Localized discussion of stimulated processes for Rindler observers and accelerated detectors

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We investigate the implications of the presence of Minkowski particles on the Rindler particle content in the accelerated frame as well as on the excitation and deexcitation of an accelerated particle detector. To obtain localized statements, the field quantization is based on wave packets. For bosonic particles, nonempty Minkowski modes imply an amplification of the number of Rindler particles in specific modes. The underlying processes reflect a nonlocal pair structure: specific pairs of modes with trajectories of the wave packet maxima passing different Rindler wedges are correlated. An elementary object of quantum optics in noninertial situations is the accelerated detector. The richer structure of the physics of its excitation and deexcitation is studied in detail. In addition to the generalizations of the inertially known excitation and deexcitation processes there are structurally new processes that are inertially forbidden. These processes reflect the nonlocal pair correlations.

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I. INTRODUCTION

It has been known since the work of Fulling [1] and Unruh [2] that the Minkowski vacuum state appears to a uniformly accelerated observer as being occupied with a thermal distribution of particles called Rindler particles. This has been the subject of a considerable amount of research. For reviews see [3–6]. The effect has also been confirmed theoretically on an operational basis by the investigation of the behavior of uniformly accelerated particle detectors [2,7]. It was found that an accelerated detector becomes spontaneously excited while moving through the Minkowski vacuum, a result called the *Unruh effect.* This effect has been analyzed in more detail by Unruh and Wald [8,9]. They concluded that the detector emits a Minkowski particle when it is spontaneously excited. They also noticed that this emitted particle is to some extent disconnected from the emitting detector. Another remarkable trait of the quantum field theory in a uniformly accelerated frame of reference is that the Minkowski vacuum appears as an entangled state of Rindler particles with nonlocal Einstein-Podolsky-Rosen-(EPR-) type correlations. This has been already found by Unruh [2] and has been discussed more explicitly in Refs. [10–12,5].

Currently there is a debate going on whether a uniformly accelerated particle detector radiates. The authors of Refs. [13–17] show that the expectation value of the stress-energy tensor of the quantum field is not increased by the presence of the detector. This result, which is seemingly at variance with the particle emission process mentioned above, has led to a discussion about the interpretation of the stress-energy tensor for a uniformly accelerated detector. We will not want to deal with the point in the present paper. A discussion of the related problems will be given in a separate paper [18].

In the context of particle creation caused by the expansion or contraction of Robertson-Walker universes it has been observed by Parker [19] that the presence of real particles tends to increase the number of created particle pairs for bosons and to decrease it for fermions. This effect has been called *gravitational amplification* or *attenuation.* It plays an important role for interacting quantum fields in Robertson-Walker universes, as has been demonstrated by Audretsch and Spangehl [20]. For a Schwarzschild black hole, Wald has shown in [21] that if bosonic particles or antiparticles are sent in during the collapse to a Schwarzschild black hole, they induce an additional creation of particle-antiparticle pairs out of the curved background. The localization of this stimulated amplification of the thermal Hawking radiation has been studied in detail by Audretsch and Müller [22] using wave packet modes.

In this paper, it is our aim to investigate the implications of the presence of Minkowski particles on the Rindler particle content in the accelerated frame on one hand and on the excitation and deexcitation of a uniformly accelerated particle detector on the other. To obtain a localization of the respective processes, we base in Sec. II the quantization of a massless bosonic field in Minkowski and in Rindler spacetime on wave packet modes and discuss the related Bogoliubov transformation. We show in Sec. III that the concept of amplification is also realized in the framework of Rindler quantum field theory. Based on the wave packet localization we are able to work out the spatio-temporal relations between the nonempty Minkowski modes and the amplification (as compared with the Minkowski vacuum state) of the particle number expectation value in specific Rindler modes. The concepts of equivalent and mirror modes and their trajectories become important, reflecting a certain pair structure in the underlying physical processes. This will directly lead to a discussion of the nonlocal correlations and their modifications caused by the presence of

Minkowski particles.

We mention that nonempty Minkowski states have been treated previously by Padmanabhan and Singh [23] and by Mishima and Nakayama [24]. They did however not give a localization of the processes or an interpretation in terms of amplification.

In Sec. IV, we turn to the excitation and deexcitation of uniformly accelerated detectors and show that their response is consistent with an interpretation in terms of amplification. We generalize the work of Unruh and Wald [8] in including ingoing Minkowski particles and in referring for the ingoing and outgoing Minkowski particles to wave packet states with fixed energy and well-defined trajectory of their maxima. It turns out that the inertial concepts of spontaneous and stimulated excitation and deexcitation are characteristically generalized. For example, there is not only the process of spontaneous detector excitation with emission instead of absorption of a Minkowski particle, but also the possibility to stimulate this excitation by incident Minkowski particles. We regard the discussion of this and similar structurally new effects as a first step in the development of a quantum optics in noninertial situations.

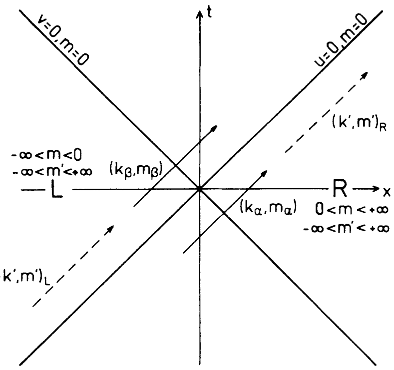

FIG. 1. Illustration of the trajectories of equivalent and mirror packet modes [Eqs. (12)–(14)] representing the pair structure: Solid lines denote Minkowski modes, dashed lines Rindler modes. Only right-moving modes ($k > 0$) are drawn. The Minkowski wave packet (k_α, m_α) is the equivalent mode to the right-wedge Rindler wave packet $(k', m')_R$. Correspondingly, (k_β, m_β) and $(-k', m')_L$ are equivalent modes. (k_α, m_α) and (k_β, m_β) are Minkowski mirror modes. $(k', m')_R$ and $(-k', m')_L$ are Rindler mirror modes.

II. WAVE PACKET QUANTIZATION IN RINDLER SPACE

A. Accelerated observer

Rindler coordinates (η', ξ') are related to Minkowski coordinates (t, x) by means of the transformation

$$t = \frac{1}{a} e^{a\xi'} \sinh a\eta', \qquad x = \frac{1}{a} e^{a\xi'} \cosh a\eta', \tag{1}$$

where $a = \text{const} > 0$ and $-\infty < \xi', \eta' < +\infty$. The coordinate lines $\xi' = \text{const}$ are the trajectories of uniformly accelerated observers with acceleration $ae^{-a\xi'}$. Throughout the paper, we shall work with the convention that Rindler quantities are marked by a prime whereas Minkowski quantities are unprimed. Without loss of generality we restrict ourselves for physical interpretations to the observer at $\xi' = 0$. We can define Rindler null coordinates $u' = \eta' - \xi'$ and $v' = \eta' + \xi'$. They are connected to the Minkowski null coordinates $u = t - x$ and $v = t + x$ by the transformation

$$u = -\frac{1}{a} e^{-au'}, \qquad v = \frac{1}{a} e^{av'}. \tag{2}$$

The Rindler coordinates (1) and (2) cover only the right wedge R in Fig. 1. The lines $u = 0$ and $v = 0$ play the role of event horizons for the accelerated observers (cf. Fig. 2).

The components of a momentum vector k_μ tangential to the null rays $u, v = \text{const}$ are (ω_k, k) in Minkowski coordinates (with $\omega_k = |k|$) and $(\omega_{k'}, k')$ in Rindler coordinates. The relation between k and k' can be obtained from the coordinate transformation (1):

$$k = \begin{cases} e^{au'} k' & \text{for } k' > 0, \\ e^{-av'} k' & \text{for } k' < 0. \end{cases} \tag{3}$$

The components in the two coordinate systems have an invariant interpretation: $\omega_{k'}$ and k' are the energy and momentum as measured by the accelerated observer at the intersection point of the null ray and observer trajectory [point P in Fig. 2 (a)], if the inertial observer measures energy ω_k and momentum k. The special relativistic relation (3) is of purely kinematical origin. It is called the *Doppler shift formula* and will play a role in the physical interpretation below.

We supply the left Rindler wedge L with a similar set of coordinates (η', ξ') in replacing (t, x) by $(-t, -x)$ in Eq. (1). The left wedge null coordinates $u' = \eta' - \xi'$ and $v' = \eta' + \xi'$ are obtained from Minkowski null coordinates by replacing (u, v) by $(-u, -v)$ in Eq. (2). In L the components of a null momentum vector transform according to

$$k = \begin{cases} -e^{-av'} k' & \text{for } k' > 0, \\ -e^{au'} k' & \text{for } k' < 0. \end{cases} \tag{4}$$

Note that in this case k and k' have different signs, which means that right-moving massless particles have $k' < 0$ in wedge L.

B. Wave packets

In order to quantize the real massless scalar field $\phi(x)$, we use a complete set of positive frequency plane wave solutions of the Klein-Gordon equation. The Minkowski mode functions are

$$f_k^M = \frac{1}{\sqrt{4\pi\omega_k}} e^{ikx - i\omega_k t} \tag{5}$$

with $\omega_k = |k|$. In the right and left Rindler wedge, we look for solutions that have the form of plane waves [3]:

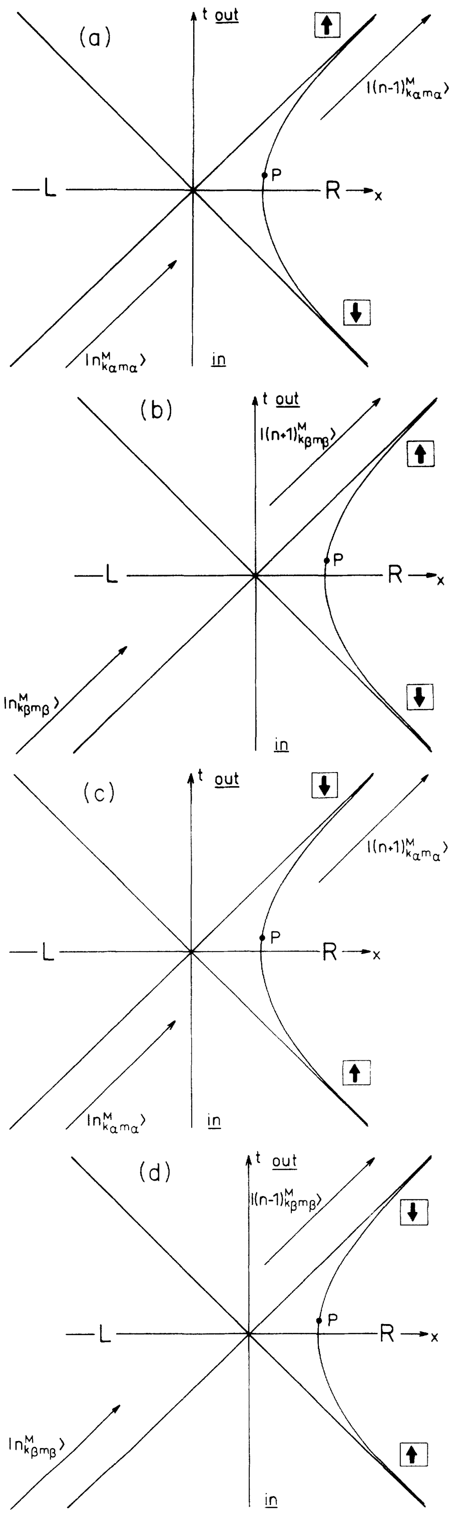

FIG. 2. Processes that can take place when the detector makes a transition: (a) detector excitation with absorption of a Minkowski particle from wedge R (inertially allowed), (b) detector excitation with emission of a Minkowski particle into wedge L (inertially forbidden), (c) detector deexcitation with emission of a Minkowski particle into wedge R (inertially allowed), (d) detector deexcitation with disappearance of a Minkowski particle from wedge L (inertially forbidden). Emission processes (b) and (c) can also occur spontaneously, independent of the presence of Minkowski particles.

$$p^R_{k'} = \frac{1}{\sqrt{4\pi\omega_{k'}}} e^{ik'\xi' - i\omega_{k'}\eta'},$$
$$p^L_{k'} = \frac{1}{\sqrt{4\pi\omega_{k'}}} e^{ik'\xi' + i\omega_{k'}\eta'}. \tag{6}$$

Both sets of Rindler mode functions are defined to be identically zero in the opposite wedge. They do not form a complete set on the whole spacetime, but only in the wedges R and L.

Because our intention is to localize the statements about the quantum processes associated with accelerated motion, we introduce wave packets as the basic mode functions for field quantization, as has been done for example in [22,5]. Following Hawking [25], we define *Minkowski wave packets* by a superposition of the plane waves (5) in a momentum interval ϵ:

$$f^M_{km}(t,x) = \frac{1}{\sqrt{\epsilon}} \int_k^{k+\epsilon} d\tilde{k}\, e^{-i\tilde{k}m} f^M_{\tilde{k}}(t,x). \tag{7}$$

For each wave packet, k/ϵ and $m\epsilon/2\pi$ are both fixed parameters which have to be integers ($-\infty < k, m < +\infty$). A particular Minkowski wave packet is specified by the set (k,m) of two packet parameters: The domain of integration in (7) shows that the momentum parameter k labels the packet energy ($\omega = |k|$) and indicates its direction of propagation. The trajectory parameter m fixes the null geodesic of the packet maximum, because, for a given m, the maximum is located at the value of u, v where the phase in the exponent of (7) becomes stationary. Using (5), we find (cf. Fig. 1)

$$u_{\max} = \text{const} = -m \quad \text{for } k > 0,$$
$$v_{\max} = \text{const} = +m \quad \text{for } k < 0. \tag{8}$$

To illustrate (7) we assume ϵ to be small. We may then consider ω_k as being constant. Integration of (7) with (5) shows that the wave packet has a form proportional to $\sin[\epsilon/2(x \mp t - m)]/(x \mp t - m)$ in this case.

By *trajectory* we will always mean the path of the packet maximum. We observe that a wave packet which passes wedge L always has $m < 0$, whereas one passing R has $m > 0$. The width in momentum of a wave packet is ϵ, while its width in space is $2\pi/\epsilon$. The wave packet modes are complete and orthonormal provided that the underlying set of mode functions is.

In the same way, we can define *Rindler wave packets* in the right and left wedges:

$$p^R_{k'm'} = \frac{1}{\sqrt{\epsilon'}} \int_{k'}^{k'+\epsilon'} d\tilde{k}' e^{-i\tilde{k}'m'} p^R_{\tilde{k}'},$$
$$p^L_{k'm'} = \frac{1}{\sqrt{\epsilon'}} \int_{k'}^{k'+\epsilon'} d\tilde{k}' e^{-i\tilde{k}'m'} p^L_{\tilde{k}'} \tag{9}$$

with $-\infty < k', m' < +\infty$. Again k' labels the propagation direction and energy as registered by the accelerated observer as described above, and m' specifies the trajectory. For the right wedge, the trajectories of the packet maxima are given by

$$\begin{aligned} u'_{\max} &= \text{const} = -m' \quad \text{for } k' > 0, \\ v'_{\max} &= \text{const} = +m' \quad \text{for } k' < 0, \end{aligned} \tag{10}$$

and for the left wedge by

$$\begin{aligned} v'_{\max} &= \text{const} = +m' \quad \text{for } k' > 0, \\ u'_{\max} &= \text{const} = -m' \quad \text{for } k' < 0. \end{aligned} \tag{11}$$

Certain pairs of Rindler and Minkowski packet modes will play a central role, representing the pair structure in the physical interpretation below. The following terminology will be useful: We define the *equivalent mode* of a right-wedge Rindler wave packet $(k',m')_R$ to be the Minkowski wave packet $(k_\alpha, m_\alpha > 0)$ traveling on the same null trajectory (see Fig. 1) with an energy obtained from the Doppler shift formula (3). We find, with (2), (8), and (10),

$$(k',m')_R \quad \overset{\text{equiv}}{\longleftrightarrow} \quad (k_\alpha, m_\alpha) := (e^{-am'}k',\ a^{-1}e^{am'}). \tag{12}$$

Furthermore we introduce to each Minkowski wave packet $(k_\alpha, m_\alpha > 0)$ an associated *Minkowski mirror mode* $(k_\beta, m_\beta < 0)$. It has the same energy, but its trajectory is obtained by "reflection" at one of the horizons $(m = 0)$ (see Fig. 1):

$$(k_\alpha, m_\alpha) \quad \overset{\text{mirror}}{\longleftrightarrow} \quad (k_\beta, m_\beta) := (k_\alpha, -m_\alpha). \tag{13}$$

Correspondingly, one can define the *Rindler mirror mode* of a Rindler wave packet. This is a Rindler wave packet of the same energy traveling in the same direction in the other wedge with a trajectory which is obtained by "reflection":

$$(k',m')_R \quad \overset{\text{mirror}}{\longleftrightarrow} \quad (-k',m')_L. \tag{14}$$

$(-k',m')_L$ is then the *equivalent mode* to (k_β, m_β). A subscript α is no tensor index but refers always to Minkowski wave packets passing the right wedge R, while β denotes Minkowski wave packets that cross the left wedge L:

$$m_\alpha > 0, \quad m_\beta < 0. \tag{15}$$

C. Wave packet Bogoliubov coefficients

The *quantization* is carried out with respect to the wave packet bases defined above. The expansion of the field operator in terms of Minkowski wave packets is

$$\phi(x) = \sum_{km} [b^M_{km} f^M_{km}(x) + b^{M\dagger}_{km} f^{M*}_{km}(x)], \tag{16}$$

defining Minkowski packet creation and annihilation operators $b^{M\dagger}_{km}$ and b^M_{km}. The Minkowski vacuum state is given by $b^M_{km}|0^M\rangle = 0$ for all values of (k,m). The expansion in terms of Rindler wave packets yields

$$\phi(x) = \sum_{k'm'} \Bigg(a^R_{k'm'} p^R_{k'm'}(x) + a^{R\dagger}_{k'm'} p^{R*}_{k'm'}(x) + a^L_{k'm'} p^L_{k'm'}(x) + a^{L\dagger}_{k'm'} p^{L*}_{k'm'}(x) \Bigg), \tag{17}$$

which leads to Rindler packet creation and annihilation operators $a^{L\dagger}_{k'm'}$, $a^L_{k'm'}$ for the left wedge, and $a^{R\dagger}_{k'm'}$, $a^R_{k'm'}$ for the right wedge.

We can gain insight into the connection between the two quantization schemes with the help of a Bogoliubov transformation. We use the general definitions and relations of Ref. [22] for a spacetime with horizon. The *wave packet Bogoliubov coefficients* are defined by

$$\alpha_{k'm'km} = (p^R_{k'm'}, f^M_{km}), \qquad \beta_{k'm'km} = -(p^R_{k'm'}, f^{M*}_{km}), \tag{18}$$

$$\gamma_{k'm'km} = (p^L_{k'm'}, f^M_{km}), \qquad \eta_{k'm'km} = -(p^L_{k'm'}, f^{M*}_{km}),$$

where the brackets denote the Klein-Gordon inner product. They can be reduced to the plane wave Bogoliubov coefficients

$$\left.\begin{matrix}\alpha_{k'm'km} \\ \beta_{k'm'km}\end{matrix}\right\} = \frac{1}{\sqrt{\epsilon\epsilon'}} \int_{k'}^{k'+\epsilon'} d\tilde{k}' \int_{k}^{k+\epsilon} d\tilde{k}\, e^{-i\tilde{k}'m' \pm ikm} \times \begin{pmatrix}\alpha_{\tilde{k}'\tilde{k}} \\ \beta_{\tilde{k}'\tilde{k}}\end{pmatrix}. \tag{19}$$

The latter can be calculated from the mode functions (5) and (6) by direct integration of the Klein-Gordon product. We obtain

$$\left.\begin{matrix}\alpha_{k'k} \\ \beta_{k'k}\end{matrix}\right\} = \mp\frac{i}{2\pi}(\omega_k\omega_{k'})^{-\frac{1}{2}} \exp\left(-i\frac{k'}{a}\ln\frac{\omega_k}{a}\right) \times e^{\pm\omega_{k'}\pi/2a}\,\Gamma\left(1 + i\frac{k'}{a}\right)\chi_{k'k} \tag{20}$$

with

$$\chi_{k'k} := \begin{cases} 1 & \text{if } k > 0, k' > 0 \\ 0 & \text{if } \operatorname{sgn}(k') \neq \operatorname{sgn}(\mathrm{k}) \\ -1 & \text{if } k < 0, k' < 0. \end{cases} \tag{21}$$

The mathematical structure of (20) is very similar to that of the Bogoliubov coefficients for the collapse to a black hole. Therefore the wave packet analysis can be carried out along similar lines as in [22].

For $|k'| \gtrsim a$, we can use Stirling's formula to approximate the phase of the gamma function. If we assume furthermore ϵ' to be small, the nonoscillating terms in the formula (19) can be removed from the integral, yielding

$$\left.\begin{matrix}\alpha_{k'm'km} \\ \beta_{k'm'km}\end{matrix}\right\} = \mp i\left(\frac{a}{2\pi\omega_{k'}}\right)^{\frac{1}{2}} e^{\pm\omega_{k'}\pi/2a}\left|\Gamma\left(1 + i\frac{k'}{a}\right)\right| \times \mathcal{G}_{\alpha/\beta}(k,m,k',m'). \tag{22}$$

Here, the functions $\mathcal{G}_{\alpha/\beta}(k,m,k',m')$ are given by

$$\mathcal{G}_{\alpha/\beta}(k,m,k',m') := \frac{1}{\sqrt{2\pi a\epsilon\epsilon'}} e^{i\frac{\pi}{4}\text{sgn}(k')} \int_k^{k+\epsilon} d\tilde{k}\, \omega_{\tilde{k}}^{-\frac{1}{2}} e^{\pm i\tilde{k}m} \times \int_{k'}^{k'+\epsilon'} d\tilde{k}' \exp\left[-i\tilde{k}'\left(m' + \frac{1}{a} + \frac{1}{a}\ln\frac{\omega_{\tilde{k}}}{\omega_{\tilde{k}'}}\right)\right] \chi_{\tilde{k}\tilde{k}'}. \tag{23}$$

Following the procedure of Ref. [22], it can be shown that

$$\sum_{km} |\mathcal{G}_{\alpha/\beta}|^2 = 1. \tag{24}$$

Analogously one can find, using the same approximations, the Bogoliubov coefficients for the left wedge. They can be obtained from (22) with (23) by substituting γ for α, η for β, and replacing $\chi_{k'k}$ by

$$\zeta_{k'k} := \begin{cases} -1 & \text{if } k>0, k'<0 \\ 0 & \text{if sgn}(k') = \text{sgn(k)} \\ 1 & \text{if } k<0, k'>0. \end{cases} \tag{25}$$

An analysis of the functions $\mathcal{G}_{\alpha/\beta}(k,m,k',m')$ based on the method of stationary phase for the integrals in (23) shows that they act as localizing functions. They are nonzero essentially only for certain parameter combinations, thus relating as in Fig. 1 the Rindler wave packet $(k',m')_R$ in R with a specific Minkowski wave packet (k,m): $\mathcal{G}_\alpha$ connects $(k',m')_R$ with the equivalent Minkowski packet (k_α, m_α), whereas $\mathcal{G}_\beta$ connects $(k',m')_R$ with the mirror mode (k_β, m_β) of its equivalent Minkowski mode.

Although we could use the full Bogoliubov coefficients (22) in the subsequent calculations, we can point out clearer their physical structure if we take this observation as a basis for an additional approximation. We write $\mathcal{G}_\alpha(k,m,k',m') \approx \delta_{kk_\alpha}\delta_{mm_\alpha}$, where $k_\alpha = k_\alpha(k',m')$ and $m_\alpha = m_\alpha(m')$ by virtue of (12). The Bogoliubov coefficients simplify to

$$\alpha_{k'm'km} \approx \alpha_{k'}\, \delta_{kk_\alpha}\delta_{mm_\alpha}, \qquad \beta_{k'm'km} \approx \beta_{k'}\, \delta_{kk_\beta}\delta_{mm_\beta}, \tag{26}$$

with (12) and (13). This leads to the Bogoliubov transformation

$$a^R_{k'm'} \approx \alpha^*_{k'} b^M_{k_\alpha m_\alpha} - \beta^*_{k'} b^{M\dagger}_{k_\beta m_\beta}. \tag{27}$$

In an analogous way, we investigate the functions $\mathcal{G}_\gamma$ and $\mathcal{G}_\eta$ to obtain

$$\gamma_{k'm'km} \approx \gamma_{k'}\, \delta_{kk_\gamma}\delta_{mm_\gamma}, \qquad \eta_{k'm'km} \approx \eta_{k'}\, \delta_{kk_\eta}\delta_{mm_\eta}. \tag{28}$$

Only the equivalent mode $(k_\gamma, m_\gamma) = (-e^{-am'}k', -a^{-1}e^{am'})$ to $(k',m')_L$ and its mirror mode $(k_\eta, m_\eta) = (-e^{-am'}k', a^{-1}e^{am'})$ contribute to the Bogoliubov coefficients (28).

The Bogoliubov coefficients (26)–(28) have a diagonal form. Within the approximation they connect, as can be read off from (27), via the respective creation and annihilation operators a Rindler wave packet $(k',m')_R$ with one particular pair of Minkowski wave packets consisting of the equivalent mode (k_α, m_α) and its mirror mode (k_β, m_β). Both are fixed with regard to the trajectories and the energies, which reflect the Doppler shift formula. There is a corresponding equation relating a Minkowski wave packet with a pair of Rindler wave packets (equivalent mode and its mirror mode):

$$b^M_{k_\beta m_\beta} \approx \gamma_{k'}\, a^L_{-k'm'} + \beta^*_{k'}\, a^{R\dagger}_{k'm'}, \tag{29}$$

where again (12) and (13) have been used. This *pair structure* will be of crucial importance for the discussion of the physical implications.

The *Bogoliubov parameters* $\alpha_{k'}, \beta_{k'}$ in (26) result from Eq. (22), where the value of $\mathcal{G}_{\alpha/\beta}$ is taken to be 1 because of (24):

$$\left.\begin{matrix}\alpha_{k'} \\ \gamma_{k'}\end{matrix}\right\} = \mp i \left(\frac{a}{2\pi\omega_{k'}}\right)^{\frac{1}{2}} e^{\omega_{k'}\pi/2a} \left|\Gamma\left(1 + i\frac{k'}{a}\right)\right|, \tag{30}$$

$$\left.\begin{matrix}\beta_{k'} \\ \eta_{k'}\end{matrix}\right\} = \pm i \left(\frac{a}{2\pi\omega_{k'}}\right)^{\frac{1}{2}} e^{-\omega_{k'}\pi/2a} \left|\Gamma\left(1 + i\frac{k'}{a}\right)\right|. \tag{31}$$

Their squares

$$|\alpha_{k'}|^2 = |\gamma_{k'}|^2 = 1 + \frac{1}{e^{2\pi\omega_{k'}/a} - 1}, \qquad |\beta_{k'}|^2 = |\eta_{k'}|^2 = \frac{1}{e^{2\pi\omega_{k'}/a} - 1}, \tag{32}$$

obey the general relation $|\alpha_{k'}|^2 - |\beta_{k'}|^2 = 1$.

III. AMPLIFICATION OF THE RINDLER PARTICLE CONTENT

A. The vacuum effect and its amplification

With the concept *particle* we will refer in the following always to the packet state quantization above, thus restricting ourselves to bosons. In this section we will specifically be concerned with the Rindler particle content of nonempty Minkowski wave packet modes. Our first question is: To what extent does the presence of particles in specific Minkowski wave packet modes influence the expectation value of the Rindler particle number for a particular Rindler wave packet, for example in the right wedge?

If the Minkowski state is the vacuum $|0^M\rangle$, the answer is well known [2]: The expectation value of the Rindler particle number operator $\hat{n}^R_{k'm'}$ referring to the wave packet mode $(k',m')_R$, shows the familiar thermal spectrum

$$\langle 0^M|\hat{n}^R_{k'm'}|0^M\rangle = |\beta_{k'}|^2 = \frac{1}{e^{2\pi\omega_{k'}/a} - 1}. \tag{33}$$

The expectation value (33) does not depend on the trajectory parameter m', indicating that Rindler wave packet states with the same energy are uniformly occupied independent of their trajectory.

Next we ask for the Rindler particle number in the mode $(k',m')_R$ if the Minkowski state $|\phi^M\rangle$ is not the vacuum. Using the formula (2.11) of [22], we obtain

$$\langle\phi^M|\hat{n}^R_{k'm'}|\phi^M\rangle = \sum_{km} n^M_{km}\,|\mathcal{G}_\alpha|^2 + |\beta_{k'}|^2 \sum_{km}\left(n^M_{km}\,|\mathcal{G}_\alpha|^2 + n^M_{km}\,|\mathcal{G}_\beta|^2\right) + |\beta_{k'}|^2, \tag{34}$$

where $n^M_{km} = \langle\phi^M|a^{M\dagger}_{km}a^M_{km}|\phi^M\rangle$ is the mean number of Minkowski particles in the mode (k,m). In the approximation of (26), this can be written

$$\langle\phi^M|\hat{n}^R_{k'm'}|\phi^M\rangle = n^M_{k_\alpha m_\alpha} + |\beta_{k'}|^2(n^M_{k_\alpha m_\alpha} + n^M_{k_\beta m_\beta}) + |\beta_{k'}|^2. \tag{35}$$

The packet parameters are again related according to the pair structure relations (12) and (13). This result shows that for a given Minkowski state $|\phi^M\rangle$, the Rindler particle content of the wave packet on the trajectory m' with momentum k' consists of several parts that contribute additively.

First, there is the last term in (35) which is identical to (33). This vacuum term is always present, independent of whether or not $|\phi^M\rangle$ is the Minkowski vacuum.

The remaining terms all go back to the presence of real Minkowski particles. The first term in (35) agrees with the number $n^M_{k_\alpha m_\alpha}$ of Minkowski particles in the mode (k_α,m_α) equivalent to $(k',m')_R$, representing a Minkowski wave packet on the same trajectory, with energy given by the Doppler shift formula (3) (see Fig. 1). These particles reappear as Rindler particles. The second term shows that, in addition to this, the same Minkowski particles induce the appearance of additional Rindler particles in the mode $(k',m')_R$. This contribution is proportional to $n^M_{k_\alpha m_\alpha}$. Furthermore this stimulated effect shows the same spectral energy distribution $|\beta_{k'}|^2$ as the vacuum effect (33).

Beyond that, there is as third term in (35) a contribution going back to Minkowski particles with the same energy but with packet maximum traveling through the left wedge along the trajectory of the mirror mode (k_β,m_β) of the mode (k_α,m_α). Like the second term, this third term is proportional to the respective number of Minkowski particles $n^M_{k_\beta m_\beta}$ in that mode and to the spectral vacuum factor $|\beta_{k'}|^2$. It represents a characteristic nonlocality which we will again encounter below. With *nonlocality* we always refer to the trajectories, i.e., to the packet maxima. One should be aware of the fact that a packet with maximum in L has a tail in R and vice versa.

The structure of the various terms in (35) is in close analogy to the corresponding expressions in the in-out scheme of induced creation of bosons in Robertson-Walker universes [19,20] and by black holes [21,22]. The two middle terms can be characterized as the *amplification terms.* Their effect is to amplify the vacuum part $|\beta_{k'}|^2$ in the wedge R when Minkowski particles are present in appropriate modes with trajectories in wedge R or even only in wedge L. The corresponding attenuation process for fermions in Robertson-Walker universes has been discussed in [20].

The physical interpretation of these amplification terms and their nonlocal character refers to the pair structure: In the Minkowski vacuum, Rindler particles occur always in pairs, belonging to Rindler mirror modes with trajectories located in different Rindler wedges. For bosonic fields, the presence of Minkowski particles stimulates an additional appearance of pairs in the respective equivalent Rindler mode and its mirror mode, leading to the amplification terms in (35). The results of the next section will confirm this interpretation.

B. Nonlocal correlations

In order to explore further the pair structure and its nonlocality that arose in the previous section, we will now look for a general transcription of Minkowski wave packet states in terms of Rindler many-particle states $|q_{k',m'}\rangle$, where q is the Rindler particle number. For the Minkowski vacuum we find (cf. [2])

$$|0^M\rangle = \prod_{\tilde{k}'\tilde{m}'} \frac{1}{|\alpha_{\tilde{k}'}|}\sum_{q=0}^{\infty}\left(\frac{\beta^*_{\tilde{k}'}}{\alpha_{\tilde{k}'}}\right)^q |q_{-\tilde{k}'\tilde{m}'}\rangle_L \otimes |q_{\tilde{k}'\tilde{m}'}\rangle_R. \tag{36}$$

The Minkowski vacuum can thus be decomposed into a set of EPR-type entangled Rindler states [5,10,11]. Equation (36) shows strong correlations between Rindler mirror states with the same particle number in the left and right wedge. Whenever one measures q particles in the wave packet $(\tilde{k}',\tilde{m}')_R$ in the right wedge (using an accelerated particle detector, for example), one will find with unit probability q particles in the respective mirror mode in the left wedge. In this sense, Rindler particles appear in pairs.

Similarly, it is possible to transform a Minkowski many-particle state into a Rindler dual representation of the type above. We consider a wave packet state $|n^M_{k_\beta m_\beta}\rangle$ with n Minkowski particles on a trajectory m_β that passes through the left wedge L (cf. Fig. 1). Applying the corresponding creation operators to (36), we find with the help of the Bogoliubov coefficients (26), whereby as usual the pair structure relations (12), (13), and (14) have to be taken into account:

$$|n^M_{k_\beta m_\beta}\rangle = \left[\prod_{\substack{\tilde{k}'\neq k'\\ \tilde{m}'\neq m'}} \frac{1}{|\alpha_{\tilde{k}'}|}\sum_{r=0}^{\infty}\left(\frac{\beta^*_{\tilde{k}'}}{\alpha_{\tilde{k}'}}\right)^r |r_{-\tilde{k}'\tilde{m}'}\rangle_L \otimes |r_{\tilde{k}'\tilde{m}'}\rangle_R\right] \times \frac{(-\alpha_{k'})^{-n}}{|\alpha_{k'}|}\sum_{q=0}^{\infty}\left(\frac{(n+q)!}{n!q!}\right)^{\frac{1}{2}}\left(\frac{\beta^*_{k'}}{\alpha_{k'}}\right)^q |(n+q)_{-k'm'}\rangle_L \otimes |q_{k'm'}\rangle_R. \tag{37}$$

Again, always a Rindler mode and its mirror mode are connected. The first line refers to the Rindler modes $(-\tilde{k}', \tilde{m}')_L$ and their mirror modes in R which are not equivalent to the Minkowski mode (k_β, m_β). For these modes we have total agreement with the vacuum expression (36). According to the second line of (37), deviations caused by the Minkowski particle content occur only for the left-wedge Rindler mode $(-k', m')_L$ equivalent to (k_β, m_β) and its mirror mode in the right wedge. Again we have the same type of nonlocal correlations as for the Minkowski vacuum. If we perform a measurement of the Rindler particle number in the equivalent mode $(-k', m')_L$ in the left wedge with result $n+q$, we can be sure to find q particles in its mirror mode in the right wedge. The Minkowski particle content contributes with the number n in L, but additional q Rindler particles appear as correlated pairs in the respective modes in L and R. Note that the probability for this result is modified as compared to the vacuum case (see below). The important point is that this probability depends on the number n of Minkowski particles.

We want to discuss these probabilities in still another way. Taking the results (36), (37) as a starting point, we can derive the probability distribution of right-wedge Rindler particles, i.e., the probability that q Rindler particles in the mode $(k'm')_R$ in R are found for a given Minkowski state, regardless of the Rindler state in L:

$$P(q^R_{k'm'}|\phi^M) = \sum_L |\langle\phi^M|\,(|L\rangle_L \otimes |q_{k'm'}\rangle_R)|^2,$$

where the sum extends over a complete set of left-wedge Rindler states. If the Minkowski state is the vacuum, we find

$$P(q^R_{k'm'}|0^M) = \frac{|\beta_{k'}|^{2q}}{|\alpha_{k'}|^{2(q+1)}} = \left(1 - e^{-2\pi\omega_{k'}/a}\right) e^{-(2\pi\omega_{k'}/a)q}. \tag{38}$$

This is a Bose-Einstein distribution. It shows that the Minkowski vacuum is a truly thermal state in the Rindler representation. Equation (38) corresponds to the probability distribution of particles created out of the vacuum in a Robertson-Walker universe [19] or, when a is replaced by the surface gravity κ, by a black hole [26] .

Turning to nonempty Minkowski states with particles in L, the corresponding probability distribution of Rindler particles can be found from (37). For n Minkowski particles in the state with (k_β, m_β), we obtain, for the probability to find q Rindler particles in the right-wedge Rindler mirror mode $(k', m')_R$,

$$P(q^R_{k'm'}|n^M_{k_\beta m_\beta}) = \frac{(n+q)!}{n!q!} \frac{|\beta_{k'}|^{2q}}{|\alpha_{k'}|^{2(n+q+1)}} = \frac{(n+q)!}{n!q!} e^{-(2\pi\omega_{k'}/a)(n+q)} \left(e^{2\pi\omega_{k'}/a} - 1\right)^n \left(1 - e^{-2\pi\omega_{k'}/a}\right). \tag{39}$$

For the remaining modes the probability agrees with the one found for the vacuum case (38). The distribution (39) is a negative binominal distribution. It shows that, as compared with the Minkowski vacuum, Minkowski particles with modes passing L have the tendency to increase the number of Rindler particles in R, in accordance with the interpretation of (35) (stimulated amplification). The result (35) with $n^M_{k_\alpha m_\alpha} = 0$ can directly be reproduced from the probability distribution (39). For the case of stimulated emission by black holes with unit absorptivity, the same distribution has been inferred by Gasperini [27] by analogy with particle creation in Robertson-Walker universes.

IV. INCIDENT PARTICLES AND THE ACCELERATED DETECTOR

A. Particle detector model

Let us now investigate the processes that occur when a particle detector is uniformly accelerating along the trajectory $\xi' = 0$ in wedge R through the Minkowski vacuum or a Minkowski many-particle wave packet state. This is important for two reasons: We want to reproduce on an operational basis the structure we have found for Rindler quantum field theory in the presence of Minkowski particles. Furthermore, the processes that occur will lead to a generalization of the concepts of spontaneous and stimulated detector excitation and deexcitation, as they are known for an inertially moving detector. This is a step toward a complete noninertial quantum optics which is still to be developed.

As a model of the accelerated particle detector we consider a two-level system with constant energy difference E, whereby the energy of the lower state is zero. Denoting the corresponding eigenstates by $|\uparrow\rangle$ and $|\downarrow\rangle$, the general form of the unperturbed Hamiltonian can be written

$$H_D = E\, d^\dagger(\eta')\, d(\eta'), \tag{40}$$

where we have introduced the operator $d := |\downarrow\rangle\langle\uparrow|$ obeying $\{d^\dagger, d\} = 1$. The free evolution of d with respect to the Rindler time variable η' can be obtained from the Heisenberg equations of motion:

$$d(\eta') = e^{-iE\eta'} d(0). \tag{41}$$

We couple the detector to the massless scalar field by an interaction of Unruh-DeWitt type [2,7]:

$$H_I = \lambda\, m(\eta')\, \phi(x(\eta')), \tag{42}$$

where the field operator $\phi(x(\eta'))$ is evaluated along the world line of the detector $x(\eta')$. λ is a coupling constant and the detector monopole moment is

$$m(\eta') = \mu(d(\eta') + d^\dagger(\eta')), \tag{43}$$

where μ is a real number. In writing (43), it has been assumed that the monopole moment has only off-diagonal components, so that the interaction couples only different detector states.

B. Rindler particles, Minkowski particles, and the detector response

In a first step, we calculate in the interaction picture the probability amplitude for a transition $|\phi_i, E_i\rangle \to |\phi_f, E_f\rangle$ of the coupled detector-field system in an in-out approach. $|E\rangle$ represents the state of the detector, $|\phi\rangle$ that of the scalar field. The final state in first order perturbation theory is

$$|\phi_f, E_f\rangle = -i \int_{-\infty}^{+\infty} d\eta'\, H_I(\eta')|\phi_i, E_i\rangle. \tag{44}$$

Let us first treat the case where the detector is initially in the ground state $|E_i\rangle = |\downarrow\rangle$: Eq. (44) gives with the decomposition (17) of the field operator into Rindler wave packets for the final state:

$$|\phi_f, E_f\rangle = -i\lambda \sum_{k'm'} \mathcal{D}_1(k', m') a^R_{k'm'} |\phi_i, \uparrow\rangle. \tag{45}$$

Here $\mathcal{D}_1(k', m')$ is defined as

$$\mathcal{D}_1(k', m') = \sqrt{\frac{\pi}{\epsilon' E}}\, \mu\, e^{-i\sigma E m'} \delta_{\epsilon'}(E, \omega_{k'}), \tag{46}$$

with $\omega_{k'} = |k'|$ and $\sigma = \mathrm{sgn}(k')$. The factor

$$\delta_{\epsilon'}(E, \omega_{k'}) = \begin{cases} 1 & \text{if } \omega_{k'} < E < \omega_{k'} + \epsilon' \\ 0 & \text{otherwise} \end{cases} \tag{47}$$

represents the *resonance condition.* Using the Bogoliubov transformation (27), we can express (45) in terms of processes referring to Minkowski particles:

$$|\phi_f, E_f\rangle = -i\lambda \sum_{k'm'} \mathcal{D}_1(k', m') \left(\alpha^*_{k'} b^M_{k_\alpha m_\alpha} - \beta^*_{k'} b^{M\dagger}_{k_\beta m_\beta}\right) \times |\phi_i, \uparrow\rangle. \tag{48}$$

It is possible to interpret Eq. (45) directly in terms of Rindler particles. The basic process underlying the transition (45) corresponds to the excitation of the detector accompanied by the absorption of a Rindler particle from the right wedge. The factor $\delta_{\epsilon'}(E, \omega_{k'})$ shows that a particle can only be absorbed if its energy coincides with the detector level spacing. Its trajectory is not fixed. The action of the annihilation operator $a^R_{k'm'}$ in Eq. (45) shows that the detector excitation probability is proportional to the number of Rindler particles with energy E in the state $|\phi_i\rangle$. In this sense, one can say that the uniformly accelerated detector is a device for measuring the Rindler particle number. It is therefore to be expected that the pair structure phenomena induced by incident Minkowski particles will be rediscovered for detectors. In fact we will find consequences with an even richer physical structure.

Correspondingly, in the second case if the detector is initially in the excited state $|E_i\rangle = |\uparrow\rangle$, the final state in first order perturbation theory is

$$|\phi_f, E_f\rangle = -i\lambda \sum_{k'm'} \mathcal{D}^*_1(k', m') a^{R\dagger}_{k'm'} |\phi_i, \downarrow\rangle, \tag{49}$$

describing a detector deexcitation with the emission of a Rindler quantum. A description in terms of Minkowski particles can again be obtained with the Bogoliubov transformation (27)

$$|\phi_f, E_f\rangle = -i\lambda \sum_{k'm'} \mathcal{D}^*_1(k', m') \left(\alpha_{k'} b^{M\dagger}_{k_\alpha m_\alpha} - \beta_{k'} b^M_{k_\beta m_\beta}\right) \times |\phi_i, \downarrow\rangle. \tag{50}$$

Before interpreting Eqs. (48) and (50), we want to demonstrate the internal coherence of the scheme. The physical results of Sec. III A can be reproduced by the present detector calculation. The total excitation probability of the detector, regardless of the final state of the quantum field, may serve as a measure of the Rindler particle content of the initial field state $|\phi_i\rangle$ [2]. The *detector response function* is obtained in summing the probability for excitation over a complete set of field states $|\phi\rangle$. This can be evaluated using formula (48) and gives, with reference to (12) and (13),

$$\sum_\phi |\langle \phi, \uparrow | \phi_f, \uparrow\rangle|^2 = \sum_{k'm'} g \left(n^M_{k_\alpha m_\alpha} + |\beta_{k'}|^2 (n^M_{k_\alpha m_\alpha} + n^M_{k_\beta m_\beta}) + |\beta_{k'}|^2\right) \delta_{\epsilon'}(E, \omega_{k'}) \tag{51}$$

with $g = \lambda^2 \mu^2 \pi / \epsilon' E$. Because of the resonance condition (47), all the equivalent modes (k_α, m_α) to modes with $\omega_{k'} = E$ and their mirror modes (k_β, m_β) contribute to (51). The structure of this expression is very similar to that of Eq. (35) for the Rindler particle number. Apart from the detector-dependent prefactor, there are two differences: It contains contributions from the modes with resonant energy $k = \pm E$, since the detector cannot discriminate between left- and right-moving Rindler particles in R. Only one of them corresponds to (35), because there we asked for the particle number in a single mode. Secondly, there is a sum over all trajectory parameters m' which is due to the fact that the detector picks up wave packets with energy E on all trajectories of the Minkowski particles.

C. Spontaneous and stimulated processes in lowest order

From now on, the concept of particles will be solely restricted to Minkowski particles, which may be detected in the usual way. Our approach generalizes the work of Unruh and Wald [8] to stimulated processes and a localization with wave packets. In specifying different initial states of the accelerated detector and the Minkowski particle field, we see from the expressions (48) and (50) that there are in lowest order four different types of processes.

1. Excitation with absorption of a Minkowski particle

We ask for the probability that the detector in R gets excited $|\downarrow\rangle \to |\uparrow\rangle$ and the number of Minkowski particles is reduced by one. From (48) we obtain for the two cases that the trajectories of the incident Minkowski packets either cross R or cross L [cf. (15)]:

$$P(|n^M_{k_\alpha m_\alpha},\downarrow\rangle \to |(n-1)^M_{k_\alpha m_\alpha},\uparrow\rangle) = n^M_{k_\alpha m_\alpha}\, g\, |\alpha_{k'}|^2 \delta_{\epsilon'}(E,\omega_{k'}) = n^M_{k_\alpha m_\alpha}\, g\, \left(1+|\beta_{k'}|^2\right) \delta_{\epsilon'}(E,\omega_{k'}), \quad (52)$$

$$P(|n^M_{k_\beta m_\beta},\downarrow\rangle \to |(n-1)^M_{k_\beta m_\beta},\uparrow\rangle) = 0. \quad (53)$$

k' is thereby derived from (12). $\omega_{k'}$ is the energy of the Rindler mode equivalent to the ingoing Minkowski mode (k_α, m_α).

The results (52) and (53) show that for the process in question the ingoing mode must intersect the path of the detector [cf. Fig. 2(a)]. There is no further restriction of the Minkowski particle trajectory. For a given trajectory the Minkowski energy must satisfy the resonance condition at the intersection point P of Fig. 2(a) [cf. (47)]. This means that the respective Doppler shifted energy $\omega_{k'}$ as registered by the accelerated observer in P must agree with the level spacing E of the detector. The resulting transition probability is then proportional to the number $n^M_{k_\alpha m_\alpha}$ of ingoing particles and to $|\alpha_{k'}|^2$.

All this is well known for the special case of an inertial detector ($a = 0$), where we have $|\alpha_{k'}|^2 = 1$ and $|\beta_{k'}|^2 = 0$. Then the process reduces to the usual absorption process. In the general case ($a \neq 0$), the only difference is that the acceleration a causes a modified absorption probability, because $|\alpha_{k'}|^2$ is different from its inertial value.

2. Excitation with emission of a Minkowski particle

We now turn to a process which happens for accelerated detectors only. We ask for the probability that a detector excitation is accompanied by an *increase* of the Minkowski particle number by one. Equation (48) leads to

$$P(|n^M_{k_\beta m_\beta},\downarrow\rangle \to |(n+1)^M_{k_\beta m_\beta},\uparrow\rangle) = (n^M_{k_\beta m_\beta}+1)\, g\, |\beta_{k'}|^2 \delta_{\epsilon'}(E,\omega_{k'}), \quad (54)$$

$$P(|n^M_{k_\alpha m_\alpha},\downarrow\rangle \to |(n+1)^M_{k_\alpha m_\alpha},\uparrow\rangle) = 0 \quad (55)$$

with (12) and (13).

From the appearance of the Bogoliubov coefficient $|\beta_{k'}|^2$ that gives rise to the thermal spectrum, we see that the process (54) is inertially forbidden. In the case $n^M_{k_\beta m_\beta} = 0$ (detector accelerating through the Minkowski vacuum), (54) reduces to the Unruh effect, which can be interpreted as spontaneous detector excitation accompanied by particle emission, as has been discussed by Unruh and Wald [8]. Equations (54) and (55) show that in addition a corresponding *stimulated* detector excitation with emission of Minkowski particles is possible if particles are going in in the mode (k_β, m_β) with trajectory of the packet maximum traversing L. See Fig. 2(b) for a right-moving packet mode ($k_\beta > 0, m_\beta$). A packet with trajectory crossing R does not cause such an effect.

For the stimulated effect the resonance factor $\delta_{\epsilon'}(E,\omega_{k'})$ demands that the Doppler-shifted energy of the inducing particles matches the detector energy at the intersection point of the empty mirror mode with the detector trajectory [cf. Fig. 2(b)].

This clearly reflects again the pair structure and its nonlocal correlations which we have seen above in Sec. III for example in connection with amplification effects which refer to the detection of particles. For a heuristic illustration one could say: If the detector is initially prepared in its ground state, the presence of Minkowski particles in a mode (k_β, m_β) with trajectory crossing R induces, as compared with the spontaneous effect, an additional appearance of Rindler particle pairs in the equivalent and its mirror mode. If thereby the Minkowski particles on the trajectory m_β have the energy demanded by the resonance condition, the Rindler particle in the mirror mode is absorbed by the detector and its pair partner appears as additional real particle in the mode (k_β, m_β) in the out-region. This process may be understood as a stimulated amplification of a corresponding noninertial spontaneous process (Unruh effect) based on the same pair structure.

3. Deexcitation with emission of a Minkowski particle

With regard to detector deexcitation we firstly discuss again the modification of the well-known inertially allowed process. With (50) we obtain, for the probability that the deexcitation is accompanied by the emission of a Minkowski particle,

$$P(|n^M_{k_\alpha m_\alpha},\uparrow\rangle \to |(n+1)^M_{k_\alpha m_\alpha},\downarrow\rangle) = (n^M_{k_\alpha m_\alpha}+1)\,g\,|\alpha_{k'}|^2\delta_{\epsilon'}(E,\omega_{k'})$$
$$= (n^M_{k_\alpha m_\alpha}+1)\,g\,\left(1+|\beta_{k'}|^2\right)\delta_{\epsilon'}(E,\omega_{k'}), \tag{56}$$
$$P(|n^M_{k_\beta m_\beta},\uparrow\rangle \to |(n+1)^M_{k_\beta m_\beta},\downarrow\rangle) = 0 \tag{57}$$

with (12). It is only nonvanishing for particles with trajectories passing the right wedge [cf. Fig. 2(c)]. The noninertial modification consists as for the absorption process (52) again in the appearance of $|\beta_{k'}|^2$. The factor $(n^M_{k_\alpha m_\alpha}+1)$ shows that spontaneous as well as stimulated emission is possible.

4. *Deexcitation with disappearance of a Minkowski particle*

A characteristic noninertial process which is structurally new is the stimulated detector deexcitation accompanied not with the appearance of an additional particle but with the *disappearance* of a Minkowski particle. Equation (50) gives, for the respective probabilities,

$$P(|n^M_{k_\beta m_\beta},\uparrow\rangle \to |(n-1)^M_{k_\beta m_\beta},\downarrow\rangle) = n^M_{k_\beta m_\beta}\,g\,|\beta_{k'}|^2\delta_{\epsilon'} \times(E,\omega_{k'}). \tag{58}$$

$$P(|n^M_{k_\alpha m_\alpha},\uparrow\rangle \to |(n-1)^M_{k_\alpha m_\alpha},\downarrow\rangle) = 0, \tag{59}$$

with (12) and (13).

The process is typically nonlocal and stimulated because it is proportional to $n^M_{k_\beta m_\beta}$ and needs incident particles with trajectories of the packet maximum passing the left wedge L and not R [cf. Fig. 2(d)]. The resonance condition refering to the Rindler mirror mode and its intersection point P has again to be satisfied. This reveals once more an underlying pair structure. In this case, in a naive picture, a particle pair consisting of an emitted and an incident particle seems to disappear because of the detector acceleration. The factor $|\beta_{k'}|^2$ shows that this amazing effect is inertially ($a=0$) forbidden.

V. CONCLUSIONS

The presence of particles in specific Minkowski wave packet modes enlarges the expectation value of the Rindler particle number for particular Rindler wave packets. This stimulated amplification and the decomposition of a Minkowski state in EPR-type entangled Rindler states reveal the nonlocal pair structure specified above.

This has consequences for quantum optics in noninertial situations. Accelerated detectors, as compared to inertial detectors show a richer structure of the physics of excitation and deexcitation. (i) The processes of stimulated excitation with absorption and spontaneous and stimulated deexcitation with emission are known from the inertial detector. The difference here is the appearance of the Bogoliubov coefficient $|\alpha_{k'}|^2 \neq 1$. (ii) The processes (2) and (4) on the other hand depend on the Bogoliubov parameters $\beta_{k'}$ and are inertially forbidden. Excitation with emission happens spontaneously (Unruh effect [2]) and furthermore in a stimulated way. In addition stimulated deexcitation with disappearance of a particle becomes possible. (iii) These $\beta_{k'}$ processes are nonlocal and the pair structure can be recovered.

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