# Baryon-number fluctuations in a quark-hadron phase transition

C. P. Singh, B. K. Patra, and Saeed Uddin
*Department of Physics, Banaras Hindu University, Varanasi-221 005, India*
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Isothermal baryon-number fluctuations arising from a first-order quark-hadron phase transition in the early Universe are obtained by including the quark-gluon interactions up to the order $g_s^3$ in the perturbative QCD coupling constant in the quark-gluon plasma (QGP) phase and the finite-size volume corrections for the hadrons in the hadron-resonance gas (HRG) and their effects on the primordial nucleosynthesis (PNS) are analyzed. The ratio of the baryon-number densities in the QGP and HRG phases at the critical temperature $T_c$ is larger than one in the range 150 MeV $< T_c <$ 260 MeV. However, it is far larger than one even when $T_c$ is outside this range, thus affecting primordial nucleosynthesis significantly.

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## I. INTRODUCTION

The nature and the critical parameters of the phase transition between a hadron resonance gas (HRG) and the quark-gluon plasma (QGP) are the object of active theoretical and experimental research [1,2]. At very high temperatures in the early Universe, we believe that the colored quarks and gluons were unconfined and the matter existed in the form of a QGP. As the Universe expanded, the temperature dropped through the critical temperature $T_c$ for the phase transition where the QGP could exist in thermal, mechanical, and chemical equilibrium with a dense hot gas of HRG. Isothermal baryon-number fluctuation is a natural consequence of a first-order phase transition between a QGP and HRG. Consequently, a large local fluctuation in the baryon-to-photon ratio results and thus alters the predictions of the standard scenario for primordial nucleosynthesis (PNS). The ratio of baryon-number densities in the two phases is represented by

$$R = n_B^{\mathrm{QGP}} / n_B^{\mathrm{HRG}} , \tag{1}$$

and the expression is evaluated at $T = T_c$ and baryon chemical potential $\mu_B \ll T_c$. When $R \gg 1$, the baryon number of the Universe preferentially remained in the QGP phase during the period of phase separation and thus affected the PNS significantly.

Several authors have calculated [3–8] the value of $R$ by assuming a QGP as an ideal thermodynamical gas of quarks and gluons and the HRG was treated as an ideal gas of protons and neutrons. Turner [3] noticed that the ratio $R$ decreases significantly when other low-lying baryon states, e.g., $\Delta$ (mass 1232 MeV, $J^P = 3/2^+$ and $I = 3/2$), $\Lambda$, $\Sigma^+$, $\Sigma^0$, and $\Sigma^-$ (strangeness $= -1$ states) were included. He found that unless $T_c < 150$ MeV, the effects of the phase transition upon PNS would not be significant. However, they found $R \ll 1$ for $T_c > 250$ MeV and that the baryon number resides predominantly in the HRG phase. Recently, Murugesan *et al.* [4] used relativistic quantum statistics for particles in both the phases and Hagedorn's correction for the finite size of the hadrons in the HRG phase. They found that these corrections lower the value of $T_c$ and, unless $T_c \le 125$ MeV and $R > 10$, the phase transition would not have any significant effect upon PNS. The purpose of this paper is to explore the results when a phase transition from a QGP with interacting quarks and gluons to a hadron gas consisting of finite-sized hadrons described by hard spherical bags is considered. We show that the incorporation of the interactions between the quarks in a QGP up to $g_s^3$ terms in the strong interaction coupling $g_s$ within perturbative QCD and the repulsive interactions between hadrons in a HRG by considering volume corrections give a unique constraint on $T_c$. We find that the PNS will significantly be affected if $T_c$ lies outside the range 150 MeV $< T_c <$ 260 MeV for which $R > 10$. Moreover, we notice that the value of $R$ is larger than 1 even in the range 150 MeV $< T_c <$ 260 MeV. This is a significant result which will affect the prediction for baryon-density inhomogeneity in the early Universe.

Recently, we have shown that the incorporation of hard-core volumes for the hadrons and treating a QGP as weakly interacting plasma substantially change the values of the parameters of the phase transition and these corrections are, in fact, essential for a realistic description of the quark-hadron phase transition [9]. We noticed [9] that the higher-order $g_s^3$ term contribution in QCD is quite significant at lower values of the baryon-chemical potential $\mu_B$ where the value of $\alpha_s (= g_s^2/4\pi)$ is large. In this paper, we want to see how these corrections affect the isothermal baryon-number fluctuations which are of a vital importance for the PNS in the early Universe.

## II. METHOD OF CALCULATION

Numerical simulation studies have revealed that the quarks and gluons in QGP are not asymptotically free states but they experience weak interactions even at temperatures $T > T_c$. Therefore, it is worthwhile to investigate the effects of the interacting plasma on the quark-

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hadron phase transition. We assume that the QGP phase can be described by perturbative QCD up to $g_s^3$ terms together with a bag constant $B$ to account for the nonperturbative influence of the vacuum. The total thermodynamic potential $\Omega$ or the partition function lnz for an interacting QGP can be written [10,11] as

$$\Omega=-Va(\tau,\mu_q,g)T^4=-T\ln z \tag{2}$$

and

$$a(T,\mu_q,g)=\frac{\pi^2}{45}\left[8+\frac{21}{4}n_f+\frac{45}{2\pi^2}\sum_{q=1}^{n_f}\left[\frac{\mu_q^2}{T^2}+\frac{\mu_q^4}{2\pi^2T^4}\right]\right]-\frac{8}{144}g_s^2\left[3+\frac{5}{4}n_f+\frac{9}{2\pi^2}\sum_{q=1}^{n_f}\left[\frac{\mu_q^2}{T^2}+\frac{\mu_q^4}{2\pi^2T^4}\right]\right]$$
$$+\frac{2}{3\pi}g_s^3\left[1+\frac{1}{6}n_f+\frac{1}{6}\sum_{q=1}^{n_f}\frac{\mu_q^2}{\pi^2T^4}\right]^{3/2}, \tag{3}$$

where $T$ is the temperature, $V$ the volume, $\mu_q$ the quark chemical potential $(=\frac{1}{3}\mu_B)$, and $n_f$ is the number of quark flavors. Following Turner [3], we assume that $\mu_q$ is flavor independent ($\mu_\mu=\mu_d=\mu_s$) and the $u,d,s$ quarks are light or massless. The running coupling constant $\alpha_s$ accounts for the interactions between the quarks and gluons. Its dependence on $\mu_q$ and $T$ is given by

$$\alpha_s(\mu_q,T)=\frac{g_s^2}{4\pi}=\frac{12\pi}{33-2n_f}\times\{\ln[(0.8\mu_q^2+15.622T^2)/\Lambda_s^2]\}^{-1}, \tag{4}$$

where $\Lambda_s$ is the scale fixing parameter in QCD. We can obtain the quark number density from (2) by using the relation

$$n_q=\frac{T}{V}\frac{\partial\ln z}{\partial\mu_q}.$$

Finally, $n_B^{\text{QGP}}=(n_q-n_{\bar{q}})/3$.

The hadronic gas consists of nucleons, antinucleons, $\Delta$, $\bar{\Delta}$, $\Lambda$, $\bar{\Lambda}$, $\Sigma$, $\bar{\Sigma}$, etc. We can use the partition function for the grand canonical ensemble of HRG as

$$\ln z=\frac{gV}{6\pi^2T}\int_0^\infty\frac{dk\,k^4}{\sqrt{k^2+m^2}}(f^++f^-), \tag{5}$$

where $g$ is the degeneracy factor, $k$ is the momentum, $m$ is the mass of the hadron, and $f^+$ and $f^-$ are the fermion and antifermion distribution functions, respectively. Thus the density of a pointlike baryon $b$ can be given as

$$n_b^{\text{pt}}=\frac{T}{V}\frac{\partial\ln z}{\partial\mu}. \tag{6}$$

Similarly, the density of pointlike antibaryons $\bar{b}$ can be obtained and, finally, the baryonic density $n_B^{\text{HRG}}$ is [12,13]

$$n_B^{\text{HRG}}=\frac{\sum_b n_b^{\text{pt}}}{1+\sum_b n_b^{\text{pt}}V_b}-\frac{\sum_{\bar{b}}n_{\bar{b}}^{\text{pt}}}{1+\sum_{\bar{b}}n_{\bar{b}}^{\text{pt}}V_{\bar{b}}}, \tag{7}$$

where summation over $b$ and $\bar{b}$ extends over all baryonic and antibaryonic states being considered and $V_b$ is the hard-core volume of the baryon $b$. We consider hard-core volume $V_b=V_{\bar{b}}=V_p=\frac{4}{3}\pi r_p^3$, where $r_p$ is the proton hard-core radius. Assuming the bag volume as the hard-core volume of the proton, we can involve the virial theorem to write $4B=m_p/V_p$ where $m_p$ is the proton mass and $B$ is the bag constant. Thus the bag constant is related to the hard-core radius of the proton. Here, one point is noteworthy. Even if HRG also consists of pions, the expression for baryon-number density is not affected in our model.

The use of Eq. (7) as the volume-corrected baryon-number density has been widely discussed in the literature [12,13]. It differs significantly from the original suggestion of Hagedorn to use the correction for the finite extension of the hadrons. Cleymans and Suhonen [12] incorporated the nonvanishing volume in the statistical mechanics formalism of the HRG and the effect of the hard core of the nucleon as an excluded volume effect was included. Recently, Kuono and Takagi [13] have considered the strong repulsive forces existing between a pair of baryons as well as a pair of antibaryons. We have used in Eq. (7) the volume correction proposed by Kuono and Takagi.

## III. RESULTS AND DISCUSSION

The early Universe evolves with a value of $\mu_B/T$ of approximately $10^{-10}$ so that only the first term in the expansion of thermodynamic quantities needs to be retained. In Fig. 1, we have shown the result for the variation of $R$ with the critical temperature $T_c$. Here we have considered $u$ and $d$ massless quarks in the QGP phase and $N$ and $\bar{N}$ in the hadronic phase. Furthermore, we have taken the value of $\Lambda_s=100$ MeV and $r_p=0.8$ fm. We have also compared our results with those of Turner [3] and Murugesan *et al.* [4], respectively. As a result of incorporations of the interaction terms in the QGP and the hard-core volume corrections for HRG, our curve lies below Turner's curve [3] but is quite a bit above the curve of Murugesan *et al.* [4]. However, at $T_c>160$ MeV, our values of $R$ exceed by a large amount from those obtained in both of these references.

In Fig. 2, we have demonstrated the effects of the inclusion of the massless $s$ quark in the QGP and strange as well as nonstrange baryons and their resonances in the HRG phase. The baryonic resonances and their parameters are shown in Table I. We obtain a very surprising result. We find that the value of $R$ decreases very rapidly from a value of 100 approximately at $T_c=100$ MeV and reaches a minimum value which is slightly larger than one at $T_c\simeq200$ MeV, and then it again increases rapidly

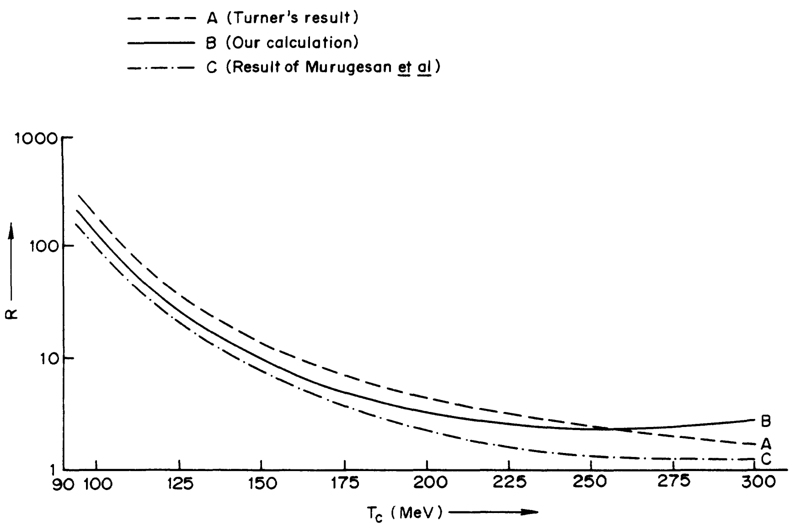


FIG. 1. The variation of the baryon-number density ratio $R$ with critical temperature $T_c$. Here the QGP phase consists of $u$ and $d$ quarks and the HRG phase consists of $N$ and $\bar{N}$ baryons. Our calculation is represented by a solid line, Turner's calculation is shown by a dashed line, whereas the dash-dotted curve represents the calculation of Murugesan *et al.*

TABLE I. List of baryonic resonances taken into account.

| Baryons | Mass (MeV) | Strangeness | Spin | Isospin | Degeneracy $(2s+1)(2I+1)$ |
|---|---|---|---|---|---|
| $N$ | 939 | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ | 4 |
| $\Delta$ | 1232 | 0 | $\frac{3}{2}$ | $\frac{3}{2}$ | 16 |
| $N$ | 1440 | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ | 4 |
| $N$ | 1520 | 0 | $\frac{3}{2}$ | $\frac{1}{2}$ | 8 |
| $N$ | 1535 | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ | 4 |
| $\Delta$ | 1620 | 0 | $\frac{1}{2}$ | $\frac{3}{2}$ | 8 |
| $N$ | 1650 | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ | 4 |
| $N$ | 1675 | 0 | $\frac{5}{2}$ | $\frac{1}{2}$ | 12 |
| $N$ | 1680 | 0 | $\frac{5}{2}$ | $\frac{1}{2}$ | 12 |
| $N$ | 1700 | 0 | $\frac{3}{2}$ | $\frac{1}{2}$ | 8 |
| $\Delta$ | 1700 | 0 | $\frac{3}{2}$ | $\frac{3}{2}$ | 16 |
| $N$ | 1710 | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ | 4 |
| $N$ | 1720 | 0 | $\frac{3}{2}$ | $\frac{1}{2}$ | 8 |
| $\Delta$ | 1905 | 0 | $\frac{5}{2}$ | $\frac{3}{2}$ | 24 |
| $\Delta$ | 1910 | 0 | $\frac{1}{2}$ | $\frac{3}{2}$ | 8 |
| $\Delta$ | 1930 | 0 | $\frac{5}{2}$ | $\frac{3}{2}$ | 24 |
| $\Delta$ | 1950 | 0 | $\frac{7}{2}$ | $\frac{3}{2}$ | 32 |
| $\Lambda$ | 1116 | $-1$ | $\frac{1}{2}$ | 0 | 2 |
| $\Lambda$ | 1405 | $-1$ | $\frac{1}{2}$ | 0 | 2 |
| $\Lambda$ | 1520 | $-1$ | $\frac{3}{2}$ | 0 | 4 |
| $\Lambda$ | 1670 | $-1$ | $\frac{1}{2}$ | 0 | 2 |
| $\Lambda$ | 1690 | $-1$ | $\frac{3}{2}$ | 0 | 4 |
| $\Lambda$ | 1820 | $-1$ | $\frac{5}{2}$ | 0 | 6 |
| $\Lambda$ | 1830 | $-1$ | $\frac{5}{2}$ | 0 | 6 |
| $\Lambda$ | 1890 | $-1$ | $\frac{3}{2}$ | 0 | 4 |
| $\Sigma$ | 1193 | $-1$ | $\frac{1}{2}$ | 1 | 6 |
| $\Sigma$ | 1385 | $-1$ | $\frac{3}{2}$ | 1 | 12 |
| $\Sigma$ | 1670 | $-1$ | $\frac{3}{2}$ | 1 | 12 |
| $\Sigma$ | 1775 | $-1$ | $\frac{5}{2}$ | 1 | 18 |
| $\Sigma$ | 1915 | $-1$ | $\frac{5}{2}$ | 1 | 18 |
| $\Xi$ | 1318 | $-2$ | $\frac{1}{2}$ | $\frac{1}{2}$ | 4 |
| $\Xi$ | 1530 | $-2$ | $\frac{3}{2}$ | $\frac{1}{2}$ | 8 |
| $\Omega$ | 1672 | $-3$ | $\frac{3}{2}$ | 0 | 4 |

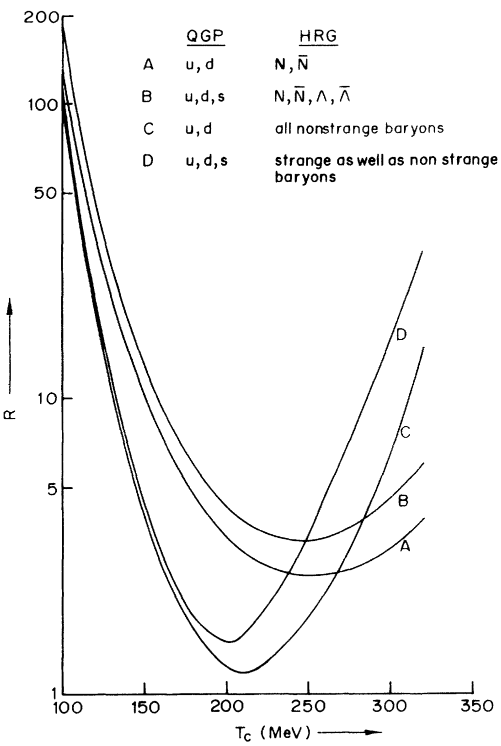


FIG. 2. Our calculation for the variation of $R$ with $T_c$ when we consider the following: (A) $u$ and $d$ quarks in QGP and $N$ and $\overline{N}$ states in HRG; (B) $u$ $d$, and $s$ in QGP and $N$, $\overline{N}$, $\Lambda$, and $\overline{\Lambda}$ in HRG; (C) $u$ and $d$ quarks in QGP and all nonstrange baryons in HRG; and (D) $u$, $d$, and $s$ quarks in QGP and strange as well as nonstrange baryons in HRG.

for $T_c > 200$ MeV. This will result in a far-reaching implication on the value of $T_c$ required for an isothermal baryon fluctuation in the early Universe. However, the second increase in the values of $R$ for $T_c > 200$ MeV is not very significant when considering $N$, $\overline{N}$, $\Lambda$, and $\overline{\Lambda}$ states only in the HRG. This amply demonstrates the effects of considering the repulsive interactions in the form of a hard-core volume correction for many baryonic resonances present in the early Universe at higher temperatures.

In order to show the above effects more clearly, we have separately shown in Fig. 3 the results of our calculations for baryonic densities $n_B$ in QGP as well as HRG phases. We have also compared our results with those of Turner [3] and Murugesan *et al.* [4]. We find that the actual difference lies in the calculation of $n_B$ in the HRG phase. We find that the values of $n_B$ in the HRG phase as obtained by us in the volume-corrected approach with 17 nonstrange resonances shown in Table I differ quite significantly from those of Turner and Murugesan *et al.* for $T_c > 200$ MeV. If we include all strange as well as nonstrange resonances in our calculation, the fall of $n_B$ in the HRG phase becomes more prominent. In order to show how the baryon density changes in our model with the increasing number of baryon resonances, we have also shown our curve with $N$, $\overline{N}$, $\Lambda$, and $\overline{\Lambda}$ states in the HRG phase. Our curves make it amply clear that the interactions in the HRG phase are important factors and altogether alter the predictions for the baryon density contrast in the early Universe.

In Fig. 4, we have shown the variation of the reduction factor $R_f = R_{N\overline{N}\Lambda\overline{\Lambda}}/R$ or $R_{N\overline{N}}/R$ with $T_c$. We have shown our results for $V_c < 1$, where

$$V_c = 1/(1 + \Sigma n_{b,\mathrm{HRG}}^{\mathrm{pt}}) .$$

Here $V_c = 1$ implies the removal of the correction factor. Again we find that our results are very different from the results of Murugesan *et al.* [4]. We find that our curve for the reduction factor with $V_c < 1$ and $\alpha_s > 0$ overlaps completely with $V_c < 1$ and $\alpha_s = 0$. It shows that in plotting the reduction factor the effects of interactions in QGP are completely compensated and thus we find that the various curves in Fig. 4 show the effects of volume corrections alone. However, in the calculation of Murugesan *et al.* [4], the two curves for $\alpha_s > 0$ and $\alpha_s = 0$, do not overlap each other. Moreover, our calculations show again a rapid decrease of $R_f$ with $T_c$ for $T_c > 200$ MeV, which can be compared with the constant

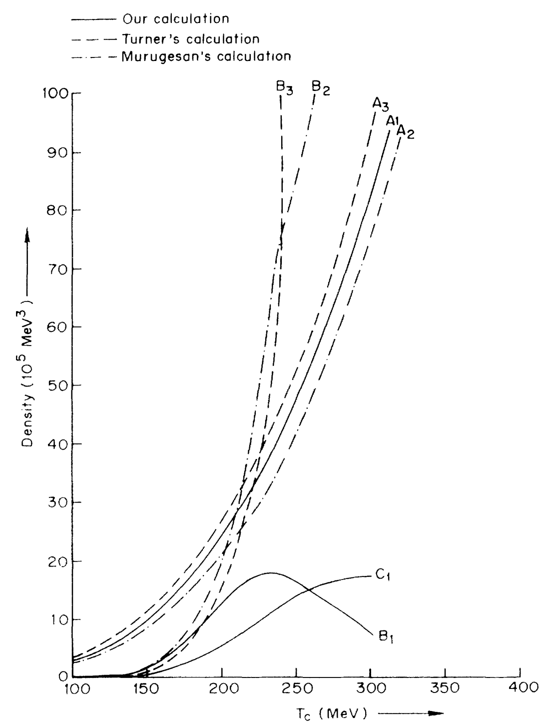


FIG. 3. Our calculations for the baryon density $n_B$ for QGP as well as HRG phases are separately shown by the curves $A_1$, $B_1$, and $C_1$ and are compared with those of Turner ($A_3$ and $B_3$) and Murugesan *et al.* ($A_2$ and $B_2$). In our calculation for the variation of density with $T_c$, ($A_1$) represents $n_B^{\mathrm{QGP}}$ with $u$, $d$, and $s$ quarks, ($B_1$) represents $n_B^{\mathrm{HRG}}$ with all nonstrange baryons, and ($C_1$) represents $n_B^{\mathrm{HRG}}$ with $N$, $\overline{N}$, $\Lambda$, and $\overline{\Lambda}$ states only. Similarly, in Turner's calculation ($A_2$) represents $n_B^{\mathrm{QGP}}$ with $u$, $d$, and $s$ quarks and ($B_2$) represents $n_B^{\mathrm{HRG}}$ with all nonstrange baryons; in Murugesan's calculation, ($A_3$) and ($B_3$) bear the same significance.

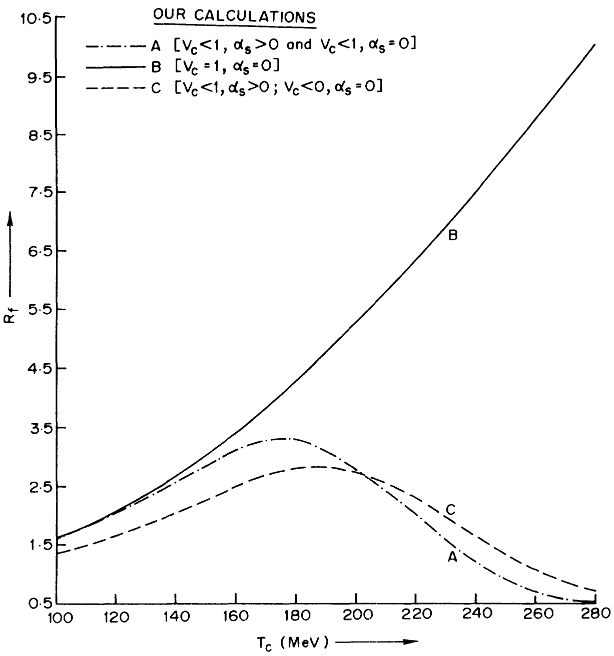


FIG. 4. Our calculations for the dependence of the reduction factor $R_f$ on $T_c$ with and/or without volume corrections as well as interactions in QGP. The curves marked (A) and (B) represent the QGP containing $u$, $d$, and $s$ quarks only, and the HRG phase containing all strange and nonstrange baryons. For this $R_f = R_{N\bar{N}\Lambda\bar{\Lambda}}/R$. For the curve (C) QGP phase consists of $u$ and $d$ quarks and the HRG phase consists of all the nonstrange baryons. For this case $R_f = R_{N\bar{N}}/R$.

value obtained by Murugesan *et al.*

In conclusion, the ratio $R$ of the baryon-number densities in the QGP and HRG phases have been obtained by incorporating the interactions up to $g_s^3$ (i.e., $\alpha_s^{3/2}$) order terms in QGP and the volume corrections, which takes care of the repulsive interactions in the dense HRG phase. Our calculation reveals that $R$ reaches a very high value ($\gg 10$) when $T_c$ does not fall in the range 150 MeV $< T_c <$ 260 MeV. This illustrates the importance of obtaining a more accurate value of $T_c$ from QCD lattice gauge calculations. In the past, calculations of $R$ have been made by several groups neglecting the interaction between quarks in QGP and hadrons in HRG. Murugesan *et al.* [4] have taken Hagedorn's finite-size corrections for the hadrons and included many hadrons in the HRG spectrum. Kapusta and Olive have considered [14] a simplified treatment for the repulsive interactions between hadrons by parametrizing the mean-field potential energy. They noticed that the baryon-density ratio $R$ is $\simeq 7$ for 160 MeV $< T_c <$ 240 MeV. We have considered the repulsive interactions between baryons as the finite hard-core volume corrections which have frequently been used by several authors in the recent past. Moreover, we find that bag constant $B$ does not remain an arbitrary parameter in our calculation [9] and is linked with the hard-core radius $r_p$ of the baryons. We also notice that merely changing the values of $r_p$ (or, in other words, $B$) as well as the strong interaction scale parameter $\Lambda_s$ does not alter the results of our calculations to any significant extent. We conclude that our present ignorance of the exact values of many relevant parameters (e.g., $T_c$) of the quark-hadron phase transition leads to much larger uncertainties in the predictions of PNS. We thus hope that a more detailed as well as realistic treatment for the calculation of the baryon-density ratio $R$ as given here will remove these uncertainties and a clear picture for the isothermal baryon-number fluctuations in the early Universe will emerge.

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