Effective Lagrangians for Z boson decay into photons

M. Stöhr and J. Hořejší
Nuclear Centre, Faculty of Mathematics and Physics, Charles University, V Holešovičkách 2, 180 00 Praha 8, Czech Republic
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The lowest-dimensional effective Lagrangians describing the decay of a neutral massive vector boson Z into three photons are considered. We restrict ourselves *a priori* to the parity-conserving interactions. The corresponding tree-level decay rate is calculated for a general effective Lagrangian of such a type. We thus get a generalization of some earlier results appearing in the literature.

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Within the standard model of electroweak interactions the process $Z\to\gamma\gamma\gamma$ is a rare decay mode as it may only proceed via one-loop (or higher) Feynman diagrams. The contribution of fermion loops has been known for some time (see [1,2]) and the corresponding boson loops have been studied recently in several papers (see [3–7]). According to these recent calculations, the branching ratio is predicted to be very small within the standard model, certainly less that 10^{-8} (see [3,4,5,7]). In addition to that, this process has also been discussed within some alternative models, in particular, assuming a scenario of compositeness, where such a decay mode might be significantly enhanced in comparison with the standard model (for a review and references, see Boudjema and Renard in Ref. [1]).

In general, an interaction of Z with three photons may be described in terms of an effective Lagrangian involving the Z boson and photon fields and their derivatives. If electromagnetic gauge invariance of such an interaction is to be maintained, one has to construct the corresponding Lagrangian by using tensors $F_{\alpha\beta}$ for photons; in order to preserve Lorentz invariance one then has to include at least one extra derivative (since the Z boson is described by a vector field). The lowest possible dimension of such an effective Lagrangian is then equal to eight, i.e., the relevant coupling constant must have a dimension of M^{-4} where M is a mass scale. This, of course, is an interaction of nonrenormalizable type, so that in renormalizable field theories (as, e.g., in the standard model) it can only be induced, as we have noted above, in a higher perturbative order, where one then gets a finite calculable result for the strength of such an induced effective interaction (cf. [1–7]).

With the above remarks in mind, let us now examine a most general effective Lagrangian of dimension 8 for the interaction $Z\gamma\gamma\gamma$. For simplicity we shall also assume invariance with respect to parity. (Note that, e.g., an effective $Z\gamma\gamma\gamma$ interaction induced within the standard model at one-loop level does satisfy such a constraint, since the only source of the parity violation might be a closed fermion loop (box) of the type $VVVA$, which, however, does not contribute in the considered electromagnetic (i.e., essentially Abelian) case—see, e.g., [2].) It is not difficult to realize that under the above-mentioned constraints there are two nontrivial independent lowest-order effective Lagrangians for the $Z\gamma\gamma\gamma$ interaction: namely,

$$\mathcal{L}_1=G_1F^{\alpha\mu}F^{\sigma\nu}\partial_\alpha F_{\mu\nu}Z_\sigma\ , \tag{1}$$

$$\mathcal{L}_2=G_2F^{\alpha\beta}F^{\nu}_{\beta}\partial_\alpha F_{\sigma\nu}Z^\sigma\ , \tag{2}$$

where $G_j, j=1,2$ are the corresponding (dimensionful) coupling constants, which may be written as

$$G_j=g_j/M^4\ , \tag{3}$$

where M is a mass scale and the g_j are dimensionless. [Needless to say, expressions (1) and (2) are automatically invariant under charge conjugation.] Concerning the "completeness" of the chosen set of effective Lagrangians, it is easy to show that all the other variants are equivalent to linear combinations of (1) and (2) if one employs partial integration (in the corresponding action) and if the equations of motion are taken into account. As a simple example of a trivial interaction term, let us, e.g., consider a modification of expression (2) which would be proportional to

$$F^{\mu\beta}F^{\nu}_{\beta}\partial_\sigma F_{\mu\nu}Z^\sigma\ .$$

It is easy to see that the last expression is identically zero if one takes into account that $F^{\mu\beta}F^{\nu}_{\beta}$ is symmetric in the indices μ and ν while $F^{\mu\nu}$ is antisymmetric. Another example would be an interaction term proportional to

$$F^{\alpha\sigma}F^{\mu\nu}\partial_\alpha F_{\mu\nu}Z_\sigma\ ,$$

which is equivalent to the interaction term (1) if one employs the second Maxwell equation (i.e., the Bianchi identity): namely,

$$\partial_\alpha F_{\mu\nu}+\partial_\mu F_{\nu\alpha}+\partial_\nu F_{\alpha\mu}=0\ .$$

We shall therefore take the sum of (1) and (2) to be the relevant lowest-dimension effective Lagrangian for describing (in the tree approximation) the decay $Z\to\gamma\gamma\gamma$:

$$\mathcal{L}_{Z\gamma\gamma\gamma}=\mathcal{L}_1+\mathcal{L}_2\ . \tag{4}$$

The results of our calculation may be summarized as follows: The decay amplitude for $Z\to\gamma\gamma\gamma$ may, in general, be written as

$$\mathcal{M}(Z\to\gamma\gamma\gamma)=\mathcal{M}_{\alpha\mu\nu\rho}\varepsilon^{\alpha}(P)\varepsilon^{*\mu}(k)\varepsilon^{*\nu}(l)\varepsilon^{*\rho}(r) \tag{5}$$

where k,l,r are the four-momenta of final-state photons, $P=k+l+r$ is the four-momentum of the decaying Z and the ε's are the corresponding polarization vectors. Using $\mathcal{L}_{Z\gamma\gamma\gamma}$ [as defined by (4), together with (1) and (2)] in the tree approximation, the "polarization tensor" $\mathcal{M}_{\alpha\mu\nu\rho}$ in (5) reads

$$\mathcal{M}_{\alpha\mu\nu\rho}=\sum_{a=1}^{6} P_a\{G_1[(k\cdot l)(l\cdot r)g_{\alpha\rho}g_{\mu\nu}-(k\cdot r)k_\nu l_\mu g_{\alpha\rho}-(k\cdot r)k_\nu l_\alpha g_{\rho\mu}+k_\nu k_\rho l_\mu r_\alpha] \\ +G_2[-(k\cdot l)(k\cdot r)g_{\alpha\mu}g_{\rho\nu}+(l\cdot r)k_\alpha k_\nu g_{\rho\mu}-(l\cdot r)k_\nu k_\rho g_{\alpha\mu}+(l\cdot r)k_\nu l_\alpha g_{\rho\mu} \\ +2(l\cdot r)k_\rho l_\mu g_{\alpha\nu}-(k\cdot r)k_\alpha l_\rho g_{\mu\nu}-k_\alpha k_\nu l_\rho r_\mu]\}\ , \tag{6}$$

where P_a denotes possible permutations $(k,\mu)\leftrightarrow(l,\nu)\leftrightarrow(r,\rho)$.

Squaring the amplitude (5) and summing over polarizations one gets first

$$\sum_{\text{pol}}|\mathcal{M}|^2=\mathcal{M}_{\alpha\mu\nu\rho}\mathcal{M}^*_{\beta\sigma\tau\omega}\left[-g^{\alpha\beta}+\frac{1}{m_Z^2}P^\alpha P^\beta\right] \\ \times(-g^{\mu\sigma})(-g^{\nu\tau})(-g^{\rho\omega})\ . \tag{7}$$

[Note that in (7) we have, of course, simplified the photon polarization sums in the standard manner, taking into account gauge invariance. One may verify explicitly that the longitudinal part of the Z boson polarization sum in fact does not contribute either.] Now substituting expression (6) into (7) and using $k^2=l^2=r^2=0$ and $P^2=m_Z^2$, quantity (7) can be expressed in terms of scalar products $k\cdot l$, $k\cdot r$, and $l\cdot r$. For brevity we denote

$$s_{12}\equiv k\cdot l\ , \\ s_{13}\equiv k\cdot r\ , \\ s_{23}\equiv l\cdot r \tag{8}$$

(this notation is inspired by [1]; note, however, that our definition differs slightly from [1] in that the s_{ij} are not dimensionless). Taking into account the identity

$$s_{12}+s_{13}+s_{23}=\tfrac{1}{2}m_Z^2$$

(which follows easily from the four-momentum conservation), the resulting expression for (7) written in terms of the variables (8) reads

$$\sum_{\text{pol}}|\mathcal{M}|^2=4G_1^2(3s_{12}^2s_{13}^2+3s_{12}^2s_{23}^2+3s_{13}^2s_{23}^2+m_Z^2s_{12}s_{13}s_{23})+16(G_2^2-G_1G_2)(s_{12}^2s_{13}^2+s_{12}^2s_{23}^2+s_{13}^2s_{23}^2+\tfrac{1}{2}m_Z^2s_{12}s_{13}s_{23})\ . \tag{9}$$

Then one may write down immediately the differential decay rate for fixed photon energies in the Z rest frame (using a form for the Dalitz plot)

$$d\Gamma=\frac{1}{(2\pi)^3}\frac{1}{8m_Z}\frac{1}{3!}\overline{|\mathcal{M}|^2}dE_1dE_2dE_3\delta(E_1+E_2+E_3-m_Z)\ , \tag{10}$$

where $\overline{|\mathcal{M}|^2}$ stands for the quantity (7) averaged over spin projections of the Z, i.e.,

$$\overline{|\mathcal{M}|^2}=\frac{1}{3}\sum_{\text{pol}}|\mathcal{M}|^2 \tag{11}$$

and E_1,E_2,E_3 are photon energies corresponding to the four-momenta k, l, r, respectively. In (10) we have also included the combinatorial factor 1/3! corresponding to the three identical particles in the final state. Using the four-momentum conservation, the relevant scalar products may be expressed simply in terms of energies E_j, $j=1,2,3$:

$$s_{12}=m_Z(\tfrac{1}{2}m_Z-E_3)\ , \\ s_{13}=m_Z(\tfrac{1}{2}m_Z-E_2)\ , \\ s_{23}=m_Z(\tfrac{1}{2}m_Z-E_1)\ . \tag{12}$$

For further manipulations it is convenient to introduce dimensionless variables

$$x=1-\frac{2E_1}{m_Z}\ , \\ y=1-\frac{2E_2}{m_Z}\ , \\ z=1-\frac{2E_3}{m_Z}\ . \tag{13}$$

In view of (13) one then has

$$s_{12}=\tfrac{1}{2}zm_Z^2\ , \\ s_{13}=\tfrac{1}{2}ym_Z^2\ , \\ s_{23}=\tfrac{1}{2}xm_Z^2\ . \tag{14}$$

It is easy to show that the allowed kinematical range for each photon energy is $(0,\tfrac{1}{2}m_Z)$; this further implies that, e.g., for a fixed value of E_1, the E_2 (or E_3) may vary from $\frac{1}{2}m_Z-E_1$ to $\frac{1}{2}m_Z$. According to (13) it means that the kinematically allowed values of x, y, and z lie between 0 and 1 and for a fixed value of x, y (or z) may vary from 0 to $1-x$. Using (9), (13), and (14) in Eq. (10), and integrating trivially over one of the energy variables (e.g., z), a corresponding differential decay probability may be written as

$$\frac{d\Gamma}{dx\,dy}=\frac{1}{(2\pi)^3}\frac{m_Z^9}{2304}\{G_1^2[2xy(1-x-y)+3x^2y^2+3(x^2+y^2)(1-x-y)^2] +4(G_2^2-G_1G_2)[xy(1-x-y)+x^2y^2+(x^2+y^2)(1-x-y)^2]\} . \tag{15}$$

Integrating Eq. (15) over the domain defined by $0<y<1-x$, $0<x<1$ we get the total decay rate

$$\Gamma=\frac{m_Z^9}{552\,960\pi^3}(2G_1^2+3G_2^2-3G_1G_2) . \tag{16}$$

In particular, if $G_1=G_2=G$, results (9) and (16) become

$$\sum_{\text{pol}}|\mathcal{M}|^2=4G^2(3s_{12}^2s_{13}^2+3s_{12}^2s_{23}^2+3s_{13}^2s_{23}^2+m_Z^2s_{12}s_{13}s_{23}) , \tag{17}$$

$$\Gamma(Z\to\gamma\gamma\gamma)=\frac{G^2m_Z^9}{276\,480\pi^3} . \tag{18}$$

In closing, let us add several remarks. First, one may observe that the two terms on the right-hand side of Eq. (15), proportional to G_1^2 and $G_2^2-G_1G_2$, respectively, exhibit different dependence on the photon energies (in fact, one has there two analogous polynomials with different coefficients.) It means that a detailed measurement of the energy distribution of the final-state photons (if it was feasible) would allow one to determine the coupling constants G_1 and G_2 [i.e., to disentangle contributions of the two effective interactions in (1), (2), and (4)].

Concerning the integrated decay rate (16) [or (18)] one may notice that it is suppressed by a large numerical factor, which is mostly due to the integration over the phase space of final-state photons. This indicates, in general, that the considered decay mode of a neutral vector boson Z can be observable if the effective coupling constants G_1, G_2 are enhanced by some particular mechanism so that the (dimensionless) quantities $G_1m_Z^4$ and/or $G_2m_Z^4$ are not too small (on the other hand, one should keep in mind that the G_1 and G_2 must always naturally involve a factor of e^3, where e is the electromagnetic coupling constant). In this context, see also Boudjema and Renard in Ref. [1]. In any case, our formula (16) enables one to make a quick estimate of the considered decay rate in any particular model predicting the values of G_1 and G_2.

Finally, let us mention that Boudjema and Renard in Ref. [1] (see pages 203–205 therein) consider an effective $Z\gamma\gamma\gamma$ interaction which looks different than our expressions (1) and (2); they discuss a particular case of a single coupling constant. Within the context of their treatment one may guess that Boudjema and Renard considered for $Z\to\gamma\gamma\gamma$ a decay amplitude of the form (using our notation for the relevant four-momenta)

$$\mathcal{M}_{\rm BR}=G[Z_{\mu\nu}(P)F^{\mu\nu}(k)F_{\alpha\beta}(l)F^{\alpha\beta}(r) +Z_{\mu\nu}(P)F^{\mu\nu}(l)F_{\alpha\beta}(k)F^{\alpha\beta}(r) +Z_{\mu\nu}(P)F^{\mu\nu}(r)F_{\alpha\beta}(k)F^{\alpha\beta}(l)] , \tag{19}$$

where $Z_{\mu\nu}(P)=P_\mu\varepsilon_\nu(P)-P_\nu\varepsilon_\mu(P)$, etc., and the G has dimension (mass)$^{-4}$. (Note that such a decay amplitude may arise, e.g., from exchange of a heavy scalar between a $Z\gamma$ pair and a pair of photons.) Comparing (19) with the general expressions (5) and (6) one finds that $\mathcal{M}_{\rm BR}$ corresponds to taking $G_1=4G$, $G_2=0$ in (6); in other words, within a model described by (19) the effective interaction term $\mathcal{L}_2$ given by (2) does not contribute at all to the considered process. Setting then $G_2=0$ in (9) one may compare the resulting photon energy distribution (15) with the previous result of Ref. [1] [notice that for $G_2=0$ our Eq. (9) becomes proportional to (17)]. We have found that the energy spectrum obtained from our formulas has the same form as that given in Ref. [1] (see formulas (41) and (42), p. 205 therein).

Thus, one may view our results as a generalization of the earlier result appearing in Ref. [1], since we have discussed explicitly the case of two different coupling constants in the independent interaction terms (1) and (2). Note that a general combination of the two structures (1) and (2) may be relevant, e.g., when the effective $Z\gamma\gamma\gamma$ interaction is induced at one-loop level in a renormalizable field theory model. In our calculation, all the algebraic manipulations were performed using the MATHEMATICA package FEYNCALC [8], which proved to be extremely useful for such a purpose.

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