Comments on information loss and remnants

Steven B. Giddings*
Department of Physics, University of California, Santa Barbara, California 93106-9530
(Received 16 November 1993)

The information loss and remnant proposals for resolving the black hole information paradox are reconsidered. It is argued that in typical cases information loss implies energy loss, and thus can be thought of in terms of coupling to a spectrum of "fictitious" remnants. This suggests proposals for information loss that do not imply Planckian energy flucutations in the low energy world. However, if consistency of gravity prevents energy nonconservation, these remnants must then be consiered to be real. In either case, the catastrophe corresponding to infinite pair production remains a potential problem. Using Reissner-Nordström black holes as a paradigm for a theory of remnants, it is argued that couplings in such a theory may give finite production despite an infinite spectrum. Evidence for this is found in analyzing the instanton for Schwinger production; fluctuations from the infinite number of states lead to a divergent stress tensor, spoiling the instanton calculation. Therefore naive arguments for infinite production fail.

PACS number(s): 04.70.Dy

I. INTRODUCTION

Although there are many variants,[1] the three basic proposals for solving the problem of information loss in black holes are fundamental information loss, remnants, or information return in the Hawking radiation. Each of these possibilities has posed serious conceptual problems, and much effort has been expended trying to overcome the difficulties for at least one of these scenarios. Two-dimensional models for black hole formation and evaporation [4] have recently served as a useful testing ground for these ideas.

In particular, two-dimensional black holes strongly suggest [5] that information return is unlikely without some new locality-violating physics. The basic argument for this rests on treatment of the two-dimensional theories in a $1/N$ expansion, where N, the number of matter fields, is large. For the information to get out the rate of information return from the black hole should be comparable its rate of energy loss for the latter part of its evolution [6]. This includes a substantial fraction of the lifetime of the black hole, where the $1/N$ approximation would appear valid. The energy flow is seen to leading order in the $1/N$ expansion, but the information flow is not, suggesting that it is suppressed by higher powers in $1/N$. If this is the case information is not returned until late in the evaporation, in the analogue of the Planck regime, when the expansion fails.

There have been several responses to this. One suggestion is that the $1/N$ expansion breaks down [6,7]. One argument for this is that fluctuations in the vicinity of the horizon become strong, and this invalidates the semiclassical reasoning [7]. This contention relies in part on the assertion that if Hawking particles are traced back to the vicinity of the horizon then they have near infinite frequency as seen by a freely falling observer. However, it is not clear why it is valid to do so. In particular, if one examines the origin of the Hawking flux, for example, by explicit computation in the soluble models of [8–10], then it is found that the Hawking radiation actually originates substantially outside the horizon where the trace anomaly becomes important. This corresponds to the known result in four dimensions that the source of the Hawking radiation cannot be localized more precisely than the wavelength of the radiation, which is approximately given by the radius of the hole.

Another response is to conjecture some new type of fundamental nonlocality in the laws of physics. One such conjecture is that of 't Hooft [11], who proposes that information within a given volume can be determined by measurements on the boundary of that volume. He suggests that this could happen if the fundamental laws of physics have some features similar to cellular automata. This would be interesting if a workable set of such laws were to be exhibited. Alternately, Susskind has advocated the viewpoint [12] that string theory has precisely the right kind of nonlocality built into it, basically from the fact that if you try to measure a string on a very short time interval then it spreads out. He argues that when a string falls into a black hole, observations of the external observer are effectively performing this type of measurement and therefore cannot resolve the location of the information on a scale less than the horizon size. One objection to this is that it is not clear what measurement can actually be performed by an outside observer to demonstrate that the string is indeed spreading out in the

*Electronic addresses: giddings@denali.physics.ucsb.edu, steve@voodoo.bitnet.

[1]For reviews see [1–3].

desired way.[2] Furthermore, there have been recent studies of causality in string theory [13]. These investigations suggest that string causality is not radically different from that in field theory. It is likely that extension of these ideas could be used to show that string theory does not allow the types of causality violation needed to get the information out of the black hole. This paper will take the point of view that such nonlocalities are not the solution.

Instead the focus will be information loss and remnants. Not long after Hawking proposed information loss [14] by generalizing the S matrix to an $\mathcal{S}$ matrix acting on density matrices, it was argued [15] that such evolution typically violates energy conservation, in so doing violently disrupting low-energy physics. In Sec. III this paper reexamines this argument, and shows that in fact the $\mathcal{S}$ matrices considered in [15] can be obtained through couplings to a hidden internal Hilbert space of oscillators, at infinite temperature, with which the Universe can exchange energy and information. This in turn suggests other proposals for information loss based on more general hidden Hilbert spaces. One particular possibility is a Hilbert space of fictitious Planck-mass remnants. This provides an example of an information loss scenario that does not necessarily cook low energy physics. This scenario does, however, share with real remnants a problem of catastrophic loss of energy through the analogue of infinite pair production. Whether one views remnants as real or fictitious, this problem requires solution. Reissner-Nordström black holes may be examples of objects that have infinitely many states[3] but are not infinitely produced, and it is therefore suggested that they serve as a viable paradigm for a theory of remnants. The remainder of the paper is devoted to investigating this possibility. In particular, in an effective theory describing such remnants' couplings to the electromagnetic field may be far from minimal. These couplings may well depend sensitively on the internal state of the remnant in a way that invalidates the argument for infinite production.[4] Such behavior seems to occur when one examines Euclidean instantons describing the analogue of Schwinger production.

This paper does not represent a detailed proposal to resolve the problem of information loss, as the form of couplings of the electromagnetic field to Reissner-Nordström black holes or similar remnants is not yet fully understood. However, I feel that great progress will be made toward solving the black hole information paradox if we can find where the logic that got us into it might fail, and even better, if there is any modification of Planck scale physics that averts it. This paper is a suggestion of where our ignorance might have allowed a resolution of the black hole information paradox to go unnoticed. I hope to return to the details of couplings in future work.

[2]One way to make measurements on an infalling string that have reasonable resolution in Schwarzschild time is to drop in, alongside the string, a particle accelerator that is probing the string with higher and higher energy particles as the string approaches the horizon. These particles can then be observed at infinity. The collisions with these probes spread the string out. But in the absence of this arrangement one is limited to observing whatever radiation is emitted from the infalling string, and this will not give the desired time resolution. In this case it appears unnecessary to conclude that the string is spread out.

[3]This has been convincingly argued in the semiclassical approximation in [16].

[4]This is in contrast with assumptions used in some formulas in [17].

II. THE EFFECTIVE APPROACH

In its basic formulation the question of information loss refers to issues involving strong spacetime curvature and Planckian physics. However, the fundamental paradox is phrased in terms of classical geometries and a definite notion of time. This has lead some to guess that perhaps the resolution to the paradox lies in proper treatment of quantum geometry and time.

This contention, however, would appear to miss the mark. Let us consider formulating the problem in terms of a fully quantum-mechanical treatment based on the Wheeler-deWitt equation, or whatever replaces it in the true theory of quantum gravity. To make contact with ordinary physics one needs a notion of time, and this is a notoriously thorny issue. However, in the present problem one has the advantage that all questions can be asked within the context of asymptotically flat space. This means that we can put a physical clock at infinity and use it to define what is meant by time [18,19]. If T is the dynamical clock variable, then the full Wheeler-DeWitt (WDW) wave function of clock plus gravitating system is of the form $\Psi[T,f,g]$ where f indicates matter fields and g the metric. Let $\mathcal{H}_c$ and $\mathcal{H}_u$ be the contributions to the WDW operator corresponding to the clock and the rest of the Universe, respectively; the WDW equation is

$$\mathcal{H}_{\mathrm{WDW}}\Psi=(\mathcal{H}_c+\mathcal{H}_u)\Psi=0\ . \tag{2.1}$$

Consider arbitrary solutions ψ, ϕ of the equations

$$\begin{aligned} i\frac{\partial}{\partial t}\phi&=\mathcal{H}_c\phi\ ,\\ i\frac{\partial}{\partial t}\psi&=\mathcal{H}_u\psi\ . \end{aligned} \tag{2.2}$$

The dependence of $\mathcal{H}_c$ on the variables g, f can be taken to be very weak by taking the clock to be very far away and very massive. In this approximation, (2.1) is separable and its general solution takes the form

$$\Psi[T,f,g]=\int_{-\infty}^{\infty}dt\ \phi(T,t)\psi[t,f,g]\ . \tag{2.3}$$

If the clock is a good one, then ϕ will be sharply peaked about $t=T$; this can be arranged by taking the mass M of the clock large. The solution then becomes

$$\Psi[T,f,g]\approx\psi[T,f,g]+O\left[\frac{1}{M}\right] \tag{2.4}$$

and satisfies a Schrödinger equation

$$i\frac{\partial}{\partial T}\Psi[T,f,g]=\mathcal{H}_u\Psi[T,f,g]+O\left[\frac{1}{M}\right]. \tag{2.5}$$

Suppose that an asymptotic observer using a time slicing specified by this clock watches diffuse dust collapse to form a black hole, and then observes the decay products from the resulting evaporation. The important question is whether or not this observer sees the scattering to be unitary. If it is unitary, one would like to know how and when the information came out. If it is not, one would like to have an effective description of what generalization of the S matrix maps the observer's initial state to the final state. In such a framework where the black hole formation and evaporation is thought of as a scattering process, and questions formulated in terms of asymptotic observations, it is hard to see how the answers could possibly get mixed up in the subtleties of time or spacetime fluctuations. Either the bottom-line S matrix is unitary or we would like to know what replaces it.

While on the topic of time, it can also be pointed out that the flexibility in choosing time slicings in quantum gravity might be used as an advantage in studying black hole formation and evaporation. In particular, suppose that one performs the exact quantization of the theory choosing one's time slicing to always stay outside what is in the semiclassical theory the black hole horizon, as indicated in Fig. 1. This allows one to avoid the region of Planckian curvature until the end of the evaporation process. Using such time slices suggests that at least up until the end point of evaporation the process can be described in terms of two coupled quantum systems. The information thrown into the black hole (and in correlation with the outgoing Hawking radiation) is all encoded in the state on the left half of the timelike slice as it approaches the horizon. Of course the dynamics on these slices becomes more and more extreme, and eventually involves Planckian physics. However, if one believes that the only place that information is truly lost is the singularity, then this can be avoided until the last instant of the evaporation.

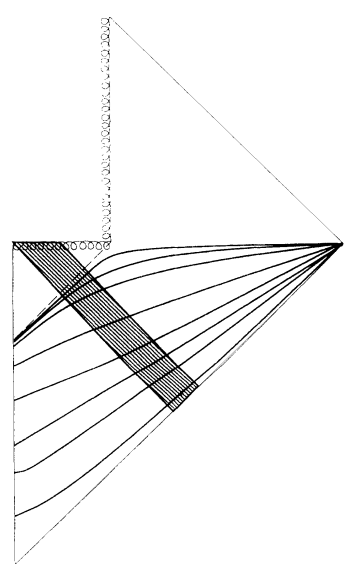

FIG. 1. Shown is a time slicing that avoids the interior of the black hole. It is plausible that evolution on this time slicing is Hamiltonian until it reaches the Planckian region near the classical singularity.

III. MODELS FOR INFORMATION LOSS

Hawking proposed [14] that information loss in quantum gravity be described by a general linear evolution law for density matrices:

$$\rho \to \$\rho , \tag{3.1}$$

with $\$$ a generalization of the usual S matrix. This proposal was investigated in more detail by Banks, Peskin, and Susskind [15], who within the context of $\$$ matrices local in time studied the constraints that the density matrix remain positive (so as to have a probabilistic interpretation) and that the entropy be nondecreasing. If one considers a finite Hilbert space and takes Q^α to be a complete set of Hermitian matrices, the infinitesimal form of (3.1) is

$$\dot\rho=\not{H}\rho=-i[H_0,\rho] -\frac{1}{2}\sum_{\alpha\beta\neq 0} h_{\alpha\beta}(Q^\beta Q^\alpha\rho+\rho Q^\beta Q^\alpha-2Q^\alpha\rho Q^\beta) , \tag{3.2}$$

here H_0 is the usual Hamiltonian. Reference [15] argues that sufficient conditions for positivity and increasing entropy are that $h_{\alpha\beta}$ be positive and real, respectively. Although it may be possible to construct other physical $\$$ matrices generated by an $\not{H}$, these clearly represent a large fraction of the interesting ones.[5]

Equation (3.2) can in fact be derived as the result of considering our system to be in contact with another quantum-mechanical system which is unobserved and therefore traced over. Let the uncoupled Hamiltonians of the two systems be H_0 and H_h, with $[H_0,H_h]=0$. Interactions between them arise from $H_i=\sum_\alpha Q^\alpha \mathcal{O}_\alpha$, where $\mathcal{O}_\alpha$ are operators acting on the "hidden" Hilbert space, and the total evolution is then governed by

$$H=H_0+H_i+H_h . \tag{3.3}$$

Consider first the case of a single Q, and let the internal system be a harmonic oscillator of frequency ω. Take the coupling to be of the form

$$H_i=\left[\frac{h\beta\omega}{2}\right]^{1/2} Qp_\omega , \tag{3.4}$$

where p_ω is the oscillator momentum. Finally, let the harmonic oscillator be in a high-temperature state:

$$\rho_h=(1-e^{-\beta\omega})\sum_n e^{-\beta\omega n}|n\rangle\langle n| , \tag{3.5}$$

[5]For example, simple examples of $\$$ matrices preserving positivity but with nonpositive $h_{\alpha\beta}$ exist [20,21].

with $\beta \to 0$. The density matrix of the observable system takes the form

$$\rho(t) = \mathrm{Tr}_h \left[T\exp\left[-i \int (H_0 + H_i) dt \right] \rho_h \otimes \rho(0) T\exp\left[i \int (H_0 + H_i) dt \right] \right] , \tag{3.6}$$

where we work in the interaction picture for the internal system. Expanding the exponential, we find, for small times,

$$\rho(\delta t) = \rho(0) - i\delta t [H_0, \rho(0)] - h \int_0^{\delta t} dt \int_0^t dt' \beta \langle [Qp(t), [Qp(t'), \rho(0)]] \rangle_\beta . \tag{3.7}$$

The thermal expectation value is easily computed:

$$\frac{\beta\omega}{2} \langle p(t) p(t') \rangle_\beta = \frac{\beta\omega e^{-\beta\omega}}{1 - e^{-\beta\omega}} \cos\omega(t - t') + \frac{\beta\omega}{2} e^{-i\omega(t-t')} \xrightarrow{\beta \to 0} \cos\omega(t - t') , \tag{3.8}$$

and we find

$$\dot\rho = -i[H_0, \rho] - h \int_0^t dt' \cos(\omega t')(Q^2\rho + \rho Q^2 - 2Q\rho Q) . \tag{3.9}$$

Therefore if we allow Q to couple to an ensemble of oscillators with a flat spectrum (that is we sum over all frequencies), (3.9) becomes

$$\dot\rho = -i[H_0, \rho] - h(Q^2\rho + \rho Q^2 - 2Q\rho Q) \tag{3.10}$$

as in (3.2).

The generalization to multiple couplings is clear: simply diagonalize $h_{\alpha\beta}$, and then introduce couplings to a family of ensembles of oscillators labeled by α. The motivation for this construction is equally clear. If one wishes to reproduce (3.2) through coupling to a hidden quantum-mechanical system, then that system should have a huge temperature so that it can raise the entropy of the visible system independent of its temperature. However, to avoid the resulting infinite exchange of energy, the coupling to the large-temperature system should fall with the inverse temperature. This limit furthermore has the desirable effect of washing out correlations that arise between interactions of our system with the hidden one at different times. Finally, note that positivity and reality of $h_{\alpha\beta}$ corresponds to positivity of the norm in the hidden Hilbert space.

In the case of a field-theoretic model we wish to reproduce the evolution law

$$\dot\rho = -i\left[\int d^3x\, H_0(x), \rho \right] - \int d^3x\, d^3y\, h_{\alpha\beta}(x-y) [\{ Q^\beta(y) Q^\alpha(x), \rho \} - 2Q^\alpha(x) \rho Q^\beta(y)] . \tag{3.11}$$

This can likewise be done by couplings to a family of oscillator ensembles. For example, in the special case $h_{\alpha\beta}(x-y) = h_{\alpha\beta}\delta^3(x-y)$, we simply need oscillators of all possible frequencies at each point in space. More generally one must introduce correlations between oscillators at different points in space, with correlation distance corresponding to the fall-off of $h_{\alpha\beta}(x-y)$.

The above example suggests two points regarding information loss. The first is that the evolution (3.2) is readily extended to more general descriptions of information loss that arise from couplings to more general hidden systems. The second is that if one expects information to be lost during a definite time interval Δt, this implies a corresponding loss of energy $\Delta E \sim 1/\Delta t$. A similar argument should hold (generalizing arguments of [15]) for information loss localized within a region of size Δx; there corresponds a momentum loss $\Delta p \sim 1/\Delta x$. To see how these statements arise, consider, for example, restricting to frequencies $\omega < \omega_0$. Then (3.9) only reduces to (3.10) if we are not capable of resolving times $\lesssim 1/\omega_0$. At shorter intervals nontrivial correlations appear, and clustering fails. Therefore if one restricts the energy loss the information loss occurs over the corresponding time scale.[6] Likewise, information loss can only be localized to a region Δx by making $h_{\alpha\beta}(x-y)$ fall off at longer scales. This implies that it carriers momenta $O(1/\Delta x)$. If this is the largest momentum loss, then information loss cannot be restricted to shorter scales.

We can now consider more general types of unitarity violating evolution, arising from coupling to various sorts of quantum-mechanical systems. Equation (3.2) corresponds to unitarity violation that is in a sense maximal. In particular, we would like to know what is likely to be the correct description of information loss for black holes, if it is indeed lost. First note that, following that argument at the end of the preceding section, we might think that a correct description is in terms of the Hilbert spaces describing states on the left and right halves of the slices of Fig. 1. When the black hole finally disappears, the Hilbert space on the left half of the slice becomes inaccessible. Therefore it is quite plausible that information loss in black holes be treated in terms of coupling to an internal Hilbert space, as in (3.3), which becomes invisible. If this is the case we may not even care if there is more fundamental nonunitarity at the singularity, as that dynamics could be totally decoupled. The hypothesis that black hole information loss can be described in terms of coupling to an internal Hilbert space fits nicely with the reformulation of Hawking's $\mathcal{S}$ matrices in terms of such couplings, as has just been outlined. Alternatively black holes might be described by more general forms of information loss arising from different internal Hilbert

[6] A sketch of a general argument for this is as follows. Consider a Hamiltonian of the form (3.3), and pass to interaction picture for the internal Hilbert space. Then $\dot\rho = -i\,\mathrm{Tr}_h\{[H_0 + H_i(t), \rho]\}$. Information loss is restricted to time interval Δt if $\mathrm{Tr}_h\{[H_i(t), \rho]\}$ vanishes outside this interval. For this to happen in general, $H_i(t)$ should vanish outside this interval. This can only be arranged if the interactions connect internal states with energies $\Delta E \gtrsim 1/\Delta t$.

spaces. For example, one might consider instead modeling their loss by assuming the existence of a family of quantum fields[7] that couple to ordinary quantum fields through operators that only become important during the final stages of black hole evaporation. These could carry the black hole's information away. Of course, in principle this information might be recoverable in couplings through the same operators that transferred it to the hidden Hilbert space. However, this by no means implies that it is recoverable in practice, as couplings to the hidden space may be small everywhere except in black holes. Indeed the reader may note that what is being discussed here is nothing more than a theory of black hole remnants, in which the remnants are not observed after they are produced. Assuming that the information is truly lost in such a picture corresponds to assuming that the remnants are fictitious—nothing more than bookkeeping devices to summarize the couplings through which we lose information. On the other hand, if the remnants are real, then the information may just be hard to find.

Let us next reassess the logic of the information loss scenario. As shown in [15], information loss via an infinitesimally generated $\mathcal{S}$ matrix also violates energy and momentum conservation. As has been explicitly described, such evolution corresponds to placing the world in contact with a fictitious Hilbert space raised to infinite temperature. This does not agree well with observation. However, one may consider more general, and more innocuous, forms of information loss. The above arguments indicate a connection between information loss and energy loss. An alternate model for information loss is to imagine that information is carried off by remnants that are fictitious in the same sense as the many-oscillator Hilbert space. These remnants also carry away energy. However, once we have such a model we eliminate the distastefulness of energy nonconservation by instead assuming the remnants to be real.

It should be noted that in order to describe formation and evaporation of near-Planck scale black holes we should consider remnants with energies up to near the Planck scale. A very plausible assumption is therefore that black hole remnants have Planck-size masses.

Such remnant models (real or fictitious) of information loss (temporary or permanent) clearly have a distinct advantage over information loss via $\mathcal{S}$ matrices: they do not offer the appearance of infinite temperature. Suppose that the remnant state is initially the vacuum. With the assumption that the remnants have Planck masses, any low energy scattering process that we perform therefore does not couple to remnants through real processes, i.e., does not see information loss. Virtual effects of remnants, although possibly important,[8] do not lead to loss of information since every remnant line must terminate in a closed loop. On-shell remnants only enter once scattering energies cross the Planckian threshold or once black holes are formed. Only in such cases is information lost.

There remains the possibility that the information could be regained through subsequent processes. However, this could be made vanishingly unlikely in ordinary circumstances if the operators to which the remnants couple only become important at the Planck scale, and because if black holes are rare, it is unlikely that a remnant from one black hole will reappear in another.

Such models are not yet immune from problems. Whether these are considered models for information loss or true remnant scenarios, one must have an infinite number of remnant species to carry off the information from a black hole as large as you can imagine. This raises the standard objection to remnants: infinite species seems to imply infinite total rates of production in any process where there is enough available energy, e.g., inside the Sun, even if individual production rates are near infinitesimal. In the case where the remnants are considered fictitious this would be interpreted as a catastrophic instability in which energy disappears at an infinite rate. These issues will be the focus of much of the rest of the paper.

It should also be noted that since they carry energy, real remnants could in principle be detected through their gravitational field. Turning the logic around, this is yet another reason to believe that remnants are real, rather than fictitious: gravity seems inconsistent in the absence of energy conservation.

In any case, since aside from energy conservation these models of information loss have the same features and drawbacks whether or not the remnants are fictitious, it makes sense to drop the extra assumption of energy nonconservation and promote the remnants to reality. This will be done in the remainder of the paper, although readers who prefer energy nonconservation may just as well imagine the remnants fictitious.

IV. A REMNANT PARADIGM

The preceding section has outlined a close relationship between remnants and information loss. $\mathcal{S}$ matrices can be thought of as arising from coupling to infinite fictitious remnant species at infinite temperature. More tame alternatives arise from a different infinite spectrum of remnants, in its vacuum. Although information loss is then in a sense more palatable (and in a sense indistinguishable from remnants), it still suffers the serious flaw corresponding to infinite remnant production.

Information loss or remnants can be saved, and the black hole information paradox solved, if an escape from this problem can be found. Although an ironclad escape has not yet been found, rather strong suggestions arise by considering extremal Reissner-Nordström black holes.

If information is not returned in Hawking evaporation, then there must be an infinite number of states of a Reissner-Nordström black hole of charge Q. These are formed by starting with any given extremal black hole state, dumping in matter carrying arbitrary information, and then allowing the black hole to evaporate back to extremality. Real Reissner-Norström black holes may well exist. Furthermore, we do not observe them to be infinitely pair produced. This indicates that they provide

[7] A related discussion appears in [22].

[8] Related effects will be discussed in subsequent sections.

an excellent arena to investigate the information paradox.[9]

Indeed, if information is not returned in Hawking radiation, then Reissner-Nordström black holes appear to give an existence proof for objects with all the desirable properties of remnants. Reissner-Nordström black holes will therefore be taken as a paradigm for a viable theory of information loss and/or remnants. We will seek to understand their essential properties that allow them to fit this role.

V. EFFECTIVE THEORIES FOR REMNANTS

In order to discuss issues of pair production and other effects of remnants it is useful to have a general framework in which to describe them. This section will take steps toward constructing an effective theory for remnants, and in particular will attempt to infer its general properties, if such a theory exists.

Remnants and their interactions should be localized in spacetime. Furthermore, a theory of remnants should also be Lorentz invariant at long distances. The only known (and possibly the only existing) way of reconciling locality, causality, and Lorentz invariance in a quantum framework is quantum field theory. Therefore at distances large as compared to the remnants and any of their interaction time scales, they should be described by a field

$$I_A(x)=\int \frac{d^3k}{(2\pi)^3 2\omega_k}[I_A(k)e^{ik\cdot x}+I_A^\dagger(k)e^{-ik\cdot x}] , \qquad (5.1)$$

where A labels the different remnant state. For simplicity we have assumed that the remnants do not carry spin, although this could be generalized. The action governing free propagation of a remnant should then be of the form

$$S_K=\int d^4x \sum_A [-\tfrac{1}{2}(\partial I_A)^2-\tfrac{1}{2}m_A^2 I_A^2] . \qquad (5.2)$$

Remnants also have couplings to the electromagnetic, gravitational, and other fields. To investigate their form, return to the case of Reissner-Nordström black holes of charge[10] $Q \gg 1$, which we will represent by complex fields I_A. First consider on which scales the dynamics can be described by effective interactions.

To begin with, recall that the infinite degeneracy originates in the fact that the extremal black hole could have been built out of matter with any initial mass $M>Q$, which is then allowed to evaporate. This gives an infinite number of possible initial states, and if information is not returned in Hawking radiation the resulting extremal hole has infinite states as well. The evaporation time is of order M^3, or very near extremality [16] Q^3. The resulting states may be truly degenerate or only nearly degenerate. Even if once the black hole nears extremality it by some mechanism begins to radiative information, the time required for all of it to escape is of order M^4, and the decay time between the states is of order M^2. Therefore for $M \gg Q$, on time scales $\gg Q^3$ and $\ll M^2$ we have essentially stationary configurations.

Now let us consider scattering electromagnetic radiation of frequency $\omega \ll 1/Q^3$ from the black hole. On time scales $\gg Q^3$ the process of absorption and reemission looks effectively pointlike, as indicated in Fig. 2. Therefore we would expect that it be summarized by an effective vertex operator at these scales. This vertex describes both the absorption of the incoming quantum and the reemission, by Hawking or other process, of the energy which leaves the black hole back at extremality. To simplify the notation we will assume the existence of a massless scalar field f and will consider only process in which the black hole adsorbs a photon and emits quanta of the scalar field. (This saves writing lots of indices but makes no essential change to the physics.) The effective vertex for such a process with n quanta emitted is of the form

$$\mathcal{A}^{\mu}_{(n)BA}(p,k,p_i)A_\mu(k)\prod_{i=1}^{n} f(p_i)I_B^\dagger(p)I_A(p') . \qquad (5.3)$$

In general this will include a minimal coupling to the electromagnetic field, although it is possible that $\mathcal{A}^{\mu}_{(0)BA}(p,k)=0$, that is the elastic scattering amplitude is zero.

These couplings will in general pair product remnants, e.g., via the analogue [23–25] of the Schwinger process for production of Reissner-Nordström black holes in a constant field. The problem of infinite production can be phrased as follows. Suppose that we consider two extremal black holes. Suppose that they both were constructed by starting with identical extremal holes, but that we have dropped the continent of Africa into one of them and then waited a time $\gg\gg Q^3$ for its energy to be reradiated and the black holes to settle down to states apparently identical from the outside. At first sight there is no obvious reason why there would not be equal production rates for these two types of black holes, and by

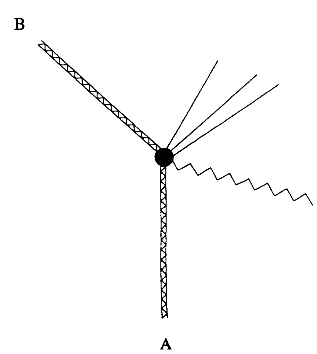

FIG. 2. Shown is a typical vertex for a photon interacting with a black hole. The photon is absorbed, but excites the black hole. The black hole then deexcites by emitting some quanta, for example, through Hawking radiation, leaving it in a different internal state.

[9]Previous advocates of this include [16].

[10]To eliminate concerns of discharge by pair production one may wish to consider magnetic charge, although the remaining discussion does not depend on this.

extension for infinitely many species. We might describe this by dividing the label A into two sets, a,α, where α parametrizes the infinite number of different states (e.g., corresponding to things that were done to the black hole in the far past) that do not give different vertices. This means that (5.3) becomes

$$\mathcal{A}^{\mu}_{(n)ba}(p,k,p_i)A_\mu(k)\prod_{i=1}^{n} f(p_i)I^{\dagger}_{b,\beta}(p)I_{a,\alpha}(p') \ . \qquad (5.4)$$

The infinite degeneracy in the states labeled by α gives the infinite production rate. In particular, note that if the coupling is dominated by the minimal term (i.e., the effective theory is weakly coupled as in [17]), then it is insensitive to the state and the production rate is infinite.

How is this problem to be avoided? Because of the difficulty in deriving the effective description of Reissner-Nordström black holes, a concrete proposal for the form of the couplings cannot yet be made. However, one can make some reasonable guesses as to what behavior is required and as to whether it emerges.

In particular, note that implicit in the argument that (5.4) is independent of α was the assumption that we are working on shell or very nearby, and with real momentum. Only in this context do the statements about irrelevance of modifications of the black hole in the far past apply. However, in calculating pair-production rates for process far below the Planck scale, one needs the couplings (5.3) off shell or at complex momenta. It is quite conceivable that in these regions α independence fails in a way that renders production finite.[11]

One motivation for this is the observation that remnants should involve Planck scale physics to describe them and their couplings to other fields. To see this consider forming one of our Reissner-Nordström remnants by throwing a large mass into a black hole and allowing it to evaporate. The remnant state is what is left; in other words the remnant can be described by taking the black hole plus Hawking radiation and acting on it by a collection of annihilation operators that eliminate the Hawking radiation. If the resulting state is evolved back in time, due to the absence of the Hawking radiation it gets very singular in the vicinity of what was the horizon.[12] The resulting strong coupling and large modification of the solution in the vicinity of the former horizon indicate that the true remnant eigenstates have large support on configurations where Planckian physics is important. This could well lead to the desired strong dependence of the remnant couplings on the momenta.

Note also that vanishing of the elastic scattering amplitudes, and thus of the minimal coupling to the electromagnetic field, seems to be required. This prevents a nonvanishing amplitude for Schwinger production with the internal state of the remnant unexcited; this would be accompanied by an overall infinite factor. It is quite plausible that the elastic amplitudes do indeed vanish. To see this, note that if we try to throw a photon of any energy at a black hole, it can be absorbed and in doing so excites the internal state above extremality. This is followed by Hawking radiation, for example of f particles. The dominance of these processes (as opposed to off-shell elastic scattering) at momenta where on-shell elastic scattering is not possible suggests that the elastic amplitudes could in fact be zero.

To illustrate these comments, consider the problem of the analogue of Schwinger pair production in this framework. The decay rate can be computed[13] from the functional integral

$$S_0[A_0^\mu]=\int \mathcal{D}\tilde{A}_\mu \mathcal{D}f\,\mathcal{D}I\, e^{iS[A_0^\mu+\tilde{A}^\mu,f,I_A]}\Big/\int \mathcal{D}\tilde{A}_\mu\mathcal{D}f\,\mathcal{D}I e^{iS[\tilde{A}^\mu,f,I_A]} \ , \qquad (5.5)$$

where the gauge field has been divided into background and fluctuation pieces. If V_4 is the four-volume in question, the rate is

$$V_4\Gamma=-2\,\mathrm{Re}\ln S_0[A_0^\mu] \ . \qquad (5.6)$$

In these expressions the action includes, in addition to the kinetic piece (5.2), coupling terms corresponding to the amplitudes (5.3). These take the position-space form

$$\sum_{AB}\sum_{n}\int d^4x\, d^4x' d^4y \prod_i^n d^4z_i f(z_i) I_B^*(x)\mathcal{A}^{\mu}_{n;BA}[x,x',y,z_i]I_A(x)A_\mu(y)\equiv\sum_{AB}\int d^4x\, d^4x' I_B^*(x)\mathcal{V}_{BA}[x,x';A^\mu,f]I_A(x') \ . \qquad (5.7)$$

In this equation $\mathcal{A}^{\mu}_{n;BA}$ also may contain derivatives acting on the fields, and in the first line we have suppressed couplings to multiple photon emission for simplicity.

The contribution of the (normalized) functional integral over I to (5.5) is

$$S_0[A,f]=\mathrm{Det}^{-1/2}\{[(-p^2-m_A^2+i\epsilon)\delta_{AB}+\mathcal{V}_{AB}]/(-p^2-m_A^2+i\epsilon)\} \ , \qquad (5.8)$$

[11]This is distinct from the suggestion [26,27] that the electromagnetic form factors vanish at large momentum transfer, since, for example, Schwinger production depends only on the form factors at small momentum transfers.

[12]This has been emphasized by Verlinde [7] in a different context.

[13]See, e.g., [28].

with the corresponding effective action w,

$$\mathrm{Im}\int d^4x\, w[A,f,x] = -2\,\mathrm{Re}\ln S_0[A,f] \ . \tag{5.9}$$

The latter can be rewritten

$$\mathrm{Im}\int d^4x\, w[A,f,x] = \int d^4x\,\mathrm{Re}\int_0^\infty \frac{ds}{s} e^{-s\epsilon} \sum_A \langle x, A | e^{-is[(p^2+m_A^2)\mathbf{1}-\mathcal{V}]} | x, A\rangle - \langle x, A | e^{-is(p^2+m_A^2)} | x, A\rangle \ , \tag{5.10}$$

or, in momentum space,

$$\mathrm{Im}\int d^4x\, w[A,f,x] = \int d^4x\,\mathrm{Re}\int_0^\infty \frac{ds}{s} e^{-s\epsilon} \int \frac{d^4p}{(2\pi)^4}\frac{d^4p'}{(2\pi)^4} e^{i(p-p')x} \sum_A \times \langle p', A | e^{-is[(p^2+m_A^2)\mathbf{1}-\mathcal{V}]} | p, A\rangle - \langle p', A | e^{-is(p^2+m_A^2)} | p, A\rangle \ . \tag{5.11}$$

If the vertex $\mathcal{V}$ corresponded merely to minimal coupling, then (5.11) can be evaluated by continuation into the complex plane. The answer arises from a sum of terms at Euclidean momenta that correspond to the Schwinger instantons, which are the Euclidean orbits in the background field. This result is then accompanied by an infinite factor from the sum over remnant states. In the example of grand unified theory (GUT) monopole production [29,30], $\mathcal{V}$ picks up corrections due to the structure of the monopole. However, these are supressed by powers of $1/M$, where $M\sim 1/R_{\rm monopole}$ is the scale for monopole excitations. In the limit of weak background fields, the contributions of these to the low-lying instantons will be suppressed by powers of eE/M. However, with the Reissner-Nordström paradigm for remnants there is no mass gap. One can no longer make the argument that the contributions of the nonminimal couplings are small, and as suggested above they may in fact be dominant. Continuation into the complex plane is no longer guaranteed to produce the Schwinger saddlepoint. Although an explicit example of such couplings is lacking, it is quite possible that they strongly depend on the state label A. With such a dependence it is possible to suppress infinite production.

Clearly it would be desirable to derive the effective couplings, both on and off shell and at complex momenta, between external fields and Reissner-Nordström black holes. This is a very difficult task. An important check to make is that interaction with a black hole must necessarily excite it; otherwise the minimal coupling is nonvanishing and infinite production likely results [17,31]. It may also be true that there is no standard effective field theory that describes such couplings. In any case, in the absence of knowing a detailed effective theory, one seeks other means to attack the pair production problem. Another approach is the study of gravitational instantons describing the production process.

VI. PAIR PRODUCTION VIA INSTANTONS

An analogue of the Schwinger process for the pair production of Reissner-Nordström black holes in a background field is described [23,24] by the Euclidean version of the Ernst metric [32]. The black hole produced by this instanton is near extremal; in fact it is just far enough above extremality so that its Hawking temperature matches its acceleration temperature. The states produced are thus in equilibrium with the Unruh radiation. The action for this metric is finite and has been computed [25]; as expected it is of the form $S_E=-\pi Q/B+O(Q^2)$. The first term gives Schwinger's rate, and the second term contains a contribution that is precisely the black hole entropy and suggests that the number of states being produced is $\exp\{S\}$.

However, it is clear that this is not the complete story. In particular, subleading corrections to the production rate also come from the fluctuation determinant, and this might be expected to incorporate the infinite number of states of the black hole.

Computing the full fluctuation determinant for arbitrary gravitational, electromagnetic, and other excitations about the instanton is a difficult problem. However, two simplifications can be made while retaining the essential flavor of the calculation. First, we will consider fluctuations only in the spectator field f. Second, we are clearly interested in fluctuations only near the horizon. In the limit of small external field, the instanton becomes effectively two dimensional in a large neighborhood of the horizon and at energies $\lesssim 1/Q$. We can thus see the essential issues by considering the low-energy states, that is, reducing to the s-wave sector so the problem is purely two dimensional.

To be more explicit, in the small B limit the Euclidean Ernst metric in the vicinity of the horizon takes the form

$$ds^2=Q^2(\sinh^2 y\, dt^2+dy^2+d\theta^2+\sin^2\theta\, d\phi^2) \ . \tag{6.1}$$

The y,t part is a solution to the reduced action

$$\frac{1}{2\pi}\int d^2\sigma\sqrt{g}\,\{e^{-2\phi}[R+2(\nabla\phi)^2]+2-2Q^2e^{2\phi}\} \ , \tag{6.2}$$

where $e^{-\phi}$ is the two-sphere radius. The fluctuations of s-wave part of the f field will be weighted using

$$S_f=\frac{1}{2}\int d^2\sigma\sqrt{g}\,(\nabla f)^2 \ , \tag{6.3}$$

where f has been rescaled by a factor proportional to $1/Q$.

For big black holes we expect to be able to work in the semiclassical limit and consider such fluctuations on the fixed background. Consider first quantizing them in the canonical framework. This is most easily done by introducing the tortoise coordinate r_* in terms of which the two-metric is conformally flat:

$$ds^2 = \frac{Q^2}{\sinh^2 r_*}(dt^2 + dr_*^2) . \tag{6.4}$$

Then the action (6.3) takes the flat-space form

$$S_f = \frac{1}{2}\int d^2\sigma_* [(\partial_t f)^2 + (\partial_* f^2)] . \tag{6.5}$$

The fluctuations are the usual left- and right-moving flat space modes, and can be quantized by introducing the standard flat space inner product and conjugate momentum. The infinite number of states arises from the infinite volume of r_*. Transition amplitudes can alternatively be converted into functional integrals by the standard procedure. Throughout only the flat metric $ds_*^2 = g_{*ab} d\sigma^a d\sigma^b = dt^2 + dr_*^2$ enters, and therefore the functional integral takes the form

$$\int \mathcal{D}_{g*} f e^{-S_f} . \tag{6.6}$$

As has been explicitly indicated, since the quantization depends only on g_* one obtains the measure regulated with respect to g_*. Equation (6.6) is infinite due to the infinite volume in g_*. This could be interpreted as yielding the infinite factor in the pair production rate.

Note, however, that there is another quantization of the fluctuations that gives a *finite* answer. This arises if one starts with the functional integral, but now regulated with respect to the Euclidean continuation of the physical metric,

$$\int \mathcal{D}_g f e^{-S_f} . \tag{6.7}$$

The volume near the horizon as measured in the metric g is finite, and the divergent factor has been eliminated.

Which answer is correct: (6.6) or (6.7)? Note that the difference between them is simply a conformal rescaling of the metric, $g_* = \exp\{2\rho\} g$. Conformal invariance of the action means that this only affects the regulator. Since we are working in the two-dimensional limit we can explicitly exhibit the difference between the functional integrals in terms of the Liouville action:

$$\int \mathcal{D}_{g*} f e^{-S_f} = e^{S_L} \int \mathcal{D}_g f e^{-S_f} , \tag{6.8}$$

with

$$S_L = \frac{1}{24\pi}\int d^2\sigma \sqrt{g}\, [(\nabla\rho)^2 + R\rho] . \tag{6.9}$$

The difference in stress tensors can likewise be computed:

$$\begin{aligned} T_{z\bar{z}} &= T_{*z\bar{z}} + \tfrac{1}{12}\partial_z \partial_{\bar{z}} \rho , \\ T_{zz} &= T_{*zz} - \tfrac{1}{12}[\partial_z^2 \rho + (\partial_z \rho)^2] . \end{aligned} \tag{6.10}$$

The stress tensor T corresponds to Hawking radiation in the Hartle-Hawking state. The difference between this and T_* gives a divergent proper flux at the horizon, as in the difference with the Boulware vacuum. Similar behavior is expected to occur more generally whenever there is a horizon.

The former corresponds to cutting off the fluctuations using a cutoff in Kruskal momentum. If on the other hand fluctuations in the remaining infinite number of states are allowed, they make an infinite contribution to the stress tensor near the horizon. In this case the back reaction on the metric becomes large and the semiclassical approximation breaks down. This means that in fact we had no right using the instanton to compute the production rate for an arbitrary state among the infinite number of possible states in the first place. It is not possible to tell if the total production rate is finite or not—to do so requires a more in-depth calculation. Because of the apparent divergence in the stress tensor this could well involve Planck physics.

We therefore cannot yet draw a concrete conclusion about the production rate. We can however see that the instanton calculation breaks down in a way that suggests relevance of strong coupling physics in the vicinity of what was the horizon. This dovetails nicely with the observations made in the preceding section; it is quite possible that this corresponds to couplings to external fields that are very different from their on-shell, real momentum values. (It alternately might indicate a breakdown of the effective approach.) This suggests that such a mechanism may prevent infinite production of Reissner-Nordström black holes. And if such a mechanism works for Reissner-Nordström black holes, one may conjecture that there exist other remnant models with the same properties.

VII. COMPARISON TO THE DILATONIC CASE

Other proposals for remnants with finite production have been made; notable is the suggestion that extremal dilatonic black holes provide a model for remnants [4,26,33], and that they have finite production rates [26,27,34]. The explanation proposed in [34] for finite production is distinct from that proposed here. In particular, [34] reasons that the rate is finite because the approximate Euclidean instanton describing pair production has finite volume, and thus corresponds to production of a finite number of states. However, it has subsequently been found that there are instantons corresponding to production of infinite volume dilatonic black holes [35,36].

Furthermore, note that merely finite volume is not necessarily sufficient to guarantee finite production. This can be illustrated with the case of Reissner-Nordström black holes, which also have an infinite spatial volume at extremality. This is not, however, the origin of the infinite number of presumed states; slightly above extremality they only have finite spatial volume, but should still have infinitely many states. In particular, since the Ernst instantons of [32,23,24] produce Reissner-Nordström black holes slightly above extremality, they have finite volume, but this should not necessarily mean that there can be only finitely many states produced. The infinite number of states are described by including fluctuations about the instanton, and arise from the infinite volume in r_*. (Reference [34] argues that finite volume means that there are only finitely many states.) However, as argued above, the fluctuations describing the production of the infinite states destroy the instanton, and so a definite conclusion cannot be drawn; production could well be supressed. The role of the massless excitations and their back reaction is thus essential. The finiteness of the volume alone does not directly imply a finite rate. It should be noted that [34] also suggested the idea that any attempt to accelerate dilatonic black holes would excite them, and advocated the view that effective field theories are therefore not useful in describing them. However, it is not clear that this happens in the dilatonic case. If one ignores the s-wave fermions, the excitation spectrum about the throat has a gap [37,33]. Therefore in the absence of fermions it is plausible that one could accelerate one of these objects without excitation. However, a detailed study of this problem is difficult due to infinite growth of the coupling in the vicinity of the black hole; as a whole the proposal also founders on the rocky shoals of strong coupling.

VIII. DISCUSSION

Remnants and certain types of information loss have been argued to be different views of the same scenario; if the remnants are truly invisible then information is lost. This suggests versions of information loss that do not violently heat the low-energy world. However, these suffer the same difficulties as real remnants, namely, the problem of infinite loss of energy to the Hilbert space of remnants.

Reissner-Nordström black holes suggest a possible paradigm for a successful remnant theory. Assuming that information is not reemitted in Hawking radiation, they should have infinitely many states yet they are not observed to be infinitely produced. This paper has made an attempt to understand how this can happen. In particular, it is suggested that the couplings of these to external fields are very nonminimal, and could depend sensitively on the internal state of the remnant. Such dependence is essential to eliminate infinite production. Unfortunately a detailed model of such dependence has not yet been found, although it is strongly suggested by the instanton calculations. Therefore this paper only represents a suggestion of how the infinite production problem might be solved.

Since Reissner-Nordström black holes do appear to offer an example of a theory in which objects have a infinite number of internal states, yet are not infinitely produced, then it is easy to imagine abstracting the essential features to a theory of Planck-scale remnants for neutral black holes. The possibility of there existing such a theory solves the black hole information paradox. It would of course still be extremely interesting to explore how one could get such a theory of remnants out of quantum gravity.

Other issues that should be confronted if such a theory is to solve the information conundrum are those of CPT and black hole thermodynamics. In particular, in the former context the Reissner-Nordström paradigm seems to suggest that there should be both "white" and "black" remnants which are CP conjugates. One would also like to understand the connection between remnants and the second law of black hole thermodynamics. If information is not returned in Hawking radiation it is difficult to interpret the Bekenstein-Hawking entropy as corresponding to the number of states inside a black hole. Another possibility is that the entropy indicates the amount of information that can be lost to the internal remnant state during the course of formation and evaporation of a black hole from an initial mass M. In this context the real entropy of the black hole is much larger than given by the Bekenstein-Hawking formula, as the hole could have been formed by evaporation from a much larger hole. Furthermore, apparent violations of the second law could be imagined from dropping such small black holes into big ones. Perhaps the second law is only valid in a limited domain, and the Bekenstein-Hawking entropy places bounds on information transfer rather than information content.

ACKNOWLEDGMENTS

This work was supported in part by DOE Grant No. DOE-91ER40618 and by NSF PYI Grant No. PHY-9157463. I wish to thank T. Banks, K. Kuchař, S. Hawking, G. Horowitz, D. Lowe, J. Polchinski, A. Strominger, L. Susskind, and S. Trivedi for helpful conversations. I would particularly like to thank M. Srednicki for valuable suggestions.

[1] S. B. Giddings, Phys. Rev. D **46**, 1347 (1992).

[2] J. Preskill, "Do black holes destroy information?" Caltech Report No. CALT-68-1819, hep-th/9209058, 1992 (unpublished).

[3] S. B. Giddings, "Black holes and quantum predictability," UCSB Report No. UCSBTH-93-16, hep-th/9306041, to appear in the [Proceedings of the 7th Nishinomiya-Yukawa Memorial Symposium, edited by K. Kikkawa and M. Ninomiya (in press)].

[4] C. G. Callan, S. B. Giddings, J. A. Harvey, and A. Strominger, Phys. Rev. D **45**, R1005 (1992).

[5] S. B. Giddings and W. M. Nelson, Phys. Rev. D **46**, 2486 (1992).

[6] D. Page, Phys. Rev. Lett. **71**, 3714 (1993).

[7] K. Schoutens, H. Verlinde, and E. Verlinde, Phys. Rev. D **48**, 2670 (1993). H. Verlinde, Talk at 1993 ITP Conference, Quantum Aspects of Black Holes (unpublished).

[8] A. Bilal and C. Callan, Nucl. Phys. **B394**, 73 (1993).

[9] S. P. deAlwis, Phys. Lett. B **289**, 278 (1992); *ibid.* **300**, 330 (1993).

[10] J. G. Russo, L. Susskind, and L. Thorlacius, Phys. Rev. D **46**, 3444 (1992).

[11] G. 't Hooft, in *Quantum Coherence,* edited by J. S. Anandan (World Scientific, Singapore, 1990), and references therein; Talk at 1993 ITP Conference, Quantum Aspects of Black Holes (unpublished); and (private communication).

[12] L. Susskind, Phys. Rev. Lett. **71**, 2367 (1993); L. Susskind

and L. Thorlacius, Phys. Rev. D **49**, 966 (1994); L. Susskind, "Strings, black holes and Lorentz contraction," Stanford Report No. SU-ITP-93-21, hep-th/930813, 1993 (unpublished).

[13] E. Martinec, "The light cone in string theory," Chicago Report No. EFI-93-21, hep-th/9304037, 1993 (unpublished).

[14] S. W. Hawking, Commun. Math. Phys. **87**, 395 (1982).

[15] T. Banks, M. E. Peskin, and L. Susskind, Nucl. Phys. **B244**, 125 (1984).

[16] A. Strominger and S. Trivedi, Phys. Rev. D **48**, 5778 (1993).

[17] S. B. Giddings, Phys. Rev. D **49**, 947 (1994).

[18] T. Banks, Nucl. Phys. **B249**, 332 (1985).

[19] J. B. Hartle, Phys. Rev. D **38**, 2985 (1988).

[20] L. Liu, "Evolution of pure states into mixed states," Stanford Report No. SU-ITP-93-1, hep-th/9301082, 1993 (unpublished).

[21] S. B. Giddings (unpublished).

[22] L. Alvarez-Gaumé and C. Gomez, Commun. Math. Phys. **89**, 235 (1983).

[23] G. W. Gibbons, in *Fields and Geometry 1986,* Proceedings of 22nd Karpacz Winter School of Theoretical Physics, Karpacz, Poland, 1986, edited by A. Jadczyk (World Scientific, Singapore, 1986).

[24] D. Garfinkle and A. Strominger, Phys. Lett. B **256**, 146 (1991).

[25] D. Garfinkle, S. B. Giddings, and A. Strominger, Phys. Rev. D **49**, 958 (1994).

[26] T. Banks, A. Dabholkar, M. R. Douglas, and M. O'Loughlin, Phys. Rev. D **45**, 3607 (1992).

[27] T. Banks and M. O'Loughlin, Phys. Rev. D **47**, 540 (1993).

[28] C. Itzykson and J.-B. Zuber, *Quantum Field Theory* (McGraw-Hill, New York, 1980).

[29] I. K. Affleck and N. S. Manton, Nucl. Phys. **B194**, 38 (1982).

[30] I. K. Affleck, O. Alvarez, and N. S. Manton, Nucl. Phys. **B197**, 509 (1982).

[31] M. Srednicki (private communication).

[32] F. J. Ernst, J. Math. Phys. **17**, 515 (1976).

[33] S. B. Giddings and A. Strominger, Phys. Rev. D **46**, 627 (1992).

[34] T. Banks, M. O'Loughlin, and A. Strominger, Phys. Rev. D **47**, 4976 (1993).

[35] F. Dowker, J. P. Gauntlett, D. Kastor, and J. Traschen, "Pair creation of dilaton black holes," Chicago, Report No. EFI-93-51, hep-th/9309075 [Phys. Rev. D (to be published)].

[36] F. Dowker, J. P. Gauntlett, S. B. Giddings, and G. Horowitz, "On pair creation of extremal black holes and Kalvza Klein Monopoles," UCSB Report No. UCSBTH-93-38, hep-th/9312172.

[37] C. F. E. Holzhey and F. Wilczek, Nucl. Phys. **B380**, 447 (1992).