Low-energy theorems in nontopological soliton models

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Three probes of the dynamical response of the nucleon are considered in the framework of nontopological soliton models: Compton scattering, pion scattering, and photoproduction. It is shown that low-energy theorems, based on gauge and chiral symmetries, are satisfied in such models. Crucial to this result is the use of a model and a framework (RPA) for calculating the response to external probes which treat the translational zero mode correctly. Processes such as the static polarizabilities and threshold pion scattering which are not constrained by low-energy theorems are also considered.

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I. INTRODUCTION

Nontopological soliton models have been used successfully to describe static baryon properties [1,2]. Excited baryon states have also been studied in such models using the random phase approximation (RPA) [3,4]. One of the advantages of these models over other relativistic quark models, in particular, bag models [5,6], is the fact that quarks are bound or confined by their interactions with fields which have their own dynamics. This should permit the calculation of dynamical processes such as the excitation of baryon resonances by photon- or pion-nucleon interactions. However, in such calculations it is important to ensure that the approximations respect the symmetries of the model, satisfying, for example, the constraints of gauge invariance. Low-energy theorems [7–9] based on gauge invariance and chiral symmetry can provide necessary tests of the approximations used in connection with such models. The standard treatments of center-of-mass motion in bag models fail to satisfy these conditions [10,11].

We study here interactions of nontopological solitons with photons and pions using a linear response approach. This is based on a RPA treatment of baryon excitations and is similar to that developed by Broniowski and Cohen for hedgehog solitons [12,13]. The version we develop is applicable to solitons with weak pion fields, for which cranking is not needed in order to generate spin-isospin eigenstates.

In the RPA, excitations are described as small-amplitude classical oscillations about the soliton. The corresponding excited states are mixtures of quark $1p$-$1h$ states and boson one-quantum states [3]. The RPA equations can be obtained by diagonalizing the Hamiltonian for small-amplitude fluctuations about the soliton. In studying the response of a nucleon to an external current, the current appears as a source term in the inhomogeneous version of these equations. The matrix element of a second current between this linear response and a ground-state nucleon yields a two-current amplitude. Such amplitudes can describe, for example, photon-nucleon scattering and the electromagnetic polarizabilities of the nucleon [12].

The spectrum of RPA excitations contains zero modes corresponding to the symmetries of the model which are broken by the mean-field soliton. There is also a conjugate cranking or boost mode for each zero mode [14]. In the cases studied here, electric dipole and s-wave pion interactions, the zero mode corresponding to motion of the nucleon as a whole together with its conjugate boost mode are the relevant ones. It is these modes which are crucial to the low-energy theorems (LET's). In some cases these modes contribute directly; in others the form of the response is constrained by the fact that the physical excitations must be orthogonal to the zero mode.

In soliton models the low-energy theorems are obtained from a combination of excited states and quark Z graphs. This is in contrast with models based on pointlike Dirac nucleons. In such models, nucleon Z graphs play an important role, for example, being responsible for producing the correct Thomson amplitude. Brodsky [15] has argued that such diagrams should be suppressed by form factors for pair creation of composite objects. Our results support the view that nucleon Z graphs should be regarded as mocking up the effects of nucleon structure [16].

The LET's are obtained by considering responses of the nucleon to static currents. The linear response method is equally applicable for nonzero frequency ω where it can be used, for example, to calculate pion scattering. The amplitude for photon scattering in the limit $\omega \rightarrow 0$ (with the Thomson term removed) can be used to determine the electric polarizability of the nucleon as in Ref. [12].

The method we describe here is applicable to any nonhedgehog soliton model. As a specific example we apply it to a chiral version of the color-dielectric model [2,17]. These pion fields in this model are sufficiently weak that the methods of the cloudy bag model [6] are appropriate. We work directly with states of definite spin and isospin, rather than having to obtain them by cranking or projection from a hedgehog state. The model embodies the appropriate chiral symmetry of QCD, as well as electromagnetic gauge invariance. LET's for both pion- and photon-nucleon interactions can therefore be studied in it.

The structure of this paper is as follows. In the rest of this section we introduce the chiral color-dielectric model. In Sec. II we briefly review the RPA equations and the forms of the translational zero mode and boost mode. The general form of the response to an external current is given in Sec. III. We study three examples in the static limit to demonstrate how the method leads to the corresponding low-energy theorems: Thomson scattering, soft pion-nucleon scattering, and photoproduction of neutral pions at threshold. In Sec. IV we calculate related quantities which are not constrained by low-energy theorems: the electric polarizabilities of the nucleons and pion-nucleon scattering at threshold. We summarize our results in Sec. V and discuss possible extensions of this approach.

A. The chiral color-dielectric model

The color-dielectric model [2,18] is, in its simplest form, a dynamical model of quarks which are confined to a localized region of space by their coupling to a color-singlet field, χ. Its form has been motivated by studies in which a process of "block-spinning" QCD is carried out [19]; this leads to an octet of "coarse-grained" gluon fields, and the singlet χ, which represents a glueball or $\bar{q}q$-glueball hybrid, and which presumably incorporates much of the nonperturbative physics of confinement.

Mesons can be introduced to the model in order to make it chirally symmetric [17]. In a hedgehog configuration the pion fields are weak [20] and so a more convenient scheme is first to solve for the basic solitonic quark and χ fields, using the mean-field approximation, and then to include mesons perturbatively [21]. This has much in common with the usual treatment of the cloudy-bag model, and many techniques developed there can be used in the color-dielectric model with only slight modifications. However, a dynamical model has the considerable advantage that the center-of-mass motion can be treated consistently, something which is not possible in the cloudy-bag model. The full Lagrangian is

$$\mathcal{L}=\bar{\psi}\left[i\partial\!\!\!/-m-\frac{g_\sigma}{\chi}(\sigma+i\gamma_5\tau\cdot\phi)\right]\psi+\tfrac{1}{2}\partial_\mu\chi\,\partial^\mu\chi-\tfrac{1}{2}M_\chi^2\chi^2 +\tfrac{1}{2}\partial_\mu\sigma\,\partial^\mu\sigma+\tfrac{1}{2}\partial_\mu\phi\,\partial^\mu\phi -\tfrac{1}{4}\lambda^2(\sigma^2+\phi^2-\nu^2)+f_\pi m_\pi^2\sigma\ . \quad (1.1)$$

The Mexican hat potential for the mesons gives the σ field a vacuum expectation value of f_π. The parameters of the potential are related to the meson masses by

$$m_\sigma^2=\lambda^2(3f_\pi^2-\nu^2)\ ,\quad m_\pi^2=\lambda^2(f_\pi^2-\nu^2)\ . \quad (1.2)$$

The coefficient of σ in the mesonic symmetry-breaking term has been chosen to satisfy PCAC (partial conservation of axial-vector current) in the meson sector. We will present results for the model both with and without the explicit quark mass term. Such terms violate the Goldberger-Treiman relation, and since the shift in the σ vacuum expectation value (VEV) produced by the symmetry-breaking term in the meson sector feeds through to the quarks there is also a degree of double counting. We have discussed elsewhere [22] a possible modification of the model which goes some way to remedying this. For the sake of simplicity, however, we have not included this possibility here.

The basic Lagrangian (with meson fields set to their vacuum expectation values and in the absence of a quark mass) has two parameters: M_χ and a dimensionless coupling constant β, where $M_\chi^2\beta^2=g_\sigma f_\pi$. For a given β we find the ground-state solution which is taken to represent the nucleon and Δ, and we fit M_χ by requiring that the rms quark radius equal the nucleon isoscalar radius, 0.72 fm, since that will be least affected by mesonic corrections. It is then found that the energy and other properties are remarkably insensitive to the value of β, and that M_χ scales as $\beta^{-2/3}$ at least for M_χ greater than around 1 GeV. These results have been shown to be exact if the χ kinetic energy can be ignored [23].

The lowest-energy solution to the Euler-Lagrange equations, with all dimensioned quantities expressed in terms of the relevant power of M_χ, consists of three quarks in an s-wave orbital and an s-wave mean χ field:

$$q_0=\begin{pmatrix} G(r) \\ i\sigma\cdot\mathbf{r}F(r)\end{pmatrix} \quad \text{and} \quad \chi=\chi_0(r)\ . \quad (1.3)$$

In this paper we have used $\beta=0.028$ which implies $M_\chi=2354$ MeV. The soliton profile for this parameter set is shown in Ref. [2].

II. THE RPA EQUATIONS

The problems that we are considering here have much in common. In particular, they can all be cast in terms of the response of the nucleon to a time-varying external probe. In each case, moreover, the inclusion of the excitation mode corresponding to the translational motion of the nucleon as a whole is crucial if low-energy theorems are to be satisfied. The appropriate framework for study of these responses within the context of a dynamical model such as the color-dielectric model is the random phase approximation (RPA), which gives the energies of the small-amplitude excitations of the soliton [3,4]. Where the ground state breaks a symmetry of the Hamiltonian (in this case translational), the RPA equations have a zero-mode solution [14]. Thus the response due to intrinsic excitations of the soliton can be distinguished from that due to the motion of the soliton as a whole, and both parts are included. This is the major difference from the treatment of the response to perturbations of the MIT bag, in which the motion of the center of mass must be corrected for in a somewhat *ad hoc* fashion [10,11].

If we write the quark and χ fields of the soliton as the mean field solution plus a small time-dependent perturbation, the linearized equations of motion in the absence of a source read

$$\begin{aligned}&\left[-i\boldsymbol{\alpha}\cdot\nabla-i\frac{\partial}{\partial t}+\gamma_0\frac{\beta^2}{\chi_0}\right]\delta q-\frac{\beta^2}{\chi_0^2}\gamma_0 q_0\delta\chi=0\ ,\\&\left[\frac{\partial^2}{\partial t^2}-\nabla^2+M_\chi^2+\frac{2\beta^2}{\chi_0^3}\sum_q\bar{q}_0q_0\right]\delta\chi\\&\qquad-\frac{\beta^2}{\chi_0^2}\sum_q(\delta\bar{q}q_0+\bar{q}_0\delta q)=0\ .\end{aligned}\tag{2.1}$$

These equations have solutions with the time dependence

$$\begin{aligned}\delta q&=q_Xe^{-i(\epsilon+\omega)t}+q_Y^*e^{-i(\epsilon-\omega)t}\ ,\\\delta\chi&=Ze^{-i\omega t}+Z^*e^{i\omega t}\ .\end{aligned}\tag{2.2}$$

The χ equation can be made linear in ω by introducing $\Pi=-i\omega Z$ as another variable, and Eqs. (2.1) can then be written in the form of an eigenvalue equation:

$$\mathbf{Mv}=\omega\boldsymbol{\eta}\mathbf{v}\ ,\tag{2.3}$$

where

$$\mathbf{v}=(q_X,q_Y,Z,\Pi)^T\ \text{ and }\ \boldsymbol{\eta}=\begin{bmatrix}1&0&0&0\\0&-1&0&0\\0&0&0&i\\0&0&-i&0\end{bmatrix}\ .\tag{2.4}$$

This equation has two classes of solution. There are those in which the term $\sum_q(\delta\bar{q}q_0+\bar{q}_0\delta q)$ in the equation for $\delta\chi$ vanishes, and the excitations correspond to excited states of the quarks in the unchanged mean χ field. These are like the excitations of the MIT bag, and are the analogues of "Goldhaber-Teller" (GT) excitations of nuclei [24]. The others are full coupled excitations of quark and χ fields in which all quarks move in the same way. We term these "isoscalar." In the GT modes there is no coupling between q_X and q_Y, and excitations will be exclusively one or the other, but the isoscalar modes will, in general, contain both. There is a symmetry of the equations under the exchange $q_X\leftrightarrow q_Y$, $\Pi\rightarrow-\Pi$, and $\omega\rightarrow-\omega$.

These RPA equations have a translational zero mode $\mathbf{P}$ and a conjugate boost mode $\mathbf{Q}$ defined by

$$\mathbf{MP}=0\ \text{ and }\ \mathbf{MQ}=-i\boldsymbol{\eta}\mathbf{P}\ ,\tag{2.5}$$

with the form

$$\begin{aligned}\mathbf{P}&=\left[\frac{\partial q_0}{\partial z},\frac{\partial q_0^*}{\partial z},\frac{\partial\chi_0}{\partial z},0\right]^T\ ,\\\mathbf{Q}&=\left[(\tfrac{1}{2}\alpha_3+i\epsilon z)q_0,(\tfrac{1}{2}\alpha_3-i\epsilon z)q_0^*,0,-\frac{\partial\chi_0}{\partial z}\right]^T\ .\end{aligned}\tag{2.6}$$

The solutions corresponding to different eigenvalues satisfy the orthogonality relations [14]

$$\begin{aligned}&\mathbf{v}_i^\dagger\boldsymbol{\eta}\mathbf{v}_j=s_i\delta_{ij}\ ,\quad\mathbf{Q}^\dagger\boldsymbol{\eta}\mathbf{P}=iM_0\ ,\\&\mathbf{v}_i^\dagger\boldsymbol{\eta}\mathbf{P}=0\ ,\quad\mathbf{P}^\dagger\boldsymbol{\eta}\mathbf{P}=0\ ,\\&\mathbf{v}_i^\dagger\boldsymbol{\eta}\mathbf{Q}=0\ ,\quad\mathbf{Q}^\dagger\boldsymbol{\eta}\mathbf{Q}=0\ ,\end{aligned}\tag{2.7}$$

where M_0 is the energy of the unperturbed soliton and $s_i=\pm1$. (The RPA matrix here is not positive definite, so there is no automatic connection between the sign of the eigenvalue and the norm of the mode.) In the zero mode the χ field is a p wave and the quarks are in a combination of $\kappa=1$ and $\kappa=-2$ states (that is, p wave upper components and s- or d-wave components, total angular momentum $j=\frac{1}{2}$ or $\frac{3}{2}$, respectively). Thus the translational zero mode does not show up in studies of breathing mode excitations, but it does contribute in the pion scattering, electroproduction, and Compton scattering processes considered in this paper.

The spectrum of excitations in the channel of interest may be found by expanding the χ and $\dot{\chi}$ fields in a Bessel-function basis, and the quark fields in a basis of MIT eigenstates with the appropriate values of κ, and diagonalizing the resultant matrix. (As the positive and negative spectra are the same, it is convenient to use a trick of Broniowski and Cohen which reduces the dimension of the matrix by a factor of 2 and gives ω^2 rather than ω [3].) Above the χ mass there are states which correspond to scattering states of the χ field.

The lack of positive definiteness of the RPA matrix shows up in complex eigenvalues above the χ threshold. These correspond to processes in which a quark drops into a state deep in the unfilled Dirac sea and emits a χ particle. These, of course, represent instabilities of the soliton which would not arise if the Pauli blocking due to the Dirac sea were properly accounted for. As discussed by Broniowski and Cohen (Appendix D of [13]) meson cloud effects should approximately reproduce the contributions of the sea to the response. However, introducing boson fields to describe the cloud cannot remove the instabilities. In the present case the χ mass is so large (it is always taken to be above 1 GeV, and in the parameter set used here it is 2354 MeV) that these complex poles have no discernable effect on low-energy processes.

III. RESPONSE TO EXTERNAL PROBES

We wish to study the response of the soliton to external probes, in order to explore processes such as pion scattering, electro-production, and Compton scattering of timelike photons (the last mentioned process will also give us the electric polarizability of the soliton). We therefore introduce a source term in the Lagrangian corresponding to an oscillating electric or π^0 field of the form $\hat{\mathbf{z}}(Se^{-i\omega t}+S^*e^{i\omega t})$. This will give rise to source terms in the RPA equations, which will in turn drive a response [12]. The electric dipole moment of the response gives the induced polarization, and the corresponding overlap with the pion source gives the pion production amplitude, in each case with energy transfer ω. The existence of the zero mode means that taking ω to zero may require special care. Without the zero mode, however, various low-energy theorems would not be satisfied.

The response $\mathbf{f}$ to a source term $\mathbf{j}$ in the RPA equations can be expanded in terms of the RPA eigenstates:

$$(\mathbf{M}-\omega\boldsymbol{\eta})\mathbf{f}=\mathbf{j}\ .\tag{3.1}$$

The solution $\mathbf{f}$ will, in general, have the form

$$\mathbf{f}=a\mathbf{P}+b\mathbf{Q}+\sum_i c_i\mathbf{V}_i\ ,\tag{3.2}$$

where

$$a=\frac{1}{M_0}\left[\frac{\mathbf{iQ}^\dagger\mathbf{j}}{\omega}-\frac{\mathbf{P}^\dagger\mathbf{j}}{\omega^2}\right], \quad b=\frac{-i\mathbf{P}^\dagger\mathbf{j}}{M_0\omega},$$

and (3.3)

$$c_i=\frac{s_i\mathbf{v}_i^\dagger\mathbf{j}}{\omega_i-\omega}.$$

[**P** and **Q** are the zero and boost modes as defined in Eq. (2.5) above.] From this we see that if the overlap of the source with the zero or boost modes is not zero, there will be a divergent piece in the response. This is the situation with an electromagnetic source term. In the physical process, Compton scattering, in which this response is measured, the amplitude has an extra factor of ω^2, so the final result is not divergent. The contribution from the zero mode is just the Thomson term, as will be detailed below.

Numerically, results for finite ω can be obtained either explicitly from Eq. (3.1), using the results of the diagonalization of the RPA matrix, or by solving the differential equations (2.1) with source terms directly with the use of a differential equation solver such as COLSYS [25]. However, since we know the analytic form of the zero mode we can demonstrate that low-energy theorems are satisfied without resorting to numerics, as we show below.

The processes in which we are interested are those in which one sort of particle (pion or photon) is absorbed on a nucleon, and the same or another sort of particle is emitted. If H_1 and H_2 are the respective interaction Hamiltonians between these particles and the field of the soliton, the scattering amplitude has the form

$$F=\langle N,2|H_2(E-H_0)^{-1}H_1|N,1\rangle, \tag{3.4}$$

where $|N,1\rangle$ is a state consisting of a nucleon and a quantum of the first field. In terms of the RPA matrix, this is proportional to the matrix element

$$M_{12}=\mathbf{j}_2^\dagger\mathbf{f}_1, \tag{3.5}$$

where $\mathbf{f}_1$ is the response of the soliton generated by $\mathbf{j}_1$, and $\mathbf{j}_1$ and $\mathbf{j}_2$ are the appropriate currents coupled to oscillating classical fields.

A. The Thomson term

In the calculation of the electric polarizability of the soliton, we calculate the induced dipole moment of the soliton in response to an oscillating electric field. Such a field gives an extra term in the Lagrangian

$$\delta\mathcal{L}=(Ee^{-i\omega t}+E^*e^{i\omega t})\sum_q(\delta q^\dagger\hat{Q}zq_0+q_0^\dagger\hat{Q}z\delta q) \tag{3.6}$$

and generates a source term on the right-hand side (RHS) of Eq. (3.1) of the form

$$\mathbf{j}_{\rm el}=(\hat{Q}zq_0,\hat{Q}zq_0^*,0,0)^T, \tag{3.7}$$

where $\hat{Q}$ is the charge operator. This source is orthogonal to the boost mode **Q**, but not to the zero mode **P**:

$$\begin{aligned}\mathbf{P}^\dagger\mathbf{j}_{\rm el}&=\left\langle N\left|\sum_q\int d^3r\,\frac{\partial q_0^\dagger}{\partial z}\hat{Q}zq_0\right|N\right\rangle+\text{c.c.}\\&=-Q_N,\end{aligned} \tag{3.8}$$

where Q_N is the charge on the nucleon, and so from Eqs. (3.2) and (3.3) the response is of the form

$$\mathbf{f}_{\rm el}=\tilde{\mathbf{f}}_{\rm el}+\frac{Q_N}{\omega^2M_0}\mathbf{P}. \tag{3.9}$$

The Compton scattering amplitude is ω^2 times the matrix element in Eq. (3.5) with the source and response appropriate to the electric field. At zero energy transfer, therefore, only the divergent part of the response will contribute in the scattering amplitude. Thus we have

$$\begin{aligned}F&=\lim_{\omega\to0}\omega^2\mathbf{j}_{\rm el}^\dagger\mathbf{f}_{\rm el}\\&=-Q_N^2/M_0,\end{aligned} \tag{3.10}$$

which is the Thomson scattering term for a nucleon of charge Q_N and mass M_0 [7].

B. Pion-nucleon scattering

The relevant low-energy theorem in the case of neutral pion-nucleon scattering relates the isospin-averaged amplitude $F^{(+)}$ at the soft point (zero four-momentum for both pions) to the σ commutator [9]:

$$\sigma_{\pi N}=-f_\pi^2F^{(+)}(0). \tag{3.11}$$

In deriving this relation, it is essential that the off-shell amplitude be defined using the divergence of the axial-vector current, $\partial_\mu A_i^\mu/f_\pi m_\pi^2$, as the interpolating pion field. In a model, the off-shell scattering amplitude calculated using the model pion field will satisfy (3.11) only if that field and the interpolating one are identical. Such ambiguities disappear for on-shell amplitudes.

In the color-dielectric model with a symmetry-breaking term in the meson sector and a quark mass term, the σ commutator is

$$\sigma_{\pi N}=\left\langle N\left|\int d^3r\left[\sum_q m\bar{q}_0q_0-f_\pi m_\pi^2(\sigma-f_\pi)\right]\right|N\right\rangle. \tag{3.12}$$

(Note that all masses and decay constants are in units of M_χ.) Here q_0 is the ground-state solution in the presence of the quark mass term. The σ field may be replaced by its source current, defined by

$$(-\nabla^2+m_\sigma^2)(\sigma-f_\pi)=j_\sigma=-\frac{\beta^2}{f_\pi\chi_0}\sum_q\bar{q}_0q_0 \tag{3.13}$$

giving

$$\sigma_{\pi N}=\left\langle N\left|\int d^3r\sum_q\left[\frac{m_\pi^2}{m_\sigma^2}\frac{\beta^2}{\chi_0}\bar{q}_0q_0+m\bar{q}_0q_0\right]\right|N\right\rangle. \tag{3.14}$$

Now the scattering amplitude has three contributions, as shown in Fig. 1. One, with an exchanged σ and momentum transfer q^2, contributes

$$F_\sigma^{(+)}(q)=\frac{1}{f_\pi}\left[1-\frac{m_\pi^2}{m_\sigma^2}+\frac{q^2}{m_\sigma^2}\right]\times\left\langle N\left|\int d^3r\, e^{i\mathbf{q}\cdot\mathbf{r}}\sum_q\frac{\beta^2}{f_\pi\chi_0}\bar{q}_0q_0\right|N\right\rangle , \qquad (3.15)$$

where we have used the relation $m_\sigma^2-m_\pi^2=2\lambda^2f_\pi^2$, and the matrix element is the nucleon-σ coupling constant.

The other two contributions are direct scattering with an intermediate excited nucleon. In the presence of a neutral pion field $P(t)$ the perturbing term in the Lagrangian is

$$\delta\mathcal{L}=-\sum_q\frac{iP(t)\beta^2}{f_\pi\chi_0}\delta\bar{q}\gamma_5\tau_3q_0+\text{c.c.} , \qquad (3.16)$$

giving a source term for unit field strength:

$$\mathbf{j}_P=\frac{\beta^2}{f_\pi}\left[-i\gamma_0\gamma_5\tau_3\frac{q_0}{\chi_0},i\gamma_0\gamma_5\tau_3\frac{q_0^*}{\chi_0},\sum_q\frac{\bar{q}_0i\gamma_5\tau_3q_0}{\chi_0^2},0\right]^T . \qquad (3.17)$$

This time we know the analytic form of the response, which is

$$\mathbf{f}_P=\frac{1}{2f_\pi}\left[-i\left[\gamma_5+\frac{m}{\epsilon}\gamma_0\gamma_5\right]\tau_3q_0 ,\ \times i\left[\gamma_5+\frac{m}{\epsilon}\gamma_0\gamma_5\right]\tau_3q_0^*,0,0\right] . \qquad (3.18)$$

Using this form in Eq. (3.5), we obtain the direct scattering amplitude for zero-momentum transfer:

$$\begin{aligned}F_d^{(+)}(0)&=-\frac{1}{f_\pi^2}\left\langle N\left|\int d^3r\sum_q\frac{\beta^2}{\chi_0}\left[\bar{q}_0q_0-\frac{m}{\epsilon}q_0^\dagger q_0\right]\right|N\right\rangle\\&=-\frac{1}{f_\pi^2}\left\langle N\left|\int d^3r\sum_q\left[\frac{\beta^2}{\chi_0}\bar{q}_0q_0-m\bar{q}_0q_0\right]\right|N\right\rangle ,\end{aligned} \qquad (3.19)$$

where we have used the equations of motion for q_0 and dropped a term of order m^2.

Setting the two contributions to $F^{(+)}$, Eqs. (3.15) and (3.19), together we obtain the soft point ($q^2=0$) scattering amplitude

$$F^{(+)}(0)=-\frac{1}{f_\pi^2}\left\langle N\left|\int d^3r\sum_q\left[\frac{m_\pi^2}{m_\sigma^2}\frac{\beta^2}{\chi^0}\bar{q}_0q_0-m\bar{q}_0q_0\right]\right|N\right\rangle . \qquad (3.20)$$

The first term is just $-1/f_\pi^2$ times the corresponding term in $\sigma_{\pi N}$, Eq. (3.14), and so the LET (3.11) is satisfied in the absence of explicit quark mass terms. If such masses are included, the second term has the wrong side for the LET; this is because the model pion field differs from the interpolating one.

The scattering amplitude calculated with the *interpolating* pion field does satisfy the LET, whether or not explicit mass terms are present in the Lagrangian. For the interpolating field, at zero four-momentum, the perturbation (3.16) should be replaced by

$$\delta\mathcal{L}=-\sum_q i\frac{P}{f_\pi}\left[\frac{\beta^2}{\chi_0}+m\right]\delta\bar{q}\gamma_5\tau_3q_0+\text{c.c.} , \qquad (3.21)$$

where the addition term comes from the contribution of the quark mass to the divergence of the axial-vector current. This term cancels the part of the response (3.18) which is proportional to m, leaving a pure chiral rotation of q_0. The scattering amplitude at zero-momentum transfer calculated using this source and response is

$$F_d^{(+)}(0)=-\frac{1}{f_\pi^2}\left\langle N\left|\int d^3r\sum_q\left[\frac{\beta^2}{\chi_0}\bar{q}_0q_0+m\bar{q}_0q_0\right]\right|N\right\rangle . \qquad (3.22)$$

Combining this with (3.15), we see that the LET is satisfied.

The analogous results also hold in a linear or nonlinear nucleon-level σ model with the corresponding symmetry-breaking terms, providing the interpolating pion field is used. In models with pseudoscalar pion coupling to Dirac nucleons the soft-pion scattering amplitude involves nucleon Z graphs. Like Thomson scattering, the amplitude (3.22) is an example of a calculation in which a sum over excited states of the soliton and quark Z graphs reproduces a result obtained at a nucleon level using Z graphs.

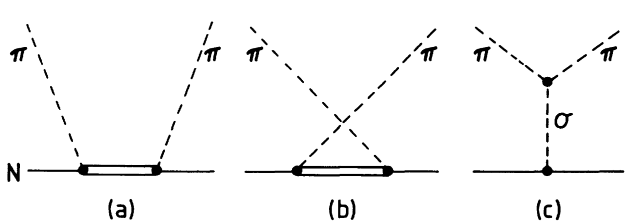

FIG. 1. The three diagrams which contribute to pion-nucleon scattering: (a) and (b) direct $F_d^{(+)}$ and (c) σ exchange $F_\sigma^{(+)}$, defined in Eqs. (3.16) and (3.20).

C. Pion photoproduction

The process in which absorption of a photon is accompanied by emission of a neutral pion can be explored in this model through a hybrid of the two processes discussed above: either an electric field [Eq. (3.6)] or a neutral pion field [Eq. (3.16)] is imposed on the soliton, and the electroproduction amplitude is related to the pionic response or the induced dipole moment, respectively. Numerically, the latter combination is preferable, since the pionic source term $\mathbf{j}_P$ is orthogonal to the zero mode $\mathbf{P}$, and so there is no divergent part to the response.

We have used here the time component of the electromagnetic current, and so the matrix element which we compute corresponds to the longitudinal multipole L_{0+}, observable in electroproduction. To the order considered here it is equivalent to the photoproduction amplitude E_{0+} [26].

Even though the overlaps of the pionic source with the zero and boost modes, $\mathbf{j}_P^\dagger\mathbf{P}$ and $\mathbf{j}_P^\dagger\mathbf{Q}$, both vanish, we cannot conclude that the response to a pion field has no zero-mode piece; provided the energy transfer ω is zero such a piece may be present, although it will not have a divergent coefficient. The response may be written

$$\mathbf{f}_P=\tilde{\mathbf{f}}_P+a\mathbf{P}\ , \tag{3.23}$$

where $\tilde{\mathbf{f}}_P$ is orthogonal to $\mathbf{P}$ and $\mathbf{Q}$ with respect to the RPA metric, and

$$\begin{aligned} a&=\mathbf{Q}^\dagger\boldsymbol{\eta}\mathbf{f}_P/\mathbf{Q}^\dagger\boldsymbol{\eta}\mathbf{P} \\ &=-g_A/2f_\pi M_0\ . \end{aligned} \tag{3.24}$$

In the absence of explicit quark mass terms, of course, the electroproduction amplitude at zero energy transfer vanishes, since the response $\mathbf{j}_P$ is simply a chiral rotation of the soliton fields, and the electromagnetic coupling is chirally invariant. It is easy to verify that $\mathbf{j}_{\rm el}^\dagger\mathbf{f}_P=0$, and hence

$$\mathbf{j}_{\rm el}^\dagger\tilde{\mathbf{f}}_P=-a\,\mathbf{j}_{\rm el}^\dagger\mathbf{P}\ . \tag{3.25}$$

However, for any finite energy transfer, however small, the coefficient a of the zero mode in the response vanishes (Eq. 3.3). Thus for infinitesimal ω the matrix element [Eq. (3.5)] is

$$\begin{aligned} M&=\mathbf{j}_{\rm el}^\dagger\tilde{\mathbf{f}}_P \\ &=-g_A Q_N/2f_\pi M_0\ . \end{aligned} \tag{3.26}$$

The (off-shell) longitudinal multipole response L_{0+} is defined in terms of the spatial components of the electromagnetic current. However, by gauge invariance we can use the time component which gives the matrix element M instead (see Appendix A for a proof that this is valid in the RPA framework):

$$L_{0+}(\omega)=\omega M(\omega)/4\pi(1+\mu)\ , \tag{3.27}$$

where $\mu=m_\pi/M_N$.

The low-energy theorem for electroproduction gives the $O(m_\pi)$ contributions to this quantity at the physical pion threshold, $\omega=m_\pi$. The predictions for the processes $\gamma p\to p\pi^0$ and $\gamma n\to n\pi^0$ are [8]

$$L_{0+}(m_\pi)=-\frac{1}{4\pi}\frac{g_A}{2f_\pi}(\tfrac{1}{2}\mu\pm\tfrac{1}{2}\mu)\ . \tag{3.28}$$

Thus our model exactly reproduces the low-energy theorem to $O(m_\pi)$. The next corrections in this framework are $O(m_\pi^3)$, since finite photon momentum, which gives $O(m_\pi^2)$ corrections, has not been incorporated. Explicit quark mass terms, with $m\propto m_\pi^2$, again contribute only at $O(m_\pi^3)$.

Crucial to obtaining the correct LET's in all three processes considered is the proper treatment of the zero mode. The Thomson scattering result requires the zero mode directly, but pion electroproduction and soft pion scattering are equally dependent on it, as obtaining the correct results requires that the rest of the excitation spectrum be orthogonal to the zero mode. This is why the cloudy-bag model cannot reproduce the LET's in these cases [26].

IV. NUMERICAL RESULTS

In obtaining the results of the last section, we relied heavily on the RPA formulation of the response to an external stimulus, but our results were analytic. There are other problems, however, to which there is no analytic answer, and for which we must actually solve the RPA equations numerically. One example of this is the proton and neutron polarizability, which is the finite part of the response left over when the zero mode is subtracted; another is pion scattering at threshold.

We have chosen to solve the differential equations (2.1) for the fields of the response q_X, q_Y, and Z directly, varying ω as required, rather than diagonalizing the RPA matrix in an appropriate basis. With the source terms in which we are interested, the quark response is a mixture of $\kappa=1$ and -2 states (negative parity; p-wave upper and s- or d-wave lower component) and the χ response is p wave; the χ momentum Π need not be introduced as a separate field. Thus we can write

$$q_X=\begin{vmatrix}\sigma\cdot\hat{\mathbf{r}}\sigma_3A_X(r)\\ i\sigma_3C_X(r)\end{vmatrix}+\begin{vmatrix}(\hat{\mathbf{z}}-\tfrac{1}{3}\sigma\cdot\hat{\mathbf{r}}\sigma_3)B_X(r)\\ i(\hat{\mathbf{z}}\sigma\cdot\hat{\mathbf{r}}-\tfrac{1}{3}\sigma_3)D_X(r)\end{vmatrix}\ ,$$

$$Z=\hat{\mathbf{z}}H(r)\ , \tag{4.1}$$

and q_Y is defined analogously to q_X in terms of the four radial functions A_Y, B_Y, C_Y, and D_Y. The inclusion of an extra σ_3 in the definition of the $\kappa=1$ piece ensures that the zero mode is the same for both spin-up and spin-down quarks.

In general, the source term will be spin or isospin dependent; in fact, both τ_3 and $\tau_3\sigma_3$ dependencies arise in the problems considered. Suppose the dependence be on a diagonal operator $\hat{O}$, and quarks in the proton or neutron fall into two classes, those with eigenvalue of $\hat{O}$ o_1, and those with o_2. Now the coupling between the quark and χ fields in the RPA equations is spin and isospin independent. It is, therefore, convenient to consider two separate collective excitations of the quarks, one in which each type is represented according to its abundance in the nucleon (an isoscalar excitation) and one in which the two types have opposite signs (a "Goldhaber-Teller" excitation). The first represents the combination which couples to the χ excitation in the second of Eq. (2.1), while the coupling of the second to the χ excitation in the first of Eq. (2.1), vanishes. Since there are no direct couplings between the two quark excitations, the equations decouple into one set of nine equations involving the χ and isoscalar quark excitations, and another involving only the GT excitation. Furthermore, in the second set, there is no coupling between the q_X and q_Y pieces, giving two sets of four equations. The zero mode is confined to the isoscalar set.

The nine isoscalar equations are

$$\frac{d}{dr}C_{\{X \atop Y\}} - \left[\epsilon \pm \omega - \frac{\beta^2}{\chi_0}\right] A_{\{X \atop Y\}} - \frac{\beta^2}{\chi_0^2} HG = j_A \langle N|\hat{O}|N\rangle ,$$

$$-\left[\frac{d}{dr}+\frac{2}{r}\right]A_{\{X \atop Y\}} - \left[\epsilon \pm \omega + \frac{\beta^2}{\chi_0}\right]C_{\{X \atop Y\}} - \frac{3\beta^2}{\chi_0^2}HG = j_B \langle N|\hat{O}|N\rangle ,$$

$$\left[\frac{d}{dr}+\frac{3}{r}\right]D_{\{X \atop Y\}} - \left[\epsilon \pm \omega - \frac{\beta^2}{\chi 0}\right]B_{\{X \atop Y\}} - \frac{3\beta^2}{\chi_0^2}HG = j_B \langle N|\hat{O}|N\rangle , \quad (4.2)$$

$$\left[-\frac{d}{dr}+\frac{1}{r}\right]B_{\{X \atop Y\}} - \left[\epsilon \pm \omega + \frac{\beta^2}{\chi_0}\right]D_{\{X \atop Y\}} + \frac{3\beta^2}{\chi_0^2}HF = j_D \langle N|\hat{O}|N\rangle ,$$

$$\left[-\frac{d^2}{dr^2}-\frac{2}{r}\frac{d}{dr}+\frac{2}{r^2}-\omega^2+1+\frac{6\beta^2}{\chi_0^3}(G^2-F^2)\right]H - \frac{\beta^2}{\chi_0^2}\{G[A_X+A_Y+\tfrac{2}{3}(B_X+B_Y)] - F[C_X+C_Y+\tfrac{2}{3}(D_X+D_Y)]\} = j_H \langle N|\hat{O}|N\rangle ,$$

where $G(r)$, $F(r)$, and $\chi_0(r)$ are the static soliton fields defined in Eq. (1.3), and we are working in units in which $M_\chi = 1$. The source terms depend on the problem under study and are listed in Table I. The GT equations differ in two respects: first, the H field is deleted, and second, $\langle N|\hat{O}|N\rangle$ in the source term is replaced by $o_1 - o_2$.

Finally, we wish to calculate the overlap of the response with a second current, which can be reduced to radial integrals and a second spin-isospin operator $\hat{O}'$ with eigenvalues $o_{1'}$ and $o_{2'}$. (We deal with cases in which the two operators $\hat{O}$ and $\hat{O}'$ commute.) The full response for a nucleon containing n_{ij} quarks with eigenvalues o_i and o'_j, respectively ($\sum n_{ij} = 3$), is

$$\tfrac{1}{3}\langle N|\hat{O}'|N\rangle M_I + \tfrac{1}{3}(n_{11}n_{22} - n_{12}n_{21})(o'_1 - o'_2)M_{\rm GT} , \quad (4.3)$$

where $\langle N|\hat{O}'|N\rangle = (n_{11}+n_{21})o'_1 + (n_{12}+n_{22})o'_2$. (If the first and second currents are the same, $n_{12} = n_{21} = 0$.) The subscripts "GT" and "I" on the matrix elements M indicate "Goldhaber-Teller" and isoscalar, respectively. A glance at the coefficients of the source terms in Eq. (4.2) confirms that the result is independent of the order of the currents (that is, $\mathbf{j}_1^\dagger \mathbf{f}_2 = \mathbf{j}_2^\dagger \mathbf{f}_1$).

TABLE I. The source terms in the RPA equations for electric and pionic sources in Eq. (4.2). The radial fields G, F, and χ are the unperturbed soliton fields.

Source	j_A	j_B	j_C	j_D	j_H
Electric	$\frac{1}{3}rG$	rG	$\frac{1}{3}rF$	rF	0
Pion	$\frac{\beta^2 F}{f_\pi \chi_0}$	0	$\frac{\beta^2 G}{f_\pi \chi_0}$	0	$-\frac{2\beta^2 GF}{f_\pi \chi_0^2}$

For the case of an electric source, the operator is simply the charge, $\frac{1}{6}+\frac{1}{2}\tau_3$, with eigenvalues $\frac{2}{3}$ and $-\frac{1}{3}$, while for the pion source it is $\sigma_3\tau_3$ with eigenvalues 1 and -1. For a spin-up proton, the numbers of quarks n_{ij} are $\frac{5}{3}$, $\frac{1}{3}$, $\frac{2}{3}$, and $\frac{1}{3}$.

A. Electric polarizability

The intrinsic polarizabilities of the proton and neutron are simply the dipole moments of the response to an external electric field, with (in the case of the proton) the divergent Thomson piece subtracted off. The pion cloud will make substantial contributions to these quantities which cannot be calculated in our current framework. Even though they should not be directly compared with experiment, the valence-quark polarizabilities are of interest since they may also contribute significantly to the observed values.

The valence quark polarizabilities are given by

$$\alpha_N = \lim_{\omega \to 0}\left[\mathbf{j}_{\rm el}^\dagger \mathbf{f}_{\rm el} + \frac{\mathbf{Q}_N^2}{\omega^2 \mathbf{M}_0}\right] . \quad (4.4)$$

In terms of the isoscalar and GT responses described above we have the following expressions for the proton and neutron polarizabilities:

$$\alpha_p = \lim_{\omega \to 0}\left[\frac{1}{3}M_I + \frac{Q_N^2}{\omega^2 M_0}\right] + \frac{2}{3}M_{\rm GT} , \quad \alpha_n = \frac{2}{3}M_{\rm GT} . \quad (4.5)$$

Since the zero mode is confined to the isoscalar response which does not contribute to the neutron polarizability, there is, of course, no Thomson term for the neutron. Numerically, the subtraction of the zero mode is not as simple as Eq. (4.4) would suggest; the numerical "zero mode" is not exactly at $\omega = 0$, nor is its strength precisely as predicted. From the raw results giving the dipole moment as a function of ω, however, it is possible to subtract the zero mode almost exactly by allowing minor alterations in these two parameters; the results are shown in Fig. 2(a). The residue in the isoscalar channel, of $-0.13M_\chi^{-3}$, is very small, and can be seen to be almost exclusively due to one very strong pole at $\omega = 0.449$. This happens to be the one mode for which the analytic form is known. If the Dirac equation has only scalar potentials and q_0 is a solution with eigenvalue ϵ, $\gamma_0\gamma_5 q_0$ is another solution with eigenvalue $-\epsilon$. This pole corresponds to such a solution, based on the ground state of the unperturbed soliton, and with no change in the χ field. The value of ω is just 2ϵ, and the response is exclusively q_Y. This GT-type pole can occur in the isoscalar channel because its coupling to $\delta\chi$ vanishes ($\bar{q}_0\gamma_0\gamma_5 q_0 = 0$). Since the analytic form is known we can check its contribution at $\omega = 0$, confirming that it contributes more than 95% of the subtracted isoscalar response.

The GT response is dominated by the lowest excitations of the system, at around $0.1M_\chi$ or 235 MeV. The lowest observed excitation in this channel is the $N(1520)$, with an excitation energy of 580 MeV. The unphysically low excitation of this model, which is shared by the MIT bag ($\Delta E = 230R^{-1}$ MeV), is a significant problem which precludes realistic results. The GT response at $\omega=0$ is $3.32M_\chi^{-3}$, [Fig. 2(b)] so our final results for the polarizabilities are

$$\alpha_p \approx \alpha_n = 13.0\times 10^{-4}\ \text{fm}^3\ ,\quad \alpha_n - \alpha_p = 0.25\times 10^{-4}\ \text{fm}^3\ . \tag{4.6}$$

The experimental polarizabilities are [27]

$$\begin{aligned} \alpha_n &= (12.0\pm 1.5\pm 2.0)\times 10^{-4}\ \text{fm}^3\ , \\ \alpha_p &= (7.0\pm 2.2\pm 1.3)\times 10^{-4}\ \text{fm}^3\ . \end{aligned} \tag{4.7}$$

The apparent agreement with experiment is probably illusory, since we have completely ignored the effects of the pion cloud. The full polarizabilities will almost certainly be too large, a result which can be traced to the low-lying excitations in this class of model. The experiments currently suggest a significant difference between the proton and neutron polarizabilities, which is not seen here. Again, pions are likely to remedy this. Such a difference has been seen in studies of elementary nucleons coupled to pions [28], and so may not be a property of the quark core. Broniowski, Banerjee, and Cohen also reproduced this splitting in a linear σ model of quarks and pions, and found that the quarks provided -5% of it [12].

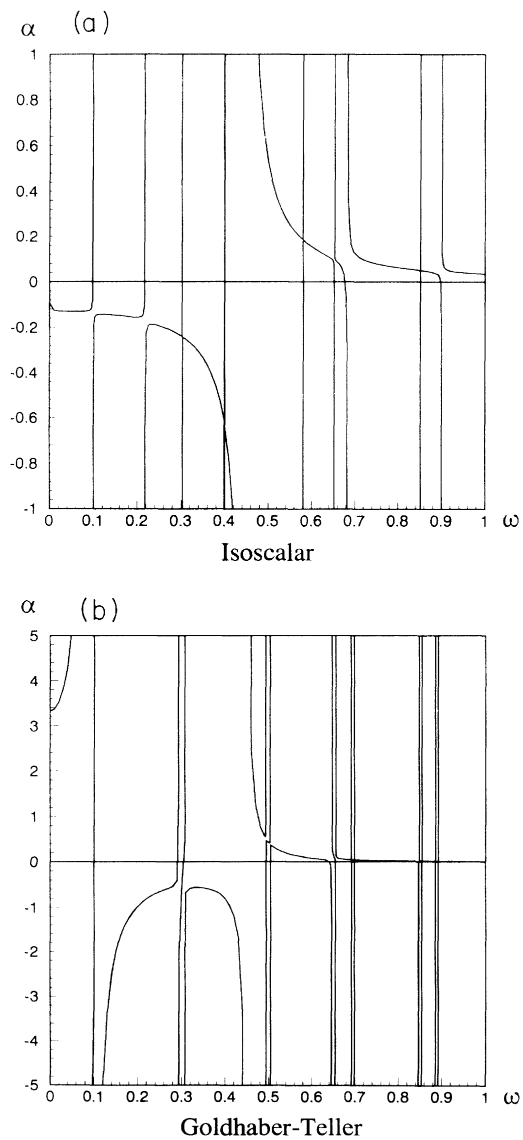

FIG. 2. (a) The isoscalar and (b) the Goldhaber-Teller contributions to the energy-dependent polarizabilities of the proton and neutron [Eq. (4.5)] in units of M_χ^{-3}.

B. Threshold pion scattering

Here we consider pion scattering at threshold, that is, with on-shell neutral pions and zero-momentum transfer. As with for soft pions, the scattering amplitude at threshold is $O(m_\pi^2)$. However, it is not fixed by any low-energy theorem. The threshold amplitude is much smaller than the soft-pion one, even though both are formally of the same order in m_π [29], but in the context of models this seems to be a contingent fact and not a consequence of any symmetry. (This was noted in Ref. [30].)

In Sec. III B we saw that the pion nucleon scattering amplitude is composed of two terms of $O(1)$ (Fig. 1), which cancel to leave terms of $O(m_\pi^2)$. Since our model Lagrangian has pointlike π-σ coupling, the σ-exchange term will not depend on the momentum squared of the pions, as long as the momentum transfer is zero. However, the direct scattering term will change, being equal to the matrix element Eq. (3.5) at $\omega = m_\pi$. (The full response is $\frac{5}{9}M_I + \frac{28}{27}M_{\text{GT}}$.) The lowest-order variation of this amplitude with ω is quadratic in m_π^2; as can be seen from Fig. 3 the change is positive. If a quark mass term is present, for example, $m=7$ MeV, and with $m_\sigma = 1200$ MeV, the soft scattering amplitude is already slightly positive and the amplitude at threshold is still larger. If one takes this as an argument against including explicit quark mass terms (the wrong-sign violation of the

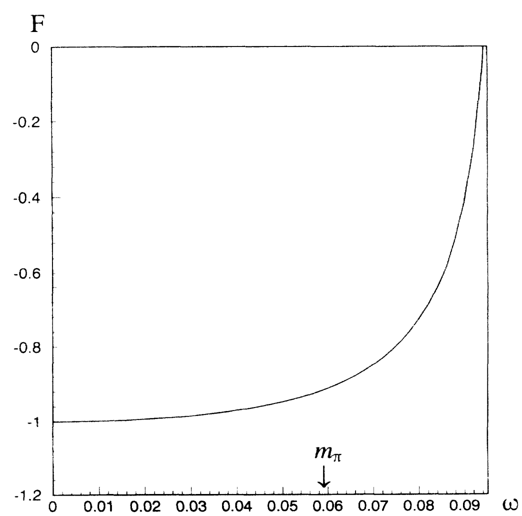

FIG. 3. The variation of the direct pion-nucleon scattering amplitude with energy, expressed as a fraction of $|F_d^{(+)}(0)|$.

Goldberger-Treiman relation is another[1]) one might hope that the increase as ω goes from 0 to m_π would cause cancellation and a small scattering length at threshold. Unfortunately, since $m_\pi = 0.059 M_\chi$ the influence of the poles at $0.1 M_\chi$ is strong. As a fraction of the direct contribution to the soft point amplitude, the total soft point amplitude in the absence of quark mass terms is -0.0134, while the change in the direct contribution between $\omega=0$ and m_π is 0.083. Thus the magnitude of the scattering amplitude at threshold is six times the soft amplitude, and the near vanishing of the physical amplitude cannot be explained in this model.

V. CONCLUSIONS

We have studied the interaction of pions and photons with nucleons using the chiral color dielectric model and a linear response approach (RPA). In this framework we have been able to explore the validity of low-energy theorems for Compton scattering, pion-nucleon scattering and pion photoproduction in this or any similar nontopological soliton model. Since these LET's depend on gauge invariance and chiral symmetry, they are important tests of the approximations used. In models with fixed boundaries, such as the cloudy bag, they are not satisfied but dynamical models can overcome these disadvantages.

We have shown that two of the three LET's are unconditionally satisfied, namely, the Thomson term in Compton scattering and the $O(m_\pi/M_N)$ prediction for the photoproduction amplitude E_{0+}. The third, namely, the relation between the soft-point isospin-averaged pion-nucleon scattering amplitude and the σ commutator, is trivially satisfied in the chirally symmetric limit by our model. For finite pion mass, however, the relation involves off-shell amplitudes, and relies on the use of the divergence of the axial-vector current as an interpolating pion field. The latter is not equal to the elementary pion field in any model which includes both that and explicit quark mass terms. Such a model does not satisfy the Goldberger-Treiman relation either. If quark mass terms are excluded, however, the color dielectric model incorporating the linear σ model for the meson sector does satisfy the LET. Even with quark mass terms the scattering amplitude and the σ commutator at the Cheng-Dashen point are correctly related, up to form factor effects.

In all three cases the presence in the excitation spectrum of a translational zero mode is crucial to satisfying the LET's. The Thomson term is due to the presence of the zero mode in the response, while the other two require an uncontaminated spectrum of finite energy excitations. We have written down analytic forms for the zero mode and its conjugate boost mode which were used in the demonstrations of the validity of the LET's.

In models with pointlike Dirac nucleons, nucleon Z graphs are essential to obtaining the LET for Compton scattering and also, at least for pseudoscalar π-N coupling, those for pionic processes. For soliton models treated in the RPA approximation, we find that a combination of quark Z graphs and excitations lead to the same results. This supports the idea that the role of nucleon Z graphs is to mock up effects of compositeness.

We have also looked at effects not constrained by symmetries, such as the electric polarizabilities of the proton and neutron, and the threshold pion-nucleon scattering amplitude. In neither case have we included pion loop effects. As in other models, we find that the observed difference between the proton and neutron polarizabilities cannot be ascribed to the quark core. Unphysically low-lying excitations mean that our polarizabilities are already as large as the experimental ones without including pions, and we fail to reproduce the experimentally observed near vanishing of the threshold scattering length.

In this paper we have looked only at probes which transfer zero-momentum to the nucleon. It is obviously important to try to extend the approach to finite momentum transfers by including recoil of the final nucleon. This would allow calculation of Compton scattering for nonzero photon energies and exploration of the order-m_π^2 contributions to the LET for pion photoproduction. Another necessary extension is the inclusion of contributions from the pion cloud to the responses. The long tail of the pion cloud is likely to make important contributions to the quantities calculated here; in particular, the electric polarizabilities of the nucleon.

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APPENDIX: GAUGE INVARIANCE

In Sec. III C we calculated the threshold electroproduction amplitude for neutral pions. The matrix element which entered could be regarded as the dipole moment induced by a neutral pion field oscillating with frequency ω. Up to a factor of the photon momentum, it is just the dipole piece of the matrix element of the time component of the electromagnetic current:

$$J^\mu = -\int d^4x\, \bar{\psi}(x)\hat{Q}\gamma^\mu\psi(x)e^{i(\mathbf{q}\cdot\mathbf{r}-\omega t)} . \tag{A1}$$

Current conservation requires that $\omega J^0 = q_i J^i$ and so the electroproduction amplitude should equally be calculable using the spatial components. Indeed, the longitudinal response is defined in terms of these, and we assumed current conservation in using ωJ^0 instead. The correct result can in fact be obtained from J^3 directly, as we show here.

Since we do not consider processes which flip the nucleon's spin, only J^0 and J^3 have nonvanishing matrix elements. Let $\mathbf{j}^\mu$ be the sources in the RPA equations generated by the above currents. To leading order in q_3, the time component is given in terms of the source defined in Eq. (3.7), $\mathbf{j}^0 = iq_3\mathbf{j}_{\rm el}$, while $\mathbf{j}^3$ is given by

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[1]In Ref. [22] we have discussed a third possibility which remedies the Goldberger-Treiman problem without excluding explicit quark mass terms, but the conclusions of this section would remain unchanged if that model were used.

$$\mathbf{j}^3=(\hat{Q}\alpha_3 q_0,\hat{Q}\alpha_3 q_0^*,0,0)^T . \tag{A2}$$

The current conservation equation thus reads

$$-i\omega q_3 \mathbf{j}_{\rm el}^\dagger \mathbf{f}_{\rm P}(\omega)=\mathbf{q}_3 \mathbf{j}^{3\dagger}\mathbf{f}_P(\omega) , \tag{A3}$$

where $\mathbf{f}_P(\omega)$ is the response to the pion source.

The zero-frequency response to $\mathbf{j}^3$ can be shown analytically to be $i\boldsymbol{\eta}\mathbf{j}_{\rm el}$, that is,

$$i\mathbf{M}\boldsymbol{\eta}\mathbf{j}_{\rm el}=\mathbf{j}^3 . \tag{A4}$$

Using this, we can rewrite the left-hand side of Eq. (A3):

$$\begin{aligned}-i\omega\mathbf{J}_{\rm el}^\dagger\mathbf{f}_P(\omega)&=i\,\mathbf{j}_{\rm el}^\dagger[\boldsymbol{\eta}(\mathbf{M}-\eta\omega)-\boldsymbol{\eta}\mathbf{M}]\mathbf{f}_P(\omega)\\&=i\,\mathbf{j}_{\rm el}^\dagger\boldsymbol{\eta}\mathbf{j}_P+\mathbf{j}^{3\dagger}\mathbf{f}_P(\omega) .\end{aligned} \tag{A5}$$

The first term vanishes since $\mathbf{j}_P$ is derived from a Hermitian current. Thus Eq. (A3) is satisfied, and our approximations preserve gauge invariance.

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