

Chiral dynamics and heavy quark symmetry in a solvable toy field-theoretic model

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We study a solvable QCD-like toy theory, a generalization of the Nambu–Jona-Lasinio model, which implements chiral symmetries of light quarks and heavy quark symmetry. The chiral symmetric and chiral broken phases can be dynamically tuned. This implies a parity-doubled heavy-light meson system, corresponding to a $(0^-, 1^-)$ multiplet and a $(0^+, 1^+)$ heavy spin multiplet. Consequently the mass difference of the two multiplets is given by a Goldberger-Treiman relation and g_A is found to be small. The Isgur-Wise function $\xi(w)$, the decay constant f_B , and other observables are studied.

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I. INTRODUCTION

In recent years, QCD applied to systems containing a single very massive quark, where one can imagine the limit $M \rightarrow \infty$ to be a reasonable physical approximation, has been the subject of considerable attention [1–5]. The pseudoscalar and vector mesons containing one very massive and one light quark become degenerate in the $M \rightarrow \infty$ limit, due to a heavy quark spin symmetry again valid to $1/M$. Moreover, Isgur and Wise [1] pointed out that transition amplitudes, such as weak decays, involving heavy quarks are described by a flavor-independent function of the invariant difference in four-velocities, $\xi(v' \cdot v)$, and therefore a heavy quark spin-flavor symmetry, $SU(2N_f)$ exists, valid to order $1/M$. Georgi has given a useful field-theoretic construction of this limit [5], and has studied the consequences and phenomenological applications of the theory, such as the computation of the QCD anomalous dimension which controls the perturbative evolution of $\xi(v' \cdot v)$ with scale for $v' \cdot v < 1$.

For many purposes one must also implement the chiral symmetries of the light quarks, in addition to the heavy quark symmetry. The heavy quark (HQ) and chiral light quark (LQ) symmetries together control the interactions of heavy-light (HL) mesons with pions and K mesons, etc. Several authors have written down model-independent chiral Lagrangians which involve these symmetries at the meson level [6–12]. A number of studies of the phenomenological applications of these chiral Lagrangians have been undertaken, such as the computation of the chiral log radiative corrections to $\xi(v' \cdot v)$ [7] associated with $SU(3) \times SU(3)$ -breaking terms, the study of radiative and meson decays of heavy mesons [11], and chiral dynamics including the effects of excited heavy mesons [12].

The chiral Lagrangian introduced by Wise [6]

represents a straightforward implementation of the heavy quark and light flavor symmetries in the nonlinear *current form*. In this form one needs only identify the linear flavor symmetries, such as isospin or $SU(3)$, and the chiral effective Lagrangian, to leading order in the momentum expansion, is automatically determined, up to an unspecified axial-vector coupling constant g_A . This effective Lagrangian is then manifestly invariant under the usual global flavor symmetries, and the full set of chiral transformations are local gaugelike transformations which are functionals of the pions. The underlying chiral representations of the heavy mesons need never be specified. Model-independent approaches are clearly the most reliable way in which to minimally implement the physical symmetries.

We may wish, however, to go closer to the underlying chiral dynamics than the model independent approaches allow. We may pose additional questions within dynamical models which can reveal additional physical consequences to the real world. For example, is there a more primitive *chiral form* of the Lagrangian in which the explicit chiral representations of the heavy mesons are identified? A related question in the broken phase is—what is the analogue of the Goldberger-Treiman relation in the heavy meson system, i.e., what receives mass from the chiral condensate's mass gap? In the case of the nucleon-meson system we can similarly write the chiral Lagrangian in the nonlinear current form, never having to specify the precise chiral representation of nucleons. However, if we ask for the *linear* chiral form we also know the answer: the left-handed [right-handed] nucleon is assigned to a $(0, \frac{1}{2})$ [$(\frac{1}{2}, 0)$] representation under $SU(2)_R \times SU(2)_L$. Most of the nucleon mass arises from the chiral condensate, or the vacuum expectation value (VEV) of Σ which is $(\frac{1}{2}, \frac{1}{2})$. We know this because the Goldberger-Treiman relation yields the pion-nucleon coupling constant in terms of the nucleon mass $g_{NN\pi} \approx m_N / f_\pi$, and $G_A \approx 1$.

In the case of heavy mesons, however, it is clear that the meson mass arises primarily from the mass of the

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heavy constituent quark, such as the b quark, and the chiral mass gap is a perturbation. Our question then is related to the outcome of a gedanken experiment: what happens to the heavy-light meson spectrum if we could somehow restore the chiral symmetry, maintaining the other features of confining QCD? While the nucleon mass goes to zero in this gedanken limit, leaving degenerate (approximately) massless left- and right-handed states, the heavy meson masses must remain (approximately) unaffected. Yet, the explicit linear chiral symmetry $SU(2)_L \times SU(2)_R$ must somehow be realized in the heavy meson mass spectrum in this limit. This leads to the conclusion that the ground state must become doubly degenerate with even and odd parity mesons B_1 and B_2 , respectively, and these must form the $SU(2)_L \times SU(2)_R$ representations in the linear combinations $(B_1 \pm B_2)/\sqrt{2}$. Therefore, the breaking of the chiral symmetry leads to a mass gap between these parity partners and associated pionic transitions between parity partners will occur (in Appendix B 2 we give a brief schematic discussion of parity doubling).

It is difficult to imagine that a simple potential model can capture this phenomenon. The chiral symmetry limit is relativistic, and the chiral symmetry breaking is a dynamical rearrangement of the vacuum. Thus, the naive picture of a heavy meson as a bound state of a heavy quark and a constituent quark will miss those aspects of the physics which involve the necessary mixing of the parity doubled states. This will show up in the present analysis in the meaning and quantitative estimate of g_A , and the analogue to the Goldberger-Treiman relation.

Thus, to better understand these issues it is interesting, if not essential, to study simple, solvable, strongly coupled toy field-theoretical models in which both heavy quark and chiral symmetries are present at the fundamental quark level, and the dynamics of chiral symmetry breaking is made explicit. We consider presently the simplest such scheme. We emphasize at the outset that this toy model is unrealistic and is intended only to convey the schematics of QCD chiral dynamics in heavy-light mesons (although we will brazenly attempt a fit to data). The simple model we consider is based upon a local gluonic current-current interaction Lagrangian:

$$-\frac{g^2}{\Lambda^2} \sum_i \bar{\psi}_i \gamma_\mu \frac{\lambda^A}{2} \psi_i \sum_j \bar{\psi}_j \gamma^\mu \frac{\lambda^A}{2} \psi_j, \quad (1)$$

where the (i, j) sums extend over all of the fundamental fermion flavors, both heavy and light, and we sum over the octet color index A . We view Eq. (1) as essentially a QCD-inspired generalization of the Nambu–Jona-Lasinio model. For small g Eq. (1) corresponds to the low-energy perturbative interaction generated by the exchange of a “massive gluon” of mass $\Lambda/\sqrt{2}$. We propose to study this model using the technique of the large- N expansion, or equivalently, the fermion bubble approximation, with a cutoff at Λ . The model is exactly solvable in leading order.

Solving the theory in the leading large- N approximation is equivalent to factorizing Eq. (1) into auxiliary

fields describing the composite pions and heavy-light mesons, at the scale Λ and integrating out the heavy and light quarks to generate the effective Lagrangian at a scale $\mu < \Lambda$. The hadrons in our model, both light and heavy, appear as dynamically generated bound states. In the light quark, meson sector we recover the chiral quark model of Manohar and Georgi [13] (with $g_A^q = 1$). In the heavy meson sector we produce various bound states of the heavy quark and the light quarks, and the full effective Lagrangian of these heavy meson bound states coupled to light mesons is determined. The effective Lagrangian is manifestly heavy quark spin and chirally symmetric.

We will make certain further simplifying assumptions, keeping only terms in the renormalized effective Lagrangian that are ~ 1 or $=O(\mu \ln(\Lambda/\mu)/\Lambda)$, while dropping subleading terms $\sim O(\mu/\Lambda)$. This is a drastic approximation from the point of view of the quantitative application of the model, but adequate for capturing the schematic of the chiral dynamics. We emphasize that we have in mind, presently, a hierarchy of scales, $\mu < \Lambda \ll M$, where M is the heavy quark mass scale. The momentum-space loop integrals will extend from μ to the cutoff Λ . We view in the context of the model Λ to be a physical scale below which the theory is nonperturbative in g , but above which an effective softening of the point-like approximation due to the perturbative $1/q^2$ gluon propagator takes place. While it is tempting to identify Λ with $\sim \Lambda_{\text{QCD}}$, we would hope that $\Lambda \sim 1$ GeV emerges from a fit to the physical quantities derived in the model. In fact, the simplest attempt at a fit to f_B and f_π yields $\Lambda \sim 1.35$ GeV, and most of the light sector observables are obtained within a factor of 2. μ is an infrared cutoff which we would like to identify with the scale of light constituent quark masses.

In the unbroken chiral symmetry phase the model produces the necessary degenerate parity doubling of the threshold spectrum of heavy mesons. In addition to the usual pseudoscalar and vector HL mesons [the B and B^* mesons which form a $(0^-, 1^-)$ heavy quark symmetry multiplet], there is necessarily a scalar and pseudovector HL meson bound state generated, which is a consequence of the chiral symmetry. This is identified with the $s_l^{\pi_l} = \frac{1}{2}^-$ p -wave radially excited mesons (in the D system this is distinct from the observed (D_2) states which are $s_l^{\pi_l} = \frac{3}{2}^-$, see Ming-Lu *et al.* [14]). Unfortunately, these states have not been observed and will be fairly broad resonances, but their effects may ultimately be detectable [12]. Technically, in the symmetric phase we must hold μ fixed at a nonzero value to protect from infrared singularities.

While the HQ symmetry maintains the degeneracy within the $(0^-, 1^-)$ and $(0^+, 1^+)$ multiplets, unbroken chiral symmetry implies the degeneracy of the two multiplets themselves. As we vary the model’s coupling parameter to dynamically induce the chiral symmetry breaking, the theory develops a mass gap. This leads to a calculable mass splitting, elevating the $(0^+, 1^+)$ HQ multiplet and depressing the $(0^-, 1^-)$. The mass gap between the ground state mesons and the resonances is constant in

the $M \rightarrow \infty$ limit and is given essentially by $\sim g f_\pi$. This is the analogue of the Goldberger-Treiman relation of the theory, and is probably more general than our specific toy model result. Moreover, as a general result of the parity doubling, the axial-vector coupling constant g_A is not necessarily expected to be close to unity. In fact, g_A tends to be small based upon our fit, ≈ 0.32 [see Appendix B 2; it occurs here as a term of order $\ln(\Lambda/\mu)/\Lambda$, which is subleading to 1]. This is a prediction which is thus far consistent with the upper limit in processes such as $D^* \rightarrow D + \pi$, though a measurement of the full D^* width is still lacking to date. In the limit of very low q^2 pion emission we can decouple the heavier parity doubling states to return to the effective chiral Lagrangian for the $(0^-, 1^-)$ ground state mesons of Ref. [6]. There remain in the low energy effective Lagrangian potentially important effects of the heavy resonances in chiral perturbation theory [12].

Thus, a key result we find is that the chiral mass gap, and hence an analogue Goldberger-Treiman relation, refers to the *splitting between parity conjugate heavy meson multiplets* in a heavy-light meson theory; i.e., *heavy meson chiral theory is a parity doubled implementation of chiral symmetry*. There are other issues of the applicability of the chiral theory and its consequences which the present analysis will attempt to address. Although not entirely realistic, the model is completely solvable for various observables. f_B is determined in terms of the short-distance cutoff on the theory and the Isgur-Wise function is computed. The Isgur-Wise function result in the present model involves issues of going beyond the chiral logs, which arise also in matching composite mesons onto QCD. We will discuss this issue which is related to consideration of reparametrization invariance [14–17].

II. TOY MODEL FIELD THEORY WITH CHIRAL AND HEAVY QUARK SYMMETRY

A. The light quark chiral dynamics

Let us write the effective Lagrangian in the light quark sector, including the current-current form of the light fermion interaction Lagrangian of Eq. (1):

$$\mathcal{L} = \bar{\psi}(i\partial - m_q)\psi - \frac{g^2}{\Lambda^2} \bar{\psi} \gamma_\mu \frac{\lambda^A}{2} \psi \bar{\psi} \gamma^\mu \frac{\lambda^A}{2} \psi. \quad (2)$$

For concreteness we will take $\psi = (u, d)$, λ^A are color matrices, and in the limit that the quark mass matrix $m_q \rightarrow 0$ we have an exact chiral $SU(2)_L \times SU(2)_R$ invariant Lagrangian. This is a single gluon exchange potential, generated by a fake, massive gluon of mass $\Lambda/\sqrt{2}$. We treat the physics on scales $q^2 < \Lambda^2$ using Eq. (2), in a fermion bubble approximation, imposing a UV loop momentum cutoff of Λ . Well above the scale Λ we would imagine the potential to soften into a $1/q^2$ perturbative gluon exchange; hence, Λ plays the role of a matching scale between strong infrared physics and weak ultraviolet QCD. Finally, the “theory” in the light sector consists of integrating out the fermions down to an infrared scale μ , keeping induced terms of order Λ^2 , and $\ln(\Lambda/\mu)$ in the

unrenormalized Lagrangian (we will discard perturbative terms that are finite, thus subleading, in the infinite Λ limit as a simplifying approximation). This generates an effective Lagrangian of composite particles. This is our essential approximation to the infrared strong coupling behavior of QCD, or the “brown muck” of heavy-light physics. Overall, this is certainly a drastic approximation. Truncating on dimension-6 operators is, in a sense, a pure s -wave approximation to QCD, and cannot dynamically confine the quarks and discarding the subleading terms will limit the quantitative reliability of the model (the model could easily be improved). The physical value of Λ is determined in principle by fitting to the derived phenomenological parameters. The theory will contain the dynamical chiral symmetry breaking, and will determine a chiral Lagrangian of the heavy-light system.

Upon Fierz rearrangement it is seen that the interaction Lagrangian of Eq. (2) contains the Nambu–Jona-Lasinio model. The subsequent analysis is standard. We can factorize Eq. (2) into a Yukawa theory with a static auxiliary field $\Sigma = \frac{1}{2}(\sigma + i\pi^a \tau^a)$ on the scale $\mu \sim \Lambda$ and then integrate out the fermions to determine the effective Lagrangian at scales $\mu < \Lambda$. The field Σ is 2×2 complex at this stage, which implies parity doubling of the π and the parity partner of σ , the η is also present. This analysis is summarized in Appendix B.

The light sector effective Lagrangian at scales $\mu < \Lambda$ can be identified with a linear σ model:

$$\begin{aligned} \mathcal{L}_L = & \bar{\psi}(i\partial - m_q)\psi - \bar{g} \bar{\psi}_L \Sigma_r \psi_R - \bar{g} \bar{\psi}_R \Sigma_r^\dagger \psi_L \\ & + \text{Tr}(\partial_\mu \Sigma_r^\dagger \partial^\mu \Sigma_r) - m_\sigma^2 \text{Tr}(\Sigma_r^\dagger \Sigma_r) \\ & + \kappa \text{Tr}(m_q \Sigma_r + \text{H.c.}) + \lambda \text{Tr}(\Sigma_r^\dagger \Sigma_r \Sigma_r^\dagger \Sigma_r). \end{aligned} \quad (3)$$

Σ_r describes the renormalized composite light mesons. We have written the renormalized effective Lagrangian, so that $\bar{g} = g/\sqrt{Z_2}$. $Z_2 = (g^2 N/16\pi^2) \ln(\Lambda^2/\mu^2)$ is the finite, induced wave-function renormalization constant of the Σ field.

A $(\text{Tr} \Sigma_r^\dagger \Sigma_r)^2$ term could be included in Eq. (3), though it is subleading in N_c , and for $SU(2) \times SU(2)$ with (σ, π) real this is equivalent to the quartic term we have included. The theory can be tuned by choosing a sufficiently large coupling g to develop a chiral-symmetry-breaking condensate, thus generating a constituent quark mass. The chiral breaking lifts of the isovector, $0^+[\text{Im}(\pi)]$ states. The $0^-[\text{Re}(\pi)]$ pion becomes the Nambu-Goldstone mode. In QCD the residual $U(1)$ symmetry is broken by anomalies and the effects of instantons. This generates additional terms such as an extra 't Hooft determinant, $\det \Sigma + \text{H.c.}$ term, which elevates $\text{Im}(\sigma) = \eta$. Any additional necessary Wess-Zumino terms should be incorporated as well.

Since the light sector dynamics is not our principal concern in the subsequent analysis we will henceforth assume that the fields (σ, π^a) comprising Σ are real, so Σ henceforth contains only the $0^- \pi^a$ isotriplet and the real σ isosinglet. Therefore, Eq. (3) becomes a linear version of the chiral quark model in the manner of Georgi, Manohar, and Holdom [14]. Nonetheless, we can

dynamically put the model either in a symmetric phase, $m_\sigma^2 > 0$, by choosing $g^2 N / 4\pi^2 < 1$, or in a chiral symmetry breaking phase $m_\sigma^2 < 0$ with $g^2 N / 4\pi^2 > 1$. The critical bare coupling corresponds to $m_\sigma^2 = 0$ as $\mu_0 \rightarrow 0$. For further discussion of the light quark sector see Appendix B.

B. The heavy-light quark dynamics

Now we focus on the dynamics of mesons containing one light and one heavy quark. The model produces one bound state per channel in the fermion bubble approximation. We can conveniently solve the theory by factorizing the heavy-light (HL) interaction into auxiliary background interpolating fields with Yukawa couplings to heavy and light quark vertices. The original four-fermion interaction is recovered when the auxiliary fields are integrated out. Upon integrating out the quarks on scales Λ to μ , the auxiliary fields acquire induced kinetic terms on the scale μ and thus become dynamical heavy-light mesons (“ B mesons”). In this way we derive the effective Lagrangian for the HL mesons coupled to the dynamical pions.

The heavy-light fermion sector interaction Lagrangian, together with the HQ kinetic term, involves the HL cross term of Eq. (1) and can be written as

$$\mathcal{L}_{\text{HL}} = \bar{Q}(i\partial - M)Q - \frac{2g^2}{\Lambda^2} \bar{Q}\gamma_\mu \frac{\lambda^A}{2} Q \bar{\psi}\gamma^\mu \frac{\lambda^A}{2} \psi. \quad (4)$$

Here we may generally take $Q = (t, b, c)$ to be a multiplet of N_H heavy quarks, and M the heavy quark mass matrix. We will presently consider, however, just a single heavy flavor in the following discussion. g should be viewed as the effective coupling at the scale Λ in both the light sector and the HL sector of our model. (In a more detailed discussion one might wish to distinguish the coupling constant in the heavy-light effective action from that of the light-light action at Λ ; we ignore this possibility in this paper.)

Upon Fierz rearrangement of the interaction, again keeping only leading terms in $1/N_C$ and writing in terms of color singlet densities, Eq. (4) takes the form

$$\begin{aligned} \mathcal{L}_{\text{HL}} = & \bar{Q}_v i v^\mu \partial_\mu Q_v + g \bar{Q}_v \psi_i \bar{B}_v^i + i g \bar{Q}_v \gamma^5 \psi_i \bar{B}_v^{5i} + g \bar{Q}_v \gamma_\mu \frac{1-\not{v}}{2} \psi_i \bar{B}_v^{i\mu} - i g \bar{Q}_v \gamma_\mu \frac{1-\not{v}}{2} \gamma_5 \psi_i \bar{B}_v^{5\mu} + \text{H.c.} \\ & - 2\Lambda^2 (\bar{B}_v^{i5} B_{vi}^5 + \bar{B}_v^i B_{iv}) + 2\Lambda^2 (\bar{B}_v^{i\mu 5} B_{iv\mu}^5 + \bar{B}_v^{i\mu} B_{iv\mu}). \end{aligned} \quad (9)$$

Upon integrating out the B fields in Eq. (9) we reproduce Eq. (8). (Note that the B fields do not yet have canonical dimension of heavy meson fields; see Appendix A.)

Equation (9) is a heavy-spin symmetric form. We can assemble the auxiliary fields into complex $\mathbf{4}$ multiplets

$$\begin{aligned} \mathcal{L}_{\text{HL}} = & \bar{Q}(i\partial - M)Q \\ & + \frac{g^2}{\Lambda^2} (\bar{Q}^a \psi_i \bar{\psi}^i Q_a - \bar{Q}^a \gamma^5 \psi_i \bar{\psi}^i \gamma^5 Q_a \\ & - \frac{1}{2} \bar{Q}^a \gamma_\mu \psi_i \bar{\psi}^i \gamma^\mu Q_a \\ & - \frac{1}{2} \bar{Q}^a \gamma_\mu \gamma_5 \psi_i \bar{\psi}^i \gamma^\mu \gamma_5 Q_a), \end{aligned} \quad (5)$$

where i are the isospin indices, and a the heavy flavor indices.

In the heavy quark limit we introduce a projection onto a heavy quark field with a well-defined four-velocity u_μ . Presently we rewrite the full theory identically in terms of a single four-velocity sector, corresponding to the four-velocity of the heavy constituent quark or equivalently the bound state heavy mesons:

$$Q \rightarrow \frac{1+\not{v}}{2} \exp(-iMv \cdot x) Q(x)_v. \quad (6)$$

Note that $(1+\not{v})Q_v/2 = Q_v$, i.e., the field Q_v always carries an implicit factor of $(1+\not{v})/2$. The HQ kinetic term then takes the form

$$\bar{Q}_v^a i v^\mu \partial_\mu Q_{va}. \quad (7)$$

The Isgur-Wise flavor symmetry is just the group of $SU(N_H)$ rotations on Q_v^a , and is now a manifest symmetry of our Lagrangian. We will consider just a single heavy flavor in the following.

We now rewrite the terms of Eq. (5) in a manifestly heavy spin symmetric form, letting $Q \rightarrow Q_v$ and further rearranging γ matrices. Then, Eq. (5) takes the form

$$\begin{aligned} \mathcal{L}_{\text{HL}} = & \bar{Q}_v i v^\mu \partial_\mu Q_v \\ & + \frac{g^2}{2\Lambda^2} \left[\bar{Q}_v \psi_i \bar{\psi}^i Q_v - \bar{Q}_v \gamma^5 \psi_i \bar{\psi}^i \gamma^5 Q_v \right. \\ & - \bar{Q}_v \gamma_\mu \frac{1-\not{v}}{2} \psi_i \bar{\psi}^i \frac{1-\not{v}}{2} \gamma^\mu Q_v \\ & \left. + \bar{Q}_v \gamma_\mu \frac{1-\not{v}}{2} \gamma_5 \psi_i \bar{\psi}^i \gamma_5 \frac{1-\not{v}}{2} \gamma^\mu Q_v \right]. \end{aligned} \quad (8)$$

We have now brought the interaction to a form which can be factorized by introducing heavy static auxiliary fields, (B, B') . To do so we must introduce four independent fields: B (B^5) are 0^+ (0^-) scalars, while B_μ (B_μ^5) are 1^- (1^+) vectors. These form a minimal complete set of auxiliary fields needed to factorize Eq. (8) in the HQ limit. Equation (8) then becomes

under $O(4) = SU(2)_h \times SU(2)_l$, where $SU(2)_h$ ($SU(2)_l$) is the little group of rotations on Q_v (ψ and gluons) which preserves u_μ . One heavy spin $\mathbf{4}$ multiplet consists of the 0^+ scalar together with the abnormal parity (1^+) vector as $(B, B^{5\mu})$ (the four-velocity label v , and isospin i indices

are understood):

$$\begin{aligned} B' &= (i\gamma^5 B + \gamma_\mu B^{5\mu}), \\ \mathcal{B}' &= (i\gamma^5 B + \gamma_\mu B^{5\mu}) \left[\frac{1+\not{v}}{2} \right]. \end{aligned} \quad (10)$$

The other 4 multiplet consists of the usual 0^- scalar and a 1^- vector (B^5, B^μ):

$$\begin{aligned} B &= (i\gamma^5 B^5 + \gamma_\mu B^\mu), \\ \mathcal{B} &= (i\gamma^5 B^5 + \gamma_\mu B^\mu) \left[\frac{1+\not{v}}{2} \right]. \end{aligned} \quad (11)$$

Under heavy spin $O(4) = \text{SU}(2)_h \times \text{SU}(2)_l$ rotations the $(B, B^{5\mu})$ mix analogously to (B^5, B^μ) . Note that $v_\mu B^\mu = 0$ always. We have introduced the calligraphic \mathcal{B} and \mathcal{B}' with the explicit projection factors. Falk has previously written similar effective ‘‘superfields’’ for excited mesons in model-independent analyses; he includes an extra factor of γ^5 (relative to us) in his writing of effective fields for the $(0^+, 1^+)$ multiplet in a model-independent approach [12]; for us the field \mathcal{B}' has overall odd parity while \mathcal{B} is even.

The factorized heavy-light interaction Lagrangian then takes the compact form

$$\begin{aligned} \mathcal{L}_{\text{HL}} &= \bar{Q}_v i v^\mu \partial_\mu Q_v + g \bar{Q}_v (-i \bar{\mathcal{B}}^i \gamma^5 + \bar{\mathcal{B}}^i) \psi_i + \text{H.c.} \\ &+ \Lambda^2 [\text{Tr}(\bar{\mathcal{B}}\mathcal{B}) + \text{Tr}(\bar{\mathcal{B}}'\mathcal{B}')]. \end{aligned} \quad (12)$$

Notice that the combination $-i\gamma^5 \mathcal{B}'^i + \mathcal{B}^i$ is coupled.

$$\begin{aligned} \mathcal{L}_{\text{LH}} &= -i\frac{1}{2} \text{Tr}(\bar{\mathcal{B}}v \cdot \partial \mathcal{B}) - i\frac{1}{2} \text{Tr}(\bar{\mathcal{B}}'v \cdot \partial \mathcal{B}') - \frac{g_r}{4} [\text{Tr}(\bar{\mathcal{B}}\bar{\sigma}\mathcal{B}) - \text{Tr}(\bar{\mathcal{B}}'\bar{\sigma}\mathcal{B}') + \text{Tr}(\bar{\mathcal{B}}'\pi \cdot \tau \mathcal{B}) + \text{Tr}(\bar{\mathcal{B}}\pi \cdot \tau \mathcal{B}')] \\ &+ \frac{h_r}{4\Lambda} [\text{Tr}(\bar{\mathcal{B}}(\bar{\sigma}^2 + \pi^2)\mathcal{B}) + \text{Tr}(\bar{\mathcal{B}}'(\bar{\sigma}^2 + \pi^2)\mathcal{B}')] \\ &+ \frac{k_r}{4f_\pi} \text{Tr}[\bar{\mathcal{B}}\gamma^5(\bar{\sigma}\pi \cdot \tau)\mathcal{B} - \bar{\mathcal{B}}'\gamma^5(\bar{\sigma}\pi \cdot \tau)\mathcal{B}' - \bar{\mathcal{B}}'\gamma^5(\bar{\sigma}\sigma)\mathcal{B} - \bar{\mathcal{B}}\gamma^5(\bar{\sigma}\sigma)\mathcal{B}'] + \Delta [\text{Tr}(\bar{\mathcal{B}}\mathcal{B}) + \text{Tr}(\bar{\mathcal{B}}'\mathcal{B}')]. \end{aligned} \quad (13)$$

The light quark PCAC (partial conservation of axial-vector current) masses are contained in the ‘‘shifted’’ σ field, $\bar{\sigma} = \sigma + 2m_q \sqrt{Z_2}/g$. The parameters of this Lagrangian are determined as

$$\begin{aligned} g_r &= \frac{g}{\sqrt{Z_2}}, \quad h_r = \frac{2g^2 \sqrt{Z_2} \Lambda}{Z_1}, \\ k_r &= \frac{2gf_\pi \sqrt{Z_2}}{Z_1}, \\ \Delta &= \frac{1}{Z_1} [\Lambda^2 - Z_1(\Lambda + \mu)/2\pi], \end{aligned} \quad (14)$$

where

We emphasize that Eq. (12) is exactly equivalent to the full four-fermion theory in the heavy quark symmetric and leading large- N limit Eq. (4). The theory forces a parity doubling of the heavy mesons upon us because the chiral symmetry is controlled dynamically by g . For weak g the linear chiral invariance is realized and the theory must contain parity doubled meson states. Heavy spin symmetry organizes the parity partners into heavy spin 4 multiplets. The effect of chiral symmetry breaking on the spectrum can now be investigated by solving the theory and choosing the broken phase.

See Appendix A 3 for a discussion of normalization conventions.

III. FULL EFFECTIVE LAGRANGIAN

We now proceed to ‘‘solve’’ the theory. The full effective Lagrangian for the heavy mesons is derived by integrating out the heavy and light quarks in Eq. (12) over momentum scales $\mu < k < \Lambda$, keeping the leading induced terms. Details of the explicit calculations are given in Appendix A. We begin the discussion with the use of the linearly realized chiral symmetry form, $\Sigma = \frac{1}{2}(\sigma + i\pi \cdot \tau)$, and we derive the nonlinear realization subsequently below. The loop integrations result in an unrenormalized effective Lagrangian. By performing a conventional wave-function renormalization and several field redefinitions we arrive at the full effective action valid to $O((\mu/\Lambda)^2)$:

$$Z_1 = \frac{g^2 N}{8\pi} (\Lambda - \mu), \quad Z_2 = \frac{g^2 N}{16\pi^2} [\ln(\Lambda^2/\mu^2)]. \quad (15)$$

The parameters defined above arise from the loop calculations of Figs. 1 and 2 and are presented in Appendix A. The g_r , h_r , and k_r are dimensionless. They are determined in principle by fitting the observables of the model as in Sec. IV. We will generally take μ to be of order the light quark constituent mass, and it will henceforth be neglected in the expression for Z_1 . Note that terms such as $\bar{\mathcal{B}}'\gamma^5(\sigma\bar{\sigma})\mathcal{B}$ are potentially induced, but they are sub-leading as $O(1/\ln(\Lambda/\mu))$, relative to the terms we keep.

We now identify the chiral representations of the composite fields in the effective theory. This can easily be done by returning to Eq. (12) and examining which heavy meson linear combinations couple to ψ_L and ψ_R . If we define the combinations

$$\mathcal{B}_1 = \frac{1}{\sqrt{2}}(\mathcal{B} - i\mathcal{B}'), \quad \mathcal{B}_2 = \frac{1}{\sqrt{2}}(\mathcal{B} + i\mathcal{B}'), \quad (16)$$

inspection of Eq. (12) reveals that $\mathcal{B}_1[\mathcal{B}_2]$ couples to $\psi_R = (1 + \gamma^5)\psi/2$ [$\psi_L = (1 - \gamma^5)\psi/2$]. Thus, the chiral

representation of $\mathcal{B}_1[\mathcal{B}_2]$ must be $(0, \frac{1}{2})$ [$(\frac{1}{2}, 0)$]. Writing in terms of $\Sigma = \frac{1}{2}(\sigma + i\pi \cdot \tau)$ (we will henceforth ignore the m_q contributions which can easily be restored by shifting $\sigma \rightarrow \bar{\sigma}$), the effective Lagrangian becomes

$$\begin{aligned} \mathcal{L}_{\text{LH}} = & -i\frac{1}{2}\text{Tr}(\bar{\mathcal{B}}_1 v \cdot \partial \mathcal{B}_1) - i\frac{1}{2}\text{Tr}(\bar{\mathcal{B}}_2 v \cdot \partial \mathcal{B}_2) - \frac{g_r}{2}[\text{Tr}(\bar{\mathcal{B}}_1 \Sigma^\dagger \mathcal{B}_2) + \text{Tr}(\bar{\mathcal{B}}_2 \Sigma \mathcal{B}_1)] \\ & + \left[\Delta + \frac{h_r}{\Lambda} \Sigma^\dagger \Sigma \right] [\text{Tr}(\bar{\mathcal{B}}_1 \mathcal{B}_1) + \text{Tr}(\bar{\mathcal{B}}_2 \mathcal{B}_2)] + \frac{ik_r}{2f_\pi} \{ \text{Tr}[\bar{\mathcal{B}}_1 \gamma^5 (\partial \Sigma^\dagger) \mathcal{B}_2] - \text{Tr}[\bar{\mathcal{B}}_2 \gamma^5 (\partial \Sigma) \mathcal{B}_1] \}. \end{aligned} \quad (17)$$

Inspection of the effective Lagrangian [as well as Eq. (12)] confirms that it is manifestly invariant under $\text{SU}(2) \times \text{SU}(2)$ provided the fields transform as

$$\mathcal{B}_1 = (0, \frac{1}{2}), \quad \mathcal{B}_2 = (\frac{1}{2}, 0), \quad \Sigma = (\frac{1}{2}, \frac{1}{2}). \quad (18)$$

We now see that indeed, Eqs. (13) and (17) have a structure analogous to that of a parity doubled nucleon theory, with $\mathcal{B} \sim (n, p)_{p=+1}$, the normal even parity nucleon doublet, and $\mathcal{B}' \sim (n, p)_{p=-1}$ the odd parity doubling partner. We give a brief synopsis of such a system in Appendix B 3. The essential results are that the axial-vector current couples only through the perturbative k_r term and describes transitions between parity partners, and the parity degeneracy will be lifted by $\langle \sigma \rangle$.

Note that Eqs. (13) and (17) describe the heavy meson dynamics in either a broken or an unbroken phase; i.e., it is simply a linear σ -model form. In the spontaneously broken phase of the heavy meson theory we can pass to the nonlinear realization by replacing Σ with a unitary matrix field which is a function of angular pion fields, and σ is now decoupled. Thus, the nonlinear realization is

$$\Sigma = \frac{1}{2}f_\pi \exp(i\pi \cdot \tau / f_\pi), \quad 2\Sigma / f_\pi = \xi^2. \quad (19)$$

We can pass to the current form by performing the chiral field redefinitions

$$\mathcal{B}_1 \rightarrow \xi^\dagger \mathcal{B}_1, \quad \mathcal{B}_2 \rightarrow \xi \mathcal{B}_2. \quad (20)$$

We then have the Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{LH}} = & -\frac{1}{2}\text{Tr}(\bar{\mathcal{B}}_1 v \cdot (i\partial + \mathcal{J}_L) \mathcal{B}_1) - \frac{1}{2}\text{Tr}(\bar{\mathcal{B}}_2 v \cdot (i\partial + \mathcal{J}_R) \mathcal{B}_2) - \frac{g_r f_\pi}{4} [\text{Tr}(\bar{\mathcal{B}}_1 \mathcal{B}_2) + \text{Tr}(\bar{\mathcal{B}}_2 \mathcal{B}_1) + \text{H.c.}] \\ & + \Delta_r [\text{Tr}(\bar{\mathcal{B}}_1 \mathcal{B}_1) + \text{Tr}(\bar{\mathcal{B}}_2 \mathcal{B}_2)] - i\frac{k_r}{2} [\text{Tr}(\mathcal{B}_1 \gamma^5 \gamma_\mu \mathcal{A}^\mu \mathcal{B}_2) + \text{Tr}(\bar{\mathcal{B}}_2 \gamma^5 \gamma_\mu \mathcal{A}^\mu \mathcal{B}_1)], \end{aligned} \quad (21)$$

where

$$\Delta_r = \left[\Delta + \frac{h_r}{4\Lambda} f_\pi^2 \right] \quad (22)$$

and the chiral currents are

$$\begin{aligned} \mathcal{J}_{\mu, L} = i\xi \partial_\mu \xi^\dagger, \quad \mathcal{J}_{\mu, R} = i\xi^\dagger \partial_\mu \xi, \\ \mathcal{A}_\mu = \frac{1}{2}(\mathcal{J}_{\mu, R} - \mathcal{T}_{\mu, L}), \quad \mathcal{V}_\mu = \frac{1}{2}(\mathcal{J}_{\mu, R} + \mathcal{J}_{\mu, L}). \end{aligned} \quad (23)$$

As usual the \mathcal{J}_A are matrices acting on the isospin indices of meson fields. The mass matrix of the chirally redefined heavy mesons is at this stage nondiagonal. We should mention that if an extra γ^5 were included in the definition of the parity partner, then the axial-vector current components of the $\mathcal{J}_{\mu, L}$ and $\mathcal{J}_{\mu, R}$ terms would carry γ^5 factors, while no γ^5 would occur in the k_r term.

Note that the fields \mathcal{B}_1 and \mathcal{B}_2 are of mixed parity. The mass matrix can readily be diagonalized now that the Lagrangian is written in the current form

$$\tilde{\mathcal{B}} = \frac{1}{\sqrt{2}}(\mathcal{B}_1 + \mathcal{B}_2), \quad \tilde{\mathcal{B}}' = \frac{1}{\sqrt{2}}(\mathcal{B}_1 - \mathcal{B}_2), \quad (24)$$

with eigenvalues $2\Delta_r - g_r f_\pi/2$ and $2\Delta_r + g_r f_\pi/2$, respectively [recall that our normalization conventions imply the physical mass shift is δM if the Lagrangian contains $\frac{1}{2}\delta M(\text{tr} \tilde{\mathcal{B}} \tilde{\mathcal{B}})$; see Appendix A]. The mass eigenfields are nontrivial functionals of the pions through the absorbed ξ, ξ^\dagger factors as in Eq. (20). The Lagrangian now becomes

$$\begin{aligned} \mathcal{L}_{\text{LH}} = & -\frac{1}{2}\text{Tr}(\bar{\tilde{\mathcal{B}}} v \cdot (i\partial + \mathcal{V}) \tilde{\mathcal{B}}) - \frac{1}{2}\text{Tr}(\bar{\tilde{\mathcal{B}}}' v \cdot (i\partial + \mathcal{V}) \tilde{\mathcal{B}}') + \left[\Delta_r - \frac{g_r f_\pi}{4} \right] \text{Tr} \tilde{\mathcal{B}} \tilde{\mathcal{B}} + \left[\Delta_r + \frac{g_r f_\pi}{4} \right] \text{Tr} \tilde{\mathcal{B}}' \tilde{\mathcal{B}}' \\ & - \frac{1}{2}\text{Tr}(\bar{\tilde{\mathcal{B}}}(v \cdot \mathcal{A}) \tilde{\mathcal{B}}') - \frac{1}{2}\text{Tr}(\bar{\tilde{\mathcal{B}}}'(v \cdot \mathcal{A}) \tilde{\mathcal{B}}) - i\frac{k_r}{2} \text{Tr} \tilde{\mathcal{B}} \gamma^5 \mathcal{A} \tilde{\mathcal{B}} + i\frac{k_r}{2} \text{Tr} \tilde{\mathcal{B}}' \gamma^5 \mathcal{A} \tilde{\mathcal{B}}'. \end{aligned} \quad (25)$$

Note the appearance of the off-diagonal pionic transition terms of the form $\bar{\mathcal{B}}(v \cdot \mathcal{A})\mathcal{B}'$. At this stage it can be seen that these terms are associated with a Goldberger-Treiman relation, by taking $\mathcal{A}_\mu = \partial_\mu \pi \cdot \tau / f_\pi$, integrating by parts, and using the equations of motion. One finds that the $\mathcal{B}'\mathcal{B}\pi$ amplitude has a coupling strength $g_{\mathcal{B}\mathcal{B}'\pi} = g_r$, and this is seen to be given by $\Delta M / f_\pi$.¹

We can decouple the heavier field \mathcal{B}' to leading order in the mass gap $g_r f_\pi$ by “integrating it out” (which amounts to setting it to zero in leading order). We can then perform the residual mass redefinition: $\mathcal{B} \rightarrow \exp(-iMv \cdot x)\mathcal{B}$ where $M = 2\Delta_r - g_r f_\pi / 2$ to yield the final result

$$\mathcal{L}_{\text{LH}} = -\frac{1}{2} \text{Tr} \bar{\mathcal{B}} i v \cdot (\partial + \mathcal{V}) \mathcal{B} - i \frac{g_A}{2} \text{Tr} \bar{\mathcal{B}} \gamma^5 \mathcal{A} \mathcal{B}, \quad (26)$$

where we now discover that

$$g_A = k_r = 2g f_\pi \sqrt{Z_2} / Z_1. \quad (27)$$

Our fit to the model yields $g_A = 0.32$ [see Eq. (38) and discussion]. Equation (26) is equivalent to the point of departure taken by Ref. [6] in writing effective Lagrangians involving simultaneous heavy symmetries and chiral symmetries. Use of this effective Lagrangian is justified so long as q is small compared to the mass gap. We see that g_A here arises from the perturbative k_r term, which is subleading to unity in our expansion.

In summary, the central observation of this analysis is that the underlying chiral representations of the full HL meson theory is a parity doubled scheme. There are two general implications of such a scheme: (1) The mass gap between the parity partners arises from $\langle \sigma \rangle$. Thus a Goldberger-Treiman relation refers, not to the overall mass of the B mesons $\sim M$, but rather to the mass splitting between the even and odd parity multiplets:

$$\Delta M = g_{\mathcal{B}\mathcal{B}'\pi} f_\pi. \quad (28)$$

Here $g_{\mathcal{B}\mathcal{B}'\pi} = g_r$ is the $\mathcal{B}\mathcal{B}'\pi$ transition coupling constant

$$\begin{aligned} & \frac{igN}{2\sqrt{Z_1}} \int \frac{d^4k}{(2\pi)^4} \text{Tr}[\gamma_\mu \gamma^5 (k - \not{p} + m_q)(1 - \not{p})(i\gamma^5 B^5 + \gamma_\nu B^\nu)] \frac{1}{(k-p)^2 - \Omega^2} \frac{1}{(v \cdot k)} \\ &= \frac{2gN}{16\pi^2 \sqrt{Z_1}} v_\mu B^5 [\Lambda^2 - \mu^2 + \pi v \cdot p (\Lambda - \mu) + \frac{1}{2} \pi \bar{\sigma} \Lambda - \Omega^2 \ln(\Lambda^2 / \mu^2) + O((v \cdot p)^2)]. \quad (30) \end{aligned}$$

We see that the integral involved here is identical to I_1 of Eqs. (60) and (62), and thus the equations of motion can be used for the B^5 fields. Upon use of the equation of motion, shifting $v \cdot p \rightarrow 2\Delta + \dots = 2\Lambda_1^2 / Z_1 - \Lambda / \pi + \dots$ a large cancellation is seen to occur on the RHS of Eq.

¹The coefficient of this term corresponds to $f'' = h = 1$ and $k_r = g$ in Falk and Luke's notation [12]. Our conclusion is then that $k_r = g_A < h = 1$, the widths of the resonances are determined to be very large (see Sec. IV B), the chiral perturbative contribution of these resonances, e.g., to f_{D_s} / f_D , may be significant.

and is the analogue of the $g_{NN\pi}$ in the nucleon system. We note that the light quark constituent mass is given by $m_c \approx g_r f_\pi / 2$ so we expect $\Delta M \approx 600$ MeV; however, this must be obtained in principle from a fit of the model to all data (see Sec. IV B); unfortunately, without exceptional circumstances the width of this state is too large for direct observation). (2) g_A is *not necessarily* expected to be ~ 1 , being given by a subleading perturbative contribution k_r alone. This is especially a consequence of parity doubling and contrasts the chiral quark model in which, $g_A^q = 1$ is a leading term. The fit we present below in Sec. IV B, which is crude, yields $g_A \approx 0.32$. This result may be indicated in the D system where $D^* \rightarrow D + \pi$ gives $g_A < 0.7$ [6,11].

IV. OTHER OBSERVABLES: f_B , AND ISGUR-WISE FUNCTION

A. Heavy meson decay constant, f_B

We presently compute the heavy meson decay constant f_B . Consider the heavy-light axial-vector current $\bar{Q} \gamma_\mu \gamma^5 \psi$. We can compute the renormalized matrix element

$$\begin{aligned} & \sqrt{Z_1^{-1}} \int d^4x e^{-iMv \cdot x} \langle \mathcal{B} | \bar{Q}(x) \gamma_\mu \gamma^5 \psi(x) | 0 \rangle \\ &= f_B \sqrt{M_B} v_\mu + \dots \quad (29) \end{aligned}$$

As a consequence of the heavy quark spin symmetry, B^5 and B_μ^5 have identical decay constants for the axial-vector current, while B and B_μ have the same decay constants for the vector current. The B meson must have a properly normalized kinetic term, which includes the finite renormalization effects, $\mathcal{B} \rightarrow \sqrt{Z_1^{-1}} \mathcal{B}$. We adopt a conventional normalization in which we expect $f_B \approx 180$ MeV.

The amplitude on the left-hand side (LHS) takes the form

(30) leaving

$$\begin{aligned} & \rightarrow \frac{2gN}{16\pi^2 \sqrt{Z_1}} v_\mu B^5 [2\pi \Lambda^3 / Z_1] \\ &= \frac{2}{g} v_\mu B^5 [\Lambda^2 / \sqrt{Z_1}]. \quad (31) \end{aligned}$$

We thus obtain

$$\sqrt{M_B} f_B = (\Lambda)^{3/2} \frac{4\sqrt{2}\pi}{g^2 \sqrt{N}}. \quad (32)$$

For example, let $g^2 N / 4\pi^2 = 1$ and use $f_B = 180$ MeV, $M_B = 5$ GeV as input parameters, to find $\Lambda = 1.35$ GeV

for $N_C=3$. Remarkably, our result is insensitive to the light quark masses.

f_B is a measure of the wave function of the meson at the origin in a nonrelativistic potential model. We can compute the wave function in principle in our model by point splitting the current in Eq. (20):

$$\sqrt{Z_1^{-1}} \int d^4x e^{-iMv \cdot x} \langle \mathcal{B} | \bar{Q}(x - \epsilon/2) \not{v} \gamma^5 \psi(x + \epsilon/2) | 0 \rangle = \mathcal{N} \Psi(\epsilon), \quad (33)$$

where \mathcal{N} is a normalization factor. This essentially replaces the momentum space cutoff procedure by a point-split regulator, and $\Lambda \sim 1/\epsilon$. The wave function is singular at the origin, and is not normalizable without a spatial cutoff of the normalization integral at $1/\Lambda$ (our theory makes no sense at shorter distances than this). Thus the wave function is at the origin is given effectively by

$$|\Psi(0)| \sim \Lambda^2 / \sqrt{Z_1} \sim (\Lambda)^{3/2}. \quad (34)$$

This implies that the result for f_B is insensitive to infrared parameters such as the light quark masses in our model, and indeed we find $f_{B,s} = f_{B,u}$. This is a defect of the model, but it is an expected result of an extremely relativistic, potential dominated system. In this sense, QCD lies somewhere between this extreme result and that of a nonrelativistic potential model.

B. Fitting the model to data

While the model we have presented is not likely to be quantitatively successful, we can attempt a fit to observables, and predict some features of the HL meson system. We use as independent inputs $f_\pi = 95$ MeV, and $f_B = 180$ MeV for $M_B = 5$ GeV. The latter implies $\Lambda = 1.35$ GeV as discussed in the previous subsection. We see, owing to the smallness of the ratio $f_\pi^2/\Lambda^2 = (\kappa - 1)/g^2$, that $\kappa = g^2 N / 4\pi^2$ is very close to unity. In defining Z_2 we cutoff the renormalization-group flow at an infrared scale $\mu \sim m_c$ taken as the approximate constituent light quark mass. Then, to obtain $Z_2 = \frac{1}{4} \kappa \ln(\Lambda^2/m_c^2)$, we self-consistently solve for the constituent quark mass $m_c = \frac{1}{2} g f_\pi / \sqrt{Z_2(m_c)}$. This yields

$$\begin{aligned} \frac{g^2 N}{4\pi^2} &= 1.065, \quad g = 3.75, \\ \Lambda &= 1.35 \text{ GeV}, \\ Z_2 &= 1.11, \quad m_c = 169 \text{ MeV}. \end{aligned} \quad (35)$$

m_c is about a factor of 2 too small. We can moreover use the pion mass, m_π , to extract the light quark PCAC masses:

$$\begin{aligned} m_\pi^2 f_\pi^2 &= \omega(m_u + m_d) f_\pi, \\ \omega &= \frac{g N \Lambda^2}{8\pi^2 \sqrt{Z_2}} = 0.25 \text{ (GeV)}^2; \end{aligned} \quad (36)$$

hence, $m_u + m_d = 8.6$ MeV, which is to be compared with the conventional ~ 15 MeV, and is small. Also, $m_s \approx m_K^2(m_u + m_d)/m_\pi^2 \approx 107$ MeV is small.

The mass gap between the excited 0^+ and ground state

0^- mesons is then

$$\Delta M = g_r f_\pi \approx 2m_c \approx 338 \text{ MeV} [600 \text{ MeV}]. \quad (37)$$

The result in brackets obtains when the known constituent masses are inputted. The decay width $\Gamma(0^+ \rightarrow 0^- \pi)$ is given by $3(\Delta M/f_\pi)^2 |k_\pi|/8\pi$. This is much too large for observation of these resonances when $k_\pi \sim \Delta M \sim 600$ MeV; with the lower estimate of $\Delta M \sim 338$ MeV the width approaches 450 MeV, which is still large. Hence, the direct observation of the parity partners of the ground state is unlikely. Their effect in chiral perturbation theory is nontrivial [12]; conceivably the decay width $\Gamma(D_s(1^+) \rightarrow D_{u,d}^*(1^-) + K) \sim |k_K|$ is phase-space suppressed by the K -meson mass and the $1/M$ corrections to the D masses, which raise the 1^- and depress the 1^+ states.

We obtain the axial-vector coupling constant

$$g_A = \frac{2g f_\pi \sqrt{Z_2}}{Z_1} = \frac{4f_\pi \sqrt{\ln(\Lambda^2/m_c^2)}}{\Lambda \sqrt{N}} \approx 0.32. \quad (38)$$

We might expect both ΔM and g_A to be underestimated in this approximation, as are the light sector observables, owing largely to the short-distance singularity of our wave function.

g_A can be in principle extracted from the decay $D^{*+} \rightarrow D^0 + \pi^+$, though it is unmeasured to date. This decay partial width is given by Ref. [6], and in our conventions it takes the form

$$\Gamma = \frac{g_A^2}{12\pi f_\pi^2} |p_\pi|^3, \quad (39)$$

where $p_\pi \sim 38.9$ MeV. While this width is not yet measured directly, we can use the analysis of Ref. [19] to obtain an estimated result of $\Gamma = 53.4$ keV from the measured branching ratio of $(D^* \rightarrow D\gamma)/(D^* \rightarrow D\pi)$ and a potential model calculation of $D^* \rightarrow D\gamma$. Combining these results we find $g_A = 0.56$, which is compatible with the parity doubled interpretation, but is also not far from the naive $g_A \sim g_A^q \sim 0.8$ from the chiral quark model (note that we derive the chiral light-quark model here with $g_A^q = 1$; thus, our prediction of $g_A \sim 0.3$ represents a significant suppression). Amundson *et al.* [11] give the current experimental limit of $g_A < 0.7$ consistent with this result. Thus, our model indicates that g_A is suppressed and smaller than unity, giving the physical underlying rationale, though the situation is arguably not decisive.

Note that $Z_1 = \pi\Lambda/2 \sim 2.12$ GeV and $Z_2 \sim 1.1$. Hence, $2\Delta = 3\Lambda/\pi \approx 1.3$ GeV. Our model seems to suffer from generating a value of Λ that is slightly large. This implies $\epsilon = 4Z_2\Delta/Z_1 \approx 1.2$, suggesting that our approximation of truncating on the $Z_2(v \cdot p)^2/Z_1$ terms is probably unreliable (Appendix A).

The binding energy is determined in the model. Neglecting the light quark PCAC masses we have, in the infinite mass HQ limit,

$$M_{D,B} = M_{c,b} + \delta m, \quad \delta m = 2\Delta - m_c + \frac{2h_r m_c^2}{g_r^2 \Lambda}, \quad (40)$$

where m_c is the constituent light quark mass (the latter term is small, but non-negligible). For the fit we have presented we find $h_r = 2g^2\sqrt{Z_2}\Lambda/Z_1 = 17.9$, and $g_r = g/\sqrt{Z_2} = 3.55$. If we use a conventional charm- (b -) quark mass of 1.2–1.8 GeV (4.5–5.0 GeV) this overestimates: $M_D \approx 2.4$ –3.0 GeV ($M_B = 5.8$ –6.3 GeV). These results should be corrected for finite mass of the heavy quark. The corrected bound state mass is

$$M_B = \sqrt{M_Q^2 + 2M_Q\delta m} . \quad (41)$$

This yields a result $M_D \approx 2.0$ –2.75 GeV ($M_B = 5.65$ –6.3 GeV). This illustrates the problem of Λ being too large in the model.

The effect of the explicit SU(3)-breaking light quark masses is calculable, upon restoring these terms in Eq. (13) as contained in the shifted $\bar{\sigma}$ field. Using the full constituent quark mass $m_c = g_r\bar{\sigma}/2$ we have

$$\begin{aligned} M_{B_q} - M_{B_{q'}} &= -(m_c - m_{c'}) + (m_c^2 - m_{c'}^2) \left(\frac{2h_r}{g_r^2\Lambda} \right) \\ &= -(m_c - m_{c'}) \\ &\quad + [2.1 \times 10^{-3} (\text{MeV})^{-1}] (m_c^2 - m_{c'}^2) . \end{aligned} \quad (42)$$

For the $B_s - B_0$ mass difference we take $m_c = 450$ MeV (the storage quark constituent mass) and $m_{c'} = 300$ MeV to obtain $M_{B_s} - M_{B(u,d)} = 86.25$ MeV. (If we use the predicted $m_{c'} = 1.69$ MeV and $m_c = 276$ MeV we obtain 52.5 MeV). This compares to ≈ 100 MeV experimentally. It shows, however, that the model must include the effects of the σ^2 term in computing these differences. The $M_{D^+} - M_{D^0} = [(+0.26), (-0.3)](m_d - m_u) \approx (2.6, -3)$ MeV (using standard constituent masses in the first entries, and the model's derived constituent masses in the second). This is subject to electromagnetic corrections, estimated to be +2.0 MeV.

We have seen that f_B is insensitive to the light quark masses in this model. Thus, we obtain $f_{B_s}/f_{B_{u,d}} = 1$, while lattice results ~ 1.09 [20]. This result owes to the unrealistic non-normalizable singularity of the wave function at the origin. This is consistent with the behavior of the binding energy for small constituent quark mass, in which increasing the constituent mass actually decreases the meson mass (for large constituent mass the σ^2 terms

contribute to increase the meson mass).

C. Isgur-Wise function

The analysis of the Isgur-Wise function in the model involves a careful treatment of the cutoff procedure. We select a preferred cutoff by demanding the validity of reparametrization invariance (or the residual mass symmetry) [15–17].

We consider the transition amplitude in four-velocity defined by the matrix element $\langle B_v | \bar{Q}_v \Gamma Q_{v'} | B_{v'} \rangle$, where Γ is an arbitrary Dirac matrix:

$$\begin{aligned} I &= \frac{\bar{g}^2}{M} \int \frac{d^4k}{(2\pi)^4} \text{Tr} \left[\bar{B} \frac{1+\not{v}}{2} \frac{\not{k}}{k^2-m^2} \frac{1+\not{v}'}{2} B \Gamma \right] \\ &\quad \times \frac{1}{v \cdot k} \frac{1}{v' \cdot k} . \end{aligned} \quad (43)$$

This involves the integral

$$\begin{aligned} I_1 &= \int \frac{d^4k}{(2\pi)^4} \frac{k_\mu}{(k^2-m^2)} \frac{1}{v \cdot k} \frac{1}{v' \cdot k} \\ &= A \cdot (v^\mu + v'^\mu) , \end{aligned} \quad (44)$$

where the latter term follows by symmetry, since A can only depend upon $v \cdot v'$ and is even under $v \leftrightarrow v'$. Now multiply by $v + v'$:

$$\begin{aligned} 2A \cdot (1 + v \cdot v') &= \int \frac{d^4k}{(2\pi)^4} \frac{k \cdot (v + v')}{(k^2-m^2)} \frac{1}{v \cdot k} \frac{1}{v' \cdot k} \\ &= \int \frac{d^4k}{(2\pi)^4} \frac{2}{(k^2-m^2)} \frac{1}{v \cdot k} \\ &= \frac{i}{8\pi} (\Lambda - 2m) . \end{aligned} \quad (45)$$

Therefore

$$A = \frac{i}{16\pi(1+v \cdot v')} (\Lambda - 2m) \quad (46)$$

and we conclude that the Isgur-Wise function is given by

$$\xi(v \cdot v') = \frac{2}{1 + v \cdot v'} . \quad (47)$$

This should be true if the momentum space integral is Lorentz invariant and finite. Computing the integral directly, without recourse to the symmetry argument one can obtain

$$\frac{1}{2} \xi(v \cdot v') = \int_0^{\pi/2} d\theta \frac{\cos\theta}{(1 + 2v \cdot v' \cos\theta \sin\theta)^{3/2}} \quad (48)$$

which agrees with the previous result.

This result contains a t -channel pole at $t = (M_1 + M_2)^2$, where $M_1(M_2)$ is the incoming (outgoing) heavy meson mass. One might ask if this is consistent with the slope constraint of de Rafael and Taron [18] arising from t -channel unitarity? Our slope, $\xi'(0) = -1$ is inconsistent with their lower bound of $-\frac{1}{2}$ arising from a t -channel branch cut at threshold. Grinstein and Mende [18] have pointed out that the de Raphael–Taron constraint is weakened by effects of resonance poles, as we are presently observing. However, the t -channel unitarity constraint is an interesting issue in heavy quark effective theory (HQET). In an HQET such as we have studied, the antiparticle has been discarded at the outset, and with it goes crossing symmetry and t -channel unitarity. Moreover, our cutoff theory would seem to require the bound of $Q^2 < \Lambda^2$ without a unitarization. Since $Q^2 = 2M^2(1 - v \cdot v')$ we see that this bound corresponds to the limit $v \cdot v' \rightarrow 1$ for $M \rightarrow \infty$. Nevertheless we can compute the t -channel behavior by incorporating the heavy antiquarks and computing the large- N bubble sum with the full interaction. While we do not present this analysis

here, we find, perhaps not surprisingly, a Nambu–Jona-Lasinio pole at $M_1 + M_2$ is generated, and our slope is consistent with the existence of this pole.

The previous result of Eq. (47) is, however, sensitive to the definition of the cutoff procedure, which we have taken to be a Lorentz-invariant Euclidean momentum space

$$A(1+v \cdot v') = \int \frac{d^4 k}{(2\pi)^4} \left[\frac{k_0 + v_0 k_0 - \mathbf{v} \cdot \mathbf{k}}{k_0^2 - \mathbf{k}^2 + i\epsilon} \right] \left[\frac{1}{v_0 k_0 - \mathbf{k} \cdot \mathbf{v}} \right] \left[\frac{1}{k_0} \right] \\ = \frac{i}{4\pi^2} \int_0^\infty dk \left[\frac{1}{\sqrt{(v \cdot v')^2 - 1}} \ln \left[\frac{v \cdot v' + \sqrt{(v \cdot v')^2 - 1}}{v \cdot v' - \sqrt{(v \cdot v')^2 - 1}} \right] + 2 \right]. \quad (49)$$

This result, using a non-Lorentz-invariant regularization procedure, differs significantly from Eq. (47) in which the Lorentz invariant cutoff was used.

There is however an implicit gauge invariance in heavy quark effective theories associated with the “residual mass ambiguity.” One is free to add a term $\chi \bar{Q}_v Q_v$ to the effective Lagrangian of Eq. (12). χ should be viewed as a gauge potential in the sense that if we redefine the heavy quark mass $M \rightarrow M + \mu$, and thus $Q \rightarrow \exp(i\mu v \cdot x)Q$ we can compensate this gauge transformation by shifting $\chi \rightarrow \chi + \mu$. Hence $iv \cdot \partial + \chi$ is a covariant derivative. This is essentially the demand that the global zero of energy of a classical theory be arbitrary. This symmetry and its implications will be discussed elsewhere, however we can see immediate implications for our present problem.

We can observe that the non-Lorentz invariant regularization procedure violates the χ symmetry. Consider the integral involved in our calculation of the Isgur-Wise function:

$$I = \int \frac{d^4 k}{(2\pi)^4} \frac{1}{[(k+p)^2 - m^2](v \cdot k + \mu)}. \quad (50)$$

We have chosen to route the external momentum p through the light fermion line. The χ symmetry applies to the external heavy mesons and requires that the following shift in $v \cdot p$ and μ be a symmetry of integral:

$$v \cdot p \rightarrow v \cdot p + \chi, \quad \mu \rightarrow \mu + \chi. \quad (51)$$

This is readily seen to be a symmetry in the case of the momentum p routed through the heavy fermion line.

In the present example we can implement this by shift-

$$\int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 - m^2)(v \cdot k)^2} = \frac{-i}{8\pi^2} \int_0^\pi \sin\theta d\theta \int_0^\infty \frac{k^2 dk}{(k^2 + m^2)^{3/2}(v_0 - |\mathbf{v}|u(k)\cos\theta)}, \quad (55)$$

where $u(k) = k/\sqrt{k^2 + m^2}$ and $v_\mu = (v_0, \mathbf{v})$, and thus $v_0^2 - \mathbf{v}^2 = 1$. Notice that if either $m \rightarrow 0$ or if $\mathbf{v} \rightarrow 0$ then $S \rightarrow 0$. Let us expand in m^2 , using the latter results, to find, for S

$$S = \frac{i}{4\pi^2} (1 - v_0^2) \int_0^\infty \frac{x^2 dx}{(1 + x^2)^{5/2}} \\ = \frac{i}{12\pi^2} [1 - (v \cdot \eta)^2]. \quad (56)$$

cutoff. Different results follow if the energy integrations are first performed by residues, and then a three-momentum cutoff procedure is used. To see this let us compute directly with a three-momentum cutoff. Let $v' = (1, \mathbf{0})$ and $v = (v_0, \mathbf{v})$. First we perform the energy integration by closing dk_0 below to pick up the single pole:

ing $p \rightarrow p + v\chi$. Therefore the shift in the integral is

$$\frac{\delta}{\delta\chi} I = \int \frac{d^4 k}{(2\pi)^4} \left[\frac{-2v \cdot k}{[(k+p)^2 - m^2]^2 (v \cdot k + \mu)} - \frac{1}{[(k+p)^2 - m^2](v \cdot k + \mu)^2} \right]. \quad (52)$$

The symmetry condition is $(\delta/\delta\chi)I = 0$, and is equivalent to demanding that the integral generates no nontrivial surface term upon shifting $k \rightarrow k + a$. For simplicity we consider the surface term

$$S = \int \frac{d^4 k}{(2\pi)^4} \left[\frac{-2v \cdot k}{(k^2 - m^2)^2 (v \cdot k)} - \frac{1}{(k^2 - m^2)(v \cdot k)^2} \right]. \quad (53)$$

If we evaluate S using the covariant cutoff we find that $S = 0$.

Now consider computing S by first performing dk_0 by residues, then the residual three-momentum integration with a cutoff. We find

$$\int \frac{d^4 k}{(2\pi)^4} \frac{v \cdot k}{(k^2 - m^2)^2 (v \cdot k)} = \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 - m^2)^2} \\ = \frac{i}{4\pi^2} \int_0^\infty \frac{k^2 dk}{(k^2 + m^2)^{3/2}}. \quad (54)$$

Consider now

Here we introduce a four-vector $\eta_\mu = (1, \mathbf{0})$ which is the direction of the dk_0 line integration.

This latter result implies that the χ symmetry is broken when the k_0 line integral is not parallel to v_μ . For the computation of the Isgur-Wise function where $v \neq v'$ then the χ symmetry can never be present in the residue computation. However, utilizing the Lorentz invariant cutoff we see that the χ symmetry can be maintained. The χ symmetry therefore requires that we reject the result of

Eq. (49) in favor of Eq. (47) which is consistent with the absence of momentum space surface terms, and the attendant symmetry.

V. CONCLUSIONS

We have presented perhaps the simplest, solvable, strongly coupled toy field-theoretic model in which both heavy quark and chiral symmetries are present at the fundamental quark level, and the dynamics of chiral symmetry breaking is made explicit. We find that the chiral representations of the heavy mesons are parity doubled. This has a well-defined meaning in the toy model because we can tune the coupling constant to restore the spontaneously broken chiral symmetry. In the symmetry limit the ground state is a degenerate system of $(0^-, 1^-)$ and $(0^+, 1^+)$ heavy mesons. When chiral symmetry is broken the degeneracy is lifted, elevating the $(0^+, 1^+)$ and depressing the $(0^-, 1^-)$ heavy meson multiplets. We obtain the full chiral Lagrangian containing the parity doubled composite HL mesons together with the composite pions. The mass gap between the multiplets is given by gf_π , and the analogue of the Goldberger-Treiman relation of the system reflects this, $g_{BB'\pi} = \Delta M / f_\pi$. We are able in the broken phase to pass to a nonlinearly realized chiral symmetry, and to write a purely derivatively coupled pion effective Lagrangian. We can then decouple the heavier parity doubling field to arrive at the conventional low energy effective chiral Lagrangian for \mathcal{B} .

We believe that the general phenomenon of the parity doubled chiral representations of heavy mesons is inherent to QCD itself. We emphasize at the outset that this toy model is only intended to convey the schematics of QCD chiral dynamics in heavy-light mesons. The model is designed to imitate these dynamical features of QCD, rather than provide a detailed phenomenological fit. Nonetheless, the simplest fit seems to agree within a factor of 2 to the expected values of physical quantities, and is predictive. While we would be inclined to trust the result $g_A = 0.32$ only to within a factor of 2, the model suggests that g_A is smaller than might be naively expected on the basis of the simple constituent quark model in which. The direct observation of the parity partners of the groundstate mesons is unlikely owing to their large widths. It would be interesting to extend these results to the heavy quark containing baryons where similar conclusions must hold.

Our analysis achieves the basic systematics of chiral symmetry in these systems where we might expect potential models to fail. The chiral symmetry limit is relativistic, and the chiral symmetry breaking is a dynamical rearrangement of the vacuum, two features which would be hard to realize in any potential model treatment. One must be careful in estimating the value of g_A in a naive potential model unless the mixing with the parity doubled states is under control. As we have observed in Eq. (21), the g_A term is a transition matrix element between the 0^+ and 0^- states in the mixed parity basis appearing there. In a basis in which the g_A term is diagonal, the mass matrix must be correspondingly diagonal. There remains the transition amplitude term between the parity

partners (some authors include an extra factor of γ^5 in the odd parity fields, and this transition term can then be mistaken for the g_A term in a mixed parity basis). In our model, the constituent quarks are found to have $g_A^q = 1$, and yet the value of g_A obtained in the Lagrangian of Ref. [6] is suppressed to ~ 0.32 . This is a subtlety of parity doubling which must be treated with some care. The resonances may have important contributions in chiral perturbation theory to quantities such as f_{D_s}/f_D and flavor ratios of Isgur-Wise functions [12] (in the notation of Falk, $h = 1$ and $g_A = g$, and thus $h^2 \gg g^2$ in our model, so the resonance contributions are significant).

We have studied the physical predictions of this system. The wave function of the theory is too singular at the origin to represent a realistic QCD wave function. This is a consequence of the strong coupling of the pointlike four-fermion interaction term. While it is a defect of the model, it indicates the trend in a theory in which the potential term is dominating the dynamics. For example, we obtain the unrealistic $f_{B_s}/f_{B_{u,d}} = 1$, because the singular short-distance behavior of the wave function becomes insensitive to the infrared parameters of the theory. This contrasts lattice results, indicating $f_{B_s}/f_{B_{u,d}} = 1.1$ [20]. However, a weakly coupled potential model would give a larger result $\sqrt{m_s/m_d} \sim 1.2$ [21].

In our analysis we fix $\Lambda \sim 1.35$ GeV from f_B and examine the relationship with the cutoff wave function at the origin. Inputting also f_π fixes g , and marginal results (within a factor of 2) obtain for $\Delta M, g_A$ and the light quark sector. A defect, related to the short-distance singularity of the wave function, is the fact that for small light quark constituent mass, the ground state mass is actually depressed as the light quark constituent mass is increased from small constituent mass. Nonetheless, the common h_r term is sufficiently large for $m_c \sim 300$ MeV that a reasonable result for $M_{B_s} - M_{B_{u,d}}$ emerges from the fit.

Of further interest is the Isgur-Wise function, which is associated with an ambiguous linearly divergent integral in the present scheme. The ambiguity is resolved by invoking ‘‘residual mass invariance’’ [15,16], or equivalently, ‘‘reparametrization invariance,’’ and enforcing an associated Ward identity [17]. The simple Isgur-Wise function corresponds to a t -channel threshold pole at $(M_1 + M_2)^2$. This pole is beyond the cutoff scale of our model, but it may be indicative of a Nambu–Jona-Lasinio result when the $Q\bar{Q}$ system is studied. In fact, the fundamental issues raised by de Rafael and Taron can in principle be explored in this scheme [18]. We will defer this discussion to another place.

We believe there remains much to do in dynamical analyses of this kind for heavy-light systems and their interactions. Our model has inherent shortcomings. While the agreement of this crude model with observation is marginal at best, it suggests that improvements, such as a Pagels-Stokar approximation, Holdom’s approach [22,23], or ‘‘Russian sum-rule’’ methods [24], will lead to more reliable estimates of crucial heavy meson observables. The singular behavior of the wave function is not expected in a more realistic scheme. Replacing our pure

s-wave dynamics by QCD ladder approximation is clearly of some interest. For example, pinning down a prediction of g_A or the Isgur-Wise function from such models would be quite interesting. The full range of phenomenological applications of generalized models would seem to be an interesting direction for future research. This toy scheme is a first step in that direction and highlights the challenges and advantages for more elaborate approaches.

Note added. After completion of this work we became aware of the contemporaneous, extensive treatment of the light-quark sector in the NJL model by Bijnens, Bruno, and de Raphael [25]. We have tacitly dropped the effects of the ρ and the A_1 in our analysis for sake of simplicity, which is the essential difference between our results. These authors argue that g_A in the light sector (Georgi-Manohar) is reduced to ~ 0.62 by mixing induced by the A_1 . Our suppression of the g_A as in Eq. (27) is a *different effect*, arising from the parity doubling chiral structure of the heavy meson spectrum. We have not examined the effect upon the heavy meson g_A discussed here due to the A_1 mixing, and might expect this to be a further and important suppression effect. In the limit $G_V=0$ the work of Bijnens *et al.* would appear to coincide with our light-quark sector, though they do not consider the parity doubling of the (π, σ) , working directly in the current form (nonlinear σ model) of the theory.

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APPENDIX A: FERMION LOOP APPROXIMATION

1. Zero momentum pions

Let us now integrate out the heavy and light fermion fields in Eq. (12) to produce an effective Lagrangian for B

$$\begin{aligned} i\mathcal{S}_{BB} &= -g^2 N \int \frac{d^4 k}{(2\pi)^4} \text{Tr} \left[(-i\bar{\mathcal{B}}'\gamma^5 + \bar{\mathcal{B}}) \left(\frac{\not{k} - \not{p} + m_q + g\Sigma^5}{(k-p)^2 - \Omega^2} \right) (-i\gamma^5 \mathcal{B}' + \mathcal{B}) \left(\frac{1 + \not{p}}{2} \right) \right] \frac{1}{v \cdot k} \\ &= -g^2 N \text{Tr} [(-i\bar{\mathcal{B}}'\gamma^5 + \bar{\mathcal{B}}) I_1 (-i\gamma^5 \mathcal{B}' + \mathcal{B})] \end{aligned} \quad (\text{A3})$$

and

$$I_1 = \int \frac{d^4 k}{(2\pi)^4} \left[\frac{(\not{k} - \not{p}) + m_q + g\Sigma^5}{(k-p)^2 - \Omega^2} \right] \frac{1}{v \cdot k}. \quad (\text{A4})$$

We carry out a ‘‘block-spin’’ integration over heavy and light quark modes between the scales μ and Λ in Euclidean momentum space. The integrals are evaluated with a Euclidean four-momentum cutoff:

$$\begin{aligned} \int \frac{d^4 k}{(2\pi)^4} \frac{1}{[(k-p)^2 - \Omega^2] v \cdot k} \\ = \frac{i}{16\pi} (\Lambda - \mu) + \frac{2i}{16\pi^2} v \cdot p [\ln(\Lambda^2/\mu^2)], \end{aligned}$$

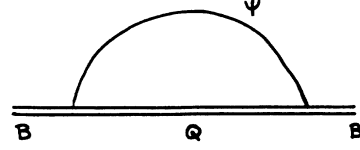


FIG. 1. Integrating out light fermions (ψ) and heavy fermion (Q) yields the leading large- N contribution to the effective Lagrangian for the heavy-light meson B .

and B' . This can be viewed as a block-spin renormalization of the theory of Eq. (12) defined at the scale $\mu = \Lambda$, to a new scale $\mu < \Lambda$, and is analogous to the treatment of the light quark dynamics in Appendix B. We begin in the approximation of treating the σ and π fields as zero-momentum (constant in spacetime) backgrounds (small momentum π amplitudes are considered subsequently). We note that the fermion propagators take the form

$$\begin{aligned} S_{\text{HQ}}(k) &= \frac{i}{v \cdot k} \left[\frac{1 + \not{p}}{2} \right], \\ S_{\text{LQ}}(k) &= i \left[\frac{\not{k} + m_q + g\Sigma^5}{k^2 - \Omega^2} \right], \end{aligned} \quad (\text{A1})$$

where

$$\begin{aligned} \Omega^2 &= (m_q + g\sigma/2)^2 + g^2\pi^2/4, \\ \pi^2 &= \pi^a \pi^a, \\ \Sigma^5 &= \frac{1}{2}\sigma + i\frac{1}{2}\gamma_5 \pi \cdot \tau. \end{aligned} \quad (\text{A2})$$

We obtain, from the diagram of Fig. 1 [recall that the B contain $(1 + \not{p})/2$ projection factors],

$$\begin{aligned} \int \frac{d^4 k}{(2\pi)^4} \frac{1}{[(k-p)^2 - \Omega^2]} \\ = -\frac{i}{16\pi^2} [\Lambda^2 - \mu^2 - \Omega^2 \ln(\Lambda^2/\mu^2)], \end{aligned} \quad (\text{A5})$$

$$\begin{aligned} \int \frac{d^4 k}{(2\pi)^4} \frac{k_\mu}{(k^2 - \Omega^2) v \cdot k} \\ = -i \frac{v_\mu}{16\pi^2} [\Lambda^2 - \mu^2 - \Omega^2 \ln(\Lambda^2/\mu^2)], \end{aligned}$$

$$\int \frac{d^4 k}{(2\pi)^4} \frac{k_\mu}{(k^2 - \Omega^2)^2 v \cdot k} = i \frac{v_\mu}{16\pi^2} [\ln(\Lambda^2/\mu^2)].$$

Then $\text{Tr}(I_1)$ can be written as [note $\text{Tr}(\bar{\mathcal{B}}\gamma^5 \mathcal{B}) = \text{Tr}(\bar{\mathcal{B}}' \mathcal{B}) = 0$, etc.]

$$\begin{aligned}
ig^2N \text{Tr}(\bar{\mathcal{B}}I_1\mathcal{B}) &= -\frac{1}{2}\text{Tr}(\bar{\mathcal{B}}\mathcal{B})[(v\cdot p + g\bar{\sigma}/2)(Z_1 + 4Z_2v\cdot p) + Z_1(\Lambda + \mu)/\pi - 2|\Omega|^2Z_2], \\
ig^2N \text{Tr}(\bar{\mathcal{B}}'(-i\gamma^5)I_1(-i\gamma^5)\mathcal{B}') &= -\frac{1}{2}\text{Tr}(\bar{\mathcal{B}}'\mathcal{B}')[(v\cdot p - g\bar{\sigma}/2)(Z_1 + 4Z_2v\cdot p) + Z_1(\Lambda + \mu)/\pi - 2|\Omega|^2Z_2], \\
ig^2N[\text{Tr}(\bar{\mathcal{B}}'(-i\gamma^5)I_1\mathcal{B}) + \text{Tr}(\bar{\mathcal{B}}I_1(-i\gamma^5)\mathcal{B}')] &= -\frac{1}{2}\text{Tr}[\bar{\mathcal{B}}'(g\pi\cdot\tau/2)\mathcal{B} + \bar{\mathcal{B}}(g\pi\cdot\tau/2)\mathcal{B}'](Z_1 + 4Z_2v\cdot p),
\end{aligned} \tag{A6}$$

where we let $g\bar{\sigma} = g\sigma + 2m_q$ and

$$Z_1 = \frac{g^2N}{8\pi}(\Lambda - \mu), \quad Z_2 = \frac{g^2N}{16\pi^2}[\ln(\Lambda^2/\mu^2)] \tag{A7}$$

(note that the expression for Z_1 contains a factor of $1/\pi$, not $1/\pi^2$).

2. The g_A term

Now consider small, but nonzero (σ, π) momentum q_μ , with $q^2 \approx 0$. We compute the effective Lagrangian, where the (σ, π) are coupled through Σ^5 . We then have the amplitude of Fig. 2:

$$i\mathcal{S}_{BB\Sigma} = \frac{1}{2}g^3N \int \frac{d^4k}{(2\pi)^4} \text{Tr} \left[(-i\bar{\mathcal{B}}'\gamma^5 + \bar{\mathcal{B}}) \frac{(k + \not{q}/2)(\sigma + i\pi\cdot\tau\gamma^5)(k - \not{q}/2)}{[(k + q/2)^2 - \Omega^2][(k - q/2)^2 - \Omega^2]v\cdot(k-p)} (-i\gamma^5\mathcal{B}' + \mathcal{B}) \right]. \tag{A8}$$

We are interested in the divergent terms of order q , since the $q=0$ term has previously been computed:

$$\begin{aligned}
&\approx -\frac{1}{4}g^3N \int \frac{d^4k}{(2\pi)^4} \text{Tr} \left[(-i\bar{\mathcal{B}}'\gamma^5 + \bar{\mathcal{B}}) \frac{[k, \not{q}](\sigma - i\pi\cdot\tau\gamma^5)}{(k)^2(k)^2(v\cdot k)} (-i\gamma^5\mathcal{B}' + \mathcal{B}) \right] \\
&= -\frac{i}{4}g^3N \frac{1}{16\pi^2} \text{Tr} [(-i\bar{\mathcal{B}}'\gamma^5 + \bar{\mathcal{B}})[\not{v}, \not{q}](\sigma - i\pi\cdot\tau\gamma^5)(-i\gamma^5\mathcal{B}' + \mathcal{B}) \ln(\Lambda^2/\mu^2)] \\
&= -\frac{i}{4}gZ_2 \text{Tr} [(-i\bar{\mathcal{B}}'\gamma^5 + \bar{\mathcal{B}})[\not{v}, \not{q}](\sigma - i\pi\cdot\tau\gamma^5)(-i\gamma^5\mathcal{B}' + \mathcal{B})].
\end{aligned} \tag{A9}$$

If we now expand the result of Eq. (A9) we observe some simplifications, e.g., $\bar{\mathcal{B}}[\not{v}, \not{q}]\mathcal{B} = 0$, and we obtain

$$\begin{aligned}
&= \frac{i}{2}gZ_2 \text{Tr} [\bar{\mathcal{B}}\not{q}(i\pi\cdot\tau\gamma^5)\mathcal{B} - \bar{\mathcal{B}}'\not{q}(i\pi\cdot\tau\gamma^5)\mathcal{B}' \\
&\quad + i\bar{\mathcal{B}}'\gamma^5\not{q}\sigma\mathcal{B} + \bar{\mathcal{B}}i\gamma^5\not{q}\sigma\mathcal{B}'].
\end{aligned} \tag{A10}$$

This implies an operator in the effective Lagrangian of the form

$$\begin{aligned}
&= \frac{1}{2}gZ_2 (\text{Tr}[\bar{\mathcal{B}}\gamma^5\gamma_\mu\tau^a\mathcal{B} - \bar{\mathcal{B}}'\gamma^5\gamma_\mu\tau^a\mathcal{B}']\partial^\mu\pi^a \\
&\quad - \text{Tr}[\bar{\mathcal{B}}'\gamma^5\gamma_\mu\mathcal{B} + \bar{\mathcal{B}}\gamma^5\gamma_\mu\mathcal{B}']\partial^\mu\sigma).
\end{aligned} \tag{A11}$$

3. Normalization conventions

Consider a complex scalar field Φ with the Lagrangian

$$\partial_\mu\Phi^\dagger\partial^\mu\Phi - (M + \delta M)^2\Phi^\dagger\Phi. \tag{A12}$$

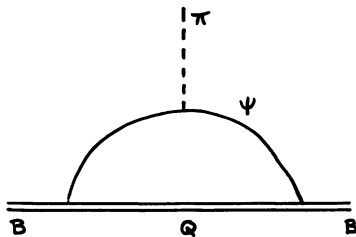


FIG. 2. Induced single pion emission vertex for \mathcal{B} from ψ coupled to the classical background pion field.

Define $\Phi' = \sqrt{2M} \exp(iMv\cdot x)\Phi$ (Φ' destroys incoming momentum $Mv_\mu + p_\mu$) and the Lagrangian becomes, to order $1/M$,

$$iv_\mu\Phi'^\dagger\partial^\mu\Phi' - \delta M\Phi'^\dagger\Phi'. \tag{A13}$$

Now let $\mathcal{B}_v = \frac{1}{2}(1 - \not{v})i\gamma^5\Phi'$ and write in terms of traces (the field \mathcal{B}_v with these conventions annihilates an incoming meson state $|B\rangle$):

$$-i\frac{1}{2}\text{Tr}(\bar{\mathcal{B}}_v v\cdot\partial\mathcal{B}) + \delta M\frac{1}{2}\text{Tr}(\bar{\mathcal{B}}_v\mathcal{B}_v). \tag{A14}$$

Thus when the Lagrangian is written in terms of \mathcal{B} and \mathcal{B}' the normal sign conventions are those of the vector mesons, and opposite those of scalars, i.e., the term in the Lagrangian $+\frac{1}{2}\delta M \text{Tr}(\bar{\mathcal{B}}\mathcal{B})$ an increase in the B^5 mass by an amount δM . A properly normalized kinetic term is $-i\frac{1}{2}\text{Tr}(\bar{\mathcal{B}}v\cdot\partial\mathcal{B})$, with the overall minus sign and $\frac{1}{2}$.

One must take care of using HQET propagators, since the direction of momentum routing is fixed. Ultimately, the veracity of Eq. (A3) is best checked by computing with finite M , routing Mv_μ through the Q propagator, and p through the light quark propagator, and then taking the $M \rightarrow \infty$ limit. Note that $v_\mu \rightarrow -v_\mu$ and $p_\mu \rightarrow -p_\mu$ is a symmetry of the final expressions. Hence, \mathcal{B}_v can be viewed as annihilating incoming particles, (ψQ) , or creating outgoing antiparticles $(\bar{\psi} Q)$.

4. Structure of the effective Lagrangian

The heavy meson effective Lagrangian therefore takes the form

$$\begin{aligned}
\mathcal{L}_{\text{LH}} = & -i\frac{1}{2}Z_1 \text{Tr}(\overline{\mathcal{B}}v \cdot \partial \mathcal{B}) - i\frac{1}{2}Z_1 \text{Tr}(\overline{\mathcal{B}'}v \cdot \partial \mathcal{B}') + 2Z_2 \text{Tr}(\overline{\mathcal{B}}(v \cdot \partial)^2 \mathcal{B}) + 2Z_2 \text{Tr}(\overline{\mathcal{B}'}(v \cdot \partial)^2 \mathcal{B}') \\
& - \frac{gZ_1}{4} [\text{Tr}(\overline{\mathcal{B}}\overline{\sigma}\mathcal{B}) - \text{Tr}(\overline{\mathcal{B}'}\overline{\sigma}\mathcal{B}')] - igZ_2 [\text{Tr}(\overline{\mathcal{B}}\overline{\sigma}v \cdot \partial \mathcal{B}) - \text{Tr}(\overline{\mathcal{B}'}\overline{\sigma}v \cdot \partial \mathcal{B}')] \\
& - \frac{gZ_1}{4} [\text{Tr}(\overline{\mathcal{B}}\overline{\pi} \cdot \tau \mathcal{B}) + \text{Tr}(\overline{\mathcal{B}'}\overline{\pi} \cdot \tau \mathcal{B}')] - igZ_2 [\text{Tr}(\overline{\mathcal{B}}\overline{\pi} \cdot \tau v \cdot \partial \mathcal{B}) + \text{Tr}(\overline{\mathcal{B}'}\overline{\pi} \cdot \tau v \cdot \partial \mathcal{B}')] \\
& + \frac{g^2Z_2}{4} [\text{Tr}(\overline{\mathcal{B}}(\overline{\sigma}^2 + \pi^2)\mathcal{B}) + \text{Tr}(\overline{\mathcal{B}'}(\overline{\sigma}^2 + \pi^2)\mathcal{B}')] \\
& + (\Lambda^2 - Z_1(\Lambda + \mu)/2\pi) [\text{Tr}(\overline{\mathcal{B}}\mathcal{B}) + \text{Tr}(\overline{\mathcal{B}'}\mathcal{B}')] \\
& + \frac{Z_2}{2} g \text{Tr}[\overline{\mathcal{B}}\gamma^5 \not{\partial}(\pi \cdot \tau)\mathcal{B} - \overline{\mathcal{B}'}\gamma^5 \not{\partial}(\pi \cdot \tau)\mathcal{B}' - \overline{\mathcal{B}}\gamma^5 \not{\partial}(\sigma)\mathcal{B} - \overline{\mathcal{B}'}\gamma^5 \not{\partial}(\sigma)\mathcal{B}'] .
\end{aligned} \tag{A15}$$

If we define

$$\mathcal{T} = [1 + (4Z_2/Z_1)iv \cdot \partial]^{-1/2} \tag{A16}$$

then Eq. (A16) becomes, more compactly,

$$\begin{aligned}
\mathcal{L}_{\text{LH}} = & -i\frac{1}{2}Z_1 \text{Tr}(\overline{\mathcal{T}}\mathcal{B}v \cdot \partial \mathcal{T}\mathcal{B}) - i\frac{1}{2}Z_1 \text{Tr}(\overline{\mathcal{T}}\mathcal{B}'v \cdot \partial \mathcal{T}\mathcal{B}') - \frac{gZ_1}{4} [\text{Tr}(\overline{\mathcal{T}}\mathcal{B}\overline{\sigma}\mathcal{T}\mathcal{B}) - \text{Tr}(\overline{\mathcal{T}}\mathcal{B}'\overline{\sigma}\mathcal{T}\mathcal{B}')] \\
& - \frac{gZ_1}{4} [\text{Tr}(\overline{\mathcal{T}}\mathcal{B}\overline{\pi} \cdot \tau \mathcal{T}\mathcal{B}) + \text{Tr}(\overline{\mathcal{T}}\mathcal{B}'\overline{\pi} \cdot \tau \mathcal{T}\mathcal{B}')] + \frac{g^2Z_2}{4} [\text{Tr}(\overline{\mathcal{T}}\mathcal{B}(\overline{\sigma}^2 + \pi^2)\mathcal{T}\mathcal{B}) + \text{Tr}(\overline{\mathcal{T}}\mathcal{B}'(\overline{\sigma}^2 + \pi^2)\mathcal{T}\mathcal{B}')] \\
& + (\Lambda^2 - Z_1(\Lambda + \mu)/2\pi) [\text{Tr}(\overline{\mathcal{T}}\mathcal{B}\mathcal{T}\mathcal{B}) + \text{Tr}(\overline{\mathcal{T}}\mathcal{B}'\mathcal{T}\mathcal{B}')] \\
& + \frac{Z_2}{2} g \text{Tr}[\overline{\mathcal{T}}\mathcal{B}\gamma^5 \not{\partial}(\pi \cdot \tau)\mathcal{T}\mathcal{B} - \overline{\mathcal{T}}\mathcal{B}'\gamma^5 \not{\partial}(\pi \cdot \tau)\mathcal{T}\mathcal{B}' - \overline{\mathcal{T}}\mathcal{B}\gamma^5 \not{\partial}(\sigma)\mathcal{T}\mathcal{B} - \overline{\mathcal{T}}\mathcal{B}'\gamma^5 \not{\partial}(\sigma)\mathcal{T}\mathcal{B}'] .
\end{aligned} \tag{A17}$$

To simplify the subsequent analysis we will assume that the subleading terms of order $Z_2 v \cdot p / Z_1$ are negligible, and take $\mathcal{T} = 1$. Since these terms arise upon expanding the loop integrals in powers of $1/\Lambda$, we cannot self-consistently use the effective Lagrangian in this form unless this condition is at least approximately valid. We see that other terms, such as the last one in Eq. (A17) which leads to g_A , are leading in this order and describe various physical processes. Thus, we expect the amplitudes these terms describe to be small. If $4Z_2 v \cdot p / Z_1$ is large, then we must retain full analytic expressions for the loop integrals to fit the theory.

We see that there is thus an induced kinetic term for the \mathcal{B} and \mathcal{B}' fields with a common wave-function normalization. We absorb the factor Z_1 into the fields as $\mathcal{B} \rightarrow \sqrt{Z_1^{-1}} \mathcal{B}$. Thus, with the field redefinition we then have the full effective Lagrangian:

$$\begin{aligned}
\mathcal{L}_{\text{LH}} = & -i\frac{1}{2}\text{Tr}(\overline{\mathcal{B}}v \cdot \partial \mathcal{B}) - i\frac{1}{2}\text{Tr}(\overline{\mathcal{B}'}v \cdot \partial \mathcal{B}') - \frac{g}{4} [\text{Tr}(\overline{\mathcal{B}}\overline{\sigma}\mathcal{B}) - \text{Tr}(\overline{\mathcal{B}'}\overline{\sigma}\mathcal{B}')] - \frac{g}{4} [\text{Tr}(\overline{\mathcal{B}}\overline{\pi} \cdot \tau \mathcal{B}) + \text{Tr}(\overline{\mathcal{B}'}\overline{\pi} \cdot \tau \mathcal{B}')] \\
& + \frac{g^2Z_2}{4Z_1} [\text{Tr}(\overline{\mathcal{B}}(\overline{\sigma}^2 + \pi^2)\mathcal{B}) + \text{Tr}(\overline{\mathcal{B}'}(\overline{\sigma}^2 + \pi^2)\mathcal{B}')] + \Delta [\text{Tr}(\overline{\mathcal{B}}\mathcal{B}) + \text{Tr}(\overline{\mathcal{B}'}\mathcal{B}')] \\
& + \frac{Z_2}{2Z_1} g \text{Tr}[\overline{\mathcal{B}}\gamma^5 \not{\partial}(\pi \cdot \tau)\mathcal{B} - \overline{\mathcal{B}'}\gamma^5 \not{\partial}(\pi \cdot \tau)\mathcal{B}' - \overline{\mathcal{B}}\gamma^5 \not{\partial}(\sigma)\mathcal{B} - \overline{\mathcal{B}'}\gamma^5 \not{\partial}(\sigma)\mathcal{B}'] ,
\end{aligned} \tag{A18}$$

where

$$\Delta = \frac{1}{Z_1} [\Lambda^2 - Z_1(\Lambda + \mu)/2\pi] . \tag{A19}$$

The equation of motion in momentum space is $v \cdot p = 2\Delta + \dots$ and 2Δ is the mass difference between the heavy meson and the heavy quark in the chiral symmetric phase:

$$M_B = 2\Delta + M_Q . \tag{A20}$$

Note that $\Delta > 0$ ($\Delta < 0$) for $g^2 N / 16\pi^2 < 1$ ($g^2 N / 16\pi^2 > 1$).

APPENDIX B: LIGHT QUARK DYNAMICS

1. Deriving the constituent quark model

The effective Lagrangian in the light quark sector is

$$\mathcal{L} = \overline{\psi}(i\not{\partial} - m_q)\psi - \frac{g^2}{\Lambda^2} \overline{\psi}\gamma_\mu \frac{\lambda^A}{2} \psi \overline{\psi}\gamma^\mu \frac{\lambda^A}{2} \psi . \tag{B1}$$

For concreteness we will take $\psi = (u, d)$, and in the limit that the quark mass matrix $m_q \rightarrow 0$, we have an exact chiral $\text{SU}(2) \times \text{SU}(2)$ invariant Lagrangian. This can be

viewed as a single gluon exchange potential, where we assume a “gluon mass” $\Lambda/\sqrt{2}$, and we have written the form of the effective Lagrangian at $q^2 \sim \Lambda^2$, integrating out the massive gluon, and truncating on dimension-6

$$\begin{aligned} \mathcal{L}_L = & \bar{\psi}(i\partial - m_q)\psi + \frac{g^2}{\Lambda^2}(\bar{\psi}_L\psi_R\bar{\psi}_R\psi_L + \bar{\psi}_L\tau^A\psi_R\bar{\psi}_R\tau^A\psi_L - \frac{1}{8}\bar{\psi}\gamma_\mu\tau^A\psi\bar{\psi}\gamma^\mu\tau^A\psi - \frac{1}{8}\bar{\psi}\gamma_\mu\gamma_5\tau^A\psi\bar{\psi}\gamma^\mu\gamma_5\tau^A\psi \\ & - \frac{1}{8}\bar{\psi}\gamma_\mu\psi\bar{\psi}\gamma^\mu\psi - \frac{1}{8}\bar{\psi}\gamma_\mu\gamma_5\psi\bar{\psi}\gamma^\mu\gamma_5\psi), \end{aligned} \quad (\text{B2})$$

where $\psi_L = (1 - \gamma_5)\psi/2$, $\psi_R = (1 + \gamma_5)\psi/2$. Here τ^A are Pauli matrices acting upon the isospin indices.

For the present analysis we will truncate Eq. (B2) on the pure Nambu–Jona-Lasinio terms, since the (vector)² and (axial-vector)² terms play no significant role in the chiral dynamics (they are associated with the formation of virtual ρ and A_1 vector mesons in the model). Hence we take

$$\begin{aligned} \mathcal{L}_L = & \bar{\psi}(i\partial - m_q)\psi + \frac{g^2}{\Lambda^2}(\bar{\psi}_L\psi_R\bar{\psi}_R\psi_L \\ & + \bar{\psi}_L\tau^A\psi_R\bar{\psi}_R\tau^A\psi_L). \end{aligned} \quad (\text{B3})$$

We can solve the light-quark dynamics in large N in the usual way by writing an equivalent effective Lagrangian of the form

$$\mathcal{L} = \bar{\psi}(i\partial - m_q)\psi - g\bar{\psi}_L\Sigma\psi_R - g\bar{\psi}_R\Sigma^\dagger\psi_L - \frac{1}{2}\Lambda^2\text{Tr}(\Sigma^\dagger\Sigma), \quad (\text{B4})$$

where

$$\Sigma = \frac{1}{2}\sigma I_2 + i\pi^a \frac{\tau^a}{2} \quad (\text{B5})$$

is an auxiliary field. We emphasize that at this stage Σ is a 2×2 complex field, so both σ and π^a are complex (otherwise, with σ and π real there would be unwanted contributions from $\langle T\Sigma\Sigma \rangle = \langle T\Sigma^\dagger\Sigma^\dagger \rangle \neq 0$ in integrating out Σ).

Thus there is parity doubling at this stage, $\text{Im}(\sigma)$ is the fourth Goldstone boson associated with the U(1) problem, and $\text{Im}(\pi^a)$ is the 0^+ isotriplet. The restriction to real π^a will emerge dynamically at very low energies, since the induced $\text{Tr}(\Sigma^\dagger\Sigma\Sigma^\dagger\Sigma)$ term will lift the degeneracy of the $\text{Re}(\pi)$ and $\text{Im}(\pi)$. We ultimately must add a $\det(\Sigma) + \text{H.c.}$ term to get rid of the $\text{Im}(\sigma)$ mode.

We now integrate out the fermion fields on scales $\Lambda^2 > q^2 > \mu^2$, keeping only the leading large- N_C fermion loop contributions. We use the massless fermion propagator, treating Σ as a classical background field. Thus we arrive at an effective field theory at the scale μ :

$$\begin{aligned} \mathcal{L} = & \bar{\psi}(i\partial - m_q)\psi - g\bar{\psi}_L\Sigma\psi_R - g\bar{\psi}_R\Sigma^\dagger\psi_L \\ & + Z_2\text{Tr}(\partial_\mu\Sigma^\dagger\partial^\mu\Sigma) - V(\Sigma), \end{aligned} \quad (\text{B6})$$

where

operators.

Upon Fierz rearrangement of the interaction Lagrangian, keeping only leading terms in $1/N_C$, Eq. (B1) takes the form

$$\begin{aligned} Z_2 = & \frac{g^2 N}{16\pi^2} \ln(\Lambda^2/\mu^2), \\ V(\Sigma) = & \left[\frac{1}{2}\Lambda^2 - \frac{g^2 N}{8\pi^2}(\Lambda^2 - \mu^2) \right] \text{Tr}(\Sigma^\dagger\Sigma) \\ & - \frac{gN}{8\pi^2}(\Lambda^2 - \mu^2)\text{Tr}(m_q\Sigma + \text{H.c.}) \\ & + \frac{g^4 N}{16\pi^2} \ln(\Lambda^2/\mu^2)\text{Tr}(\Sigma^\dagger\Sigma\Sigma^\dagger\Sigma). \end{aligned} \quad (\text{B7})$$

We see that $Z_2 \rightarrow 0$ as $\mu \rightarrow \Lambda$, reflecting the compositeness of the Σ field. Let us now renormalize the Σ field,

$$\Sigma \rightarrow \sqrt{Z_2}\Sigma, \quad (\text{B8})$$

and we have the properly normalized effective Lagrangian at the scale μ (this is proper normalization for real σ and π):

$$\begin{aligned} \mathcal{L} = & \bar{\psi}(i\partial - m_q)\psi - \bar{g}\bar{\psi}_L\Sigma\psi_R - \bar{g}\bar{\psi}_R\Sigma^\dagger\psi_L \\ & + \text{Tr}(\partial_\mu\Sigma^\dagger\partial^\mu\Sigma) - \bar{V}(\Sigma), \end{aligned} \quad (\text{B9})$$

where

$$\begin{aligned} \bar{g} & \equiv 1/\sqrt{Z_2}, \\ \bar{V}(\Sigma) = & m_\sigma^2 \text{Tr}(\Sigma^\dagger\Sigma) - \omega \text{Tr}(m_q\Sigma + \text{H.c.}) \\ & + \lambda \text{Tr}(\Sigma^\dagger\Sigma\Sigma^\dagger\Sigma), \\ m_\sigma^2 = & \left[\frac{1}{Z_2} \right] \left[\frac{1}{2}\Lambda^2 - \frac{g^2 N}{8\pi^2}(\Lambda^2 - \mu^2) \right], \\ \lambda = & \frac{16\pi^2}{N \ln(\Lambda^2/\mu^2)} = \bar{g}^2, \\ \omega = & \frac{\bar{g}gN}{8\pi^2}(\Lambda^2 - \mu^2). \end{aligned} \quad (\text{B10})$$

The effective Lagrangian is seen to be a linear σ model at scales $\mu < \Lambda$. As the scale $\mu \rightarrow 0$ we see that the theory is trivial, since $\bar{g} \rightarrow 0$. However, these evolution results apply only to a scale μ_0 corresponding to a mass scale for the fermion. Nonzero m_q will block the evolution into the far infrared, but we will neglect this presently. The theory will develop a chiral instability (a constituent quark mass) provided that m_σ^2 becomes tachyonic (negative) at some scale μ_0 . By tuning the bare coupling constant g^2 we can put the model in a symmetric phase, $m^2 > 0 \rightarrow g^2 N/4\pi^2 < 1$, or in a chiral symmetry breaking phase: $m^2 < 0 \rightarrow g^2 N/4\pi^2 > 1$, where the critical bare

coupling corresponds to $m_\sigma^2 = 0$ as $\mu_0 \rightarrow 0$.

In the broken phase (ignoring m_q) the σ field develops a vacuum expectation value $\langle \sigma \rangle = f_\pi = \sqrt{2} |m_\sigma| / \sqrt{\lambda}$. We see that the renormalized σ field develops a vacuum expectation value given by

$$\begin{aligned} \langle \sigma \rangle_r^2 &= Z_2 \left[\frac{16\pi^2 \Lambda^2}{g^5 N \ln(\Lambda^2 / \mu^2)} \right] \left[\frac{g^2 N}{4\pi^2} - 1 \right] \\ &= \left[\frac{\Lambda^2}{g^2} \right] \left[\frac{g^2 N}{4\pi^2} - 1 \right] \end{aligned} \quad (\text{B11})$$

In the broken phase we can then write $\sigma = f_\pi + \hat{\sigma}$, and the physical mass² of the $\hat{\sigma}$ is readily seen to be $m_{\hat{\sigma}}^2 = 2|m_\sigma^2|$, while the fermion mass becomes $m_0 = \frac{1}{2} f_\pi \bar{g}$. Thus, using Eqs. (B10) to relate $\hat{g}^2 = \lambda$, we obtain the usual Nambu–Jona-Lasinio result: $m_\sigma = 2m_0$.

The solution to the theory can thus be written as a chiral quark model in which we have both constituent quarks described by ψ and the mesons described by Σ . In the broken phase it is useful to pass to a nonlinear σ model and write

$$\Sigma \rightarrow \frac{1}{2} f_\pi \exp(i\pi^a \tau^a / f_\pi) \quad (\text{B12})$$

and

$$\begin{aligned} \mathcal{L} &= \bar{\psi}(i\partial - m_q)\psi - m_0 \bar{\psi}_L \exp(i\pi^a \tau^a / f_\pi) \psi_R \\ &\quad - m_0 \bar{\psi}_R \exp(-i\pi^a \tau^a / f_\pi) \psi_L \\ &\quad + \text{Tr}(\partial_\mu \Sigma^\dagger \partial^\mu \Sigma) + \omega \text{Tr}(m_q \Sigma + \text{H.c.}), \end{aligned} \quad (\text{B13})$$

where $m_0 = \frac{1}{2} \bar{g} f_\pi$ is the constituent quark mass. Note, in our present normalization conventions that $f_\pi = 93$ MeV. By a chiral redefinition of the fields, $\psi_R \rightarrow \xi \psi_R$ and $\psi_L \rightarrow \xi^\dagger \psi_L$ we arrive at the Georgi-Manohar Lagrangian [their Eq. (2.9)] with $g_A = 1.0$ [note that they fit $G_A / G_V = (5/3)g_A$ and obtain $g_A = 0.75$, consistent with our large- N approximation].

When the σ and π fields are slowly varying in space, the light quark propagator of the chiral quark model is given by (in terms of the unrenormalized fields)

$$\begin{aligned} S_F &= i(\not{p} - m_q - g \frac{1}{2} \sigma - i g \frac{1}{2} \gamma_5 \pi \cdot \tau)^{-1} \\ &= i \left[\frac{\not{p} + m_q + g \Sigma^5}{p^2 - \Omega^2} \right], \end{aligned} \quad (\text{B14})$$

where we define

$$\Omega^2 = (m_q + g\sigma/2)^2 + g^2 \pi^2 / 4, \quad \pi^2 = \pi^a \pi^a, \quad (\text{B15})$$

$$\Sigma^5 = \frac{1}{2} \sigma + i \frac{1}{2} \gamma_5 \pi \cdot \tau. \quad (\text{B16})$$

In the broken phase we replace $\sigma = f_\pi$ and $\Sigma \rightarrow \frac{1}{2} f_\pi \exp(i\pi^a \tau^a / f_\pi)$. For future ease of writing we can often replace $g\bar{\sigma}/2 = g\sigma/2 + m_q \sqrt{Z_2}$ since it is easy to restore the explicit chiral-symmetry-breaking quark mass terms.

2. Schematic discussion of a parity doubled nucleon

Consider a ‘‘nucleon’’ doublet N with the $SU(2)_L \times SU(2)_R$ assignments $N_L \sim (\frac{1}{2}, 0)$, $N_R \sim (0, \frac{1}{2})$. Also, we introduce a partner, K , of opposite parity with assignments $K_L \sim (0, \frac{1}{2})$, $K_R \sim (\frac{1}{2}, 0)$. A typical renormalizable linear σ -model effective matter Lagrangian (not including the Σ kinetic and potential terms) is then

$$\begin{aligned} \mathcal{L} &= \bar{N} i \not{\partial} N + \bar{K} i \not{\partial} K \\ &\quad - M_1 \bar{N}_L \Sigma N_R - M_2 \bar{K}_L \Sigma^\dagger K_R - M_0 \bar{N}_L K_R \\ &\quad - M_0' \bar{N}_R K_L + \text{H.c.} \end{aligned} \quad (\text{B17})$$

Parity symmetry requires $M_0 = M_0'$. We consider the special case $M_1 = M_2 = M$, which is the analogue of our model, but this is not generally required by symmetries. Now perform the redefinitions, $N_L \rightarrow \xi N_L$, $K_L \rightarrow \xi^\dagger K_L$, $N_R \rightarrow \xi^\dagger N_R$, $K_R \rightarrow \xi K_R$. Thus, the Lagrangian becomes

$$\begin{aligned} \mathcal{L} &= \bar{N}(i\partial + \bar{V} + \gamma^5 \mathcal{A})N + \bar{K}(i\partial + \mathcal{V} - \gamma^5 \mathcal{A})K \\ &\quad - M \bar{N} N - M \bar{K} K - M_0 \bar{N} K - M_0 \bar{K} N + \text{H.c.} \end{aligned} \quad (\text{B18})$$

Upon diagonalizing, the mass eigenfields are just $(N \pm K) / \sqrt{2}$, with mass eigenvalues $M \pm M_0$. We can decouple the heavier state by setting $(N + K) / \sqrt{2} = 0$, where the light effective Lagrangian for $Q = (N - K) / \sqrt{2}$ is

$$\mathcal{L} = \bar{Q}(i\partial + \mathcal{V})Q - (M - M_0) \bar{Q} Q. \quad (\text{B19})$$

We see that $g_A = 0$. Hence, g_A is not generally of order unity as is the case of a nonparity doubled nucleon. [This is also a consequence of the special case $M_1 = M_2$; more generally $g_A = \sin(2\theta)$ where θ is the mass mixing angle.] With $g_A = 0$ the only nontrivial Goldberger-Treiman relation refers to the pionic transition amplitude between the ground state Q and the parity partner $Q' = (N + K) / \sqrt{2}$.

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