# Radiative kaon decays $K^{\pm}\to\pi^{\pm}\pi^{0}\gamma$ and direct $CP$ violation

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It is stressed that a measurement of the electric dipole amplitude for direct photon emission in $K^{\pm}\to\pi^{\pm}\pi^{0}\gamma$ decays through its interference with inner bremsstrahlung is important for differentiating among various models. The effects of amplitude $CP$ violation in the radiative decays of the charged kaon are analyzed in the standard model in conjunction with the large-$N_c$ approach. We point out that gluon and electromagnetic penguin contributions to the $CP$-violating asymmetry between the Dalitz plots of $K^{\pm}\to\pi^{\pm}\pi^{0}\gamma$ are of equal weight. The magnitude of $CP$ asymmetry ranges from $2\times10^{-6}$ to $1\times10^{-5}$ when the photon energy in the kaon rest frame varies from 50 MeV to 170 MeV.

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In a recent paper [1] we studied $CP$ violation in the radiative kaon decay $K_L\to\pi^+\pi^-\gamma$. We conclude that the direct $CP$-violating effect originating from the electromagnetic penguin diagram is only of the order of $(10^{-3}-10^{-4})\epsilon$, depending on the region of the Dalitz plot under consideration. On the contrary, it has been advocated that direct $CP$-violating asymmetry in the radiative decays $K^{\pm}\to\pi^{\pm}\pi^{0}\gamma$ defined by

$$\Delta_\Gamma=\frac{\Gamma(K^+\to\pi^+\pi^0\gamma)-\Gamma(K^-\to\pi^-\pi^0\gamma)}{\Gamma(K^+\to\pi^+\pi^0\gamma)+\Gamma(K^-\to\pi^-\pi^0\gamma)}\,, \tag{1}$$

arising from the same electromagnetic penguin mechanism, can be large enough for experimental interest; explicitly, $\Delta_\Gamma\le 9\times10^{-4}$ is obtained in Ref. [2]. If this estimate is correct, it will be on the verge of the capability of the $\phi$ factory DA$\Phi$NE[3], and could be detected at future high-statistics facilities. The purpose of this Brief Report is to reexamine this $CP$-violating effect in the standard model in conjunction with the $1/N_c$ approach.

The general amplitude of the decay

$$K^+(k)\to\pi^+(p_+)\pi^0(p)\gamma(q,\varepsilon)$$

is of the form

$$A(K^+\to\pi^+\pi^0\gamma)=-eA(K^+\to\pi^+\pi^0)(p_+\cdot\varepsilon/p_+\cdot q-p\cdot\varepsilon/p\cdot q)e^{i\delta_0^2}+M[ie\epsilon_{\mu\nu\rho\sigma}p_+^{\mu}p^{\nu}q^{\rho}\varepsilon^{\sigma}]e^{i\delta_1^1}$$
$$+Ee[(p_+\cdot\varepsilon)(p\cdot q)-(p\cdot\varepsilon)(p_+\cdot q)]e^{i\delta_1^1}\,, \tag{2}$$

where we have included the isospin phase shifts $\delta_J^I$ with $I$ being the isospin of the two pions and $J$ the total angular momentum, which are necessary for generating the decay-rate $CP$ asymmetry. To the leading multipole expansion, the direct emission (DE) amplitude $M$ corresponds to a magnetic dipole ($M1$) transition, while $E$ an electric dipole ($E1$) transition. From time to time this decay mode has received a constant attention both theoretically [2–14] and experimentally [15–17]. Outside of the framework of chiral perturbation theory (ChPT), various techniques such as the short-distance effective weak Hamiltonian, the vector-meson-dominance model and current algebra have been employed to study the DE of $K^+\to\pi^+\pi^0\gamma$ decay. These methods are plagued with some fundamental problems. For example, the short-distance effective Hamiltonian utilized in Refs. [5,7] does not explicitly couple to the external photon field. Hence, after the usage of the factorization approximation, one has to appeal to the soft-pion theorem to evaluate the matrix element $\langle\pi^0\gamma|\bar s\gamma_\mu(1-\gamma_5)u|K^+\rangle$, for instance. However, it has been shown [8] that the soft-pion technique is no longer applicable to the magnetic transition amplitude as in the case of $\pi^0\to\gamma\gamma$. Also, it is known that the short-distance effective weak Hamiltonian is far from being adequate to describe the kaon $\Delta I=\frac{1}{2}$ rule.

In ChPT, the most general $p^4$ $CP$-invariant $\Delta S=1$ nonanomalous electroweak chiral Lagrangian with one external photon field which satisfies the constraints of chiral and $CPS$ symmetry (the latter being the product of ordinary $CP$ with a switching symmetry $S$, which switches the $d$- and $s$-quark fields) has the expression [18]

$$\mathcal{L}^{\Delta S=1}_{\rm nonanom}=i(2/f_\pi^2)g_8eF^{\mu\nu}[\omega_1{\rm Tr}(\lambda_6L_\mu L_\nu Q)+\omega_2{\rm Tr}(\lambda_6L_\nu QL_\mu)+\omega_3{\rm Tr}(\lambda_6UR_\mu R_\nu QU^\dagger)$$
$$+\omega_4{\rm Tr}(\lambda_6UQR_\mu R_\nu U^\dagger)+\omega_5{\rm Tr}(\lambda_6UR_\nu QR_\mu U^\dagger)] \tag{3}$$

for normal intrinsic parity transitions, while the anomalous Lagrangian terms for the odd intrinsic parity sector are [10,12]

$$\mathcal{L}^{\Delta S=1}_{\rm anom}=ia(2/f_\pi^2)g_8e\tilde F^{\mu\nu}{\rm Tr}(QL_\mu){\rm Tr}(\lambda_6L_\nu)$$
$$+ib(2/f_\pi^2)g_8e\tilde F^{\mu\nu}{\rm Tr}(QR_\mu){\rm Tr}(\lambda_6L_\nu)$$
$$+ic(2/f_\pi^2)g_8e\tilde F^{\mu\nu}{\rm Tr}(\lambda_6[UQU^\dagger,L_\mu L_\nu])\,, \tag{4}$$

where $\tilde F_{\mu\nu}\equiv\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$, $Q={\rm diag}(\frac{2}{3},-\frac{1}{3},-\frac{1}{3})$, $L_\mu\equiv(D_\mu U)U^\dagger$ with $D_\mu U=\partial_\mu U-ieA_\mu[Q,U]$, $R_\mu\equiv U^\dagger(D_\mu U)$, and $U=\exp[2i(\phi/f_\pi)]$ with $f_\pi=132$ MeV, $\phi\equiv(1/\sqrt2)\phi^a\lambda^a$. In Eqs. (3) and (4), $g_8$ is the octet weak-coupling constant appearing in the lowest-order $CP$ invariant $\Delta S=1$ weak chiral Lagrangian $\mathcal{L}_W^{(2)}=-g_8{\rm Tr}(\lambda_6L_\mu L^\mu)$ and is fixed to be [19]

 

$$g_8 = -0.26\times 10^{-8} m_K^2 \qquad (5)$$

from the experimental measurement of $K^0 \to \pi\pi$ decay rates. The coupling constants $\omega_i$, $a$, $b$, $c$ depend on the choice of the renormalization scale $\mu$ as divergences of chiral loops are absorbed by the counterterms which have the same structure as $\mathcal{L}^{\Delta S=1}_{\text{nonanom}}$ and $\mathcal{L}^{\Delta S=1}_{\text{anom}}$. Therefore, those coupling constants, in principle, can be determined only empirically from various low-energy hadronic processes. However, in the limit of large $N_c$ ($N_c$ being the number of quark color degrees of freedom), these couplings become $\mu$ independent and are theoretically manageable at least to the zeroth order of $\alpha_s$ [10]. It is found that, in the large-$N_c$ approach [10],[1]

$$\begin{aligned} &\omega_1=\omega_2=N_c/12\pi^2, \quad \omega_3=\omega_4=\omega_5=0 \ , \\ &a=2b=4c=N_c/12\pi^2 \ . \end{aligned} \qquad (6)$$

Note that the couplings $a$, $b$, and $c$ are determined by chiral anomalies and hence are free of gluonic corrections in the large-$N_c$ limit.

There are two different contributions to the direct emission of $K^\pm \to \pi^\pm\pi^0\gamma$: contact term contributions induced by $\mathcal{L}^{\Delta S=1}_{\text{nonanom}}$ and $\mathcal{L}^{\Delta S=1}_{\text{anom}}$, and three long-distance pole diagrams with the $\pi\pi\pi\gamma$ vertex governed by the anomalous Wess-Zumino-Witten term. As shown in Ref. [10], the results are (since we are working in the leading order in $1/N_c$ expansion, chiral loops can be neglected)

$$\begin{aligned} &E(K^\pm \to \pi^\pm\pi^0\gamma) = E_{\text{contact}} = \frac{\sqrt{2}g_8}{\pi^2 f_\pi^5}(2) \ , \\ &M(K^\pm \to \pi^\pm\pi^0\gamma) = M_{\text{contact}} + M_{\text{pole}} = \mp \frac{\sqrt{2}g_8}{\pi^2 f_\pi^5}(2+3) \ . \end{aligned} \qquad (7)$$

The *constructive* interference between pole and direct-transition $M1$ amplitudes for $K^\pm \to \pi^\pm\pi^0\gamma$ decays is a prominent feature different from the decay $K_L \to \pi^+\pi^-\gamma$ where a large and *destructive* interference for $M1$ transitions is required to explain the data [1,10]. Experimentally, the DE rates are extracted in the charged-pion kinetic-energy range of 55–90 MeV [15–17]. With this experimental condition, the branching ratio of direct emission is given by [10]

$$\begin{aligned} B(K^\pm \to \pi^\pm\pi^0\gamma)_{\text{DE}} &= 1.32\times 10^5(|E|^2+|M|^2)\ \text{GeV}^6 \\ &= 2.02\times 10^{-5} \ , \end{aligned} \qquad (8)$$

which is in agreement with the experimental values

$$B(K^\pm \to \pi^\pm\pi^0\gamma)_{\text{DE}} = \begin{cases} (1.56\pm 0.35\pm 0.5)\times 10^{-5} & [1972]\ (\text{Ref. [15]}), \\ (2.3\pm 3.2)\times 10^{-5} & [1976]\ (\text{Ref. [16]}), \\ (2.05\pm 0.46^{+0.39}_{-0.23})\times 10^{-5} & [1987]\ (\text{Ref. [17]}). \end{cases} \qquad (9)$$

Previous calculations [5,7,8] based on the short-distance effective weak Hamiltonian predict a smaller branching ratio. This is attributed to the fact that, as noted in passing, only short-distance corrections to the Wilson coefficient functions are taken into account in the approach of the effective weak Hamiltonian, which are not sufficient to explain the $\Delta I=\frac{1}{2}$ rule in kaon decays (see Ref. [19] for a review). We note that although the DE rate is dominated by magnetic transitions[2] due to additional constructive contributions from the pole diagrams, the $E1$ contribution is nevertheless non-negligible. Experimentally, the DE electric dipole amplitude can be measured from the interference of inner bremsstrahlung (IB) with $E1$ transitions. Thus far, there is only one experiment (done 2 decades ago) measuring this interference and only a limit is obtained [15]. Evidently, a measurement of the $E1$ amplitude is important for understanding the underlying mechanism of $K^\pm \to \pi^\pm\pi^0\gamma$ decays and their direct $CP$ violation.

We next turn to examine $CP$-violating effects in the decays $K^\pm \to \pi^\pm\pi^0\gamma$. The simplest way of observing $CP$ nonconservation is through the measurement of the decay-rate $CP$ asymmetry parameter $\Delta_\Gamma$ as defined in Eq. (1). If photon polarization is not measured, there is no interference between $E1$ and $M1$ amplitudes. Consequently, when photon polarizations are summed over, a nonvanishing $\Delta_\Gamma$ must arise from the interference of IB with the $E1$ amplitude of DE. Although $CP$-violating asymmetry had been studied intensively in late 1960s [22], a modern analysis in the framework of the standard model was carried out only recently in Refs. [2,7]. However, the result is somewhat controversial: While $\Delta_\Gamma$ arising from the gluon penguin diagram is estimated to be of order $10^{-6}$ by McGuigan and Sanda [7], it is claimed by Dib and Peccei [2] that a large $CP$ asymmetry can be induced from the electromagnetic penguin diagram, namely, $\Delta_\Gamma \le 9\times 10^{-4}$. Naively, it is expected that the amplitude $CP$ violation coming from the electromagnetic penguin diagram with a photon radiated from the loop quark or from the $W$ boson is of equal weight as that from the QCD penguin diagram with a photon emitted from the external quark lines or from the $W$ boson. Therefore, a resolution of this discrepancy is called for.

We will follow Ref. [7] to consider $CP$ asymmetry between the Dalitz plots of $K^+ \to \pi^+\pi^0\gamma$ and $K^- \to \pi^-\pi^0\gamma$

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[1]The predictions for the anomalous couplings $a$, $b$, and $c$ were confirmed recently by Ref. [12]. A different large-$N_c$ prediction $\omega_1=\omega_2=8L_9$ is obtained in Ref. [20]. This is ascribed to the fact that a different short-distance effective weak Hamiltonian is employed in [20]. Our results for $\omega_i$'s also differ from Ref. [21] in which the authors claimed that the factorization approximation is the same as the weak deformation model except for a different overall fudge factor $k_f$.

[2]Precisely, $|E/M|^2=0.16$ is predicted in the large-$N_c$ approach. Most earlier calculations yield even smaller ratio for $|E/M|^2$. In Ref. [14] this ratio is calculated to be $5.1\times 10^{-3}$. In ChPT, the coupling constants of $\mathcal{L}^{\Delta S=1}_{\text{nonanom}}$ and $\mathcal{L}^{\Delta S=1}_{\text{anom}}$ are expected to be of the same order of magnitude [see Eq. (6)]. Therefore, unlike the decay $K_L \to \pi^+\pi^-\gamma$, it seems to us that there is no reason to have a severe suppression on the electric dipole amplitude of the DE in $K^\pm \to \pi^\pm\pi^0\gamma$ decays.

rather than in the total decay rates,

$$\Delta=\frac{|A(K^+\to\pi^+\pi^0\gamma)|^2-|A(K^-\to\pi^-\pi^0\gamma)|^2}{|A(K^+\to\pi^+\pi^0\gamma)|^2+|A(K^-\to\pi^-\pi^0\gamma)|^2}\ , \tag{10}$$

so that a larger $CP$ asymmetry can be obtained in certain particular regions of the Dalitz plot. Since, under $CPT$ invariance,

$$A(K^-\to\pi^-\pi^0\gamma)=-eA(K^-\to\pi^-\pi^0)(p_-\cdot\varepsilon/p_-\cdot q-p\cdot\varepsilon/p\cdot q)e^{i\delta_0^2}-M^*[ie\epsilon_{\mu\nu\rho\sigma}p_-^{\mu}p^{\nu}q^{\rho}\varepsilon^{\sigma}]e^{i\delta_1^1}$$
$$+E^*e[(p_-\cdot\varepsilon)(p\cdot q)-(p\cdot\varepsilon)(p_-\cdot q)]e^{i\delta_1^1}\ , \tag{11}$$

it follows that

$$\Delta=\frac{-2|E|z\sin\phi_E\sin(\delta_1^1-\delta_0^2)}{A(K^+\to\pi^+\pi^0)/m_K^4+2|E|z\cos\phi_E\cos(\delta_1^1-\delta_0^2)+(|E|^2+|M|^2)z^2}\ , \tag{12}$$

where $E=-|E|e^{i\phi_E}$ [recall that our $E$ is negative; see Eqs. (5) and (7)] and $z\equiv(q\cdot p_\pm)(q\cdot p)/m_K^4$.

The main task is to estimate the $CP$-odd phase $\phi_E$ of the $E1$ amplitude. There are two different contributions to the imaginary part of $E$: one from the gluon penguin diagram, and the other from the electromagnetic penguin diagram. As to the former, following the prescription presented in Ref. [1], we find in the $1/N_c$ approach that

$$E=(2\sqrt{2}/\pi^2f_\pi^5)(g_8+ig_8')\ , \tag{13}$$

where $g_8'$ is the $CP$-violating coupling constant appearing in the lowest-order $CP$-odd $\Delta S=1$ weak chiral Lagrangian $\mathcal{L}_W^-=-ig_8'\mathrm{Tr}(\lambda_7L_\mu L^\mu)$ and is dominated by the short-distance QCD penguin diagram. Hence,

$$(\sin\phi_E)_{\rm gluon}=g_8'/g_8=\mathrm{Im}A_0/\mathrm{Re}A_0\ , \tag{14}$$

where $A_0\equiv A(K^0\to\pi\pi(I=0))$, and use of

$$A(K_2\to\pi\pi(I=0))/A(K_1\to\pi\pi(I=0))=ig_8'/g_8 \tag{15}$$

has been made. The calculation of $\mathrm{Im}A_0$ in the standard model is standard and is given by [23]

$$\mathrm{Im}A_0=-(G_F/\sqrt{2})(\mathrm{Im}\lambda_t)y_6\langle(\pi\pi)_{I=0}|Q_6|K^0\rangle\ , \tag{16}$$

where $y_6=\mathrm{Im}c_6/\mathrm{Im}\tau$, $\tau=-\lambda_t/\lambda_u$, $\lambda_i=V_{is}^*V_{id}$, $Q_6$ is a penguin operator, and $c_6$ is the corresponding Wilson coefficient. The $K-\pi\pi$ matrix element of $Q_6$ evaluated in the large-$N_c$ approach is known to be [19]

$$\langle(\pi\pi)_{I=0}|Q_6|K^0\rangle=-i4\sqrt{3}f_\pi v^2(m_K^2-m_\pi^2)/\Lambda_\chi^2\ , \tag{17}$$

where

$$v=\frac{m_\pi^2}{m_u+m_d}=\frac{m_{K^+}^2}{m_u+m_s}=\frac{m_{K^0}^2}{m_d+m_s} \tag{18}$$

characterizes the quark order parameter $\langle\bar{q}q\rangle$, and $\Lambda_\chi\sim1$ GeV is a chiral-symmetry-breaking scale. Since $\mathrm{Re}A_0=i4.69\times10^{-7}$ GeV [19][3] and

$$\mathrm{Im}(V_{ts}^*V_{td})\simeq s_{13}s_{23}\sin\delta_{13} \tag{19}$$

in the Chau-Keung parametrization of the quark mixing matrix [24], we find numerically

$$(\sin\phi_E)_{\rm gluon}=-8.1\times10^{-5}\sin\delta_{13}\ , \tag{20}$$

where uses have been made of $m_s=175$ MeV, $s_{23}=0.044$, $s_{13}/s_{23}=0.1$, and $y_6=-0.057$ for $m_t=150$ GeV [23].

Following Ref. [1], the DE amplitude of $K^+\to\pi^+\pi^0\gamma$ induced by the electromagnetic penguin diagram is given by

$$A(K_L\to\pi^+\pi^-\gamma)_{\rm DE}^{\rm em}=iG_F\frac{e}{16\pi^2}\mathrm{Im}(V_{ts}^*V_{td})F(x_t)\langle\pi^+\pi^0\gamma|Q_T|K^+\rangle\ , \tag{21}$$

with

$$Q_T=i[m_s\bar{s}\sigma_{\mu\nu}(1-\gamma_5)d+m_d\bar{s}\sigma_{\mu\nu}(1+\gamma_5)d]F^{\mu\nu}\ ,$$
$$F(x)=\frac{(8x^2+5x-7)x}{12(x-1)^3}-\frac{(3x-2)x^2}{2(x-1)^4}\ln x\ , \tag{22}$$

and $x_t=m_t^2/M_W^2$. By working out the chiral realization of the tensor operator $Q_T$ as in Ref. [2] (see also Ref. [1]), we obtain

$$E_{\rm em}=i(G_Fm_s/2\sqrt{2}\pi^2f_\pi^2)F(x_t)(s_{13}s_{23}\sin\delta_{13})\ . \tag{23}$$

It follows from Eqs. (23) and (7) that

$$(\sin\phi_E)_{\rm em}=\frac{-iE_{\rm em}}{E}=\frac{1}{8}\frac{G_Fm_sF_\pi^3}{g_8}F(x_t)(s_{13}s_{23}\sin\delta_{13})\ . \tag{24}$$

Numerically,

$$(\sin\phi_E)_{\rm em}=-6.0\times10^{-5}\sin\delta_{13}\ , \tag{25}$$

where we have applied Eq. (5). It is evident that gluon and electromagnetic penguin contributions to the $CP$-odd phase of the electric dipole DE amplitude are equally important, as it should be.

At this point we would like to comment on our work in relation to the study in Ref. [2]. Dib and Peccei first calculated $CP$ asymmetry for charged kaon decay into two pions and then applied the $CPT$ relation[4]

[3]The experimental values of $K\to\pi\pi$ amplitudes are usually expressed in terms of real numbers. However, model calculations show that the amplitude of $K\to2\pi$ contains a factor of $i$. This means that $\mathrm{Re}A_0$ given in Ref. [19] should be multiplied by a factor of $i$ when compared with $\langle\pi\pi|Q_6|K^0\rangle$.

[4]In principle, one should also include the decay rates of $K\to3\pi$ decays, namely, $\Gamma(K^+\to\pi^+\pi^+\pi^-)+\Gamma(K^+\to\pi^+\pi^0\pi^0)$, to the LHS of Eq. (26) and their charge conjugate to the RHS.

$$\Gamma(K^+\to\pi^+\pi^0)+\Gamma(K^+\to\pi^+\pi^0\gamma)$$
$$=\Gamma(K^-\to\pi^-\pi^0)+\Gamma(K^-\to\pi^-\pi^0\gamma) \quad (26)$$

to estimate $\Delta_\Gamma$ for $K^\pm\to\pi^\pm\pi^0\gamma$ decays. On the contrary, we compute *CP*-violating asymmetry directly for the radiative decays of $K^\pm$. Therefore, the strong-interaction phase difference necessary for generating *CP*-odd asymmetry is $\sin(\delta_1^1-\delta_0^2)$ in our case, while it is $\sin(\delta_\gamma-\delta_0^2)$ in Ref. [2], where $\delta_\gamma$ is the strong-interaction phase shift for $K^\pm\to\pi^\pm\pi^0$ amplitudes involving a $\pi\pi\gamma$ intermediate state. Apart from this, it seems to us that the numerical discrepancy between the present work and Ref. [2] lies mainly in the fact that a factor of $1/(4\pi)$ is missing in Eq. (15) of Ref. [2] for the effective Lagrangian of electromagnetic penguins. Consequently, $\Delta_\Gamma$ is overestimated by a factor of $(4\pi)^2$; in other words, the predicted upper bound for $\Delta_\Gamma$ in Ref. [2] should read $5.6\times10^{-6}$ instead of $9\times10^{-4}$. Nevertheless, Dib and Peccei did point out the importance of the electromagnetic penguin diagram, which is no longer negligible for $m_t>M_W$. From Eqs. (20) and (25) we see that gluon and electromagnetic penguin diagrams contribute constructively to $\phi_E$. As a result,

$$\sin\phi_E=(\sin\phi_E)_{\rm gluon}+(\sin\phi_E)_{\rm em}$$
$$=-1.4\times10^{-4}\sin\delta_{13}\ . \quad (27)$$

It remains to work out the quantity $z$ defined in Eq. (12). It can be recast in terms of the variables $x=2k\cdot q/m_K^2$ and $y=2k\cdot p_+/m_K^2$:

$$z=\tfrac{1}{4}[x(1-y)-(1-y)^2]\ . \quad (28)$$

Note that in the kaon rest frame $x=2E_\gamma/m_K$, $y=2E_+/m_K$. It is easily seen from Eq. (28) that the maximum $z$ for a given photon energy $E_\gamma$ in the c.m. is given by

$$(z)_{\max}=\tfrac{1}{4}(E_\gamma/m_K)^2\ . \quad (29)$$

Since $(\delta_1^1-\delta_0^2)\sim10°$ [15,16] and [19] $A(K^+\to\pi^+\pi^0)=1.829\times10^{-8}$ GeV, it follows from Eqs. (7), (12), (25), and (29) that

$$\Delta(E_\gamma)=\frac{0.75\times10^{-5}(E_\gamma/100\ \text{MeV})^2}{1+0.31(E_\gamma/100\ \text{MeV})^2}\ . \quad (30)$$

This *CP*-odd asymmetry ranges from $2\times10^{-6}$ to $1\times10^{-5}$ when $E_\gamma$ varies from 50 MeV to its highest value of 170 MeV.

To conclude, we have shown in the $1/N_c$ approach that the $E1$ amplitude of DE in $K^\pm\to\pi^\pm\pi^0\gamma$ decays is not negligible. Therefore, a measurement of the interference of inner bremsstrahlung with electric dipole transitions is important for differentiating between various models. We also pointed out that *CP*-violating asymmetry between the Dalitz plots of $K^\pm\to\pi^\pm\pi^0\gamma$ decays receive equally important contributions from gluon and electromagnetic penguin diagrams. The magnitude of *CP* asymmetry ranges from $2\times10^{-6}$ to $1\times10^{-5}$ when the c.m. photon energy varies from 50 to 170 MeV.

*Note added.* After this work was completed, we learned of a paper by G. Ecker, H. Neufeld, and A. Pich [Report Nos. CERN-TH-6920/93 and UWThPh-1993-22] in which the decay $K^+\to\pi^+\pi^0\gamma$ is analyzed and a potentially sizable electric amplitude interfering with bremsstrahlung is emphasized.

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