle \Big|_{m_0=0} = \left[\frac{dT_c}{dm_0} \text{disc } s + \frac{d\mu_c}{dm_0} \text{disc } n \right] \Big|_{m_0=0}, \tag{4.15}$$

where $s = S/V = \partial p/\partial T$ is the entropy density, $n = \langle N \rangle / V = \partial p/\partial \mu$ is the particle number density, and all the discontinuities of the physical quantities are evaluated at a critical point. However, as discussed in Ref. [11], if we take into account that dT_c/dm_0 and $d\mu_c/dm_0$ are not independent quantities, as all the possible points (μ_c, T_c) have to lie on some $L_I^{m_0}$, we can rewrite Eq. (4.15) in the form

$$\begin{aligned} \text{disc}\langle \bar{\psi}\psi \rangle \Big|_{m_0=0} &= \left[\frac{\partial T_c}{\partial m_0}\Big|_\mu \text{disc } s \right] \Big|_{m_0=0} \\ &= \left[\frac{\partial \mu_c}{\partial m_0}\Big|_T \text{disc } n \right] \Big|_{m_0=0}, \end{aligned} \tag{4.16}$$

where the subscripts $|_\mu$ and $|_T$ mean that the derivatives are taken for fixed μ and T, respectively.

Equations (4.16) are similar to the Clausius-Clapeyron equations for a liquid-vapor transition, as remarked by Leutwyler in Ref. [10]. In addition, as already shown in Ref. [11], one can express the latent heat (for vanishing m_0)

$$\text{disc } \varepsilon|_{m_0=0} = (T_c \text{disc } s + \mu_c \text{disc } n)|_{m_0=0} \tag{4.17}$$

as

$$\text{disc } \varepsilon|_{m_0=0} = \left[\left(\frac{1}{\partial(\ln T_c)/\partial m_0|_\mu} + \frac{1}{\partial(\ln \mu_c)/\partial m_0|_T} \right) \times \text{disc}\langle \bar{\psi}\rangle\psi\rangle \right] \Big|_{m_0=0}. \tag{4.18}$$

This type of relation among the various discontinuities, as remarked in Ref. [10], can be especially useful whenever the equation of state is not known analytically (as in lattice simulations) and one wants, for instance, to evaluate the latent heat through the discontinuity of $\langle \bar{\psi}\psi \rangle$ rather than by using the discontinuity of the entropy density. Nevertheless, they can also be more generally predictive. In fact, one also has further relations such as [11]

$$\frac{\text{disc}\langle \bar{u}u \rangle}{\partial T_c/\partial m_u|_\mu} = \frac{\text{disc}\langle \bar{d}d \rangle}{\partial T_c/\partial m_d|_\mu} = \frac{\text{disc}\langle \bar{s}s \rangle}{\partial T_c/\partial m_s|_\mu} \tag{4.19}$$

or analogous relations with the partial derivatives of the critical chemical potential at fixed T.

As one clearly expects that [10]

$$\frac{\partial T_c}{\partial m_{u,d}}\Big|_\mu > \frac{\partial T_c}{\partial m_s}\Big|_\mu, \tag{4.20}$$

one finds from Eq. (4.19) that the discontinuity for the lightest quarks is certainly bigger than that of the strange quark. This result, especially when considering the ratio of these discontinuities to the different condensates, allows us to draw the same conclusions discussed for the case of continuous transitions, namely, that chiral symmetry restoration is a phenomenon relevant only for the two lightest quarks.

In our model we have limited ourselves to study the variation of the critical temperatures (or chemical potentials) with respect to a quark bare mass m_0. Actually, the numerical analysis we have done shows that the critical temperatures (or chemical potentials) are more sensitive to the light masses than to the heavy one according to Eq. (4.20).

V. ZERO TEMPERATURES AND FINITE DENSITIES

Let us now consider the limiting situation of zero temperature and finite chemical potential. We have to distinguish between the case in which Eq. (2.20) has real roots and that in which it has complex roots.

In the first case it is possible to get the analytic expression for the density correction to the effective potential by taking the $r \to 0$ limit in Eq. (2.14). The terms which require some straightforward calculations to be evaluated are

$$\mathcal{V}_s^{\log} \equiv \sum_{k=1}^{3} \mathcal{V}_{s,k}^{\log}$$

$$= \sum_{k=1}^{3} \theta(s^2 - z_k) \left[\frac{s}{3}\sqrt{s^2 - z_k}\left(\frac{5}{2}z_k - s^2\right) + \frac{z_k^2}{2}\ln\left|\frac{\sqrt{z_k}}{s+\sqrt{s^2-z_k}}\right|\right] \tag{5.1}$$

and

$$z(s) = 2c(s) - \int_0^1 dt \frac{\hat{\chi}^2(s)t^2}{1-t+\hat{\chi}^2(s)t^3} - 2\sum_{k=1}^{3}\frac{\theta(s^2-z_k^0)}{(1-3z_k^0)}\left[s\sqrt{s^2-z_k^0} + \ln\left|\frac{\sqrt{z_k^0}}{s+\sqrt{s^2-z_k^0}}\right|\right] \tag{5.2}$$

when the z_k's are the real solutions of Eq. (2.20).

On the other hand, when Eq. (2.20) admits one real and two complex roots we obtain, after some algebra,

$$\mathcal{V}_s^{\log} = \mathcal{V}_{s,1}^{\log} + 8\theta\left(s^2 - \frac{z_{2R}}{2} - \frac{|z_2|}{2}\right)\left[-\frac{1}{3}s\left(s^2 - z_{2R} - \frac{z_{2I}^2}{4s^2}\right)^{3/2} + \frac{1}{\sqrt{2}}\int_0^{\sqrt{s^2 - z_{2R} - z_{2I}^2/4s^2}} dx\, x^2\left[\sqrt{(x^2+z_{2R})^2+z_{2I}^2} + (x^2+z_{2R})\right]^{1/2}\right] \tag{5.3}$$

for the one-loop term and the expression for the counterterm is modified as

$$z(s) = 2c(s) - \int_0^1 dt \frac{\hat{\chi}^2(s)t^2}{1-t+\hat{\chi}^2(s)t^3} - 2\frac{\theta(s^2-z_1^0)}{(1-3z_1^0)}\left[s\sqrt{s^2-z_1^0} + \ln\left|\frac{\sqrt{z_1^0}}{s+\sqrt{s^2-z_1^0}}\right|\right]$$

$$-2\sqrt{2}\theta\left(s^2 - \frac{z_{2R}^0}{2} - \frac{|z_2^0|}{2}\right)\int_0^{\sqrt{s^2 - z_{2R}^0 - (z_{2I}^0)^2/4s^2}} dx$$

$$\times\left\{\frac{x^2}{\left[\sqrt{(x^2+z_{2R}^0)^2+(z_{2I}^0)^2}+(x^2+z_{2R}^0)\right]^{1/2}}\frac{2}{(1-3z_{2R}^0)^2+(3z_{2I}^0)^2}\right.$$

$$\left.\times\left[(1-3z_{2R}^0)\left(1+\frac{(x^2+z_{2R}^0)}{\sqrt{(x^2+z_{2R}^0)^2+(z_{2I}^0)^2}}\right)+\frac{3(z_{2I}^0)^2}{\sqrt{(x^2+z_{2R}^0)^2+(z_{2I}^0)^2}}\right]\right\}, \tag{5.4}$$

where z_{2R} (z_{2I}) and z_{2R}^0 (z_{2I}^0) are the real (imaginary) parts of z_2 and z_2^0, respectively. As we are moving along the horizontal axis in Fig. 1, by studying the evolution of the absolute minimum $\bar{\chi}(\mu)$ for growing chemical potentials, we find that CS restores through a first-order phase transition, and that the transition proceeds in the following typical steps: first there are two symmetrical minima and the origin is a relative maximum. At a certain value μ_γ the curvature at the origin changes, but $\bar{\chi}(\mu)$ is still located away from the origin. The critical point μ_c occurs when the two minima (in the origin and off the origin) are degenerate. Finally, we can evaluate also the value μ_δ beyond which the effective potential is a convex function, which we can call the ending point of metastable phases. This structure is exhibited for any value of the parameter ζ in Eq. (2.13) that we have considered (i.e., in the interval (0,1]), except for $\zeta=0$, where numerical studies show that in such a case we still find the corresponding value μ_γ, but also that the minimum out of the origin remains almost degenerate, but always deeper than for $\chi=0$. Thus we do not find CS restoration in this particular case, which corresponds to neglecting the running of the coupling with the chemical potential.

In Table I, we list the critical values obtained with the calculations presented here. Even if for more quantitative analysis we would like to improve our model, a suggestion for the order of magnitude of the parameter ζ can be obtained by looking at the baryonic density in the chiral case. In this case the baryonic density for $\mu \geq \mu_c$ is that of a gas of free and massless quarks, as it can be verified by taking the derivative of the effective potential V with respect to μ at the minimum [we recall that the effective potential evaluated at the minimum is nothing but that the pressure changed on sign, see Eq. (2.23)]. Because of the fact that the baryonic density is related to the quark density by $n_B = n_Q/3$, and through Eq. (2.14) and the relation between V and $\mathcal{V}$, we find

$$n_B = -\frac{1}{3}\frac{\partial V}{\partial \mu}\bigg|_{\bar{\chi}=\bar{\chi}(\mu\geq\mu_c)=0}$$

$$= \frac{Nn_f}{9}\frac{\mu^3}{\pi^2}. \tag{5.5}$$

TABLE I. The points μ_γ, μ_c, and μ_δ (MeV) vs ζ.

ζ	μ_γ	μ_c	μ_δ
0.0	158		
0.1	156	465	975
0.2	153	338	675
0.3	151	286	555
0.4	149	238	485
0.5	147	223	440
0.6	145	213	410
0.7	143	204	390
0.8	141	197	375
0.9	139	192	360
1.0	137	186	345

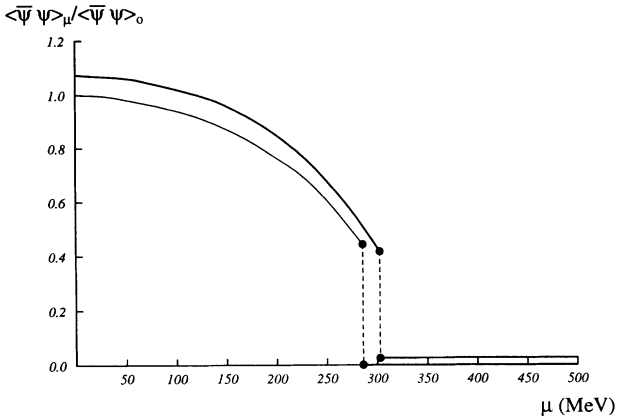

FIG. 4. Normalized condensate (as in Fig. 2) vs chemical potential at $T=0$ for $m_0=0$ (lowest curve) and $m_0=9.5$ MeV (highest curve). Here $\zeta=0.3$ and the critical chemical potential changes from $\mu_c=286$ to 303 MeV.

The previous equation suggests quite a small value for the parameter ζ if we require that the ratio $n_B[\mu_c(\zeta)]/n_{B_0}$ be of order unity where $n_{B_0}\simeq 0.15$ fm^{-3} is the ordinary nuclear density. For instance, by using the values for μ_c given in Table I we find that this ratio will change from $n_B[\mu_c(\zeta)]/n_{B_0}\simeq 9$ to $n_B[\mu_c(\zeta)]/n_{B_0}\simeq 2$ when ζ varies from $\zeta=0.1$ to 0.3. Finally, in Fig. 4 we compare (with $\zeta=0.3$), the behavior of the condensate for $m_0=0$ ($\mu_c=286$ MeV) and for $m_0=(m_u+m_d)/2=9.5$ MeV ($\mu_c=303$ MeV).

IV. OUTLOOK AND CONCLUSIONS

The analysis we have done by using the effective potential approach to study the phase diagram of the theory is, in our opinion, especially relevant only in the regions near the critical points. We note that our effective potential contains only the constant field χ and its chiral partner π related to the scalar and pseudoscalar condensates, respectively. On the other hand, one expects that at low temperature and zero density the main contribution to the partition function comes from the dynamical pions [9]. It would thus be suspicious, in our opinion, to extrapolate our results to the region of low T or μ. To correctly study such a region one would have to evaluate the effective action (and not the effective potential) for the pions and the nucleons, with their parameters derived from the basic underlying theory in terms of quarks, and then analyze the thermodynamics of such a physical system.

In conclusion, we have studied chiral transitions in QCD in the framework of a variational approach to the effective potential for composite operators. For massless quarks, the phase diagram (in the plane of chemical potential μ and temperature T), derived from the effective potential, shows the existence of a tricritical point $t=(\mu_t,T_t)$, dividing the critical line in two parts L_{II} and L_I: by crossing them the chiral symmetry is restored through a second- or first-order phase transition, respectively. Phase diagrams with similar structures have been found in other theories such as the Gross-Neveu model but they have never been suggested for QCD.

In the realistic case in which a quark mass term is present, for small enough mass, the order parameter (related to the quark condensate) maintains the main features of the massless case up to the critical points. However, because of the current quark mass, the quark condensate does not vanish any more for large T and/or μ and approaches a constant value depending on the current quark mass term itself. Furthermore, the behavior of the condensate close to the phase transition points is sensibly affected by variation of the mass parameter m_0. The current quark mass plays a role analogous to that of an external magnetic field in the ferromagnetic transition, as it explicitly violates the chiral symmetry whose restoration characterizes the phase transition.

All these effects can be easily studied if we treat separately the points of L_{II} and those of L_I. Actually, the effective potential admits a Landau expansion around the critical line L_{II} and thus the behavior of the condensate, with m_0 inserted, is well reconstructed knowing the coefficients (which are infrared safe) and the critical exponents. In the case of first order phase transitions, it turns out that a discontinuity of the condensate survives for small m_0, and that the relevant effects are described in terms of Clausius-Clapeyron-like relations.

Analogous results can be found for other related physical quantities such as $f_\pi(T,\mu)$, showing again the typical behavior coming from a mean field theory near the critical point for a second-order phase transition.

ACKNOWLEDGMENTS

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