Three model-dependent estimates of $\mathcal{B}(D_s^+ \to \phi\pi^+)$

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We present three calculations of the absolute branching ratio for $D_s^+ \to \phi\pi^+$. The average of the three results is $\mathcal{B}(D_s^+ \to \phi\pi^+) = (3.6 \pm 0.6)\%$. We also derive an upper limit of $\mathcal{B}(D_s^+ \to \phi\pi^+) < 4.8\%$ (90% C.L.) based only on experimentally measured relative branching fractions.

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I. INTRODUCTION

The D_s^+ meson, formed of a charm quark and an antistrange quark, plays an important role in many physical processes. Information from D_s^+ decays, however, has been limited by the lack of a definitive experimental measurement of any absolute branching ratio. Since the $\phi\pi^+$ decay mode is the easiest to observe experimentally, it is this mode that all other channels have been normalized to. In this paper we give three different model-dependent estimates of $\mathcal{B}(D_s^+ \to \phi\pi^+)$ and an upper limit derived purely from data.

II. $\mathcal{B}(D_s^+ \to \phi\pi^+)$ FROM MEASUREMENTS OF THE D^{**0} AND D_s^{**+} 1^+ STATES

The absolute $\phi\pi^+$ branching ratio can be estimated if one knows the inclusive cross section for the sum of D^0 and D^+ mesons, the product of cross section times branching ratio for $D_s^+ \to \phi\pi^+$, $\sigma\mathcal{B}$, and the relative probability for "popping" an $s\bar{s}$ pair from the vacuum with respect to $u\bar{u}$ plus $d\bar{d}$ pairs. Then

$$\frac{\sigma(D_s^+)}{\sigma(D^0)+\sigma(D^+)} = \frac{\sigma\mathcal{B}/\mathcal{B}(D_s^+ \to \phi\pi^+)}{\sigma(D^0)+\sigma(D^+)} = \frac{s\bar{s}}{u\bar{u}+d\bar{d}} \ . \quad (1)$$

One measurement of the relative quark popping probability has been made using the fact that the 1^+ D_s^{**+} state does not decay into D_s^+. Instead, the 1^+ D_s^{**+} state decays into $D^{*0}K^+$ and $D^{*+}K^0$, while the 1^+ D^{**0} state decays into $D^{*+}\pi^-$ and $D^{*0}\pi^0$ [1]. CLEO has measured the production cross section of both decays [2]. Using the assumption that the production process depends only on the light quark flavor, they reported a value for the $s\bar{s}/u\bar{u}$ quark-antiquark popping probability of 0.24 ± 0.06.

The production cross sections for D^0 and D^+ mesons have been measured in continuum e^+e^- annihilations by the CLEO [3] and ARGUS [4] groups, who have measured, in addition, $\sigma\mathcal{B}$ for $D_s^+ \to \phi\pi^+$ [5]. The data are shown in Table I. For the D^0 branching ratio, we use the new CLEO measurement of $\mathcal{B}(D^0 \to K^-\pi^+) = (3.91 \pm 0.08 \pm 0.17)\%$ [6], which is in agreement with an updated measurement by ALEPH [7]. For the $D^+ \to K^-\pi^+\pi^+$ branching ratio, we use the Mark III measurement of $(9.1 \pm 1.3 \pm 0.4)\%$ [8] (see Table III). The total cross section $\sigma(D^0)+\sigma(D^+)$ equals 1.76 ± 0.16 nb.

To apply Eq. (1), we need to assume that the $d\bar{d}$ popping probability is equal to the $u\bar{u}$ popping probability. The right-hand side of the equation thus becomes equal to 0.12 ± 0.03. We also need to correct for excited D_s^{**+} states which decay into D^0 and D^+ final states rather than D_s^+, which we call cross feed. The 1^+ D_s^{**+} cross section of 12.3 ± 2.0 pb is measured by CLEO [2]. We also assume the existence of a charmed strange partner D_{s2}^{**+} of the $D^{**0}(2460)$ meson that is produced with the same $s\bar{s}$ popping probability and use the CLEO measurement for the production ratio for $D^{**0}(2460)/D^+$ of 0.10 ± 0.03 to estimate a D_{s2}^{**+} cross section of 12.7 ± 5.3 pb. The sum of the D_s^{**+} and D_{s2}^{**+} cross sections is then applied as a correction to the left-hand side of Eq. (1); i.e., the cross feed is subtracted from the denominator $\sigma(D^0)+\sigma(D^+)$ and added to the numerator $\sigma(D_s^+)$. Thus the total D_s^+ production cross section is estimated to be 184 ± 56 pb. Using the $D_s^+ \to \phi\pi^+$ cross section times branching ratio $\sigma\mathcal{B} = 8.5 \pm 1.0$ pb, we derive

$$\mathcal{B}(D_s^+ \to \phi\pi^+) = (4.6 \pm 1.5)\% \ .$$

The cross-feed correction has the effect of raising $\mathcal{B}(D_s^+ \to \phi\pi^+)$ by 15%. Possible cross feeds from higher-resonance states are not taken into account, but would probably further increase the branching ratio. We will show in Sec. V that there is an upper limit on $\mathcal{B}(D_s^+ \to \phi\pi^+)$ which this estimate is already close to, thus implying that higher-resonance states have a small effect.

TABLE I. Charm meson production measurements.

	$D^0 \to K^-\pi^+$	$D^+ \to K^-\pi^+\pi^+$	$D_s^+ \to \phi\pi^+$
CLEO $\sigma\mathcal{B}$ (pb)	52.0 ± 6.4	51.0 ± 7.3	8.9 ± 1.2
ARGUS $\sigma\mathcal{B}$ (pb)	43.8 ± 5.6	50.1 ± 6.9	7.8 ± 1.5
Average $\sigma\mathcal{B}$ (pb)	47.3 ± 4.2	50.5 ± 5.0	8.5 ± 1.0
Average σ (nb)	1.21 ± 0.12	0.55 ± 0.10	

III. $\mathcal{B}(D_s^+ \to \phi\pi^+)$ FROM FACTORIZATION IN B DECAY AND THE CLEO MEASUREMENT OF $\Gamma(D_s^+ \to \mu^+\nu)/\Gamma(D_s^+ \to \phi\pi^+)$

The decay rate of purely leptonic decays of charged mesons is sensitive to the wave function overlap of the two quarks at zero spatial separation, which is parametrized as the meson decay constant. The decay

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rate for D_s^+ is given by the formula [9]

$$\Gamma(D_s^+ \to l^+\nu) = \frac{1}{8\pi} G_F^2 f_{D_s}^2 m_l^2 M_{D_s} \left[1 - \frac{m_l^2}{M_{D_s}^2}\right]^2 |V_{cs}|^2 , \quad (2)$$

where F_{D_s} is the D_s^+ decay constant, M_{D_s} is the D_s^+ mass, m_l is the mass of the final-state lepton, and V_{cs} is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element, taken to have a value of 0.9744 [1].

CLEO has recently measured the leptonic decay mode of the D_s^+ into a muon and neutrino [10]. The width is normalized to the $\phi\pi^+$ decay mode:

$$\Gamma(D_s^+ \to \mu^+\nu)/\Gamma(D_s^+ \to \phi\pi^+) = 0.245 \pm 0.052 \pm 0.074 . \quad (3)$$

This ratio and Eq. (2) allow us to relate the decay constant f_{D_s} to the $D_s^+ \to \phi\pi^+$ branching ratio as

$$f_{D_s} = (344 \pm 64) \left[\frac{\mathcal{B}(D_s \to \phi\pi^+)}{3.7\%}\right]^{1/2} \text{MeV} . \quad (4)$$

We can get another relationship between f_{D_s} and the $D_s^+ \to \phi\pi^+$ branching ratio using factorization in two-body $B \to D^* D_s^-$ decays. Consider two-body $\bar{B}^0$ decays where there is a D^{*+} and another hadron h^- in the final state. Factorization is the assumption that these decays can be expressed as the product of two amplitudes, one that describes $\bar{B}^0 \to D^{*+}$ and can be measured in the semileptonic decay $\bar{B}^0 \to D^{*+} l^- \bar{\nu}_l$, and another one that couples the h^- to the virtual W^-. This assumption has been shown to be approximately true for light hadrons by Bortoletto and Stone [11] and subsequently by CLEO [12].

Factorization can be applied to the case where the h^- is a D_s^- [11–13]. This leads to the relation

$$\frac{\Gamma(\bar{B}^0 \to D^{*+} D_s^-)}{(d\Gamma/dq^2)(\bar{B}^0 \to D^{*+} l^- \bar{\nu}_l)|_{q^2 = M_{D_s}^2}} = 6\pi^2 |V_{cs}|^2 f_{D_s}^2 \delta , \quad (5)$$

where $\delta = 0.41$. Treating $\mathcal{B}(D_s^+ \to \phi\pi^+)$ as an unmeasured parameter and combining the available ARGUS and CLEO data shown in Table II, Eq. (5) leads to the relationship [14]

$$f_{D_s} = (288 \pm 64) \left[\frac{3.7\%}{\mathcal{B}(D_s^+ \to \phi\pi^+)}\right]^{1/2} \text{MeV} . \quad (6)$$

TABLE II. $\mathcal{B}(B \to D^* D_s^-)$ for $\mathcal{B}(D_s^+ \to \phi\pi^+) = 3.7\%$.

Mode	Experiment	$\mathcal{B}$ (%)
$D^{*+} D_s^-$	CLEO	1.2 ± 0.8
$D^{*+} D^-$	ARGUS	0.8 ± 0.6
$D^{*0} D_s^-$	ARGUS	0.8 ± 0.6
Average		0.9 ± 0.4

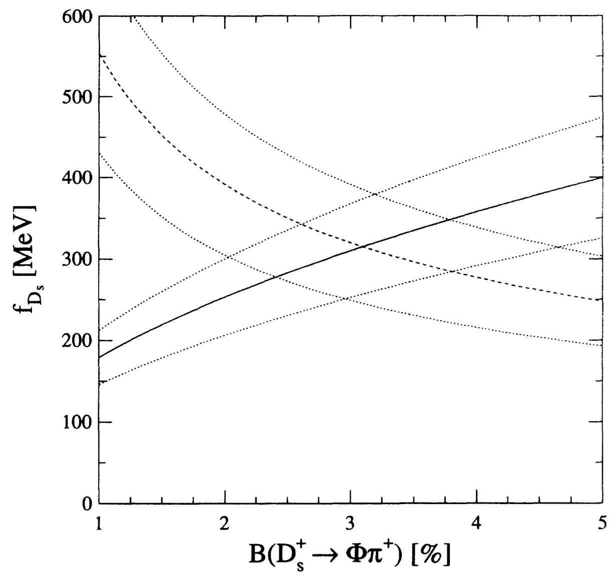

FIG. 1. Predictions for f_{D_s} obtained using $B \to D^* D_s^-$ data and the factorization assumption as a function of $\mathcal{B}(D_s^+ \to \phi\pi^+)$ (dashed curve) and the result of the CLEO measurement of $\Gamma(D_s^+ \to \mu^+\nu)/\Gamma(D_s^+ \to \phi\pi^+)$ (solid curve). The parallel top and bottom curves show the 1σ error limits.

Equations (4) and (6) both depend on $\mathcal{B}(D_s^+ \to \phi\pi^+)$, but in an opposite manner. Plots of the two relationships are shown in Fig. 1. Solving the two equations gives

$$f_{D_s} = (315 \pm 46) \text{ MeV} \quad (7a)$$

and

$$\mathcal{B}(D_s \to \phi\pi^+) = (3.1 \pm 0.9)\% . \quad (7b)$$

IV. $\mathcal{B}(D_s^+ \to \phi\pi^+)$ FROM PREDICTION OF THE SEMILEPTONIC WIDTH OF $D_s^+ \to \phi l^+\nu$

The absolute branching ratio for $D_s^+ \to \phi\pi^+$ can be estimated by measuring the decay width relative to the $\phi l^+\nu$ mode:

$$\mathcal{B}(D_s^+ \to \phi l^+\nu) = \mathcal{F}\mathcal{B}(D^+ \to \bar{K}^{*0} l^+\nu) \frac{\tau_{D_s^+}}{\tau_{D^+}} , \quad (8)$$

where $\mathcal{F}$ is a theoretical correction to account for differences in the $D^+ \to \bar{K}^{*0} l^+\nu$ and $D_s^+ \to \phi l^+\nu$ widths, which are nominally equal. Two models give different predictions for $\mathcal{F}$. Scora predicts $\mathcal{F} = 1.02$, whereas a value of 0.83 is given by Wirbel *et al.* [15]. We average the two values and assign an error of ± 0.1.

This relation was first used by E691 to set a lower limit on $\mathcal{B}(D_s^+ \to \phi\pi^+)$ from their upper limit on $D_s^+ \to \phi l^+\nu$ decays [16]. There now are, however, several observations which supercede the E691 result (see Table III). CLEO [17], ARGUS [18], and a new E687 [19] measurement average to

$$\Gamma(D_s^+ \to \phi l^+\nu)/\Gamma(D_s^+ \to \phi\pi^+) = 0.54 \pm 0.11 .$$

The ratio of the decay widths

TABLE III. Experimental data.

Experiment	R^{*+}
E691 [20]	$0.49\pm0.04\pm0.5$
E653 [21][a]	$0.46\pm0.07\pm0.08$
E687 [22][a]	$0.56\pm0.04\pm0.06$
CLEO [23]	$0.67\pm0.09\pm0.07$
Average	0.54 ± 0.04
	$\mathcal{B}(D^+\to K^-\pi^+\pi^+)$ (%)
Mark III [8]	$9.1\pm1.3\pm0.4$
	$\Gamma(D_s^+\to\phi l^+\nu)/\Gamma(D_s^+\to\phi\pi^+)$
E691 [16]	<0.45 (90% C.L.)
CLEO [17]	$0.49\pm0.10^{+0.10}_{-0.14}$
ARGUS [18]	$0.57\pm0.15\pm0.15$
E687 [19][a]	$0.58\pm0.17\pm0.07$
Average	0.54 ± 0.11

[a]These semimuonic width measurements are scaled up by 1.04 before averaging to correspond to the semielectronic width.

$$R^{*+}=\Gamma(D^+\to\bar{K}^{*0}l^+\nu)/\Gamma(D^+\to K^-\pi^+\pi^+)$$

has been measured by E691 [20] and E653 [21] (see Table III). Including new E687 [22] and CLEO [23] measurements, we get an average value of $R^{*+}=0.54\pm0.04$. For the absolute branching ratio of the decay $D^+\to K^-\pi^+\pi^+$, we take the MARK III [8] value of $(9.1\pm1.3\pm0.4)\%$. Using the new E687 [24] measurement for the D_s^+ lifetime $\tau_{D_s^+}=(4.75\pm0.21)\times10^{-13}$ s to calculate the ratio of the lifetimes $\tau_{D_s^+}/\tau_{D^+}=0.45\pm0.02$ [1], we arrive at

$$\mathcal{B}(D_s^+\to\phi\pi^+)=(3.7\pm1.0)\% \ .$$

V. $\mathcal{B}(D_s^+\to\phi\pi^+)$ UPPER LIMIT FROM SUMMING ALL KNOWN DECAY CHANNELS

All known hadronic branching ratios of the D_s^+ mesons are measured relative to the decay mode $D_s^+\to\phi\pi^+$. The well-established modes are shown in Table IV. These include mostly two-body modes. The

TABLE IV. Established D_s^+ branching ratios [1,25].

Decay mode	Branching ratio relative to $\phi\pi^+$
$\phi\pi^+$	1
$\bar{K}^0K^+$	1.01 ± 0.16
$K^{*+}\bar{K}^0$	1.2 ± 0.25
$\bar{K}^{*0}K^+$	0.95 ± 0.10
$K^{*+}\bar{K}^{*0}$	1.8 ± 0.5
$(K^-K^+\pi^+)_{\rm NR}$	0.29 ± 0.09
$\phi\pi^-\pi^+\pi^+$	0.42 ± 0.12
$f_0(975)\pi^+$	0.28 ± 0.10
$(\pi^-\pi^+\pi^+)_{\rm NR}$	0.29 ± 0.09
$\eta\pi^+$	$0.54\pm0.09\pm0.06$
$\eta'\pi^+$	$1.20\pm0.15\pm0.11$
$\eta\rho^+$	$2.86\pm0.38^{+0.36}_{-0.38}$
$\eta'\rho^+$	$3.44\pm0.62^{+0.44}_{-0.46}$
$\phi\rho^+$	$1.86\pm0.26^{+0.29}_{-0.40}$

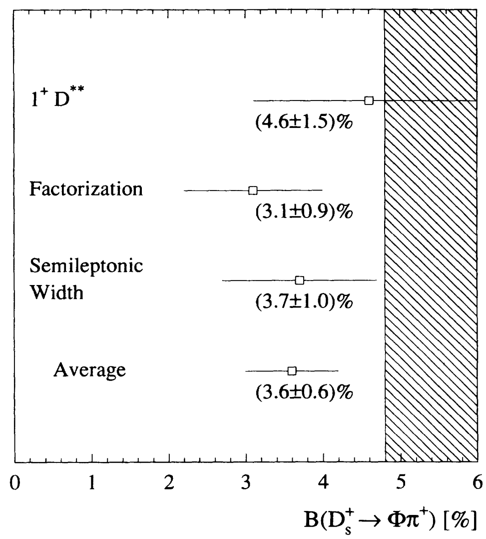

FIG. 2. Compilation of the obtained values of $\mathcal{B}(D_s^+\to\phi\pi^+)$ (%) for the three different methods described in the text. The shaded area is forbidden by the upper limit described in the text.

branching ratios of modes including an η, an η', or a ρ^+ in the final state are taken from a recent CLEO measurement [25]. The sum of measured hadronic modes is $(17.14\pm1.23)\mathcal{B}(D_s^+\to\phi\pi^+)$.

Assuming that the semileptonic widths of charmed mesons are equal, which is observed within experimental errors for the D^0 and D^+ semileptonic widths, and using the measured D_s^+ lifetime [24], we estimate that $\mathcal{B}(D_s^+\to Xl^+\nu)=(7.5\pm1.0)\%$ [26].

In addition, we use the new CLEO measurement [10] of the purely leptonic decay [see Eq. (3)] to estimate the total leptonic branching ratio to be

$$\mathcal{B}(D_s^+\to\tau^+\nu+\mu^+\nu)=(2.45\pm0.90)\mathcal{B}(D_s^+\to\phi\pi^+) \ .$$

Thus the sum of all known decay modes is $(15\pm2)\%$ from semileptonic decays, plus $(19.6\pm1.5)\mathcal{B}(D_s^+\to\phi\pi^+)$ from hadronic and leptonic decays. This implies [27]

$$\mathcal{B}(D_s^+\to\phi\pi^+)<4.8\% \quad \text{at 90\% C.L.} \tag{9}$$

This result is consistent with an older Mark III upper limit of $\mathcal{B}(D_s^+\to\phi\pi^+)<4.1\%$ at 90% C.L. [28]. The latter result was based on measurements of relative branching ratios which have subsequently decreased, possibly raising this limit.

VI. CONCLUSIONS

We have calculated $\mathcal{B}(D_s^+\to\phi\pi^+)$ using three independent theoretical hypotheses and distinct sets of data. We average the three results and obtain

$$\mathcal{B}(D_s^+\to\phi\pi^+)=(3.6\pm0.6)\% \ .$$

We also have derived an upper limit of the branching ratio of $\mathcal{B}(D_s^+\to\phi\pi^+)<4.8\%$ at 90% C.L., using the sum of all seen D_s^+ decays modes plus some clearly inferred modes. The branching ratios and the limit are shown on Fig. 2. If we use $\mathcal{B}(D_s^+ \to \phi\pi^+) = (3.6 \pm 0.6)\%$, we can account for $(86 \pm 13)\%$ of all D_s^+ decay modes, where the error is dominated by the error on the $D_s^+ \to \phi\pi^+$ branching ratio.

The agreement between the methods is good, but it may be fortuitous given the large errors. Assuming factorization in two-body $B \to D^* D_s^-$ decays, we find $f_{D_s} = (315 \pm 46)$ MeV. In any case, if even one of the three theoretical assumptions used turns out to be valid, then the result for $\mathcal{B}(D_s^+ \to \phi\pi^+)$ would imply quite a large value for f_{D_s} using the CLEO measurement of $D_s^+ \to \mu^+ \nu$.

On the other hand, a good experimental determination of $\mathcal{B}(D_s^+ \to \phi\pi^+)$ would allow independent tests of the theoretical assumptions made here.

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[1] Particle Data Group, K. Hikasa *et al.*, Phys. Rev. D **45**, S1 (1992).

[2] For the D^{**0} see CLEO Collaboration, P. Avery *et al.*, Phys. Rev. D **41**, 774 (1990); ARGUS Collaboration, H. Albrecht *et al.*, Phys. Lett. B **232**, 398 (1989). For the D_s^{**+}, see J. P. Alexander *et al.*, *ibid.* **303**, 377 (1993); ARGUS, H. Albrecht *et al.*, *ibid.* **230**, 162 (1989).

[3] D. Bortoletto *et al.*, Phys. Rev. D **37**, 1719 (1988); **39**, 1471(E) (1989).

[4] H. Albrecht *et al.*, Z. Phys. C **52**, 353 (1991).

[5] W. Y. Chen *et al.*, Phys. Lett. B **226**, 192 (1989); H. Albrecht *et al.*, Phys. C **53**, 361 (1992). The CLEO result has been extrapolated to $x_p = 0$.

[6] D. S. Akerib *et al.*, Phys. Rev. Lett. **71**, 3070 (1993).

[7] The updated ALEPH result is $\mathcal{B}(D^0 \to K^- \pi^+) = (3.89 \pm 0.29 \pm 0.16)\%$; see P. Burchat, "Charm Physics," in Proceeding of the 5th International Symposium on Heavy Flavour Physics, Montreal, 1993 (in press, Editions Frontières).

[8] J. Adler *et al.*, Phys. Rev. Lett. **60**, 89 (1988).

[9] J. L. Rosner, in *Particles and Fields 3*, Proceedings of the 1988 Banff Summer Inst., Banff, Alberta, Canada, edited by A. N. Kamal and F. C. Khanna (World Scientific, Singapore, 1989), p. 395.

[10] CLEO Collaboration, F. Muheim, "First Measurement of $\Gamma(D_s^+ \to \mu^+ \nu)/\Gamma(D_s^+ \to \phi\pi^+)$," in Proceedings of the 5th International Symposium on Heavy Flavour Physics, Montreal, 1993 (in press, Editions Frontières).

[11] D. Bortoletto and S. Stone, Phys. Rev. Lett. **65**, 2951 (1990).

[12] S. Stone, "Hadronic B Decays," in Proceedings of the 5th International Symposium on Heavy Flavour Physics, Montreal, 1993 (in press, Editions Frontières).

[13] J. L. Rosner, Phys. Rev. D **42**, 3732 (1990).

[14] Measurements in Table II and the denominator of Eq. (6), $(d\mathcal{B}/dq^2)(\bar{B}^0 \to D^{*+} l^- \bar{\nu}_l)|_{q^2 = M_{D_s}^2} = (4.4 \pm 0.4) \times 10^{-3}$ GeV^{-2}, taken from Ref. [11], have been scaled to the new CLEO $D^{*+} \to D^0 \pi^+$ and $D^0 \to K^- \pi^+$ branching ratio measurements; see F. Butler *et al.*, Phys. Rev. Lett. **69**, 2041 (1992), and Ref. [6].

[15] N. Isgur *et al.*, Phys. Rev. D **39**, 799 (1989); D. Scora, in *Particles and Nuclei*, in Proceedings of the Twelfth International Conference, Cambridge, Massachusetts, 1990, edited by J. L. Matthews *et al.* [Nucl. Phys. **A527** (1991)]; M. Wirbel *et al.*, Z. Phys. C **29**, 269 (1985).

[16] J. C. Anjos *et al.*, Phys. Rev. Lett. **64**, 2885 (1990).

[17] J. Alexander *et al.*, Phys. Rev. Lett. **65**, 1531 (1990).

[18] H. Albrecht *et al.*, Phys. Lett. B **255**, 634 (1991).

[19] P. Frabetti *et al.*, Phys. Lett. B **313**, 253 (1993).

[20] J. C. Anjos *et al.*, Phys. Rev. Lett. **62**, 722 (1989).

[21] K. Kodama *et al.*, Phys. Lett. B **286**, 187 (1992).

[22] P. Frabetti *et al.*, Phys. Lett. B **307**, 262 (1993).

[23] A. Bean *et al.*, Phys. Lett. B **317**, 647 (1993).

[24] P. Frabetti *et al.*, Phys. Rev. Lett. **71**, 827 (1993).

[25] M. Daoudi *et al.*, Phys. Rev. D **45**, 3965 (1992).

[26] This is consistent with the Mark III upper limit of $\mathcal{B}(D_s^+ \to X l^+ \nu) < 20\%$ at 90% C.L.; see Z. Bai *et al.*, Phys. Rev. Lett. **65**, 686 (1990).

[27] We expect about 5% of the D_s^+ decays will be into Cabibbo suppressed channels, which have not been observed. Since we do not have an accurate rate estimate of these modes, we do not include them in the upper-limit calculation.

[28] J. Adler *et al.*, Phys. Rev. Lett. **64**, 169 (1990).